Recent Advances in the 2HDM and Beyond

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Based on:
- N. Darvishi, AP, PRD99 (2019) 115014; PRD101 (2020) 095008
- C Birch-Sykes, N Darvishi, Y Peters, AP, NPB960 (2020) 115171
- R Battye, AP, D Viatic, JHEP2101 (2021) 105
- R Battye, AP, D Viatic, PRD102 (2020) 123536

Outline:

- Accidental Symmetries in 2HDM, 2HDMEFT, and multi-HDMs
- Maximal Symmetry and Quartic Coupling Unification
- Vacuum Topology of the 2HDM
- Charge-Violating Domain Walls in the 2HDM
- Conclusions

• Accidental Symmetries in 2HDM, 2HDMEFT, and multi-HDMs

• 2HDM potential

[TD Lee '73; AP, C Wagner '99;
Review: Branco, Ferreira, Lavoura, Rebelo, Sher, Silva '12.]

$$
\begin{aligned}
\mathrm{V} \ = \ & -\mu_1^2(\phi_1^\dagger\phi_1) \ - \ \mu_2^2(\phi_2^\dagger\phi_2) \ - \ m_{12}^2(\phi_1^\dagger\phi_2) \ - \ m_{12}^{*2}(\phi_2^\dagger\phi_1) \\
& + \lambda_1(\phi_1^\dagger\phi_1)^2 \ + \ \lambda_2(\phi_2^\dagger\phi_2)^2 \ + \ \lambda_3(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2) \ + \ \lambda_4(\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1) \\
& + \frac{\lambda_5}{2}(\phi_1^\dagger\phi_2)^2 \ + \ \frac{\lambda_5^*}{2}(\phi_2^\dagger\phi_1)^2 \ + \ \lambda_6(\phi_1^\dagger\phi_1)(\phi_1^\dagger\phi_2) \ + \ \lambda_6^*(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_1) \\
& + \lambda_7(\phi_2^\dagger\phi_2)(\phi_1^\dagger\phi_2) \ + \ \lambda_7^*(\phi_2^\dagger\phi_2)(\phi_2^\dagger\phi_1) \ .
\end{aligned}
$$

• Physical spectrum (CP-conserving limit):

CP-even Higgs bosons $\boldsymbol{H}$ and $\boldsymbol{h}$; CP-odd scalar $\boldsymbol{a}$; charged scalars $\boldsymbol{h}^{\pm}$.

• Higgs coupling to gauge bosons $V = W, Z$:

$$
g_{HVV} \ = \ \cos(\beta - \alpha) \ , \qquad g_{hVV} \ = \ \sin(\beta - \alpha) \ ,
$$

where $\tan\beta = \langle\phi_2\rangle/\langle\phi_1\rangle$ and α diagonalizes the CP-even mass matrix.

References (*an incomplete list on symmetries in the 2HDM*)

- **Spontaneous CP Violation:** T. D. Lee, Phys. Rev. D8 (1973) 1226.
- **$\mathbf{Z}_2$ symmetry:** S. L. Glashow, S. Weinberg, Phys. Rev. D15 (1977) 1958.
- **Inert $\mathbf{Z}_2$ symmetry:** N. G. Deshpande, E. Ma, Phys. Rev. D18 (1978) 2574.
- **PQ U(1) symmetry:** R. D. Peccei, H. R. Quinn, Phys. Rev. Lett. 38 (1977) 1440.
- **Custodial $\mathbf{SU(2)}_L$-preserving symmetry:**
  P. Sikivie, L. Susskind, M. B. Voloshin, V. I. Zakharov, Nucl. Phys. B173 (1980) 189.
- **Bilinear formalism:**
  M. Maniatis, A. von Manteuffel, O. Nachtmann, F. Nagel, EPJC48 (2006) 805;
  C. C. Nishi, Phys. Rev. D74 (2006) 036003.
- **$\mathbf{SU(2)}_L \otimes \mathbf{U(1)}_Y$-preserving symmetries:** 6
  I. P. Ivanov, Phys. Rev. D75 (2007) 035001;
  P. M. Ferreira, H. E. Haber, J. P. Silva, Phys. Rev. D79 (2009) 116004.
- **Hypercustodial $\mathbf{SU(2)}_L$-preserving symmetries:** +7
  R. A. Battye, G. D. Brawn, AP, JHEP1108 (2011) 020.
- **On completeness and uniqueness of classification:**
  AP, Phys. Lett. B706 (2012) 465.

• Symmetries of the 2HDM Potential

[R. A. Battye, G. D. Brawn, AP, JHEP1108 (2011) 020.]

Introduce the $\mathrm{SU}(2)_L$-covariant 8D complex field multiplet

$$\mathbf{\Phi} \;=\; \begin{pmatrix} \phi_1 \\ \phi_2 \\ i\sigma^2\phi_1^* \\ i\sigma^2\phi_2^* \end{pmatrix}, \quad \text{with} \;\; U_L \in \; \mathrm{SU}(2)_L : \; \mathbf{\Phi} \;\mapsto\; \mathbf{\Phi}' \;=\; U_L\,\mathbf{\Phi}\,.$$

$\mathbf{\Phi}$ satisfies the **Majorana constraint**

$$\mathbf{\Phi} \;=\; \mathsf{C}\,\mathbf{\Phi}^*\,,$$

where C is the **charge conjugation** 8D matrix

$$\mathsf{C} \;=\; \sigma^2 \otimes \sigma^0 \otimes \sigma^2 \;=\; \begin{pmatrix} \mathbf{0}_4 & \mathbf{1}_4 \\ -\,\mathbf{1}_4 & \mathbf{0}_4 \end{pmatrix} \otimes (-i\sigma_2)\,.$$

• The SO(1,5) Bilinear Formalism

Introduce the *null* 6-Vector

$$\mathsf{R}^A \;=\; \mathbf{\Phi}^\dagger\, \Sigma^A\, \mathbf{\Phi} \;=\; \begin{pmatrix} \phi_1^\dagger\phi_1 + \phi_2^\dagger\phi_2 \\ \phi_1^\dagger\phi_2 + \phi_2^\dagger\phi_1 \\ -i\left[\phi_1^\dagger\phi_2 - \phi_2^\dagger\phi_1\right] \\ \phi_1^\dagger\phi_1 - \phi_2^\dagger\phi_2 \\ \phi_1^{\mathsf{T}} i\sigma^2\phi_2 - \phi_2^\dagger i\sigma^2\phi_1^* \\ -i\left[\phi_1^{\mathsf{T}} i\sigma^2\phi_2 + \phi_2^\dagger i\sigma^2\phi_1^*\right] \end{pmatrix},$$

with $A = \mu,\ 4,\ 5$, and

$$\Sigma^\mu \;=\; \frac{1}{2}\begin{pmatrix} \sigma^\mu & \mathbf{0}_2 \\ \mathbf{0}_2 & (\sigma^\mu)^{\mathsf{T}} \end{pmatrix} \otimes \sigma^0\,,$$

$$\Sigma^4 \;=\; \frac{1}{2}\begin{pmatrix} \mathbf{0}_2 & i\sigma^2 \\ -i\sigma^2 & \mathbf{0}_2 \end{pmatrix} \otimes \sigma^0\,, \qquad \Sigma^5 \;=\; \frac{1}{2}\begin{pmatrix} \mathbf{0}_2 & -\sigma^2 \\ -\sigma^2 & \mathbf{0}_2 \end{pmatrix} \otimes \sigma^0\,.$$

• The 2HDM Potential in the SO(1,5) Formalism

$$V_{\mathsf{2HDM}} \;=\; -\,\frac{1}{2}\,M_A\,\mathsf{R}^A \;+\; \frac{1}{4}\,L_{AB}\,\mathsf{R}^A\,\mathsf{R}^B\ ,$$

with

$$M_A \;=\; \big(\ \mu_1^2+\mu_2^2\,,\quad 2\mathrm{Re}(m_{12}^2)\,,\quad -2\mathrm{Im}(m_{12}^2)\,,\quad \mu_1^2-\mu_2^2\,,\quad 0\,,\quad 0\ \big)\ ,$$

$$L_{AB} \;=\; \begin{pmatrix} \lambda_1+\lambda_2+\lambda_3 & \mathrm{Re}(\lambda_6+\lambda_7) & -\mathrm{Im}(\lambda_6+\lambda_7) & \lambda_1-\lambda_2 & 0 & 0 \\ \mathrm{Re}(\lambda_6+\lambda_7) & \lambda_4+\mathrm{Re}(\lambda_5) & -\mathrm{Im}(\lambda_5) & \mathrm{Re}(\lambda_6-\lambda_7) & 0 & 0 \\ -\mathrm{Im}(\lambda_6+\lambda_7) & -\mathrm{Im}(\lambda_5) & \lambda_4-\mathrm{Re}(\lambda_5) & -\mathrm{Im}(\lambda_6-\lambda_7) & 0 & 0 \\ \lambda_1-\lambda_2 & \mathrm{Re}(\lambda_6-\lambda_7) & -\mathrm{Im}(\lambda_6-\lambda_7) & \lambda_1+\lambda_2-\lambda_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

• Unitary Field Transformations:

[AP, Phys. Lett. B706 (2012) 465.]

$\mathsf{Sp}(4):\quad \mathbf{\Phi}\ \mapsto\ \mathbf{\Phi}'\ =\ U\,\mathbf{\Phi}\,,\qquad$ with $\ U \in \mathsf{U}(4)$ <u>and</u> $\ U\mathsf{C}U^{\mathsf{T}} = \mathsf{C}$

$\mathsf{SO}(5):\quad \mathsf{R}^{\mathsf{I}}\ \mapsto\ \mathsf{R}'^{\mathsf{I}}\ =\ \mathsf{O}^{\mathsf{I}}{}_{\mathsf{J}}\,\mathsf{R}^{\mathsf{J}}\,,\qquad$ with $\ \mathsf{O}\in\mathsf{SO}(5)\ \subset\ \mathsf{SO}(1,5)$

$\Longrightarrow\qquad \mathsf{SO}(5)\ \sim\ \mathsf{Sp}(4)/\mathbf{Z}_2$

• Symmetry Breaking Scenarios and pseudo-Goldstone Bosons

[AP, Phys. Lett. B706 (2012) 465.]

No	Symmetry	Generators $T^a \leftrightarrow K^a$	Discrete Group Elements	Maximally Broken SO(5) Generators	Number of Pseudo-Goldstone Bosons
1	$Z_2 \times O(2)$	T^0	D_{CP1}	–	0
2	$(Z_2)^2 \times SO(2)$	T^0	D_{Z_2}	–	0
3	$(Z_2)^3 \times O(2)$	T^0	D_{CP2}	–	0
4	$O(2) \times O(2)$	$T^3,\ T^0$	–	T^3	1 (a)
✓ 5	$Z_2 \times [O(2)]^2$	$T^2,\ T^0$	D_{CP1}	T^2	1 (h)
✓ 6	$O(3) \times O(2)$	$T^{1,2,3},\ T^0$	–	$T^{1,2}$	2 $(h,\ a)$
7	$SO(3)$	$T^{0,4,6}$	–	$T^{4,6}$	2 $(h^\pm)$
8	$Z_2 \times O(3)$	$T^{0,4,6}$	$D_{Z_2} \cdot D_{CP2}$	$T^{4,6}$	2 $(h^\pm)$
9	$(Z_2)^2 \times SO(3)$	$T^{0,5,7}$	$D_{CP1} \cdot D_{CP2}$	$T^{5,7}$	2 $(h^\pm)$
10	$O(2) \times O(3)$	$T^3,\ T^{0,8,9}$	–	T^3	1 (a)
11	$SO(4)$	$T^{0,3,4,5,6,7}$	–	$T^{3,5,7}$	3 $(a,\ h^\pm)$
12	$Z_2 \times O(4)$	$T^{0,3,4,5,6,7}$	$D_{Z_2} \cdot D_{CP2}$	$T^{3,5,7}$	3 $(a,\ h^\pm)$
✓ 13	$SO(5)$	$T^{0,1,2,\dots,9}$	–	$T^{1,2,8,9}$	4 $(h,\ a,\ h^\pm)$

✓: Natural SM Alignment $\mapsto$ [Dev, AP, JHEP1412 (2014) 024.]

• Symmetries in 2HDMEFTs

[C Birch-Sykes, N Darvishi, Y Peters, AP, NPB960 (2020) 115171]

$$\mathrm{V}_{\mathrm{2HDMEFT}} = -\frac{1}{2} M_A \mathsf{R}^A + \frac{1}{4} L_{AB} \mathsf{R}^A \mathsf{R}^B + \frac{1}{\Lambda^2} K_{ABC} \mathsf{R}^A \mathsf{R}^B \mathsf{R}^C + \frac{1}{\Lambda^4} Z_{ABCD} \mathsf{R}^A \mathsf{R}^B \mathsf{R}^C \mathsf{R}^D + \cdots$$

No. of couplings: $\boldsymbol{N^{(\mathrm{dim}=2n)} = \frac{1}{6}(n+1)(n+2)(n+3)}$

$$N^{(\mathrm{dim}\leq 4)} = 14, \quad N^{(\mathrm{dim}\leq 6)} = 34, \quad N^{(\mathrm{dim}\leq 8)} = 69, \quad \ldots, \quad N^{(\mathrm{dim}\leq 20)} = 1000$$

Symmetry restrictions:

$$M_A [T^a]^{A'}_{\ A} = 0, \quad L_{A'B} [T^a]^{A'}_{\ A} + L_{AB'} [T^a]^{B'}_{\ B} = 0,$$

$$K_{A'BC} [T^a]^{A'}_{\ A} + K_{AB'C} [T^a]^{B'}_{\ B} + K_{ABC'} [T^a]^{C'}_{\ C} = 0,$$

$$Z_{A'BCD} [T^a]^{A'}_{\ A} + Z_{AB'CD} [T^a]^{B'}_{\ B} + Z_{ABC'D} [T^a]^{C'}_{\ C} + Z_{ABCD'} [T^a]^{D'}_{\ D} = 0\,,$$

where $T^a \in \mathfrak{g}$ are the generators of the symmetry subgroup $G \subseteq \mathrm{SO}(5)$.

No.	Symmetry	Non-zero parameters of Symmetric 2HDMEFT Potential	Dim
1	CP1	μ_1^2, μ_2^2, Re(m_{12}^2), λ_1, λ_2, λ_3, λ_4, Re(λ_5), Re(λ_6), Re(λ_7) $\kappa_1, \ldots, \kappa_6$, Re$(\kappa_7, \ldots, \kappa_{13})$ $\zeta_1, \ldots, \zeta_9$, Re$(\zeta_{10}, \ldots, \zeta_{22})$	$D \geq 4$
2	Z_2	μ_1^2, μ_2^2, λ_1, λ_2, λ_3, λ_4, λ_5 κ_1, κ_2, κ_3, κ_4, κ_5, κ_6, κ_8, κ_9 ζ_1, ζ_2, ζ_3, ζ_4, ζ_5 ζ_6, ζ_7, ζ_8, ζ_9, ζ_{10}, ζ_{13}, ζ_{14}, ζ_{15}, ζ_{16}	$D \geq 4$
3	Z_3	μ_1^2, μ_2^2, λ_1, λ_2, λ_3, λ_4 κ_1, κ_2, κ_3, κ_4, κ_5, κ_6, κ_7 ζ_1, ζ_2, ζ_3, ζ_4, ζ_5, ζ_6, ζ_7, ζ_8, ζ_9, ζ_{11}, ζ_{12}	$D \geq 6$
4	Z_4	μ_1^2, μ_2^2, λ_1, λ_2, λ_3, λ_4 κ_1, κ_2, κ_3, κ_4, κ_5, κ_6 ζ_1, ζ_2, ζ_3, ζ_4, ζ_5, ζ_6, ζ_7, ζ_8, ζ_9, ζ_{10}	$D \geq 8$
5	CP2	$\mu_1^2 = \mu_2^2$, $\lambda_1 = \lambda_2$, λ_3, λ_4, λ_5, $\lambda_6 = -\lambda_7$ $\kappa_1 = \kappa_2$, $\kappa_3 = \kappa_4$, $\kappa_5 = \kappa_6$, $\kappa_8 = \kappa_9$, $\kappa_{11} = -\kappa_{12}$ $\zeta_1 = \zeta_2$, ζ_3, $\zeta_4 = \zeta_5$, ζ_6, $\zeta_7 = \zeta_8$, ζ_9, ζ_{10}, $\zeta_{11} = -\zeta_{12}$, ζ_{13}, $\zeta_{14} = \zeta_{15}$, ζ_{16}, $\zeta_{17} = -\zeta_{18}$, $\zeta_{19} = -\zeta_{20}$, $\zeta_{21} = -\zeta_{22}$	$D \geq 4$
6	CP3	$\mu_1 = \mu_2$, $\lambda_1 = \lambda_2$, λ_3, λ_4 $\kappa_1 = \kappa_2$, $\kappa_3 = \kappa_4$, $\kappa_5 = \kappa_6$, κ_7 $\zeta_1 = \zeta_2$, ζ_3, $\zeta_4 = \zeta_5$, ζ_6, $\zeta_7 = \zeta_8$, ζ_9, $\zeta_{11} = \zeta_{12}$	$D \geq 6$
7	CP4	$\mu_1 = \mu_2$, $\lambda_1 = \lambda_2$, λ_3, λ_4 $\kappa_1 = \kappa_2$, $\kappa_3 = \kappa_4$, $\kappa_5 = \kappa_6$, $\kappa_8 = -\kappa_9$ $\zeta_1 = \zeta_2$, ζ_3, $\zeta_4 = \zeta_5$, ζ_6, $\zeta_7 = \zeta_8$, ζ_9, ζ_{10}, $\zeta_{14} = -\zeta_{15}$	$D \geq 6$

8	$\mathrm{U}(1)_{\mathrm{PQ}}$	$\mu_1^2,\ \mu_2^2,\ \lambda_1,\ \lambda_2,\ \lambda_3,\ \lambda_4$ $\kappa_1,\ \kappa_2,\ \kappa_3,\ \kappa_4,\ \kappa_5,\ \kappa_6$ $\zeta_1,\ \zeta_2,\ \zeta_3,\ \zeta_4,\ \zeta_5,\ \zeta_6,\ \zeta_7,\ \zeta_8,\ \zeta_9$	$D \geq 4$
9	$\mathrm{CP1} \otimes \mathrm{SO}(2)_{\mathrm{HF}}$	$\mu_1^2 = \mu_2^2,\ \lambda_1 = \lambda_2,\ \lambda_3,\ \lambda_4,\ \mathrm{Re}(\lambda_5) = 2\lambda_1 - \lambda_{34}$ $\kappa_1 = \kappa_2,\ \kappa_3 = \kappa_4,\ \kappa_5 = \kappa_6,$ $\mathrm{Re}(\kappa_8) = \mathrm{Re}(\kappa_9) = \frac{1}{2}(3\kappa_1 - \kappa_3 - \kappa_5)$ $\zeta_1 = \zeta_2,\ \zeta_3,\ \zeta_4 = \zeta_5,\ \zeta_6,\ \zeta_7 = \zeta_8,\ \zeta_9,$ $\mathrm{Re}(\zeta_{10}) = -\frac{1}{4}\mathrm{Re}(\zeta_{13}) + \frac{1}{2}\mathrm{Re}(\zeta_{14}) - \frac{1}{4}\mathrm{Re}(\zeta_{16}),$ $\mathrm{Re}(\zeta_{13}) = \frac{1}{6}(4\zeta_1 + 2\zeta_3 - 4\zeta_4 - 4\zeta_6 + 2\zeta_7 - \zeta_9),$ $\mathrm{Re}(\zeta_{14}) = \mathrm{Re}(\zeta_{15}) = \frac{1}{2}(4\zeta_1 - \zeta_4 - \zeta_7),$ $\mathrm{Re}(\zeta_{16}) = \frac{1}{2}(4\zeta_1 - 2\zeta_3 + 2\zeta_4 - \zeta_9)$	$D \geq 4$
10	$\mathrm{SU}(2)_{\mathrm{HF}}$	$\mu_1^2 = \mu_2^2,\ \lambda_1 = \lambda_2,\ \lambda_3,\ \lambda_4 = 2\lambda_1 - \lambda_3$ $\kappa_1 = \kappa_2,\ \kappa_3 = \kappa_4,\ \kappa_5 = \kappa_6 = 3\kappa_1 - \kappa_3$ $\zeta_1 = \zeta_2,\ \zeta_3,\ \zeta_4 = \zeta_5,\ \zeta_6 = 2\zeta_1 + \zeta_3 - 2\zeta_4,$ $\zeta_7 = \zeta_8 = 4\zeta_1 - \zeta_4,\ \zeta_9 = 4\zeta_1 - 2\zeta_3 + 2\zeta_4$	$D \geq 4$
11	$\mathrm{Sp}(2)_{\phi_1+\phi_2}$	$\mu_1^2,\ \mu_2^2,\ \mathrm{Re}(m_{12}^2),\ \lambda_1,\ \lambda_2,\ \lambda_3,\ \lambda_4 = \mathrm{Re}(\lambda_5),\ \mathrm{Re}(\lambda_6),\ \mathrm{Re}(\lambda_7)$ $\kappa_1,\ \kappa_2,\ \kappa_3,\ \kappa_4,\ \kappa_5 = 2\mathrm{Re}(\kappa_8),\ \kappa_6 = 2\mathrm{Re}(\kappa_9),$ $\mathrm{Re}(\kappa_7) = \frac{1}{3}\mathrm{Re}(\kappa_{10}),\ \mathrm{Re}(\kappa_{11}),\ \mathrm{Re}(\kappa_{12}),\ \mathrm{Re}(\kappa_{13})$ $\zeta_1,\ \zeta_2,\ \zeta_3,\ \zeta_4,\ \zeta_5,\ \zeta_6 = 6\mathrm{Re}(\zeta_{10}) = \frac{3}{2}\mathrm{Re}(\zeta_{13}),$ $\zeta_7 = 2\mathrm{Re}(\zeta_{14}),\ \zeta_8 = 2\mathrm{Re}(\zeta_{15}),\ \zeta_9 = 2\mathrm{Re}(\zeta_{16}),$ $\mathrm{Re}(\zeta_{11}) = \frac{1}{3}\mathrm{Re}(\zeta_{17}),\ \mathrm{Re}(\zeta_{12}) = \frac{1}{3}\mathrm{Re}(\zeta_{18}),$ $\mathrm{Re}(\zeta_{19}),\ \mathrm{Re}(\zeta_{20}),\ \mathrm{Re}(\zeta_{21}),\ \mathrm{Re}(\zeta_{22})$	$D \geq 4$

12	$S_2 \otimes \mathrm{Sp}(2)_{\phi_1+\phi_2}$	$\mu_1^2 = \mu_2^2$, $\mathrm{Re}(m_{12}^2)$, $\lambda_1 = \lambda_2$, λ_3, $\lambda_4 = \mathrm{Re}(\lambda_5)$, $\mathrm{Re}(\lambda_6)=\mathrm{Re}(\lambda_7)$ $\kappa_1 = \kappa_2$, $\kappa_3 = \kappa_4$, $\kappa_5 = \kappa_6 = 2\mathrm{Re}(\kappa_8) = 2\mathrm{Re}(\kappa_9)$, $\mathrm{Re}(\kappa_7) = \frac{1}{3}\mathrm{Re}(\kappa_{10})$, $\mathrm{Re}(\kappa_{11}) = \mathrm{Re}(\kappa_{12})$, $\mathrm{Re}(\kappa_{13})$ $\zeta_1 = \zeta_2$, ζ_3, $\zeta_4 = \zeta_5$, $\zeta_6 = 6\mathrm{Re}(\zeta_{10}) = \frac{3}{2}\mathrm{Re}(\zeta_{13})$, $\zeta_7 = \zeta_8 = 2\mathrm{Re}(\zeta_{14}) = 2\mathrm{Re}(\zeta_{15})$, $\zeta_9 = 2\mathrm{Re}(\zeta_{16})$ $\mathrm{Re}(\zeta_{11}) = \mathrm{Re}(\zeta_{12}) = \frac{1}{3}\mathrm{Re}(\zeta_{17}) = \frac{1}{3}\mathrm{Re}(\zeta_{18})$, $\mathrm{Re}(\zeta_{19}) = \mathrm{Re}(\zeta_{20})$, $\mathrm{Re}(\zeta_{21}) = \mathrm{Re}(\zeta_{22})$	$D \geq 4$
13	$\mathrm{CP2} \otimes \mathrm{Sp}(2)_{\phi_1+\phi_2}$	$\mu_1^2 = \mu_2^2$, $\lambda_1 = \lambda_2$, λ_3, $\lambda_4 = \mathrm{Re}(\lambda_5)$, $\mathrm{Re}(\lambda_6) = -\mathrm{Re}(\lambda_7)$ $\kappa_1 = \kappa_2$, $\kappa_3 = \kappa_4$, $\kappa_5 = \kappa_6 = 2\mathrm{Re}(\kappa_8) = 2\mathrm{Re}(\kappa_9)$, $\mathrm{Re}(\kappa_{11}) = -\mathrm{Re}(\kappa_{12})$ $\zeta_1 = \zeta_2$, ζ_3, $\zeta_4 = \zeta_5$, $\zeta_6 = 6\mathrm{Re}(\zeta_{10}) = \frac{3}{2}\mathrm{Re}(\zeta_{13})$, $\zeta_7 = \zeta_8 = 2\mathrm{Re}(\zeta_{14}) = 2\mathrm{Re}(\zeta_{15})$, $\zeta_9 = 2\mathrm{Re}(\zeta_{16})$ $\mathrm{Re}(\zeta_{11}) = -\mathrm{Re}(\zeta_{12}) = \frac{1}{3}\mathrm{Re}(\zeta_{17}) = -\frac{1}{3}\mathrm{Re}(\zeta_{18})$, $\mathrm{Re}(\zeta_{19}) = -\mathrm{Re}(\zeta_{20})$, $\mathrm{Re}(\zeta_{21}) = -\mathrm{Re}(\zeta_{22})$	$D \geq 4$
14	$\mathrm{U}(1)_{\mathrm{PQ}} \otimes \mathrm{Sp}(2)_{\phi_1\phi_2}$	$\mu_1^2 = \mu_2^2$, $\lambda_1 = \lambda_2 = \frac{1}{2}\lambda_3$, λ_4 $\kappa_1 = \kappa_2 = \frac{1}{3}\kappa_3 = \frac{1}{3}\kappa_4$, $\kappa_5 = \kappa_6$ $\zeta_1 = \zeta_2 = \frac{1}{6}\zeta_3 = \frac{1}{4}\zeta_4 = \frac{1}{4}\zeta_5$, ζ_6, $\zeta_7 = \zeta_8 = \frac{1}{2}\zeta_9$	$D \geq 4$
15	$\mathrm{Sp}(2)_{\phi_1} \otimes \mathrm{Sp}(2)_{\phi_2}$	μ_1^2, μ_2^2, λ_1, λ_2, λ_3 κ_1, κ_2, κ_3, κ_4 ζ_1, ζ_2, ζ_3, ζ_4, ζ_5	$D \geq 4$
16	$S_2 \otimes \mathrm{Sp}(2)_{\phi_1} \otimes \mathrm{Sp}(2)_{\phi_2}$	$\mu_1^2 = \mu_2^2$, $\lambda_1 = \lambda_2$, λ_3 $\kappa_1 = \kappa_2$, $\kappa_3 = \kappa_4$ $\zeta_1 = \zeta_2$, ζ_3, $\zeta_4 = \zeta_5$	$D \geq 4$
17	$\mathrm{Sp}(4)$	$\mu_1^2 = \mu_2^2$, $\lambda_1 = \lambda_2 = \frac{1}{2}\lambda_3$, $\kappa_1 = \kappa_2 = \frac{1}{3}\kappa_3 = \frac{1}{3}\kappa_4$ $\zeta_1 = \zeta_2 = \frac{1}{6}\zeta_3 = \frac{1}{4}\zeta_4 = \frac{1}{4}\zeta_5$	$D \geq 4$

• Symmetries of multi-HDM Potentials

[N Darvishi, AP, PRD101 (2020) 095008]

Prime bilinear invariants:

- **Maximal block:**

$$\begin{cases} \mathrm{Sp}(2n): & S_n = \Phi^+\Phi \quad \text{with } \Phi = \begin{pmatrix}\phi \\ i\sigma^2\phi^*\end{pmatrix} \\ \mathrm{SU}(n): & D_n^a = \phi^\dagger\sigma^a\phi \quad \text{and } \phi = (\phi_1, \phi_2, \cdots, \phi_n)^\mathsf{T} \\ \mathrm{SO}(n): & T_n = \phi\phi^\mathsf{T} \end{cases}$$

- **Minimal block:**

$$\begin{cases} \mathrm{Sp}(2): & \begin{cases} S_{ii} = \phi_i^\dagger\phi_i & \text{for } \begin{pmatrix}\phi_i \\ i\sigma^2\phi_i^*\end{pmatrix} \\ S_{ij} = \phi_i^\dagger\phi_j + \phi_j^\dagger\phi_i & \text{for } \begin{pmatrix}\phi_i \\ i\sigma^2\phi_j^*\end{pmatrix} \,\&\, \begin{pmatrix}\phi_j \\ i\sigma^2\phi_i^*\end{pmatrix} \end{cases} \\ \mathrm{SU(2)\times U(1)}: & \begin{cases} D_{ij}^a = \phi_i^\dagger\sigma^a\phi_i + \phi_j^\dagger\sigma^a\phi_j = D_{ji}^a & \text{for } \begin{pmatrix}\phi_i \\ \phi_j\end{pmatrix} \\ D_{ij}'^a = \phi_i^\dagger\sigma^a\phi_i + (i\sigma^2\phi_j^*)\sigma^a(i\sigma^2\phi_j^*) & \text{for } \begin{pmatrix}\phi_i \\ i\sigma^2\phi_j^*\end{pmatrix} \end{cases} \\ \mathrm{SO}(2): & \left\{ T_{ij} = \phi_i\phi_i^\mathsf{T} + \phi_j\phi_j^\mathsf{T} = T_{ji} \quad \text{for } \begin{pmatrix}\phi_i \\ \phi_j\end{pmatrix} \right. \end{cases}$$

[AP, PRD93 (2016) 075012]

The symmetric potential $\rightarrow$ $V_{\rm sym} = -\mu^2 S_n + \lambda_S S_n^2 + \lambda_D D_n^2 + \lambda_T T_n^2$

• Discrete Symmetries

[Earlier studies: Ivanov, Vdovin '12; V Keus et al '13; Ivanov, Varzielas '19, . . .]

→ **Generalized CP (GCP) transformations:**

$$\mathsf{GCP}[\phi_i] = G_{ij}\phi_j^* \qquad G_{ij} \in \mathsf{SU}(n)$$

→ **Abelian Discrete Symmetries:**

$$Z_2, \quad Z_3, \quad Z_4, \quad Z_2 \times Z_2, \quad Z_3 \times Z_3, \quad \cdots, \quad Z_n, \quad \cdots,$$

where $Z_n = \{1, \omega, \cdots, \omega^{(n-1)}\}$ with $\omega^n = 1$.

→ **Non-Abelian Discrete Symmetries**

- **Typical Non-Abelian Discrete Symmetries**

  - *Permutation group* $S_N \xrightarrow{\text{with order}} N!$

  - *Alternating group* $A_N \xrightarrow{\text{with order}} N!/2$

  - *Dihedral group* $D_N \xrightarrow{\text{isomorphic to}} Z_N \rtimes Z_2$

  - *Binary Dihedral group* $Q_{2N} \xrightarrow{\text{with order}} 4N$

  - *Tetrahedral group* $T_{N\,(\text{prime number})} \xrightarrow{\text{isomorphic to}} Z_N \rtimes Z_3$

  - *Dihedral-like groups*:

$$\Sigma(2N^2) \cong (Z_N \times Z'_N) \rtimes Z_2 \qquad \Delta(3N^2) \cong (Z_N \times Z'_N) \rtimes Z_3$$

$$\Sigma(3N^3) \cong Z_N \times \Delta(3N^2) \qquad \Delta(6N^2) \cong (Z_N \times Z'_N) \rtimes S_3$$

  - *Crystal-like groups* $\Sigma(M\phi)$, with $\phi = 1, 2, 3$:

$$\Sigma(60\phi), \quad \Sigma(168\phi), \quad \Sigma(36\phi), \quad \Sigma(72\phi), \quad \Sigma(216\phi), \quad \Sigma(360\phi)$$

No.	Symmetry	Non-zero parameters for 3HDM potentials
1	CP1	μ_1^2, μ_2^2, μ_3^2, Re(m_{12}^2), Re(m_{13}^2), Re(m_{23}^2), λ_{11}, λ_{22}, λ_{33}, λ_{1122}, λ_{1133}, λ_{2233}, λ_{1221}, λ_{1331}, λ_{2332}, Re(λ_{1212}), Re(λ_{1313}), Re(λ_{2323}), Re(λ_{1213}), Re(λ_{2113}), Re(λ_{1223}), Re(λ_{2123}), Re(λ_{1323}), Re(λ_{1332}), Re(λ_{1112}), Re(λ_{2212}), Re(λ_{3312}), Re(λ_{1113}), Re(λ_{2213}), Re(λ_{3313}), Re(λ_{1123}), Re(λ_{2223}), Re(λ_{3323})
2	Z_2	μ_1^2, μ_2^2, μ_3^2, λ_{11}, λ_{22}, λ_{33}, λ_{1122}, λ_{1133}, λ_{2233}, λ_{1221}, λ_{1331}, λ_{2332}, $\{m_{13}^2$, λ_{1212}, λ_{1313}, λ_{2323}, λ_{1232}, λ_{1113}, λ_{2213}, λ_{3313} and H.c.$\}$
$2'$	Z_2'	μ_1^2, μ_2^2, μ_3^2, λ_{11}, λ_{22}, λ_{33}, λ_{1122}, λ_{1133}, λ_{2233}, λ_{1221}, λ_{1331}, λ_{2332}, $\{m_{23}^2$, λ_{1212}, λ_{1313}, λ_{2323}, λ_{1213}, λ_{1123}, λ_{2223}, λ_{3323} and H.c.$\}$
3	$Z_2 \otimes Z_2'$	μ_1^2, μ_2^2, μ_3^2, λ_{11}, λ_{22}, λ_{33}, λ_{1122}, λ_{1133}, λ_{2233}, λ_{1221}, λ_{1331}, λ_{2332}, $\{\lambda_{1212}$, λ_{1313}, λ_{2323} and H.c.$\}$
4	Z_3	μ_1^2, μ_2^2, μ_3^2, λ_{11}, λ_{22}, λ_{33}, λ_{1122}, λ_{1133}, λ_{2233}, λ_{1221}, λ_{1331}, λ_{2332}, $\{\lambda_{1213}$, λ_{1323}, λ_{2123} and H.c.$\}$
5	Z_4	μ_1^2, μ_2^2, μ_3^2, λ_{11}, λ_{22}, λ_{33}, λ_{1122}, λ_{1133}, λ_{2233}, λ_{1221}, λ_{1331}, λ_{2332}, $\{\lambda_{1212}$, λ_{1323} and H.c.$\}$
$5'$	Z_4'	μ_1^2, μ_2^2, μ_3^2, λ_{11}, λ_{22}, λ_{33}, λ_{1122}, λ_{1133}, λ_{2233}, λ_{1221}, λ_{1331}, λ_{2332}, $\{\lambda_{1313}$, λ_{3212} and H.c.$\}$
6	^{a}U(1)	μ_1^2, μ_2^2, μ_3^2, λ_{11}, λ_{22}, λ_{33}, λ_{1122}, λ_{1133}, λ_{2233}, λ_{1221}, λ_{1331}, λ_{2332}, $\{\lambda_{1323}$ and H.c.$\}$
$6'$	^{b}U(1)$'$	μ_1^2, μ_2^2, μ_3^2, λ_{11}, λ_{22}, λ_{33}, λ_{1122}, λ_{1133}, λ_{2233}, λ_{1221}, λ_{1331}, λ_{2332}, $\{m_{12}^2$, λ_{1212}, λ_{1112}, λ_{2212}, λ_{3312}, λ_{1332} and H.c.$\}$
7	U(1) $\otimes$ U(1)$'$	μ_1^2, μ_2^2, μ_3^2, λ_{11}, λ_{22}, λ_{33}, λ_{1122}, λ_{1133}, λ_{2233}, λ_{1221}, λ_{1331}, λ_{2332}
8	$Z_2 \otimes$ U(1)$'$	μ_1^2, μ_2^2, μ_3^2, λ_{11}, λ_{22}, λ_{33}, λ_{1122}, λ_{1133}, λ_{2233}, λ_{1221}, λ_{1331}, λ_{2332}, $\{\lambda_{1212}$ and H.c.$\}$
9	CP1 $\otimes$ Sp(2)$_{\phi_3}$	μ_1^2, μ_2^2, μ_3^2, Re(m_{12}^2), λ_{11}, λ_{22}, λ_{33}, λ_{1122}, λ_{1133}, λ_{2233}, λ_{1221}, Re(λ_{1212}), Re(λ_{1112}), Re(λ_{2212}), Re(λ_{3312})
10	CP1 $\otimes Z_2 \otimes$ Sp(2)$_{\phi_3}$	μ_1^2, μ_2^2, μ_3^2, λ_{11}, λ_{22}, λ_{33}, λ_{1122}, λ_{1133}, λ_{2233}, λ_{1221}, Re(λ_{1212})
11	U(1) $\otimes$ Sp(2)$_{\phi_3}$	μ_1^2, μ_2^2, μ_3^2, λ_{11}, λ_{22}, λ_{33}, λ_{1122}, λ_{1133}, λ_{2233}, λ_{1221}
12	CP2	$\mu_1^2 = \mu_2^2$, μ_3^2, $\lambda_{11} = \lambda_{22}$, λ_{33}, λ_{1122}, λ_{1221}, $\lambda_{1133} = \lambda_{2233}$, $\lambda_{1331} = \lambda_{2332}$, Re($\lambda_{1313}$)=Re($\lambda_{2323}$), Re($\lambda_{1212}$), $\{\lambda_{1112} = -\lambda_{2212}$ and H.c.$\}$

13	CP2 $\otimes$ Sp(2)$_{\phi_3}$	$\mu_1^2=\mu_2^2$, μ_3^2, $\lambda_{11}=\lambda_{22}$, λ_{33}, λ_{1122}, λ_{1221}, $\lambda_{1133}=\lambda_{2233}$, Re($\lambda_{1212}$), $\{\lambda_{1112}=-\lambda_{2212}$ and H.c.$\}$
14	SO(2)$_{\phi_1,\phi_2}$	$\mu_1^2=\mu_2^2$, μ_3^2, $\lambda_{11}=\lambda_{22}$, λ_{33}, λ_{1122}, λ_{1221}, $\lambda_{1133}=\lambda_{2233}$, $\lambda_{1331}=\lambda_{2332}$, Re($\lambda_{1313}$) = Re($\lambda_{2323}$), Re($\lambda_{1212}$) $=2\lambda_{11}-(\lambda_{1122}+\lambda_{1221})$,
15	D_3	$\mu_1^2=\mu_2^2$, μ_3^2, $\lambda_{11}=\lambda_{22}$, λ_{33}, λ_{1122}, $\lambda_{1133}=\lambda_{2233}$, λ_{1221}, $\lambda_{1331}=\lambda_{2332}$, $\{\lambda_{2131}=-\lambda_{1232}$, λ_{1323} and H.c.$\}$
16	D_4	$\mu_1^2=\mu_2^2$, μ_3^2, $\lambda_{11}=\lambda_{22}$, λ_{33}, λ_{1122}, $\lambda_{1133}=\lambda_{2233}$, λ_{1221}, $\{\lambda_{1212}$ and H.c.$\}$, $\lambda_{1331}=\lambda_{2332}=$ Re(λ_{3231})
17	$D_3\otimes$ Sp(2)$_{\phi_3}$	$\mu_1^2=\mu_2^2$, μ_3^2, $\lambda_{11}=\lambda_{22}$, λ_{33}, λ_{1122}, $\lambda_{1133}=\lambda_{2233}$, λ_{1221}
18	$D_4\otimes$ Sp(2)$_{\phi_3}$	$\mu_1^2=\mu_2^2$, μ_3^2, $\lambda_{11}=\lambda_{22}$, λ_{33}, λ_{1122}, $\lambda_{1133}=\lambda_{2233}$, λ_{1221}, Re(λ_{1212})
19	SO(2)$_{\phi_1,\phi_2}\otimes$ Sp(2)$_{\phi_3}$	$\mu_1^2=\mu_2^2$, μ_3^2, $\lambda_{11}=\lambda_{22}$, λ_{33}, λ_{1122}, $\lambda_{1133}=\lambda_{2233}$, $\lambda_{1221}=$ Re(λ_{1212}) $=\lambda_{11}-\frac{1}{2}\lambda_{1122}$
20	SU(2)$_{\phi_1,\phi_2}$	$\mu_1^2=\mu_2^2$, μ_3^2, $\lambda_{11}=\lambda_{22}$, λ_{33}, $\lambda_{1122}=2\lambda_{11}-\lambda_{1221}$, λ_{1221}, $\lambda_{1133}=\lambda_{2233}$, $\lambda_{1331}=\lambda_{2332}$
21	SU(2)$_{\phi_1,\phi_2}\otimes$ Sp(2)$_{\phi_3}$	$\mu_1^2=\mu_2^2$, μ_3^2, $\lambda_{11}=\lambda_{22}$, λ_{33}, $\lambda_{1122}=2\lambda_{11}-\lambda_{1221}$, $\lambda_{1133}=\lambda_{2233}$, λ_{1221}
22	Sp(2)$_{\phi_1+\phi_2+\phi_3}$	μ_1^2, μ_2^2, μ_3^2, Re(m_{12}^2), Re(m_{13}^2), Re(m_{23}^2), λ_{11}, λ_{22}, λ_{33}, λ_{1122}, λ_{1133}, λ_{2233}, $\lambda_{1221}=$ Re(λ_{1212}), $\lambda_{1331}=$Re(λ_{1313}), $\lambda_{2332}=$Re(λ_{2323}), Re(λ_{1213}) = Re(λ_{2113}), Re(λ_{1223}) = Re(λ_{2123}), Re(λ_{1323}) = Re(λ_{1332}), Re(λ_{1112}), Re(λ_{2212}), Re(λ_{3312}), Re(λ_{1113}), Re(λ_{2213}), Re(λ_{3313}), Re(λ_{1123}), Re(λ_{2223}), Re(λ_{3323})
23	$Z_2\otimes$ Sp(2)$_{\phi_1+\phi_2+\phi_3}$	μ_1^2, μ_2^2, μ_3^2, Re(m_{13}^2), λ_{11}, λ_{22}, λ_{33}, λ_{1122}, λ_{1133}, λ_{2233}, $\lambda_{1221}=$ Re(λ_{1212}), $\lambda_{1331}=$Re(λ_{1313}), $\lambda_{2332}=$Re(λ_{2323}), Re(λ_{1113}), Re(λ_{2213}), Re(λ_{3313})
23′	$Z_2'\otimes$ Sp(2)$_{\phi_1+\phi_2+\phi_3}$	μ_1^2, μ_2^2, μ_3^2, Re(m_{23}^2), λ_{11}, λ_{22}, λ_{33}, λ_{1122}, λ_{1133}, λ_{2233}, $\lambda_{1221}=$ Re(λ_{1212}), $\lambda_{1331}=$Re(λ_{1313}), $\lambda_{2332}=$Re(λ_{2323}), Re(λ_{1123}), Re(λ_{2223}), Re(λ_{3323})
24	$Z_2\otimes Z_2'\otimes$ Sp(2)$_{\phi_1+\phi_2+\phi_3}$	μ_1^2, μ_2^2, μ_3^2, λ_{11}, λ_{22}, λ_{33}, λ_{1122}, λ_{1133}, λ_{2233}, $\lambda_{1221}=$ Re(λ_{1212}), $\lambda_{1331}=$Re(λ_{1313}), $\lambda_{2332}=$Re(λ_{2323})
25	$Z_4\otimes$ Sp(2)$_{\phi_1+\phi_2+\phi_3}$	μ_1^2, μ_2^2, μ_3^2, λ_{11}, λ_{22}, λ_{33}, λ_{1122}, λ_{1133}, λ_{2233}, $\lambda_{1221}=$ Re(λ_{1212})

26	(CP1 $\rtimes S_2$)$\otimes$ Sp(2)$_{\phi_1+\phi_2+\phi_3}$	$\mu_1^2=\mu_2^2$, μ_3^2, Re(m_{12}^2), Re(m_{13}^2)=Re(m_{23}^2), $\lambda_{11}=\lambda_{22}$, λ_{33}, λ_{1122}, $\lambda_{1133}=\lambda_{2233}$, λ_{1221}= Re(λ_{1212}), λ_{1331}=λ_{2332}=Re(λ_{1313})=Re(λ_{2323}), Re(λ_{1323}) = Re(λ_{1332}), Re(λ_{1223}) = Re(λ_{2123})=Re(λ_{1213}) = Re(λ_{2113}), Re(λ_{3313}) = Re(λ_{3323}), Re(λ_{1112}) = Re(λ_{2212}), Re(λ_{3312}), Re(λ_{1113}) = Re(λ_{1123}) = Re(λ_{2213}) = Re(λ_{2223})
27	$D_4\otimes$ Sp(2)$_{\phi_1+\phi_2+\phi_3}$	$\mu_1^2=\mu_2^2$, μ_3^2, $\lambda_{11}=\lambda_{22}=\frac{1}{2}\lambda_{1122}$, λ_{33}, $\lambda_{1133}=\lambda_{2233}$, λ_{1221} =Re(λ_{1212})
28	Sp(2)$_{\phi_1+\phi_2}\otimes$ Sp(2)$_{\phi_3}$	μ_1^2, μ_2^2, μ_3^2, Re(m_{12}^2), λ_{11}, λ_{22}, λ_{33}, λ_{1122}, λ_{1133}, λ_{2233}, λ_{1221}= Re(λ_{1212}), Re(λ_{1112}), Re(λ_{2212}), Re(λ_{3312})
29	Sp(2)$_{\phi_1\phi_2}$	$\mu_1^2=\mu_2^2$, μ_3^2, $\lambda_{11}=\lambda_{22}=\frac{1}{2}\lambda_{1122}$, λ_{33}, $\lambda_{1133}=\lambda_{2233}$, λ_{1221}, $\lambda_{1331}=\lambda_{2332}$
30	Sp(2)$_{\phi_1\phi_2}\otimes$ Sp(2)$_{\phi_3}$	$\mu_1^2=\mu_2^2$, μ_3^2, $\lambda_{11}=\lambda_{22}=\frac{1}{2}\lambda_{1122}$, λ_{33}, $\lambda_{1133}=\lambda_{2233}$, λ_{1221}
31	A_4	$\mu_1^2=\mu_2^2=\mu_3^2$, $\lambda_{11}=\lambda_{22}=\lambda_{33}$, $\lambda_{1122}=\lambda_{1133}=\lambda_{2233}$, $\lambda_{1221}=\lambda_{1331}=\lambda_{2332}$, $\{\lambda_{1212}=\lambda_{1313}=\lambda_{2323}$, and $\mathrm{H.c.}\}$
32	S_4	$\mu_1^2=\mu_2^2=\mu_3^2$, $\lambda_{11}=\lambda_{22}=\lambda_{33}$, $\lambda_{1122}=\lambda_{1133}=\lambda_{2233}$, $\lambda_{1221}=\lambda_{1331}=\lambda_{2332}$, Re($\lambda_{1212}$) = Re($\lambda_{1313}$) = Re($\lambda_{2323}$)
33	SO(3)	$\mu_1^2=\mu_2^2=\mu_3^2$, $\lambda_{11}=\lambda_{22}=\lambda_{33}$, $\lambda_{1122}=\lambda_{1133}=\lambda_{2233}$, $\lambda_{1221}=\lambda_{1331}=\lambda_{2332}$, Re($\lambda_{1212}$) = Re($\lambda_{1313}$) = Re($\lambda_{2323}$) = $2\lambda_{11}-(\lambda_{1122}+\lambda_{1221})$
34	$S_4\otimes$ Sp(2)$_{\phi_1+\phi_2+\phi_3}$	$\mu_1^2=\mu_2^2=\mu_3^2$, $\lambda_{11}=\lambda_{22}=\lambda_{33}=\frac{1}{2}\lambda_{1122}=\lambda_{1133}=\lambda_{2233}$, $\lambda_{1221}=\lambda_{1331}=\lambda_{2332}$=Re($\lambda_{1212}$) = Re($\lambda_{1313}$) = Re($\lambda_{2323}$)
35	$\Delta(54)$	$\mu_1^2=\mu_2^2=\mu_3^2$, $\lambda_{11}=\lambda_{22}=\lambda_{33}$, $\lambda_{1122}=\lambda_{1133}=\lambda_{2233}$, $\lambda_{1221}=\lambda_{1331}=\lambda_{2332}$, $\{\lambda_{1213}=\lambda_{2123}=\lambda_{3231}$ and $\mathrm{H.c.}\}$
36	$\Sigma(36)$	$\mu_1^2=\mu_2^2=\mu_3^2$, $\lambda_{11}=\lambda_{22}=\lambda_{33}$, $\lambda_{1122}=\lambda_{1133}=\lambda_{2233}$, $\lambda_{1221}=\lambda_{1331}=\lambda_{2332}$, Re($\lambda_{1213}$) = Re($\lambda_{1323}$) = Re($\lambda_{1232}$)= $\frac{3}{4}(2\lambda_{11}-\lambda_{1122}-\lambda_{1221})$
37	Sp(2)$_{\phi_1}\otimes$ Sp(2)$_{\phi_2}\otimes$ Sp(2)$_{\phi_3}$	μ_1^2, μ_2^2, μ_3^2, λ_{11}, λ_{22}, λ_{33}, λ_{1122}, λ_{1133}, λ_{2233}
38	Sp(4) $\otimes$ Sp(2)$_{\phi_3}$	$\mu_1^2=\mu_2^2$, μ_3^2, $\lambda_{11}=\lambda_{22}=\frac{1}{2}\lambda_{1122}$, λ_{33}, $\lambda_{1133}=\lambda_{2233}$
39	SU(3) $\otimes$ U(1)	$\mu_1^2=\mu_2^2=\mu_3^2$, $\lambda_{11}=\lambda_{22}=\lambda_{33}$, $\lambda_{1122}=\lambda_{1133}=\lambda_{2233}$, $\lambda_{1221}=\lambda_{1331}=\lambda_{2332}=2\lambda_{11}-\lambda_{1122}$
40	Sp(6)	$\mu_1^2=\mu_2^2=\mu_3^2$, $\lambda_{11}=\lambda_{22}=\lambda_{33}=\frac{1}{2}\lambda_{1122}=\frac{1}{2}\lambda_{1133}=\frac{1}{2}\lambda_{2233}$

• Maximal Symmetry and Quartic Coupling Unification

• Maximally Symmetric Two Higgs Doublet Model

[P.S.B. Dev, AP '14; N. Darvishi, AP '19]

$$G_{\mathbf{\Phi}} = \mathrm{SU}(2)_L \otimes \mathrm{Sp}(4)/\mathrm{Z}_2 \simeq \mathrm{SU}(2)_L \otimes \mathrm{SO}(5)$$

$$\mathrm{V} \ = \ - \, \mu^2 \Big(|\Phi_1|^2 + |\Phi_2|^2 \Big) + \lambda \, \Big(|\Phi_1|^2 + |\Phi_2|^2 \Big)^2 \ = \ -\frac{\mu^2}{2} \, \mathbf{\Phi}^\dagger \, \mathbf{\Phi} + \frac{\lambda}{4} \, (\mathbf{\Phi}^\dagger \, \mathbf{\Phi})^2 \, ,$$

where

$$\mathbf{\Phi} \ = \ \begin{pmatrix} \phi_1 \\ \phi_2 \\ i\sigma^2 \phi_1^* \\ i\sigma^2 \phi_2^* \end{pmatrix} \, , \quad \text{with } \ U_L \in \ \mathrm{SU}(2)_L : \ \mathbf{\Phi} \ \mapsto \ \mathbf{\Phi}' \ = \ U_L \, \mathbf{\Phi} \, ,$$

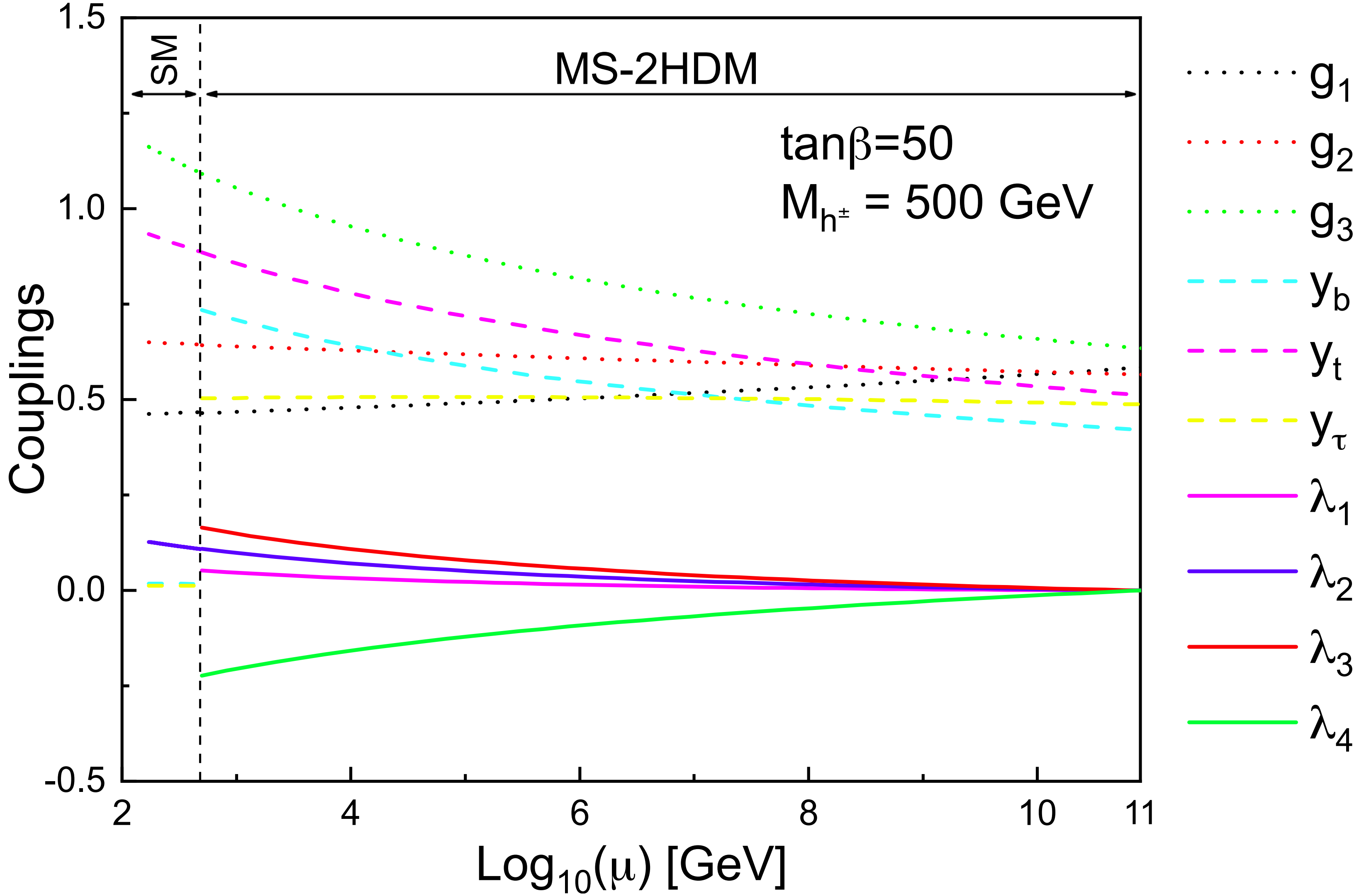
such that under **global field transformations**,

$$\mathrm{Sp}(4) : \quad \mathbf{\Phi} \ \mapsto \ \mathbf{\Phi}' \ = \ U \, \mathbf{\Phi} \, , \quad \text{with } U \in \mathrm{U}(4) \ \ \& \ \ U \mathrm{C} U^{\mathsf{T}} = \ \mathrm{C} \equiv i\sigma^2 \otimes \sigma^0$$

$\mathrm{SU}(2)_L$ gauge kinetic terms remain invariant.

Breaking Effects: $m_{12}^2 \, \phi_1^\dagger \phi_2$ (or $M_{h^\pm}$), $\mathrm{U}(1)_Y$ coupling g', Yukawa's $\mathbf{Y}^{u,d}$.

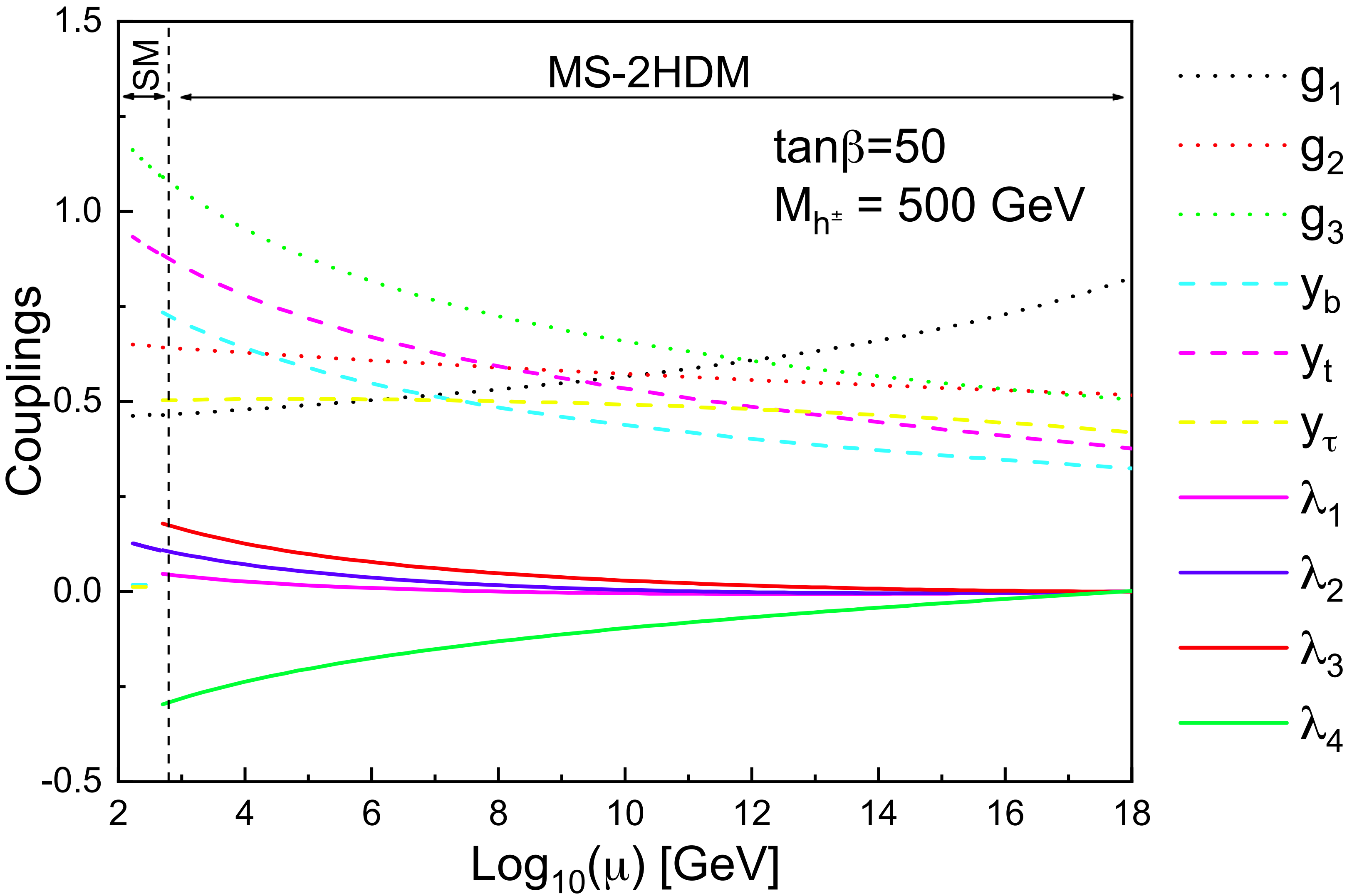
• **Quartic Coupling Unification (up to two loops)** [N. Darvishi, AP '19]



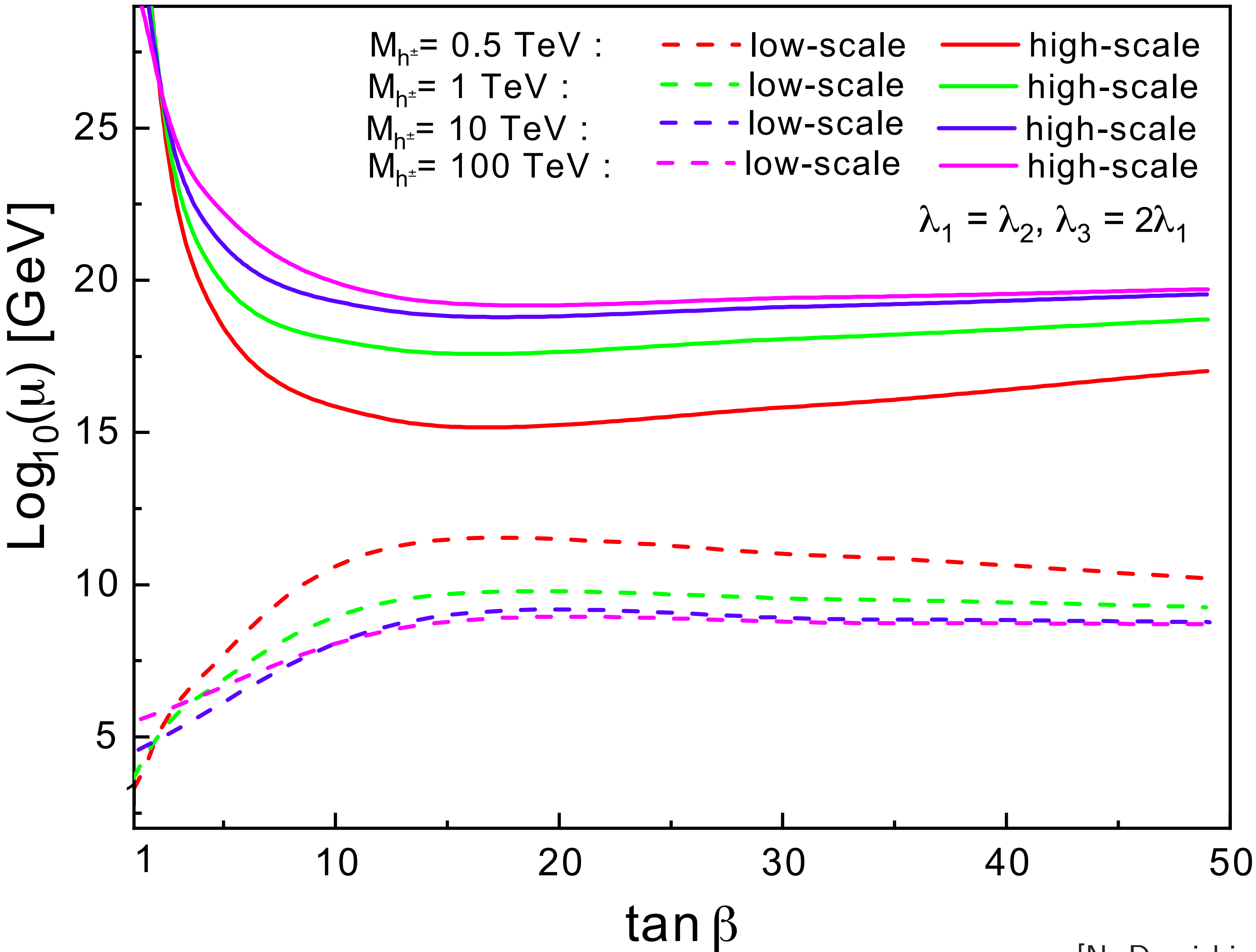
First conformal unification point: $\mu_X^{(1)} \sim 10^{11}$ GeV (of order PQ scale)

Second conformal unification point: $\mu_X^{(2)} \sim 10^{18}$ GeV (of order $m_{\rm Pl}$)

[N. Darvishi, AP '19]



Low- and high-scale quartic coupling unification: $\tan\beta$ vs $\mu_X^{(1,2)}$



[N. Darvishi, AP '19]

• Misalignment in the MS-2HDM

CP-even mass matrix in Higgs basis:

$$\mathcal{M}_{\mathcal{S}}^2 = \begin{pmatrix} \widehat{A} & \widehat{C} \\ \widehat{C} & \widehat{B} \end{pmatrix} \xrightarrow[\text{approx.}]{\text{seesaw}} M_H^2 \simeq \widehat{A} - \frac{\widehat{C}^2}{\widehat{B}} \quad \& \quad M_h^2 \simeq \widehat{B} \gg \widehat{A},\, \widehat{C}$$

Light-to-heavy scalar mixing:

$$\theta_{\mathcal{S}} \equiv \frac{\widehat{C}}{\widehat{B}} = \frac{v^2 s_\beta c_\beta \big[s_\beta^2\,(2\lambda_2 - \lambda_{34}) - c_\beta^2\,(2\lambda_1 - \lambda_{34}) \big]}{M_a^2 + 2v^2 s_\beta^2 c_\beta^2\,(\lambda_1 + \lambda_2 - \lambda_{34})} \ll 1$$

Higgs couplings to $V = W, Z$:

$$g_{HVV} \simeq 1 - \frac{1}{2}\,\theta_{\mathcal{S}}^2\,, \qquad g_{hVV} \simeq -\,\theta_{\mathcal{S}}$$

Higgs couplings to quarks:

$$g_{Huu} \simeq 1 + t_\beta^{-1}\,\theta_{\mathcal{S}}\,, \qquad g_{Hdd} \simeq 1 - t_\beta\,\theta_{\mathcal{S}}\,,$$

$$g_{huu} \simeq -\,\theta_{\mathcal{S}} + t_\beta^{-1}\,, \qquad g_{hdd} \simeq -\,\theta_{\mathcal{S}} - t_\beta\,.$$

Misalignment predictions in the MS-2HDM with low- and high-scale quartic coupling unification, assuming $M_{h^\pm} = 500$ GeV.

[N. Darvishi, AP '19]

Couplings	ATLAS	CMS	$\tan\beta = 2$	$\tan\beta = 20$	$\tan\beta = 50$
$\lvert g_{HZZ}^{\text{low-scale}} \rvert$	[0.86, 1.00]	[0.90, 1.00]	0.9999	0.9999	0.9999
$\lvert g_{HZZ}^{\text{high-scale}} \rvert$			0.9981	0.9999	0.9999
$\lvert g_{Htt}^{\text{low-scale}} \rvert$	$1.31^{+0.35}_{-0.33}$	$1.45^{+0.42}_{-0.32}$	1.0049	1.0001	1.0000
$\lvert g_{Htt}^{\text{high-scale}} \rvert$			1.0987	1.0003	1.0001
$\lvert g_{Hbb}^{\text{low-scale}} \rvert$	$0.49^{+0.26}_{-0.19}$	$0.57^{+0.16}_{-0.16}$	0.9803	0.9560	0.9590
$\lvert g_{Hbb}^{\text{high-scale}} \rvert$			0.8810	0.9449	0.9427

$\longrightarrow$ Misalignment predictions consistent with experiment

• Vacuum Topology of the 2HDM

[R Battye, G Brawn, AP, JHEP08 (2011) 020.]

$\mathrm{G}_{\mathrm{HF/CP}}$	$\mathrm{H}_{\mathrm{HF/CP}}$	$\mathcal{M}_{\Phi}^{\mathrm{HF/CP}}$	Topological Defect
Z_2	$\mathbf{I}$	Z_2	Domain Wall
$\mathrm{U(1)}_{\mathrm{PQ}} \simeq S^1$	$\mathbf{I}$	S^1	Vortex
$\mathrm{SO(3)}_{\mathrm{HF}}$	$\mathrm{SO(2)}_{\mathrm{HF}}$	S^2	Global Monopole
$\mathrm{CP1} \simeq \mathrm{Z}_2$	$\mathbf{I}$	Z_2	Domain Wall
$\mathrm{CP2} = \mathrm{Z}_2 \otimes \Pi_2$	Π_2	Z_2	Domain Wall
$\mathrm{CP1} \otimes \mathrm{SO(2)}$	$\mathrm{CP1}$	S^1	Vortex

- **Energy density of the topological defect $\phi_{1,2}(\mathbf{r})$:**

$$\mathcal{E}(\phi_1, \phi_2) \;=\; (\nabla\phi_1^\dagger)\cdot(\nabla\phi_1) + (\nabla\phi_2^\dagger)\cdot(\nabla\phi_2) + \;\mathrm{V}(\phi_1,\phi_2) \;+\; \mathrm{V}_0\,.$$

- **Gradient flow approach to numerically find $\phi_{1,2}(\mathbf{r})$**

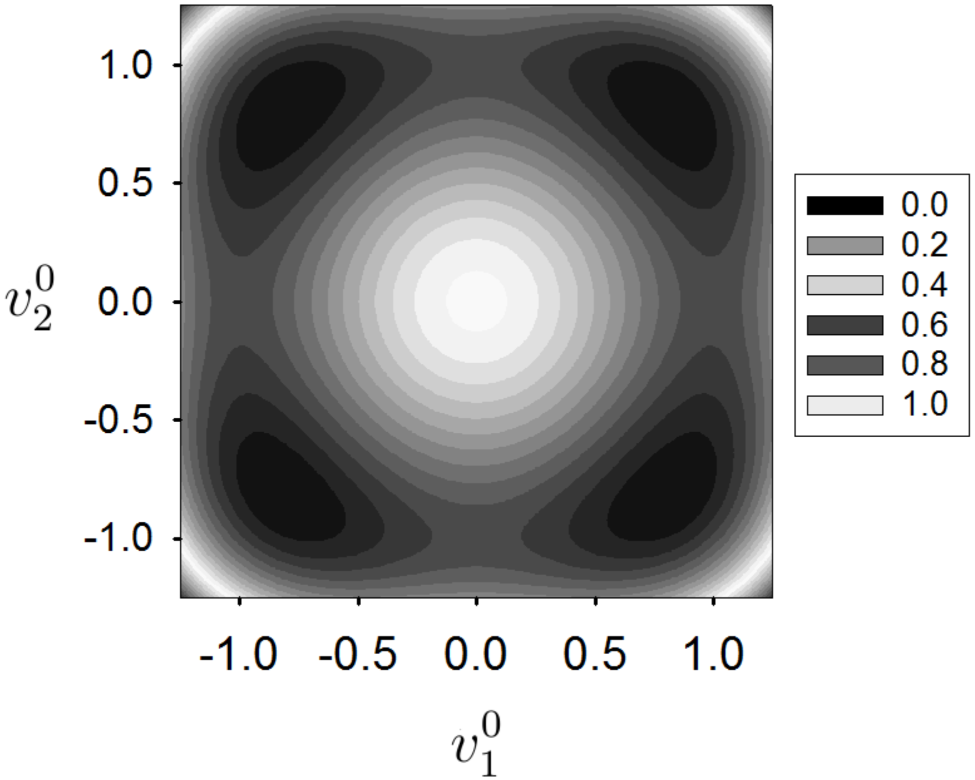
$$-\,\frac{\delta E[\phi_{1,2}]}{\delta\phi_{1,2}(\mathbf{r},\tau)} \;=\; \frac{\partial\phi_{1,2}(\mathbf{r},\tau)}{\partial\tau} \;\to\; 0\,, \qquad \text{for } \tau \gg 1\,.$$

• Z_2 Domain Walls

$$\begin{pmatrix} v_1^0 \\ v_2^0 \end{pmatrix} \overset{U(1)_Y}{\longleftrightarrow} \begin{pmatrix} -v_1^0 \\ -v_2^0 \end{pmatrix}$$

$$Z_2 \updownarrow \qquad\qquad \updownarrow Z_2$$

$$\begin{pmatrix} v_1^0 \\ -v_2^0 \end{pmatrix} \overset{U(1)_Y}{\longleftrightarrow} \begin{pmatrix} -v_1^0 \\ v_2^0 \end{pmatrix}$$

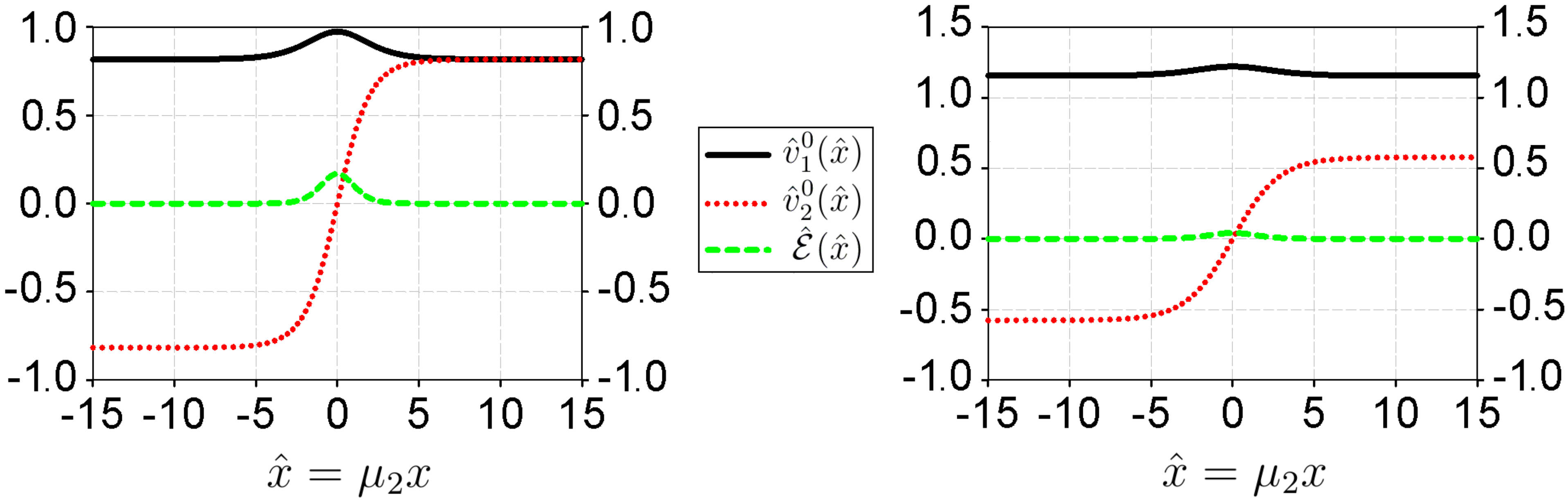


Spatial profile of the $\mathbf{Z}_2$ domain wall

[R Battye, G Brown, AP, JHEP08 (2011) 020.]

Introduce dimensionless quantities:

$$\hat{x} \;=\; \mu_2 x\,, \quad \hat{v}^0_{1,2}(\hat{x}) \;=\; \frac{v^0_{1,2}(\hat{x})}{\eta}\,, \quad \widehat{\mathrm{E}} \;=\; \frac{\lambda_2 \mathrm{E}}{\mu_2^3}\,, \quad \text{with } \eta \;=\; \frac{\mu_2}{\sqrt{\lambda_2}}\,.$$



• Charge-Violating Domain Walls in the 2HDM

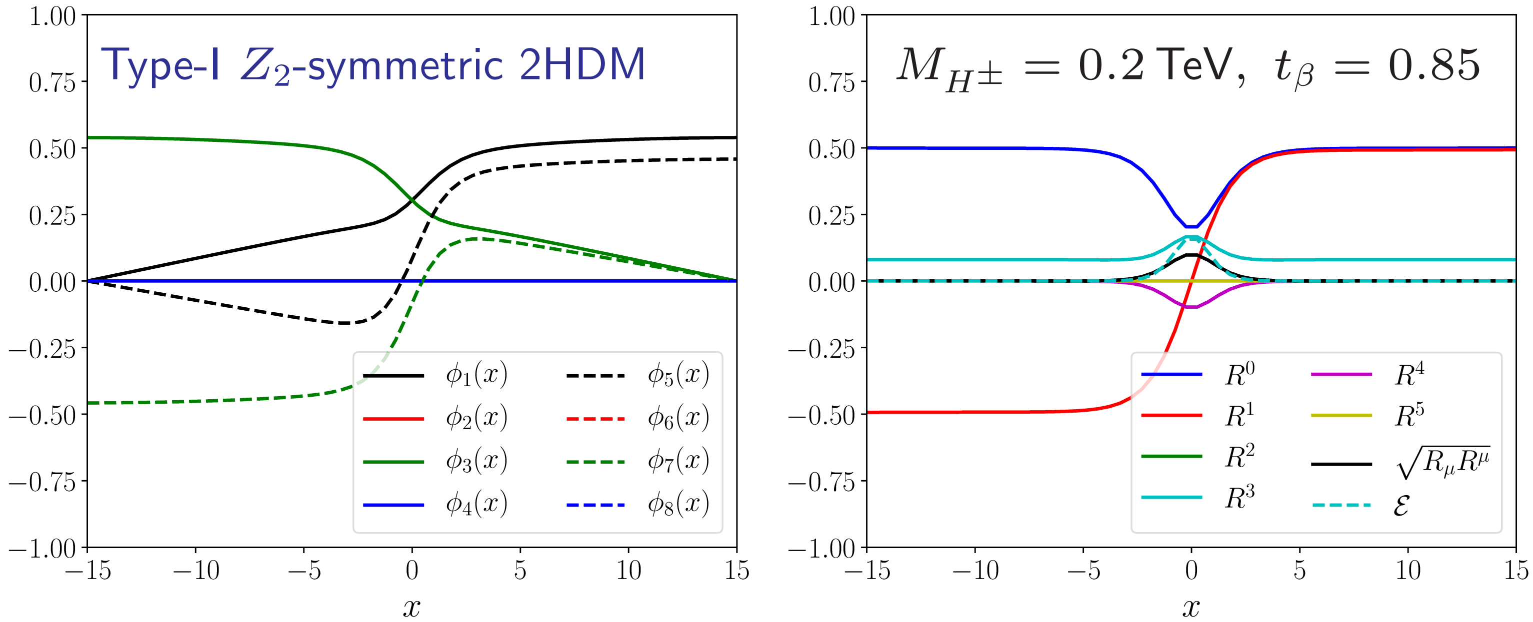
[R Battye, AP, D Viatic, JHEP2101 (2021) 105.]

– Relatively gauge-rotated vacua at the boundaries:

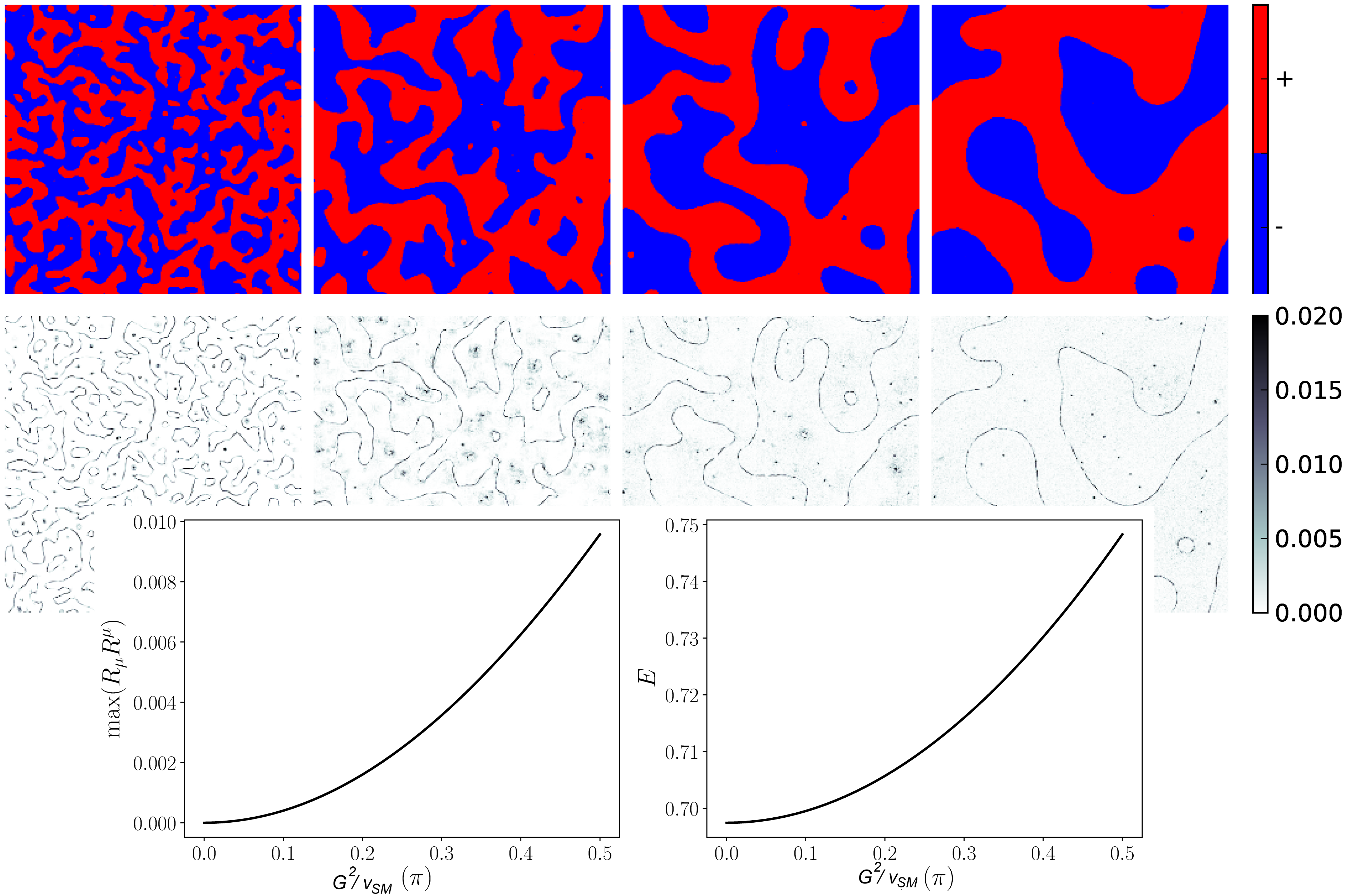
$$\Phi_1(-\infty) = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_1 \end{pmatrix}, \qquad \Phi_2(-\infty) = \frac{1}{\sqrt{2}}\begin{pmatrix} v_+ \to 0 \\ -v_2\, e^{-i\xi} \end{pmatrix},$$

$$\Phi_1(+\infty) = U(+\infty)\,\frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_1 \end{pmatrix}, \qquad \Phi_2(+\infty) = U(+\infty)\,\frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ +v_2\, e^{+i\xi} \end{pmatrix},$$

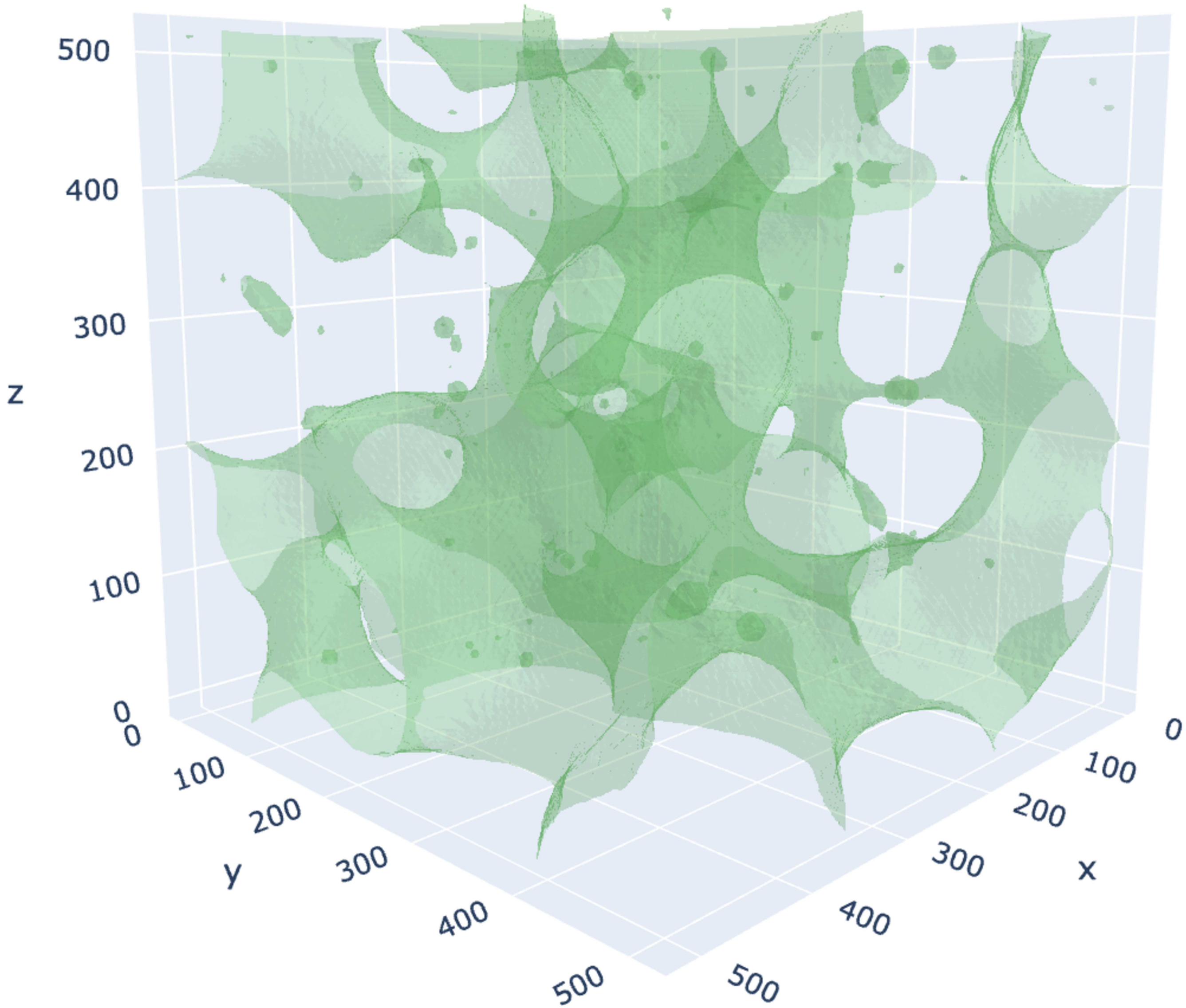
$$U(x) = e^{i\theta(x)}\,\exp\left(\frac{i\,G^i(x)}{v_{\text{SM}}}\,\frac{\sigma^i}{2}\right), \qquad \text{with} \quad U(-\infty) = \mathbf{1}_2\,.$$



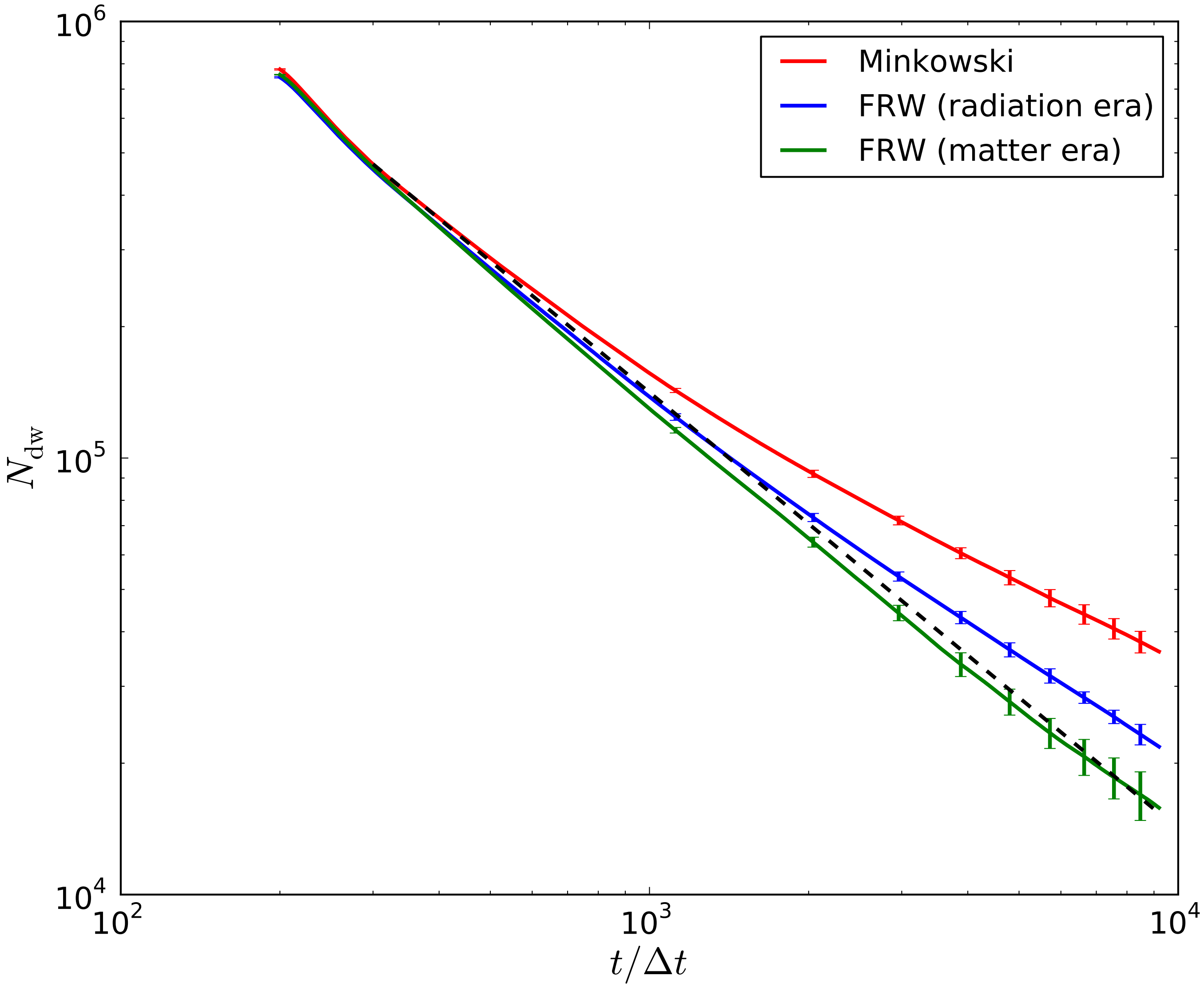
– 2D DW simulations in the Type-I Z_2-symmetric 2HDM



– 3D DW network in the Type-I Z_2-symmetric 2HDM



– Evolution of DW number $N_{\rm dw}$ in the Type-I Z_2-symmetric 2HDM



– QCD instantons in Type-II Z_2-symmetric 2HDM

[R Battye, AP, D Viatic, PRD102 (2020) 123536;
RD Peccei, HR Quinn '77]

$$V_{\rm inst} \sim \Lambda^4_{\rm QCD} \left[\left(\frac{\Phi_1^\dagger \Phi_2}{v^2_{\rm SM}} \right)^{n_{\sf G}} - \left(\frac{\Phi_1^\dagger \Phi_2\, e^{i\theta_{\sf QCD}}}{v^2_{\rm SM}} \right)^{n_{\sf G}} \right] + {\rm H.c.}$$

$$\lesssim \frac{\Lambda^4_{\rm QCD}}{v^2_{\rm SM}}\, s^2_\beta c^2_\beta \Big(1 - \cos\big(n_{\sf G}\theta_{\sf QCD}\big)\Big)\ \Phi_1^\dagger \Phi_2 \ + \ {\rm H.c.}\,,$$

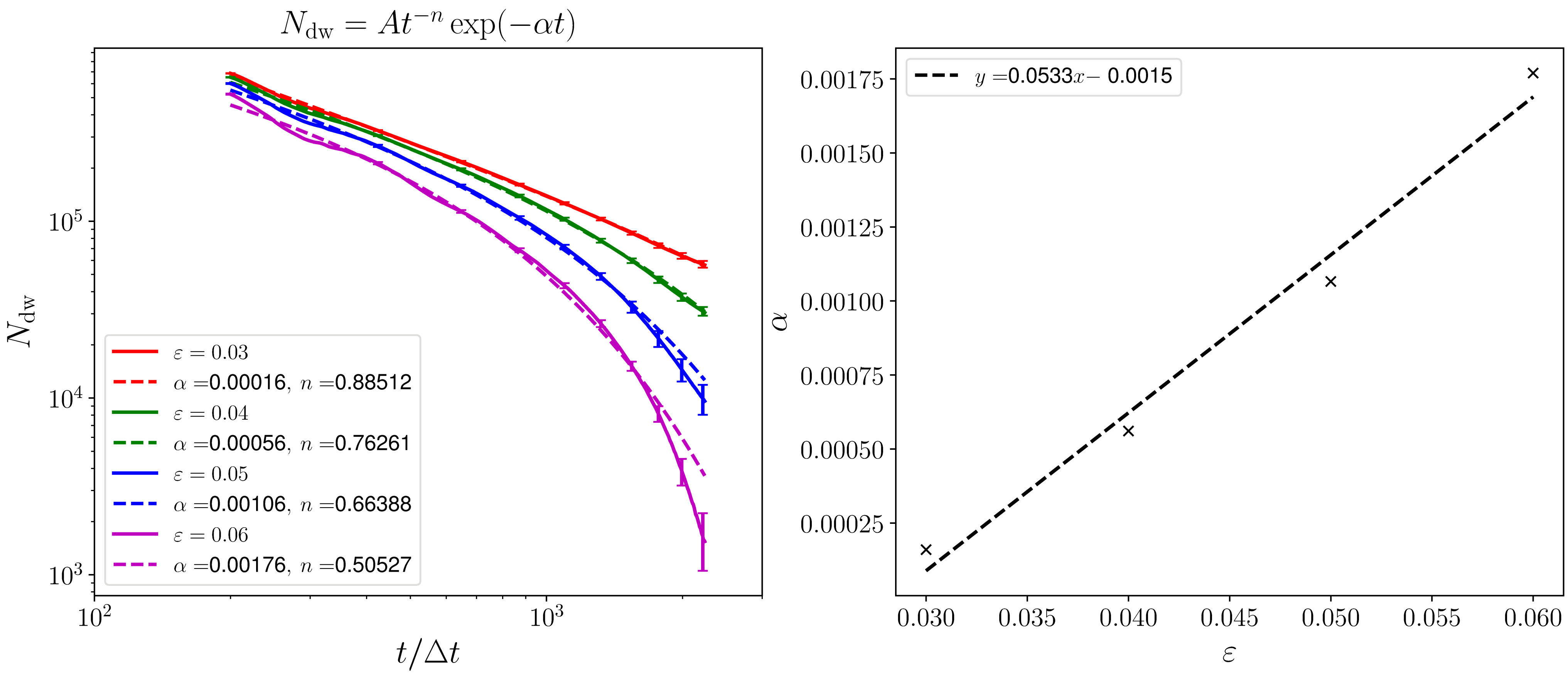
$$\Longrightarrow \quad \theta_{\sf QCD} \ \gtrsim \ \frac{10^{-11}}{\sin\beta\cos\beta}$$

From neutron EDM limit: $\theta_{\sf QCD} \ \lesssim \ 10^{-11} - 10^{-10}$

Loose constraint: $\mathbf{0.3 \lesssim \tan\beta \lesssim 3}$

– Biased initial conditions in Z_2-symmetric 2HDMs

[R Battye, AP, D Viatic, PRD102 (2020) 123536]



Avoidance of DW domination in the Universe:

$$\varepsilon \; > \; \frac{640\pi}{3}\frac{A\widehat{E}}{e}\left(\frac{v_{\rm SM}^{3/2}}{M_{\rm Pl}}\right)^2 \; \simeq \; 2.5\times 10^{-29} A\widehat{E}\ \text{GeV}, \quad \text{with } A,\ \widehat{E}\sim 1.$$

• Conclusions

- **Systematic method based on prime bilinear invariants enables to construct all accidentally symmetric scalar potentials.**
  $\Rightarrow$ Method applied to 2HDM, 2HDMEFTs and multi-HDMs:

  – 2HDM (D = 4): 13 = 6 [U$(1)_Y$] + 7 [Custodial]
  – 2HDMEFT (D = 6): 15 = 8 + 7; 2HDMEFT (D = 8): 17 = 10 + 7
  – 3HDM (D = 4): 40 = 19 [U$(1)_Y$] + 21 [Custodial]

- **Quartic coupling unification for maximally symmetric nHDMs:**

$$G_\Phi \;=\; \mathsf{SU}(2)_L \otimes \mathsf{Sp}(2n)/\mathsf{Z}_2 \qquad (\text{here } n = 2).$$

  INPUT: $M_{h^\pm}$ & $\tan\beta$ $\rightarrow$ $\mu_X^{(1)} \sim 10^{11}$ GeV & $\mu_X^{(2)} \sim 10^{19}$ GeV.
  $\Rightarrow$ RG effects give rise to *definite* misalignment predictions for *all H*-couplings to SM particles in terms of $M_{h^\pm}$ & $\tan\beta$.

- **Domain Walls in the 2HDM violate charge that delays their collapse in the early Universe.**
  Avoidance of DW domination $\Longrightarrow$ $\theta_{\mathsf{QCD}} \gtrsim 10^{-11}/(\sin\beta\cos\beta)$
  in Type-II Z_2-symmetric 2HDM $\Longrightarrow 0.3 \lesssim \tan\beta \lesssim 3$ from EDMs.

Back-Up Slides

• Quartic coupling unification in the MS-2HDM

[Dev, AP '14; N. Darvishi, AP '19]

Symmetry-breaking of Sp(4)/Z_2 $\sim$ SO(5):

- Soft breaking (e.g. through m_{12}^2):

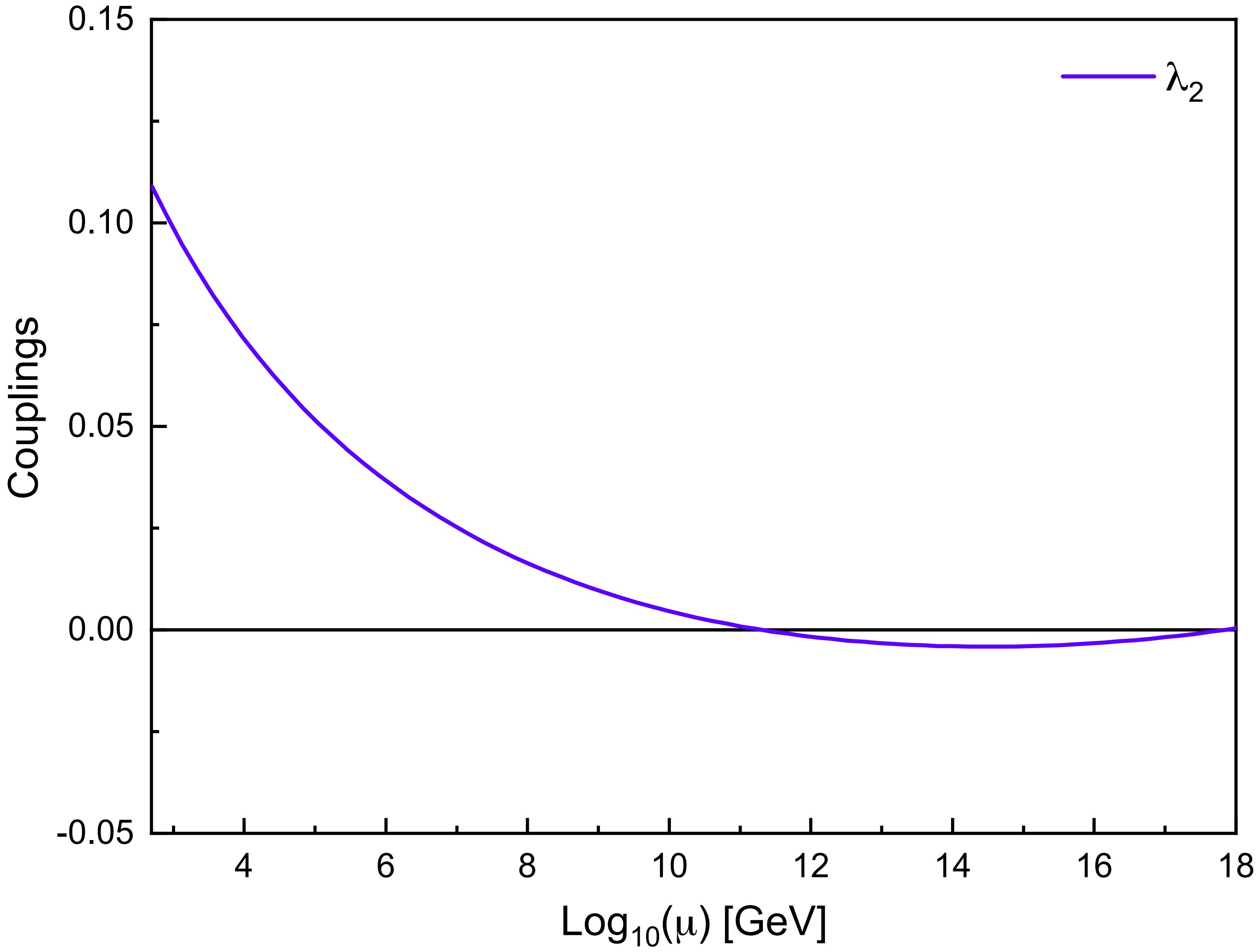
$$M_H^2 = 2\lambda_2 v^2, \qquad M_h^2 = M_a^2 = M_{h^\pm}^2 = \frac{\mathrm{Re}(m_{12}^2)}{s_\beta c_\beta}$$

Heavy Higgs spectrum is degenerate at tree level.

- Explicit breaking through RG running (two loops):

$$\begin{aligned}
\mathrm{Sp}(4)/\mathrm{Z}_2 \otimes \mathrm{SU}(2)_L \quad &\xrightarrow{g' \neq 0} \quad \mathrm{SU}(2)_{\mathrm{HF}} \otimes \mathrm{U}(1)_Y \otimes \mathrm{SU}(2)_L \\
&\xrightarrow{\mathbf{Y}^{u,d}} \quad \mathrm{U}(1)_{\mathrm{PQ}} \otimes \mathrm{U}(1)_Y \otimes \mathrm{SU}(2)_L \\
&\xrightarrow[\langle \Phi_{1,2} \rangle]{m_{12}^2} \quad \mathrm{U}(1)_{\mathrm{em}}
\end{aligned}$$

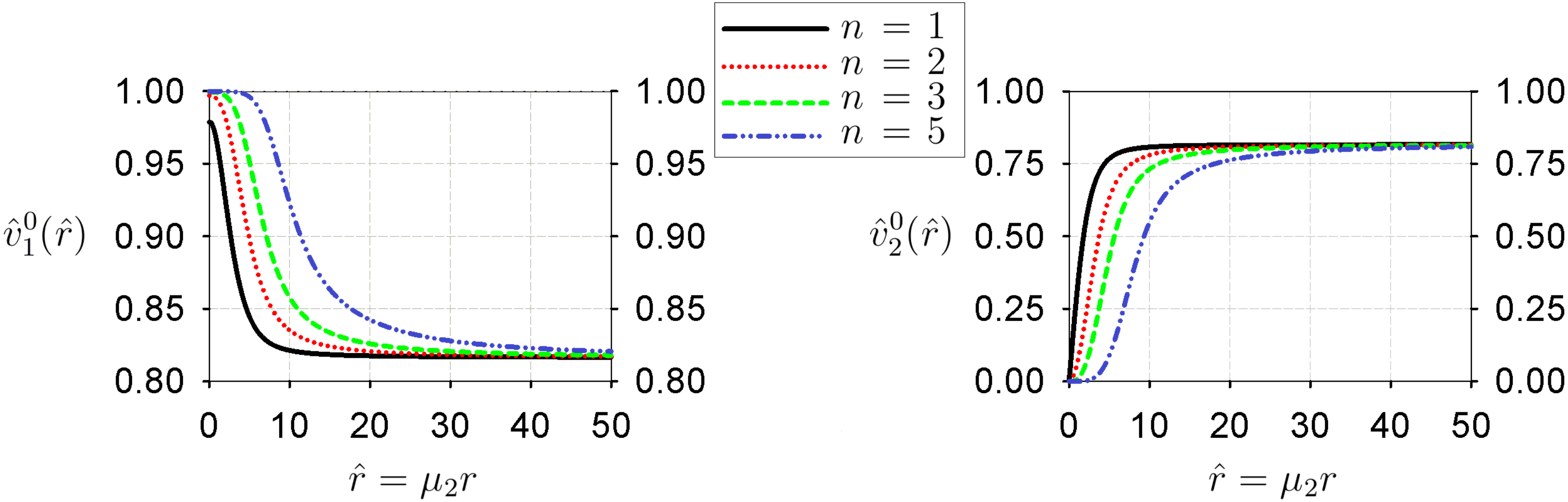
A closer look at the RG evolution of λ_2



• Other Topological Defects from the 2HDM Potential

• $\mathbf{U(1)_{PQ}}$ Vortices [R. A. Battye, G. D. Brawn, A.P., JHEP08 (2011) 020.]

$$\phi_1(r) \;=\; \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_1^0(r) \end{pmatrix}\,, \qquad \phi_2(r,\chi) \;=\; \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_2^0(r)e^{in\chi} \end{pmatrix}\,.$$

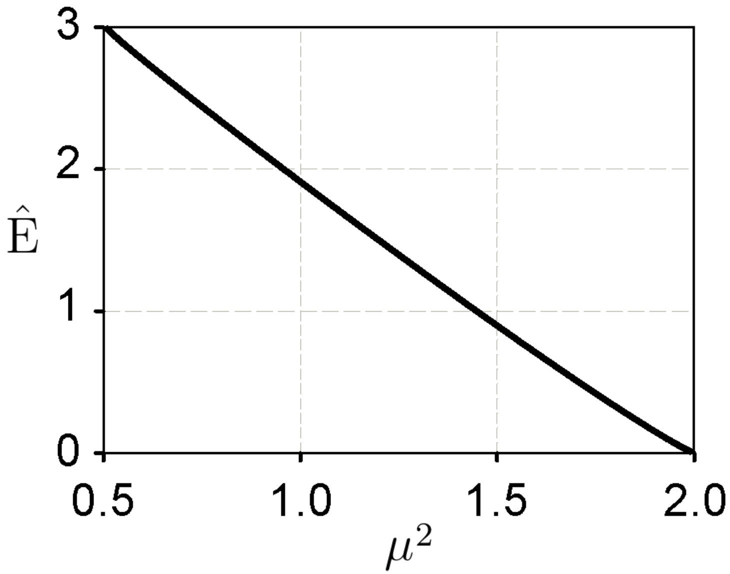


Energy dependence of the $\mathrm{U(1)_{PQ}}$ Vortex

[R. A. Battye, G. D. Brawn, A.P., JHEP08 (2011) 020.]

Energy per unit length:

$$\mathrm{E} \;=\; 2\pi \int_0^\infty r dr \;\; \mathcal{E}(\phi_1, \phi_2) \;\; ,$$



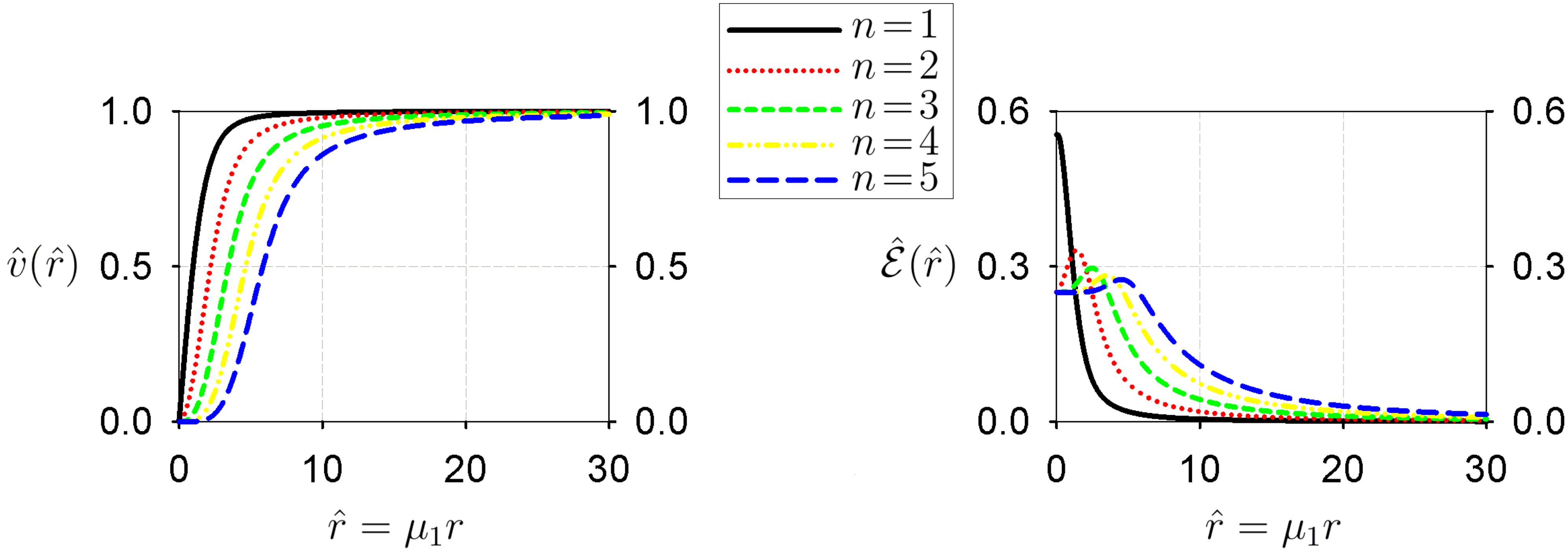
with

$$\mu^2 \;=\; \frac{\mu_1^2}{\mu_2^2} \; .$$

• CP3 Vortices

[R. A. Battye, G. D. Brawn, A.P., JHEP08 (2011) 020.]

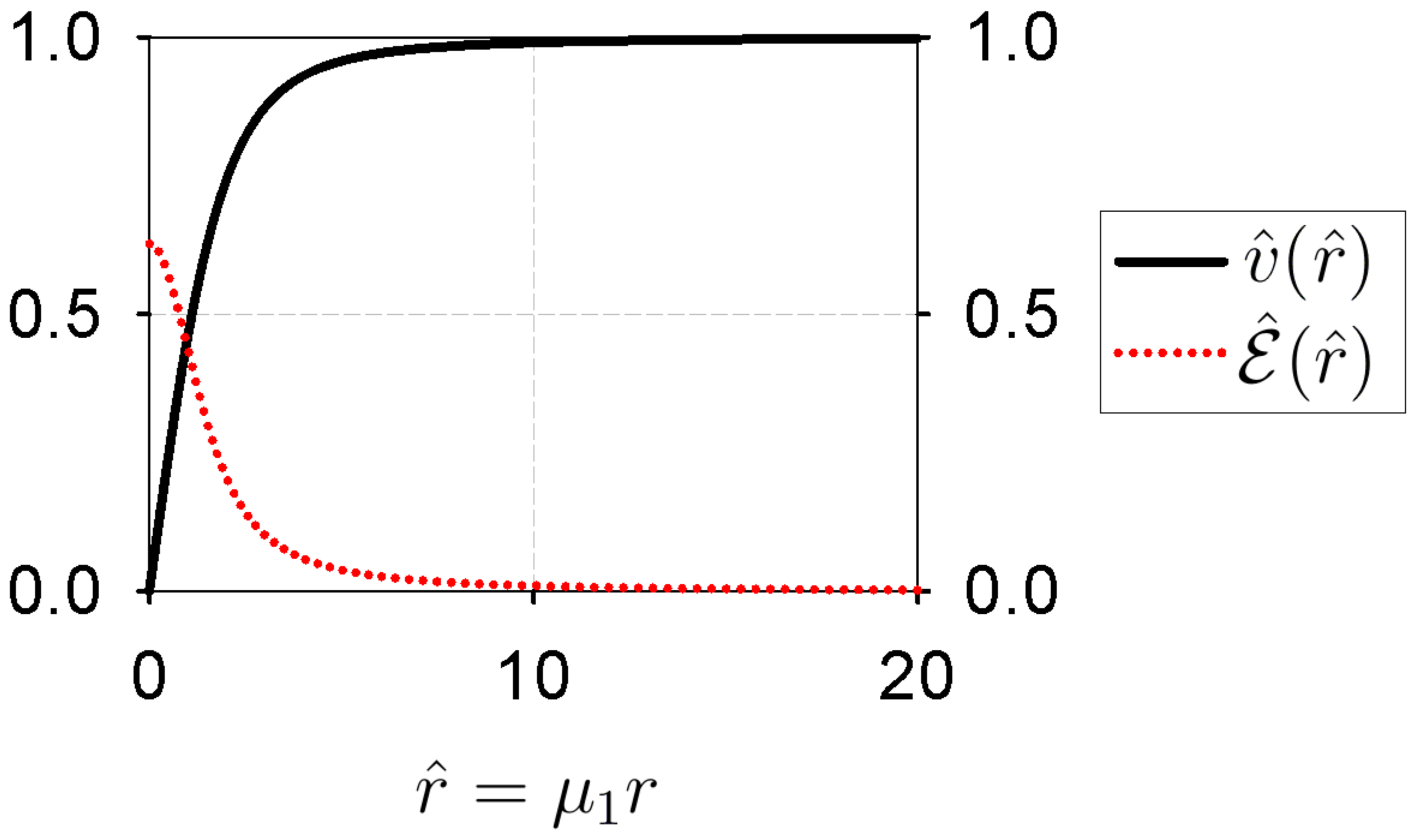
$$\phi_1(r,\chi) \;=\; \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v(r)\cos(n\chi) \end{pmatrix}, \qquad \phi_2(r,\chi) \;=\; \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ -v(r)\sin(n\chi) \end{pmatrix}.$$



• $SO(3)_{\rm HF}$ Global Monopole

[R. A. Battye, G. D. Brawn, A.P., JHEP08 (2011) 020.]

$$\phi_1(r,\chi) \;=\; \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v(r)\sin\chi \end{pmatrix}, \qquad \phi_2(r,\chi,\psi) \;=\; \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v(r)e^{i\psi}\cos\chi \end{pmatrix}.$$



• Natural Alignment Beyond the 2HDM

[AP '16]

– nHDM **potential** with m **inert scalar doublets:**

$$V_{n\mathrm{HDM}} \;=\; V_{\mathrm{sym}} \;+\; V_{\mathrm{inert}} \;+\; \Delta V_{\mathrm{soft}}\;,$$

– **3 continuous alignment symmetries** in the **field space of the active EWSB sector** ($N_H = n - m$):

(i) $\mathrm{Sp}(2N_H) \times \mathcal{D}$ (ii) $\mathrm{SU}(N_H) \times \mathcal{D}$ (iii) $\mathrm{SO}(N_H) \times \mathcal{CP} \times \mathcal{D}$,

where $\mathcal{D}$ acts on the inert sector *only*.

– **Symmetry invariants:**

$$\text{(i)}\quad S \;=\; \Phi_1^\dagger \Phi_1 \;+\; \Phi_2^\dagger \Phi_2 \;+\; \ldots \;=\; \tfrac{1}{2}\,\mathbf{\Phi}^\dagger\,\mathbf{\Phi}$$

$$\text{(ii)}\quad D^a \;=\; \Phi_1^\dagger \sigma^a \Phi_1 \;+\; \Phi_2^\dagger \sigma^a \Phi_2 \;+\; \ldots$$

$$\text{(iii)}\quad T \;=\; \Phi_1 \Phi_1^\mathsf{T} \;+\; \Phi_2 \Phi_2^\mathsf{T} \;+\; \ldots$$

– **Symmetric part of the scalar potential:**

$$V_{\mathrm{sym}} \;=\; -\,\mu^2\,S \;+\; \lambda_S\,S^2 \;+\; \lambda_D\,D^a D^a \;+\; \lambda_T\,\mathrm{Tr}\,(T\,T^*)\;.$$

– **Inert part of the scalar potential:**

$$V_{\rm inert} \;=\; \widehat{m}^2_{\hat{a}\hat{b}}\, \widehat{\Phi}^\dagger_{\hat{a}}\widehat{\Phi}_{\hat{b}} \;+\; \lambda_{\hat{a}\hat{b}\hat{c}\hat{d}}\, (\widehat{\Phi}^\dagger_{\hat{a}}\widehat{\Phi}_{\hat{b}})(\widehat{\Phi}^\dagger_{\hat{c}}\widehat{\Phi}_{\hat{d}}) \;+\; \lambda_{\hat{a}\hat{b}cd}\, (\widehat{\Phi}^\dagger_{\hat{a}}\widehat{\Phi}_{\hat{b}})(\Phi^\dagger_c\Phi_d)$$
$$+\, \lambda_{a\hat{b}\hat{c}d}\, (\Phi^\dagger_a\widehat{\Phi}_{\hat{b}})(\widehat{\Phi}^\dagger_{\hat{c}}\Phi_d) \;+\; \Big[\lambda_{a\hat{b}c\hat{d}}\, (\Phi^\dagger_a\widehat{\Phi}_{\hat{b}})(\Phi^\dagger_c\widehat{\Phi}_{\hat{d}}) \;+\; \mathrm{H.c.}\Big]$$

$$\mathbf{Z}_2^{\mathbf{I}}: \quad \Phi_a \to \Phi_a \quad (a = 1,\, 2, \ldots, N_H)\,, \qquad \widehat{\Phi}_{\hat{b}} \to -\,\widehat{\Phi}_{\hat{b}} \quad (\hat{b} = \hat{1},\, \hat{2}, \ldots, \widehat{m})$$

– **Soft-symmetry Breaking:**

$$\Delta V_{\rm soft} \;=\; m^2_{ab}\, \Phi^\dagger_a\Phi_b$$

– **Minimal Symmetry of Alignment in the Higgs basis:**

$$\mathbf{Z}_2^{\mathbf{EW}}: \quad \Phi'_1 \to \Phi'_1\,, \qquad \Phi'_{a'} \to -\,\Phi'_{a'} \qquad (a' = 2,\, 3, \ldots, N_H)$$

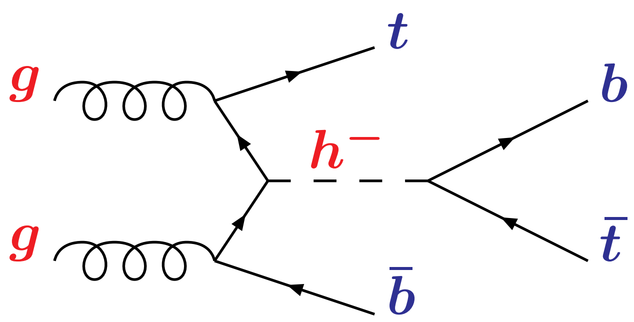
where m^2_{ab} becomes diagonal.

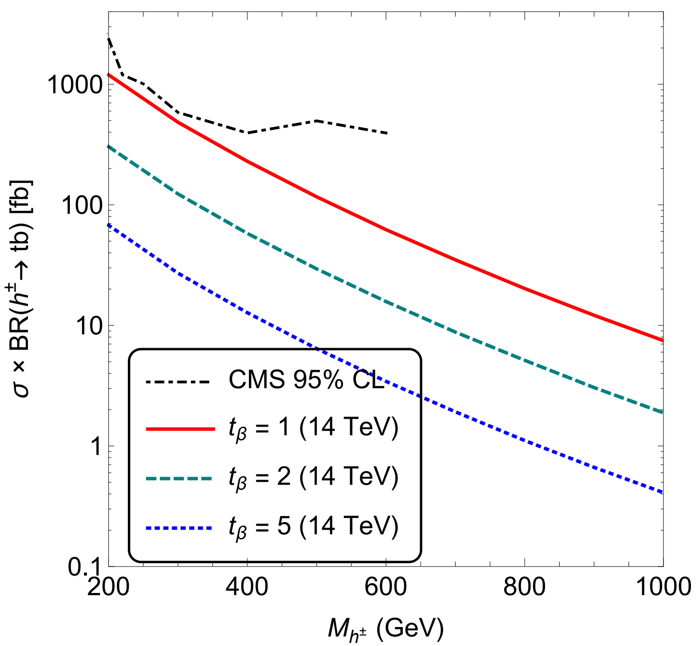
$\Longrightarrow$ **Minimal Alignment Symmetry:** $\mathbf{Z}_2^{\mathbf{EW}} \times \mathbf{Z}_2^{\mathbf{I}}$ [AP '16]

• Phenomenological implications at the LHC

Discovery channels for aligned Higgs doublets:

• $gg \to t\bar{b}h^- \to t\bar{b}\bar{t}b$ [Dev, AP '14]





$p_T^\ell > 20$ GeV,

$|\eta^\ell| < 2.5$,

$\Delta R^{\ell\ell} > 0.4$,

$M_{\ell\ell} > 12$ GeV,

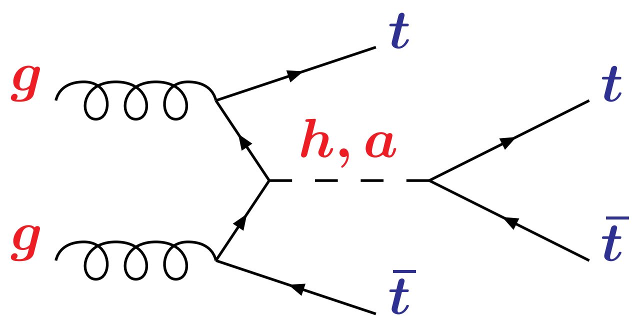
$|M_{\ell\ell} - M_Z| > 10$ GeV,

$p_T^j > 30$ GeV,

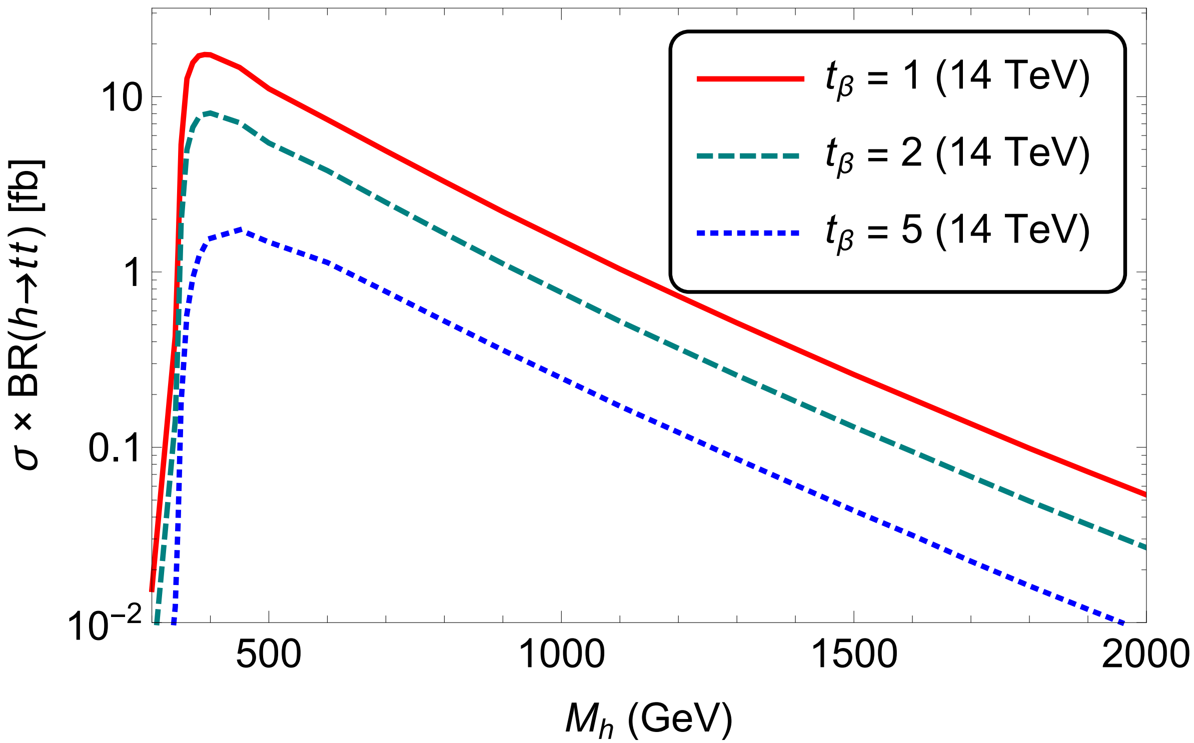
$|\eta^j| < 2.4$,

$\not{E}_T > 40$ GeV.

- $gg \to t\bar{t}(h,a) \to t\bar{t}t\bar{t}$

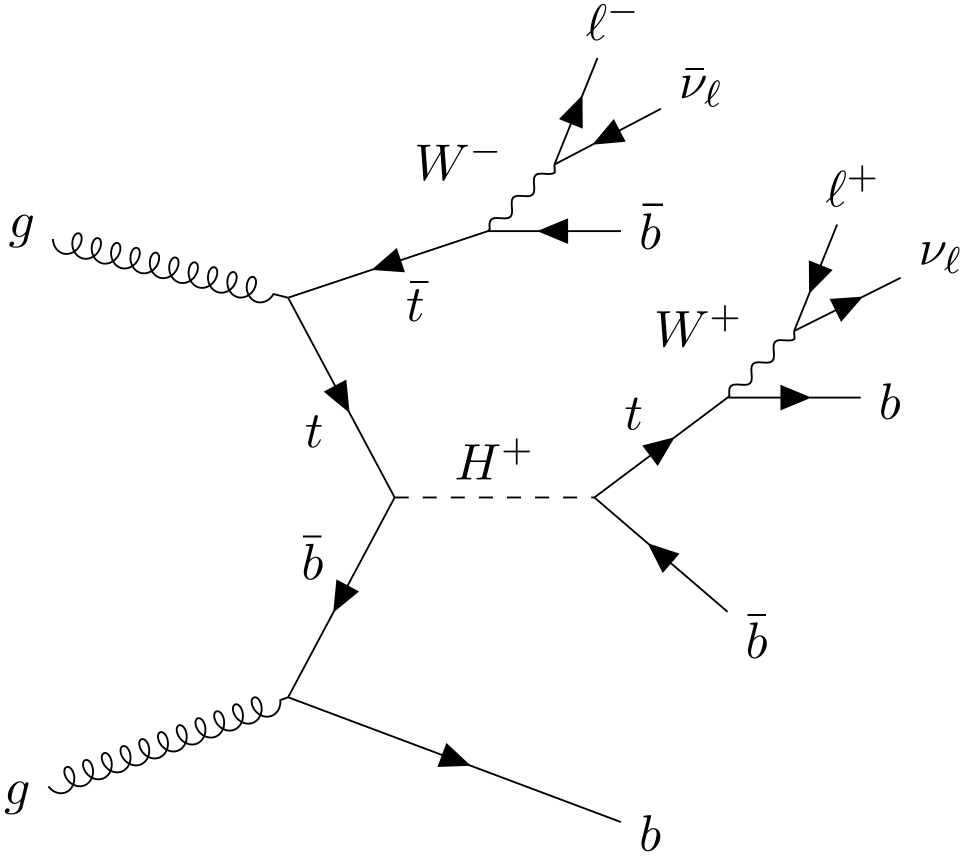


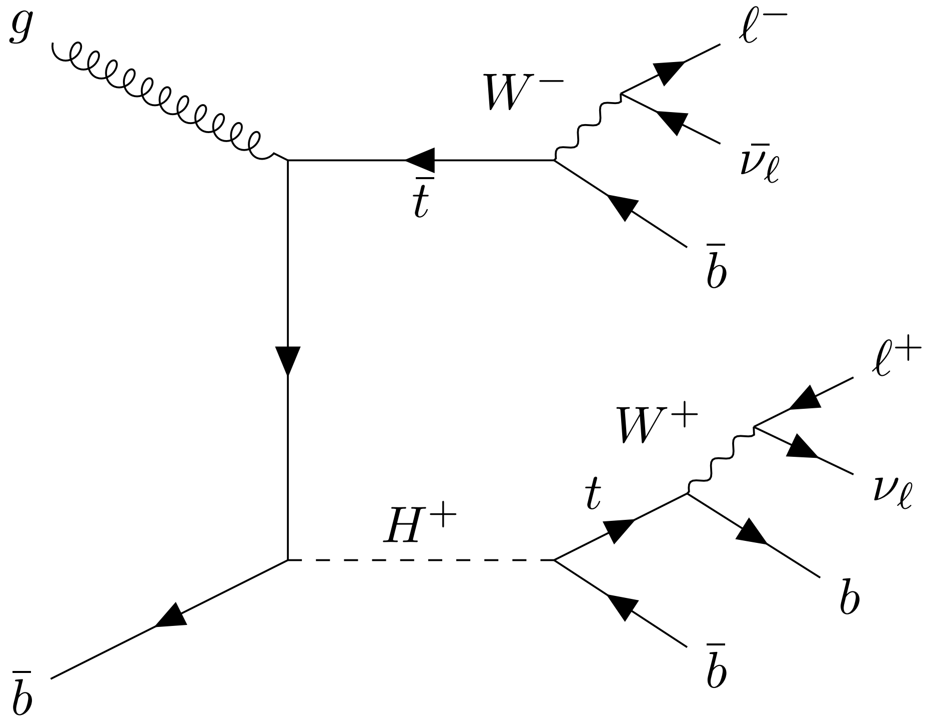
[Dev, AP '14]



Realistic simulation analysis with a reconstruction BDT

[Emily Hanson, W. Klemm, R. Naranjo, Yvonne Peters, AP, PRD100 (2019) 035026]





Reconstruction BDT trained on 57 observables:

- $\Delta R(b_i, l^a)$, $\Delta\eta(b_i, l^a)$, $\Delta\phi(b_i, l^a)$, $p_T^{b_i+l^a}$, $m(b_i, l^a)$, where $i = tH, t$ and $a = +, -$
- $\left|m(l^+, b_{tH}) - m(l^-, b_t)\right|$ and $\left|m(l^-, b_{tH}) - m(l^+, b_t)\right|$
- $p_T^{b_j}$, where $j = tH, H, t$
- $\Delta R(b_{tH}, b_k)$, $\Delta\eta(b_{tH}, b_k)$, $\Delta\phi(b_{tH}, b_k)$, $p_T^{b_{tH}+b_k}$, $m(b_{tH}, b_k)$, where $k = H, t$
- $\Delta R(t_{H^a}, b_H)$, $\Delta\eta(t_{H^a}, b_H)$, $\Delta\phi(t_{H^a}, b_H)$, $p_T^{t_{H^a}, b_H}$, $m(t_{H^a}, b_H)$, where $a = +, -$
- $\Delta R(t_{H^a}, t_c)$, $\Delta\eta(t_{H^a}, t_c)$, $\Delta\phi(t_{H^a}, t_c)$, where $(H^a, t_c) = (H^+, \bar{t})$ or (H^-, t)
- $m(H^a) - m(b_H)$, where $a = +, -$
- $m(H^+) - m(\bar{t})$ and $m(H^-) - m(t)$
- $p_T^{H^\pm + t_{\text{other}}}$
- $m(H^\pm, t_{\text{other}})$

Results

[Emily Hanson, W. Klemm, R. Naranjo, Yvonne Peters, AP, PRD100 (2019) 035026]