

# **Application of Vlasov method to instabilities driven by e-cloud in the SPS dipoles**

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e-cloud meeting  
4 December 2020

# Introduction

A linearized method for studying instabilities driven by e-cloud was recently developed, where the eigenmodes of the bunch motion are identified as a function of the strength of the e-cloud

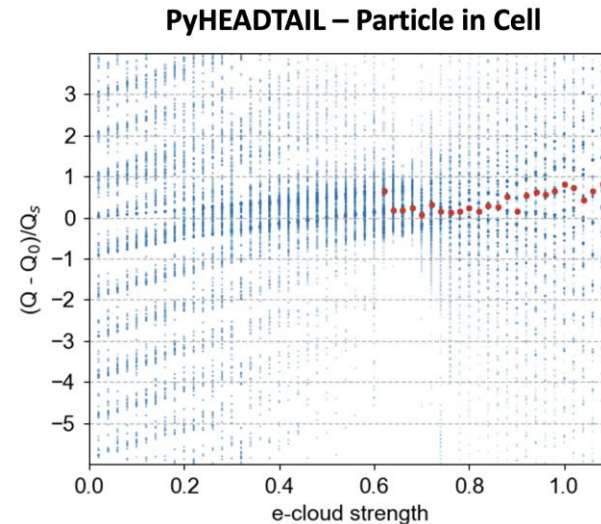
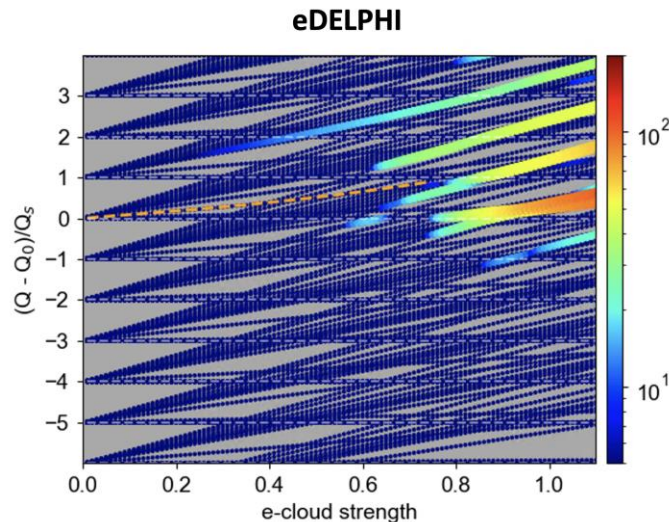
- *G. Iadarola et al., [Phys. Rev. Accel. Beams 23, 081002, 2020](#)*
- The method has been applied to the case of instabilities driven by e-cloud in the LHC dipoles and quadrupoles at injection energy
  - *G. Iadarola et al., [e-cloud meeting, Oct 2020](#)*

Here we apply the method to study the transverse instabilities driven by e-cloud in the SPS dipoles at injection

- For the e-cloud we consider the following parameters
  - The e-cloud is characterized for a density of  $2 \times 10^{12} \text{ e}^-/\text{m}^3$  (before bunch arrival)
  - The e-cloud strength is scanned in the range 0.1-2 to study the effect on stability

# Introduction

- For the studied LHC cases, results using the linearized model with the Vlasov method (eDELPHI) have been compared to macroparticle simulations
  - Good agreement was found both with macroparticle simulations using the linearized model as well as full particle-in-cell simulations



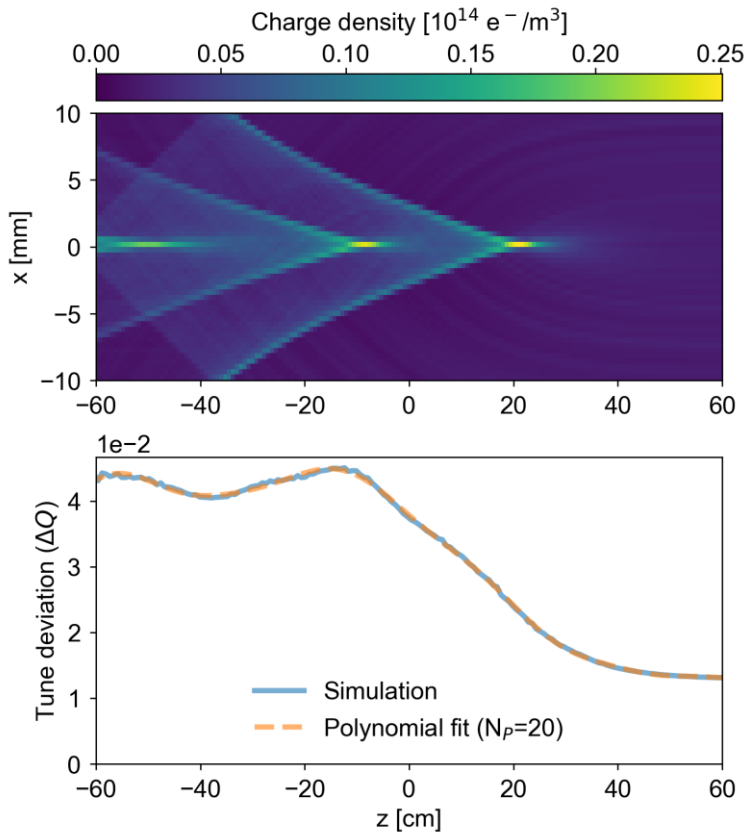
*G. Iadarola et al*

- Here we use the Vlasov method exclusively to study the transverse stability in the vertical plane in the SPS dipoles at injection, considering the effects of
  - The bunch intensity:  $1 \times 10^{11}$  vs.  $2 \times 10^{11}$  protons per bunch
  - The synchrotron tune: scanning  $Q_s$  between 0.002 and 0.02, in steps of 0.002

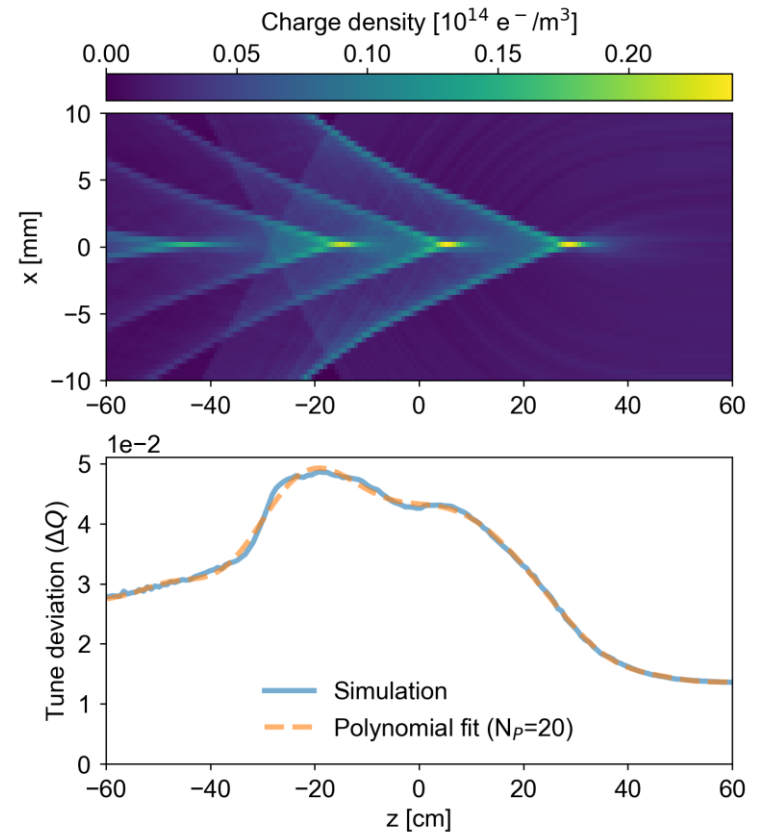
# Detuning forces from the e-cloud

The dependence of the detuning forces on the z coordinate is described by a polynomial

$$N_b = 1.0 \times 10^{11}$$



$$N_b = 2.0 \times 10^{11}$$



# Characterization of the dipolar forces

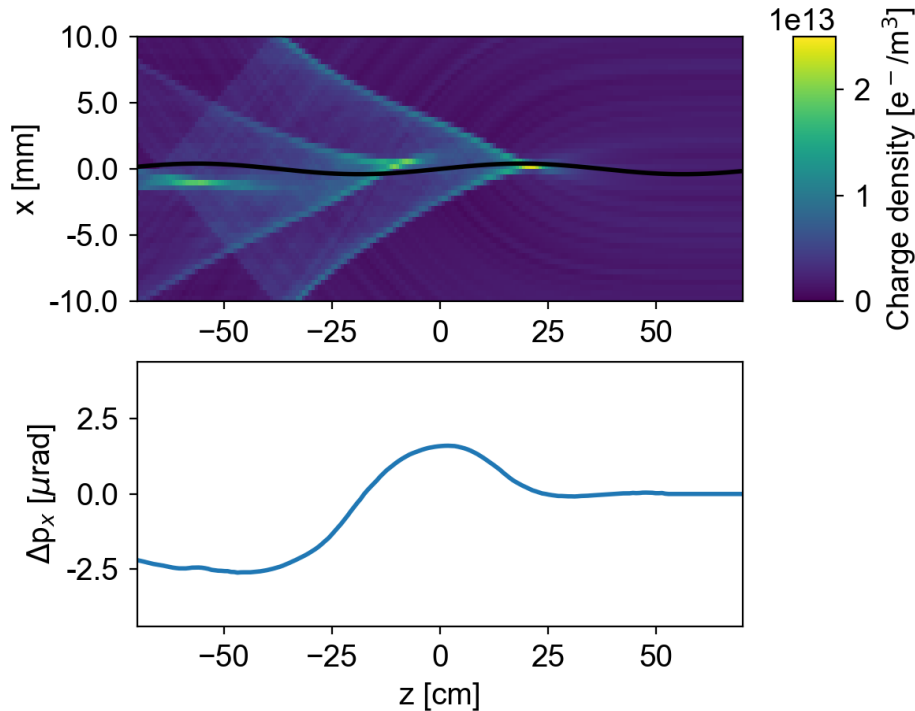
The dipolar forces are characterized by means of single-pass PyELOUD-PyHEADTAIL simulations with a set of pre-distorted bunches

**Test functions:**

$$h_n(z) = \begin{cases} \mathcal{A}_n \cos(2\pi f_n^z z), & \text{if } n \text{ is even} \\ \mathcal{A}_n \sin(2\pi f_n^z z), & \text{if } n \text{ is odd} \end{cases} \quad f_n^z = \begin{cases} \frac{n}{2} \frac{1}{L_{\text{bkt}}} & \text{if } n \text{ is even} \\ \frac{n+1}{2} \frac{1}{L_{\text{bkt}}} & \text{if } n \text{ is odd} \end{cases}$$

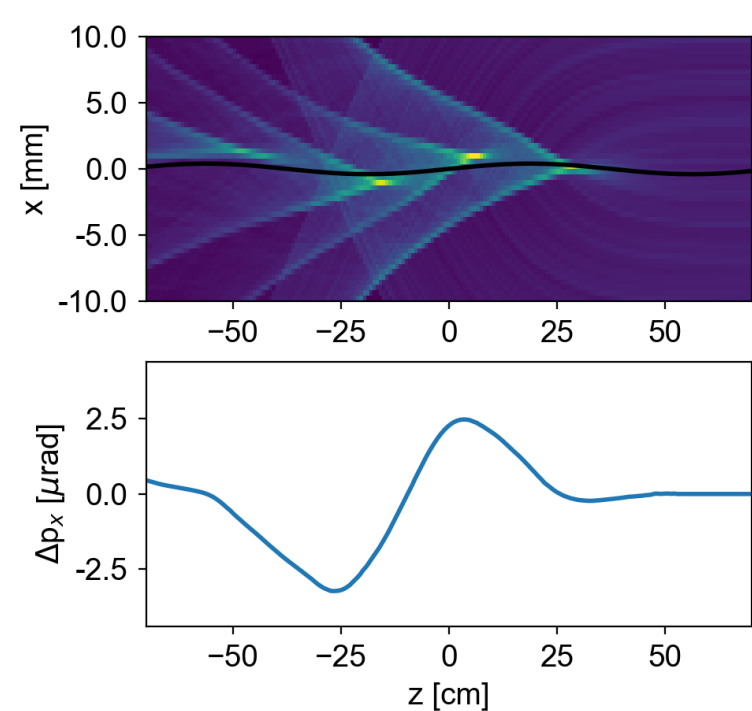
**$N_b = 1.0 \times 10^{11}$**

n=2 sine



**$N_b = 2.0 \times 10^{11}$**

n=2 sine

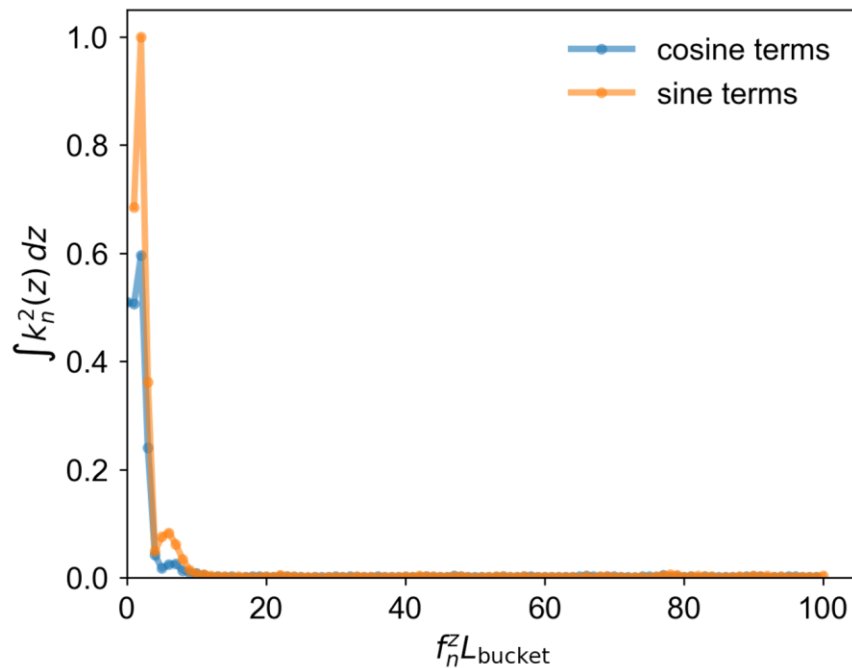


# Characterization of the dipolar forces

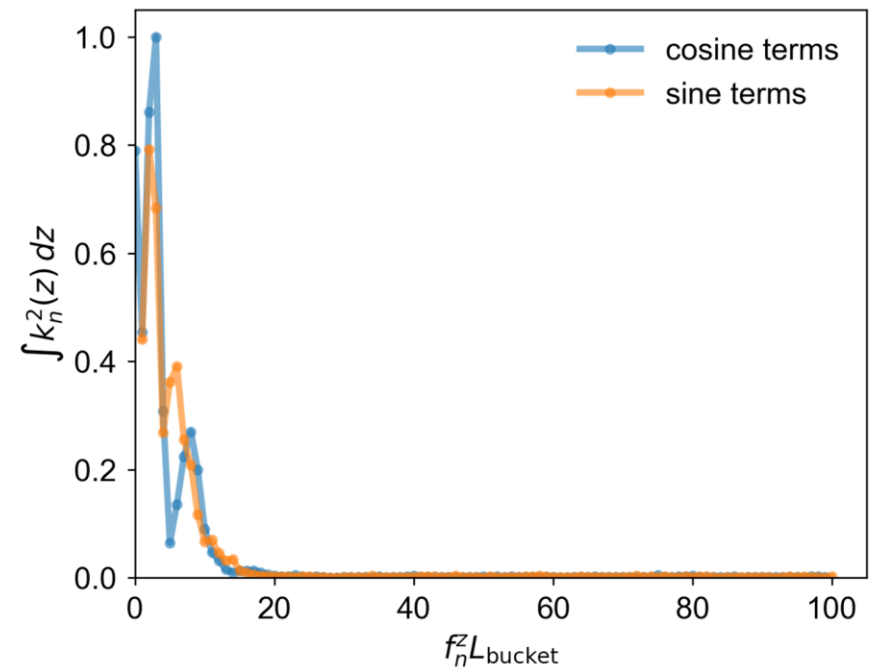
The dipolar forces are characterized by means of single-pass PyECLOUD-PyHEADTAIL simulations with a set of pre-distorted bunches

- For  $N_b = 1 \times 10^{11}$ , the response of the e-cloud is visible for frequencies up to  $10 \times f_{RF}$
- For  $N_b = 2 \times 10^{11}$ , the response of the e-cloud is visible for frequencies up to  $20 \times f_{RF}$

$N_b = 1.0 \times 10^{11}$



$N_b = 2.0 \times 10^{11}$



Each picture is normalized to its maximum

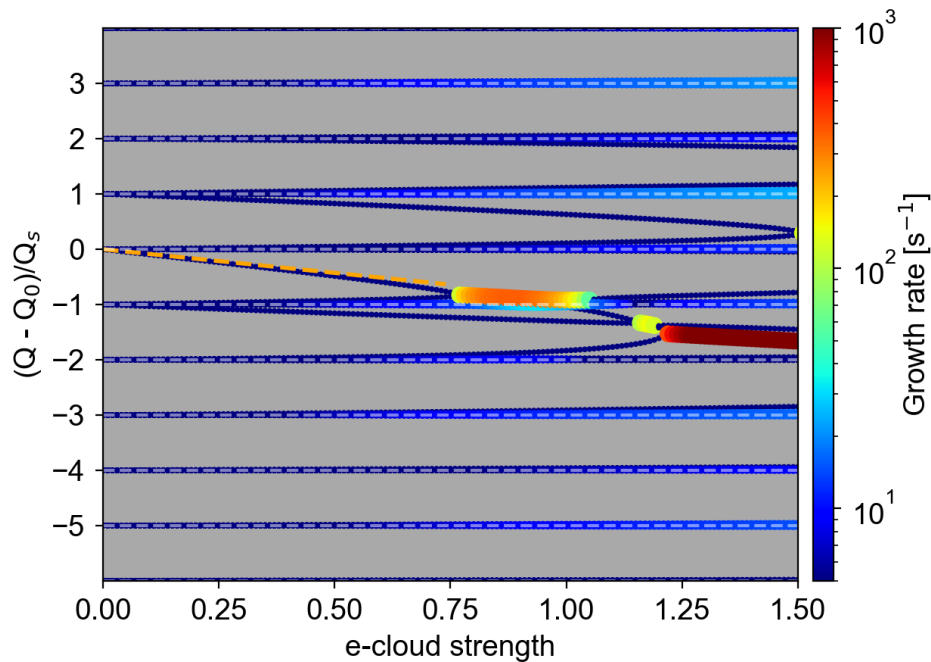
# Instability growth rates

Eigenmodes and growth rates have been determined **with dipolar forces alone**

- The instabilities appear to be triggered by mode coupling (TMCI)
- The instability threshold is slightly lower for **lower** bunch intensity

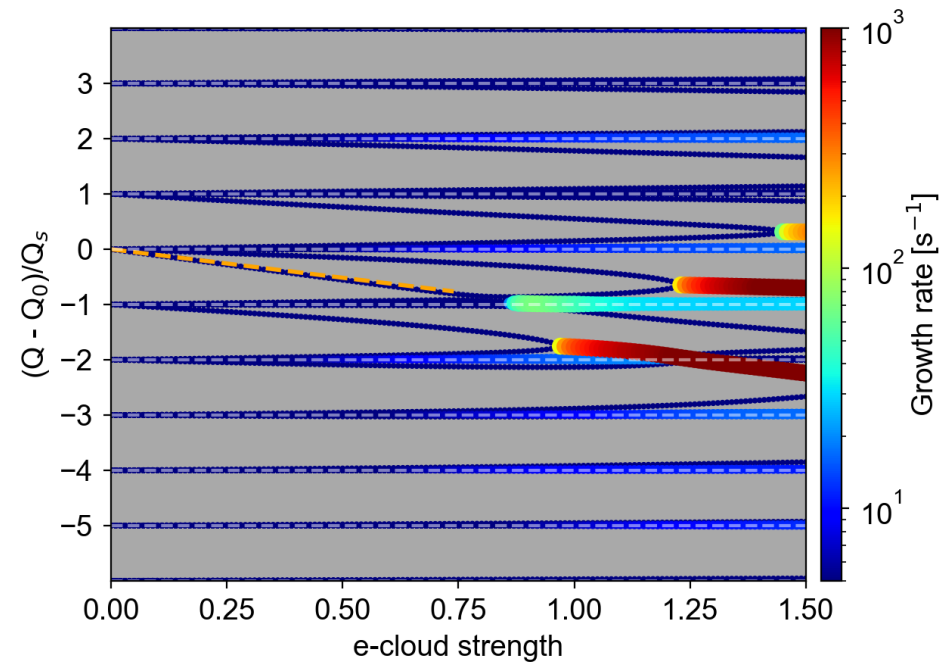
$$N_b = 1.0 \times 10^{11}$$

onlydip



$$N_b = 2.0 \times 10^{11}$$

onlydip



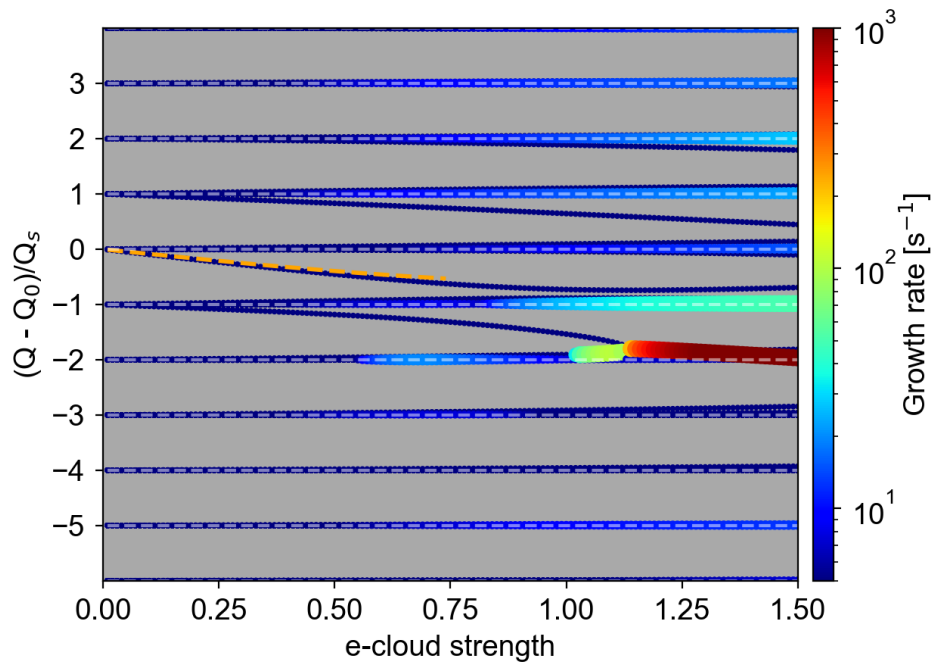
# Instability growth rates

Eigenmodes and growth rates have been determined **with dipolar forces + the phase shift from quadrupolar forces**

- The instabilities appear to be triggered by mode coupling (TMCI)
- The instability threshold is slightly lower for **higher** bunch intensity

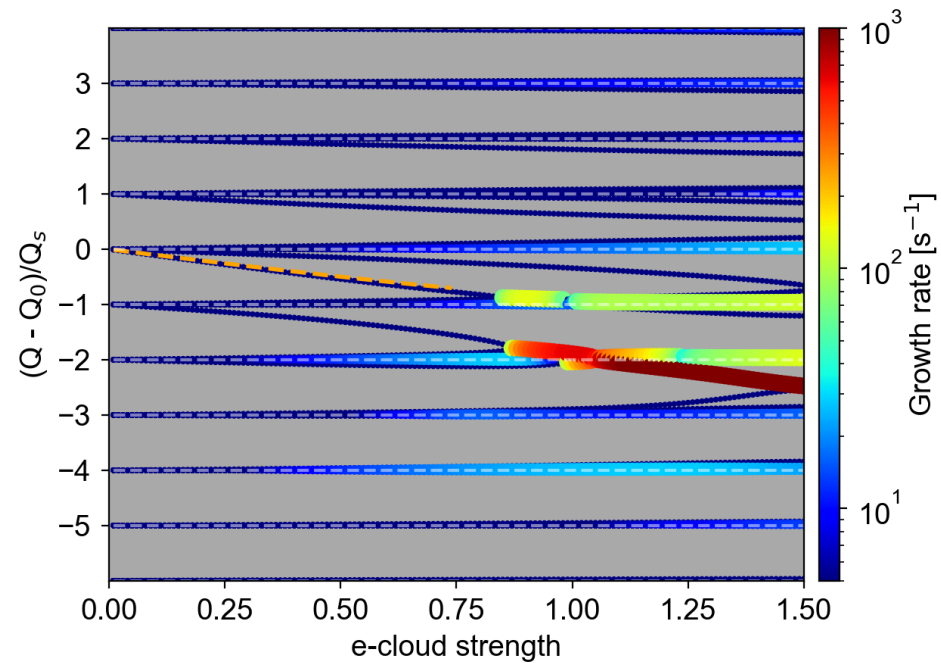
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phase



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phase





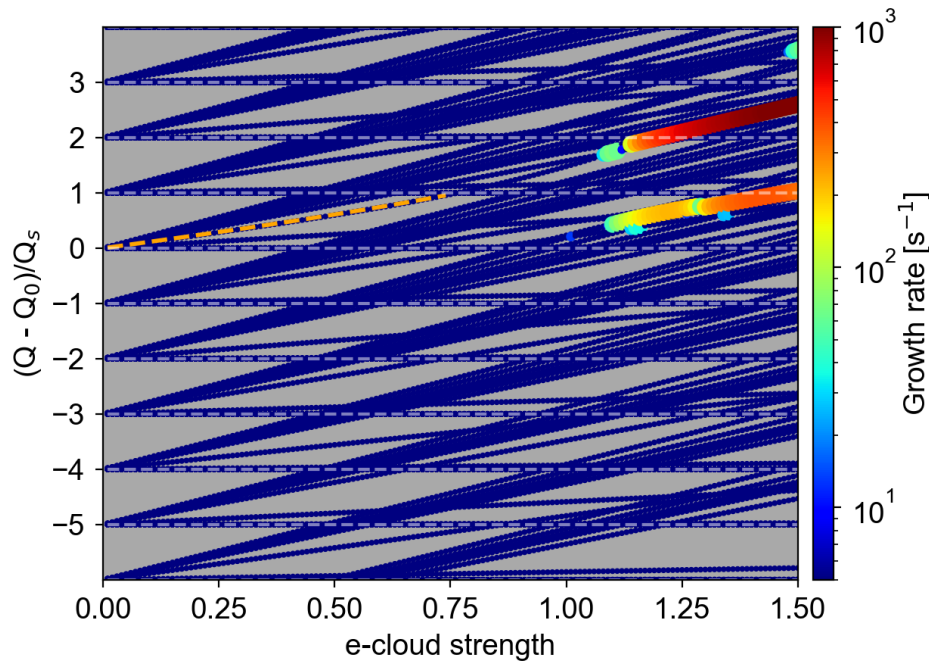
# Instability growth rates

Eigenmodes and growth rates have been determined **with dipolar forces + the full effect of quadrupolar forces**

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- The instability threshold is slightly lower for **higher** bunch intensity

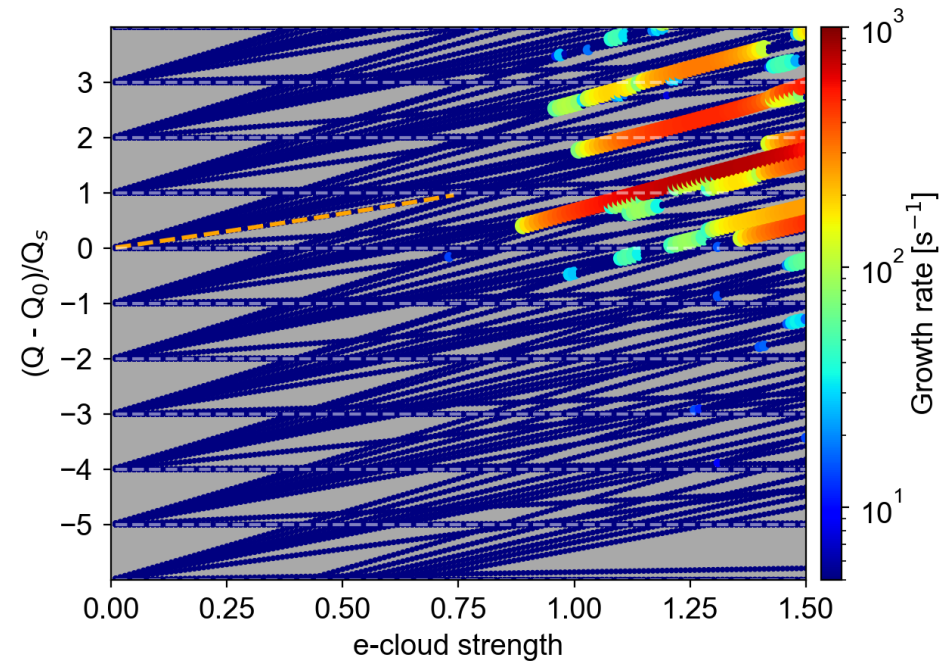
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wquad



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wquad

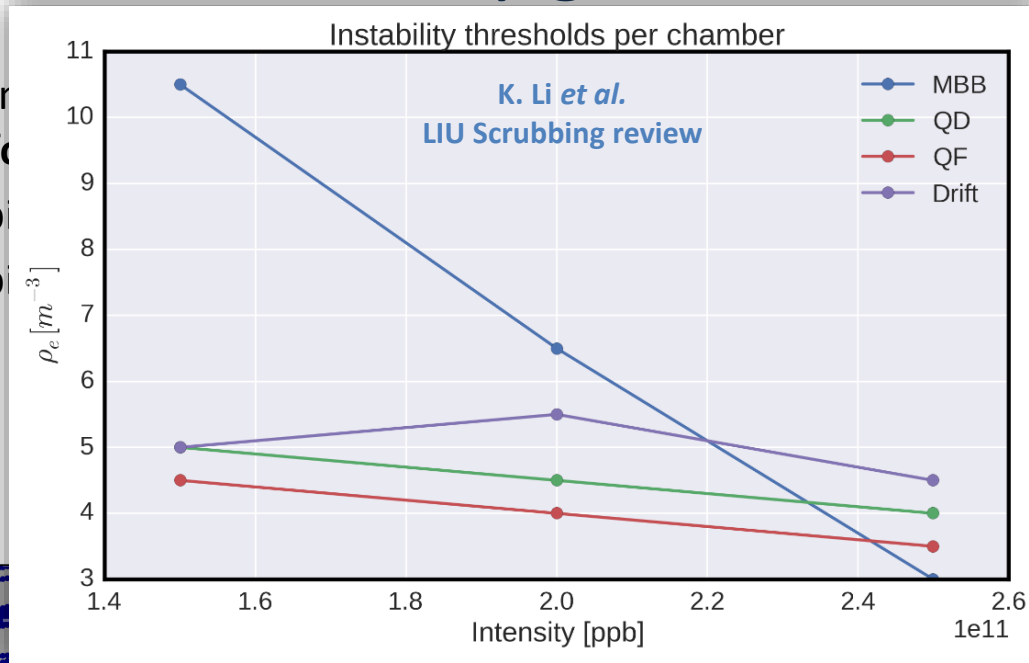


# Instability growth rates

Eigenmodes are quadrupolar for

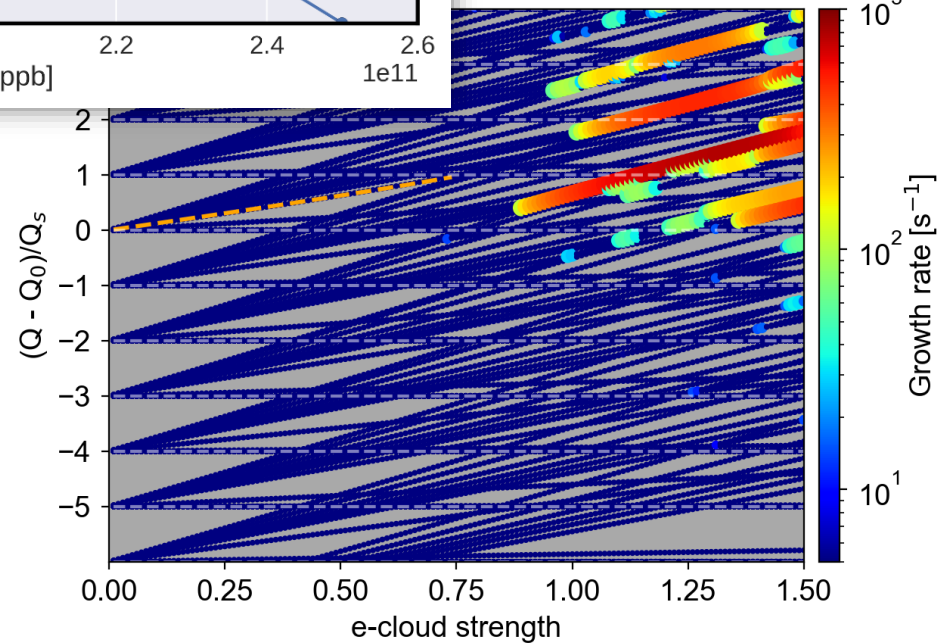
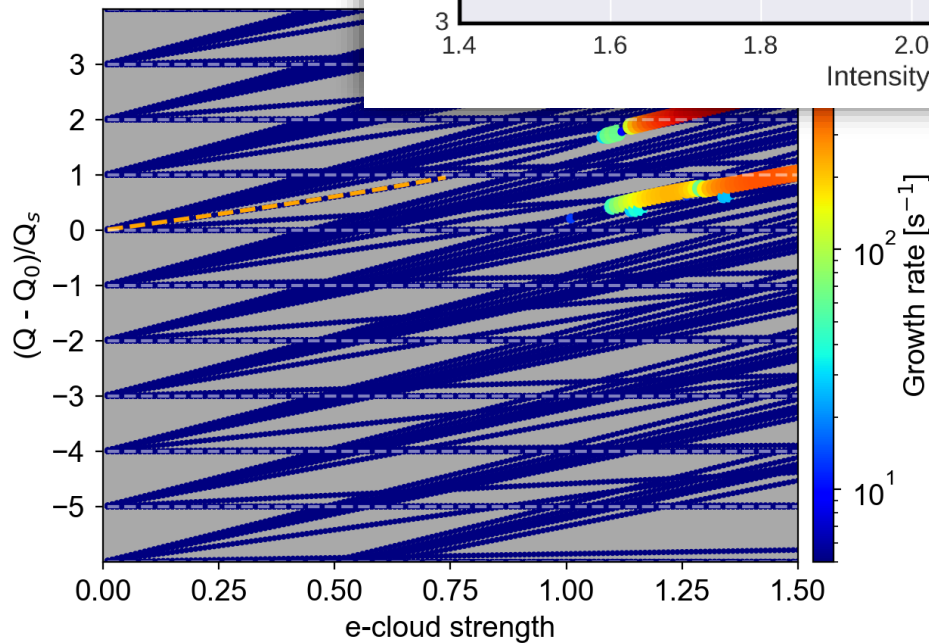
- The instability
- The instability

$N_b$



es + the full effect of

The scaling with intensity is qualitatively in agreement with results from past PyECLOUD-PyHEADTAIL studies using full PIC

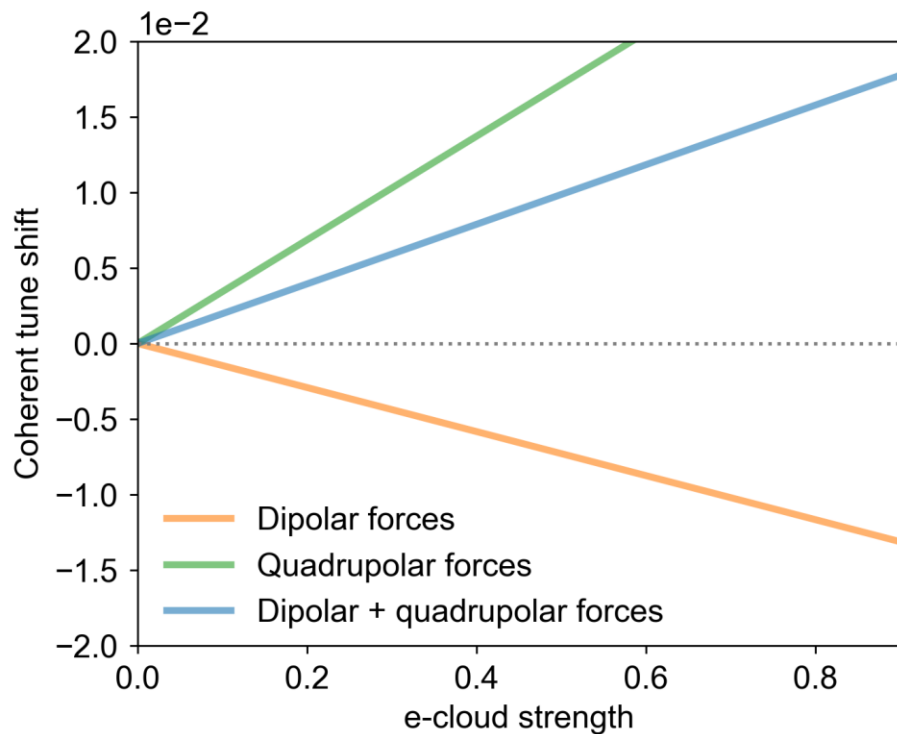


# Coherent tune shift

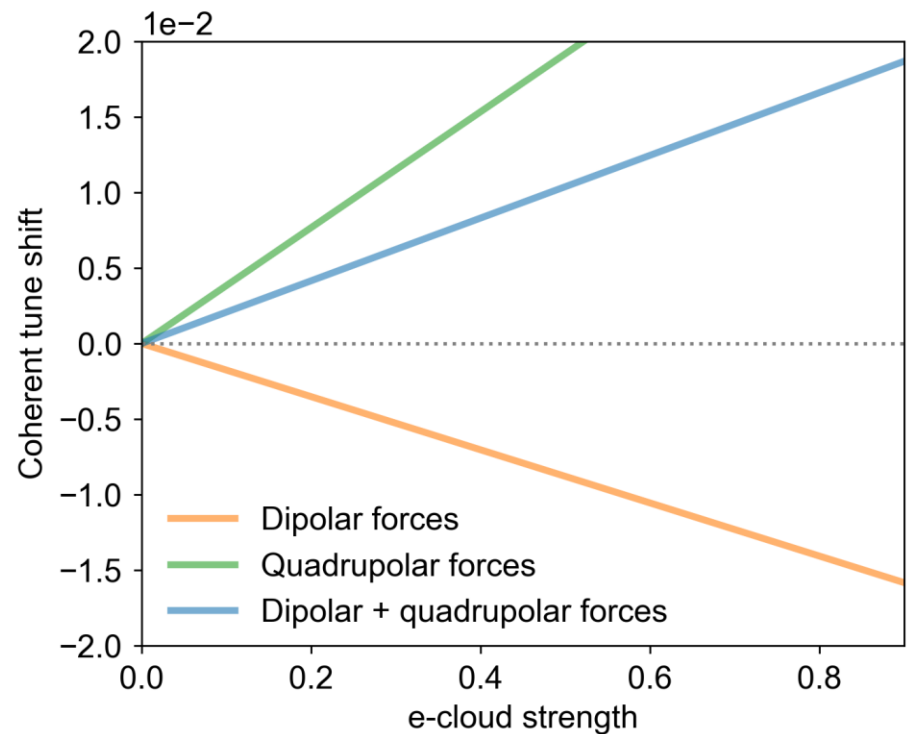
The coherent tune shift below the instability threshold can be estimated with the model

- A cancellation between the detuning from dipolar and quadrupolar forces occurs
- The tune shifts are very similar for the two bunch intensities

$N_b = 1.0 \times 10^{11}$



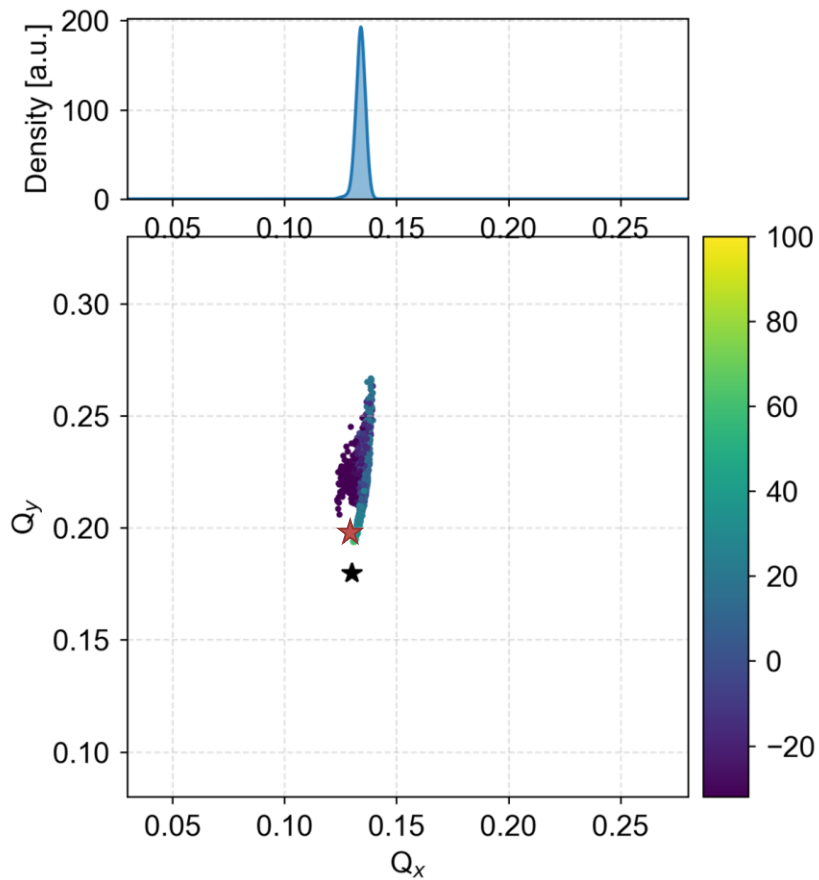
$N_b = 2.0 \times 10^{11}$



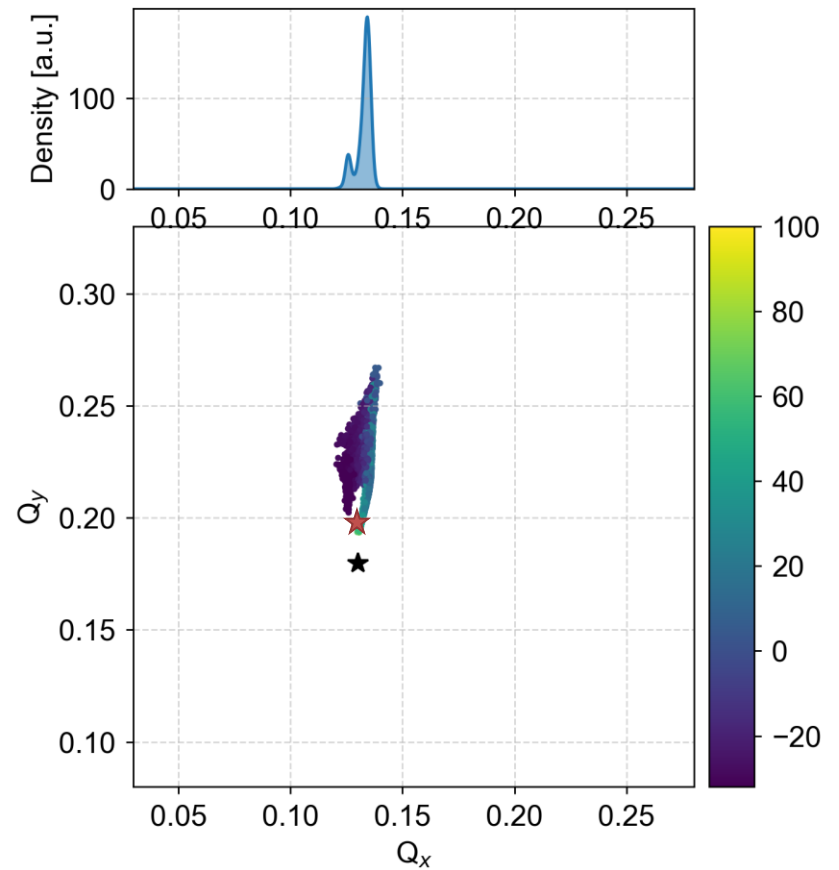
# Tune shift vs tune spread

The coherent tune shift is much smaller than the incoherent tune spread

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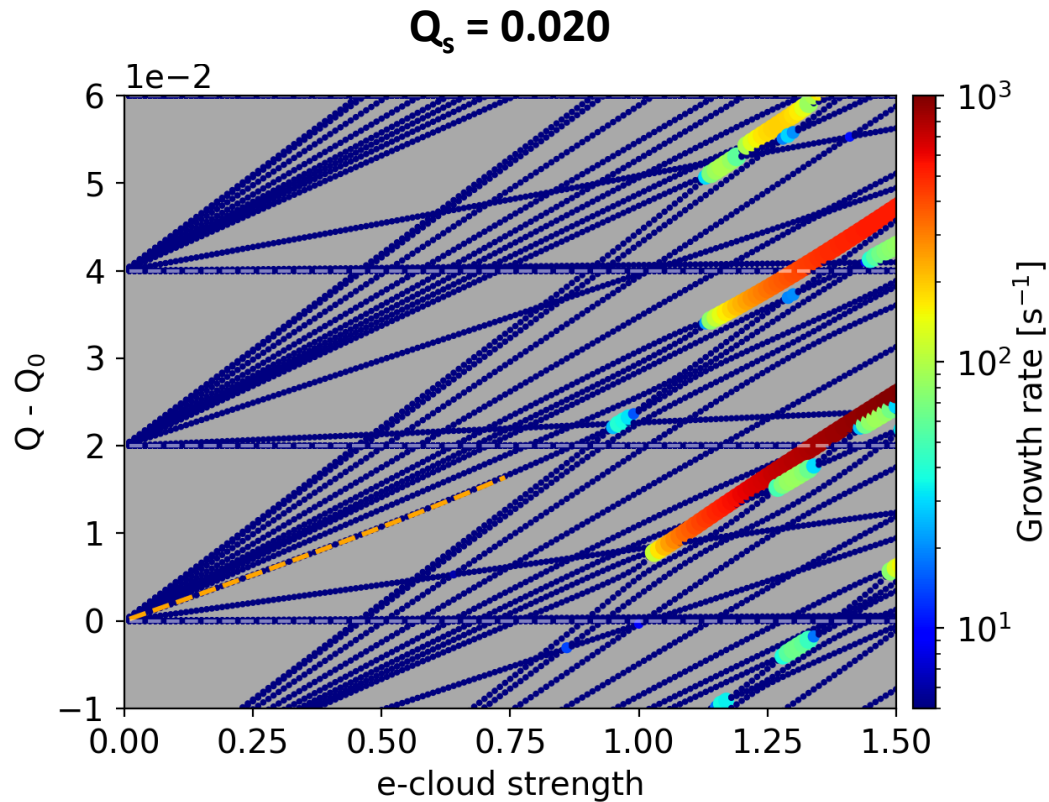


$N_b = 2.0 \times 10^{11}$



# Effect of synchrotron tune

The effect of the synchrotron tune has been studied with bunch intensity  $N_b = 2 \times 10^{11}$

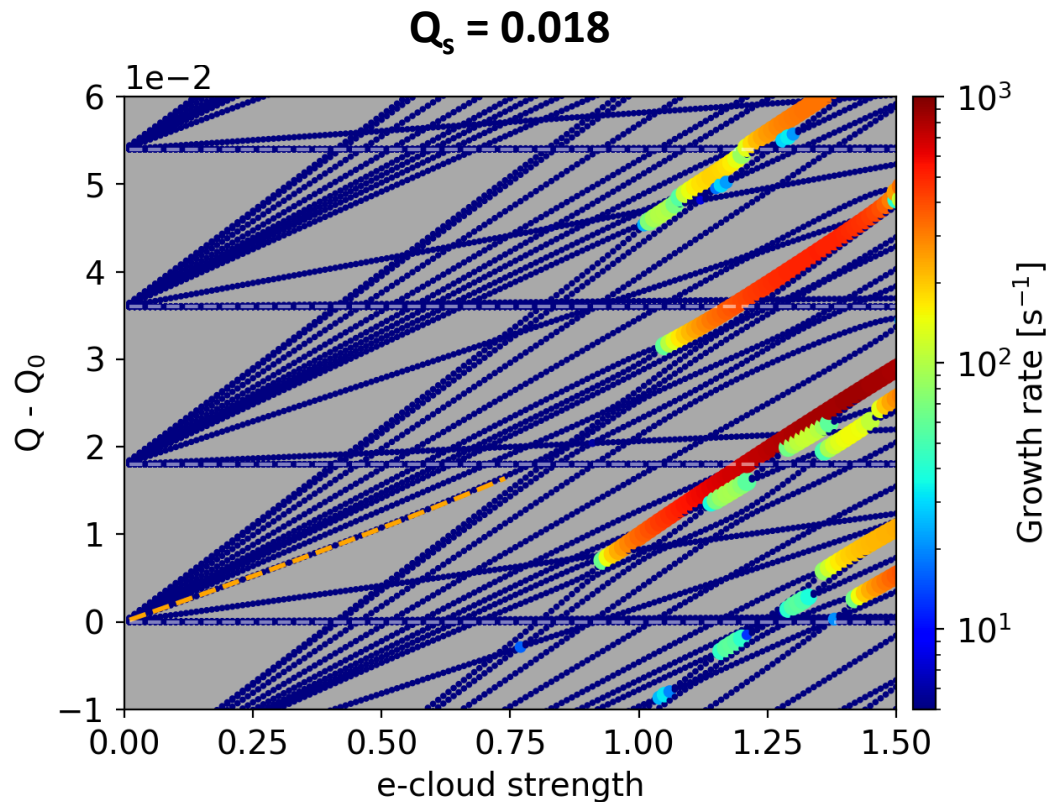


Q20:  $Q_s = 0.017$   
Q26:  $Q_s = 0.0059$

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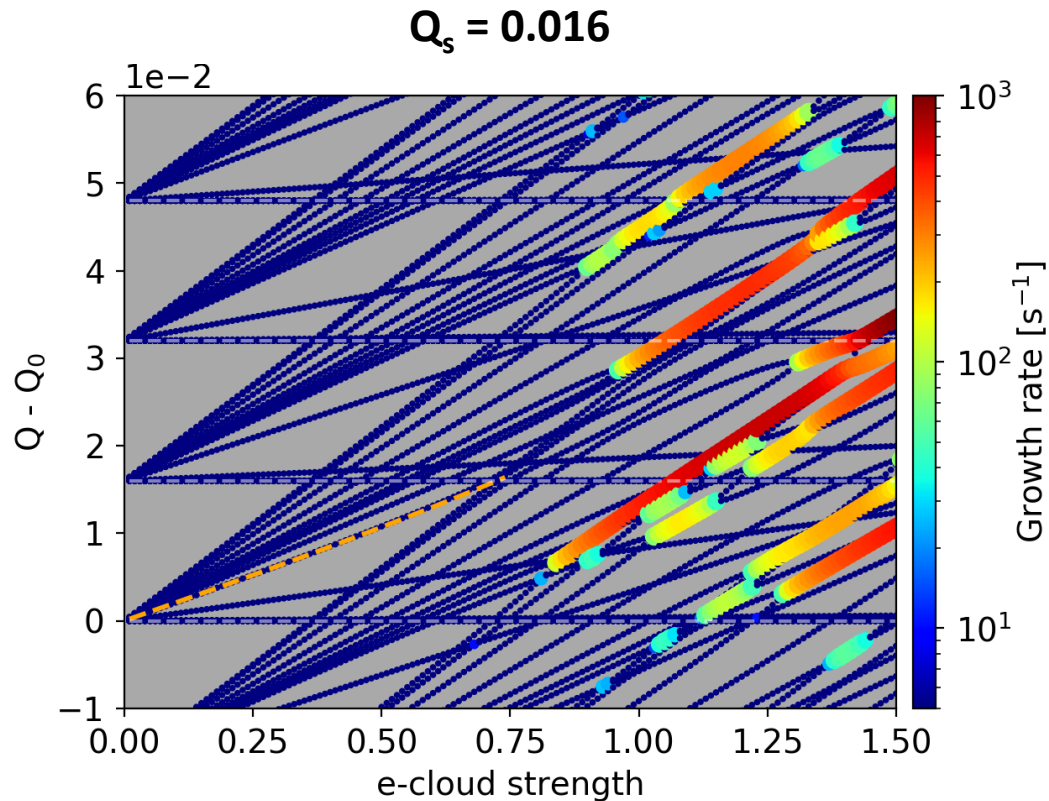


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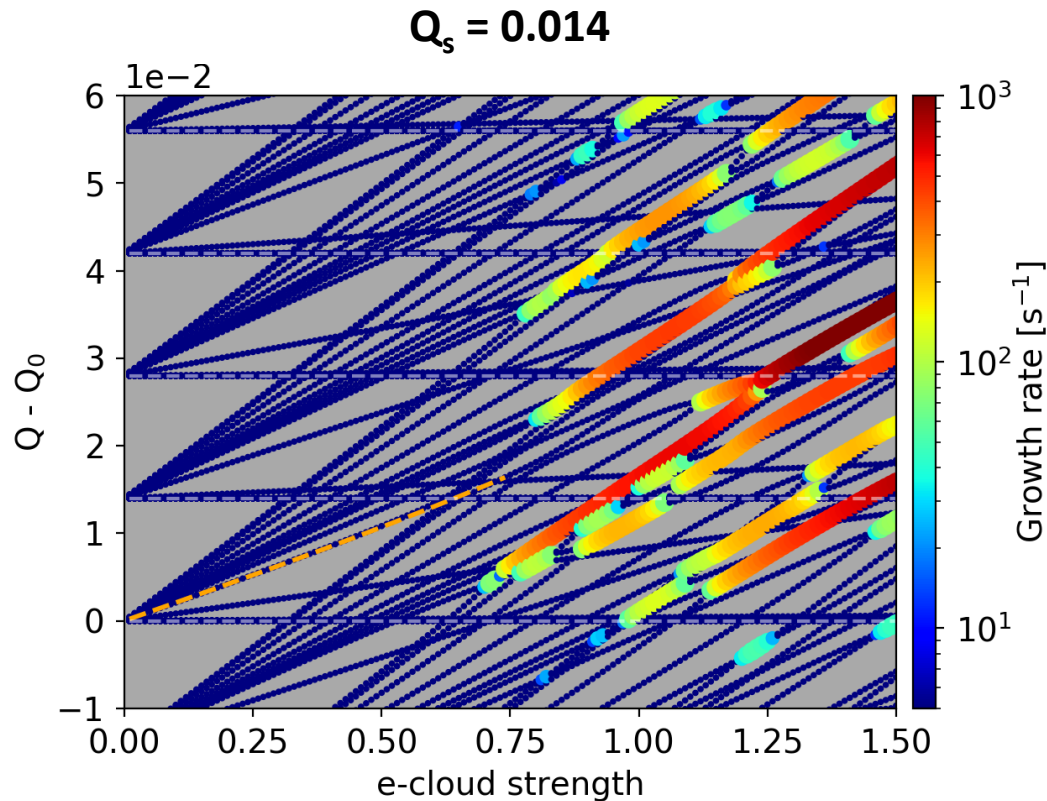
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The effect of the synchrotron tune has been studied with bunch intensity  $N_b = 2 \times 10^{11}$

- When  $Q_s$  decreases the synchrotron sidebands move closer together  
→ A smaller e-cloud strength is sufficient to trigger instabilities



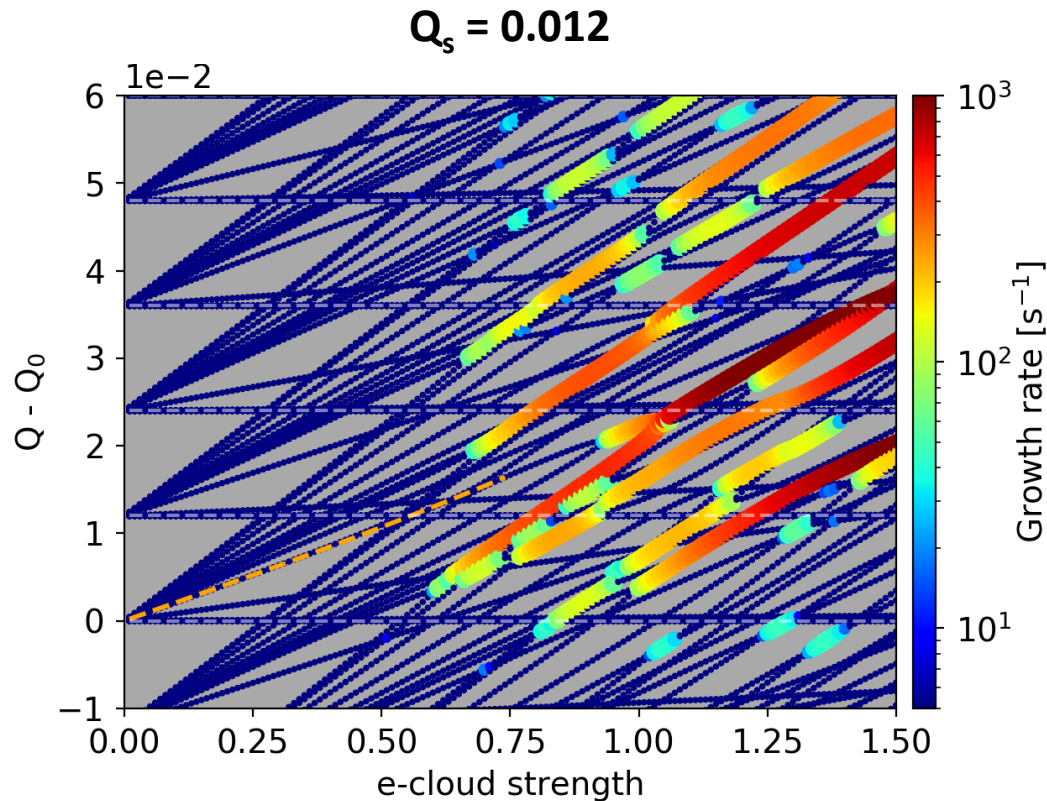
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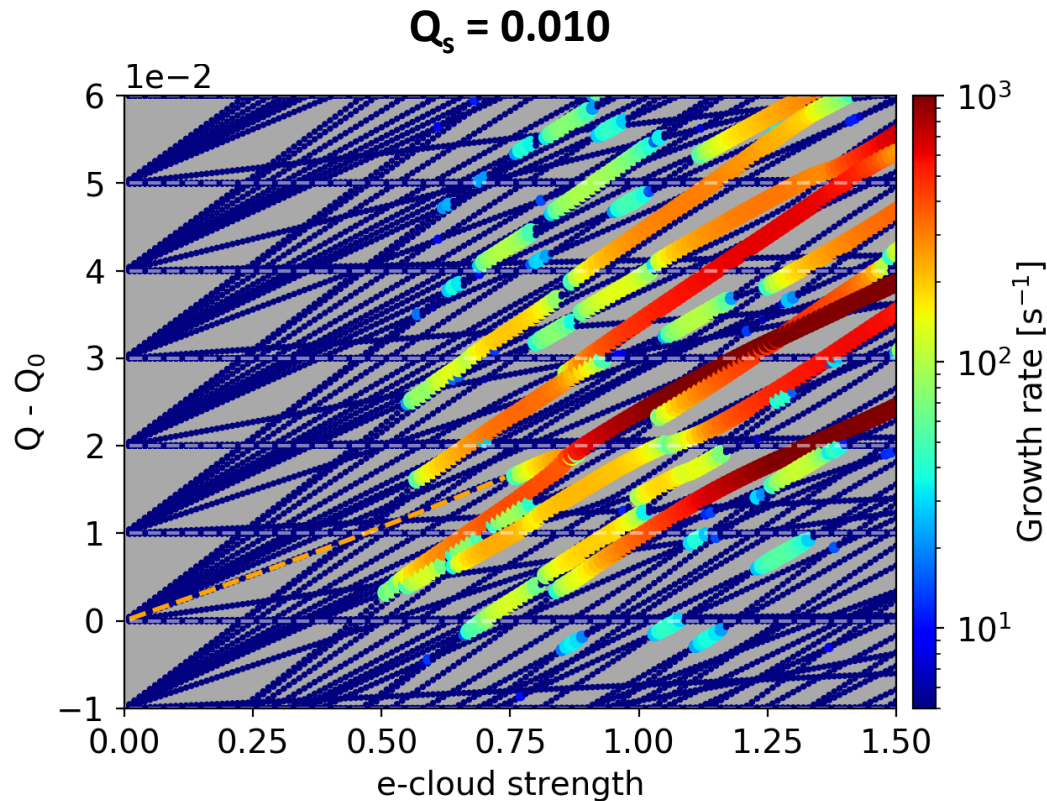


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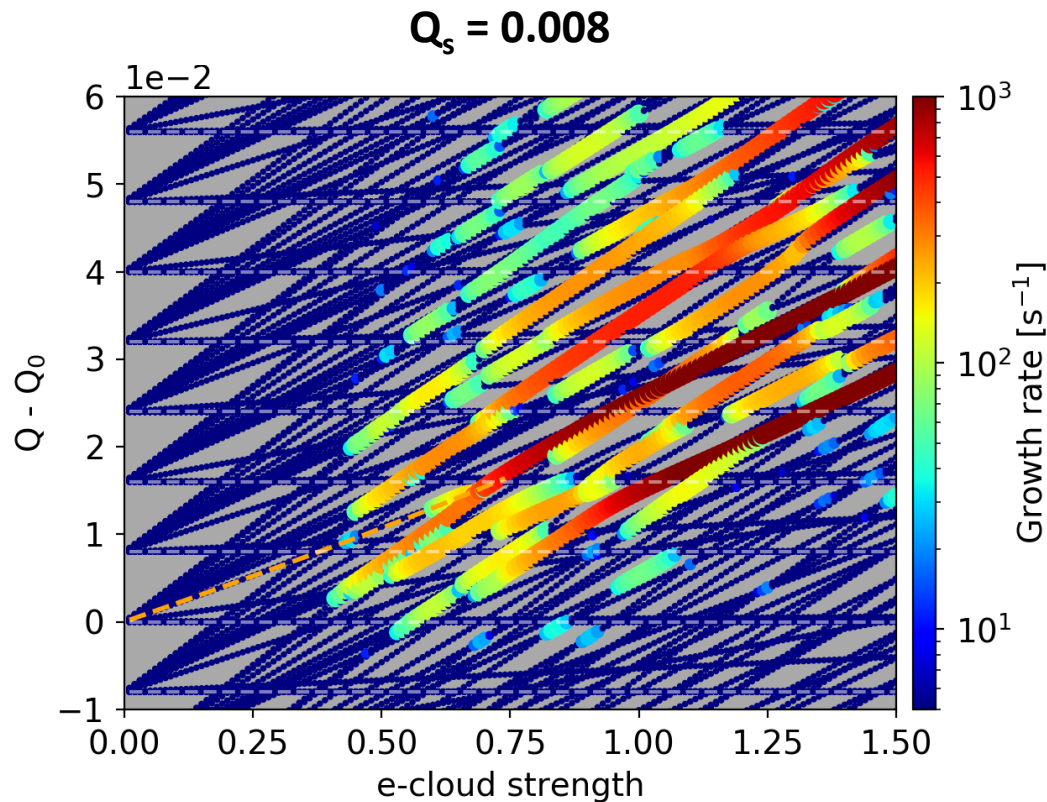


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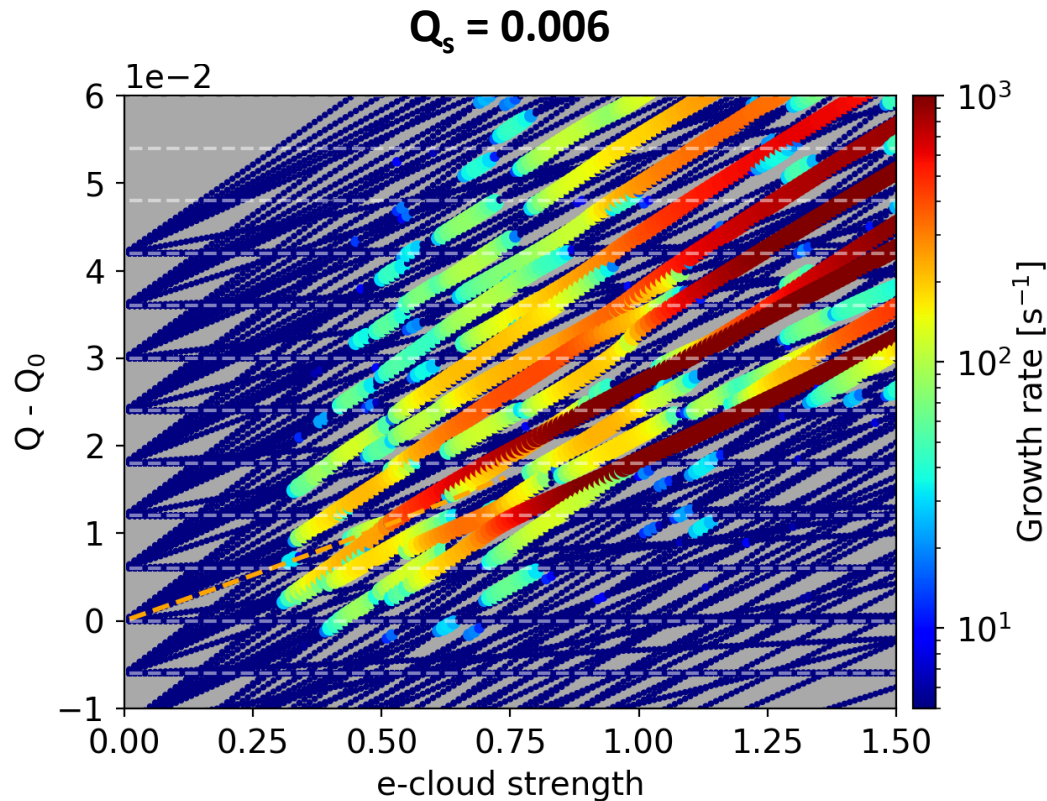


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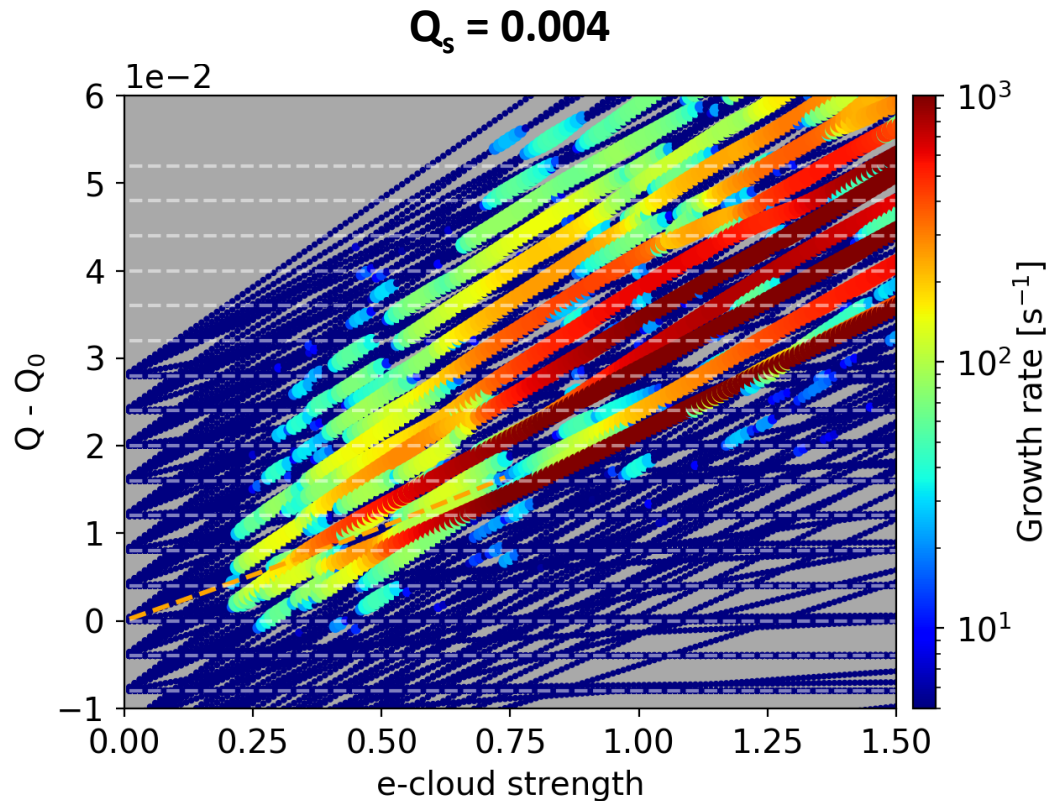


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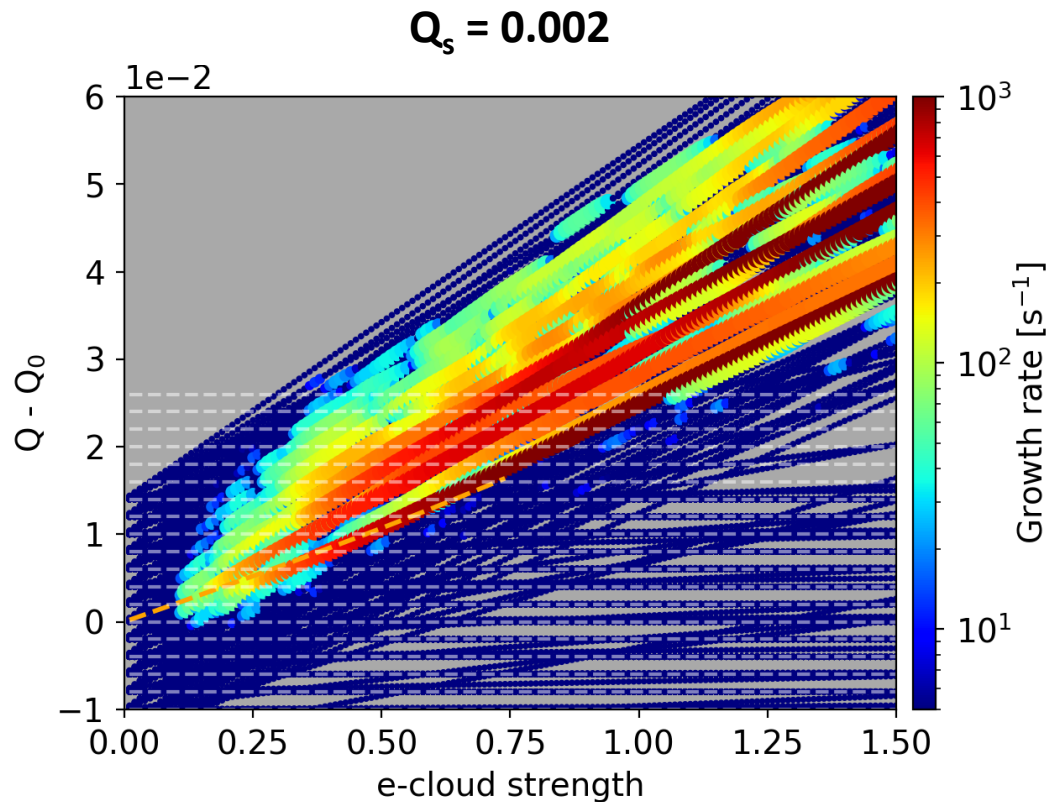


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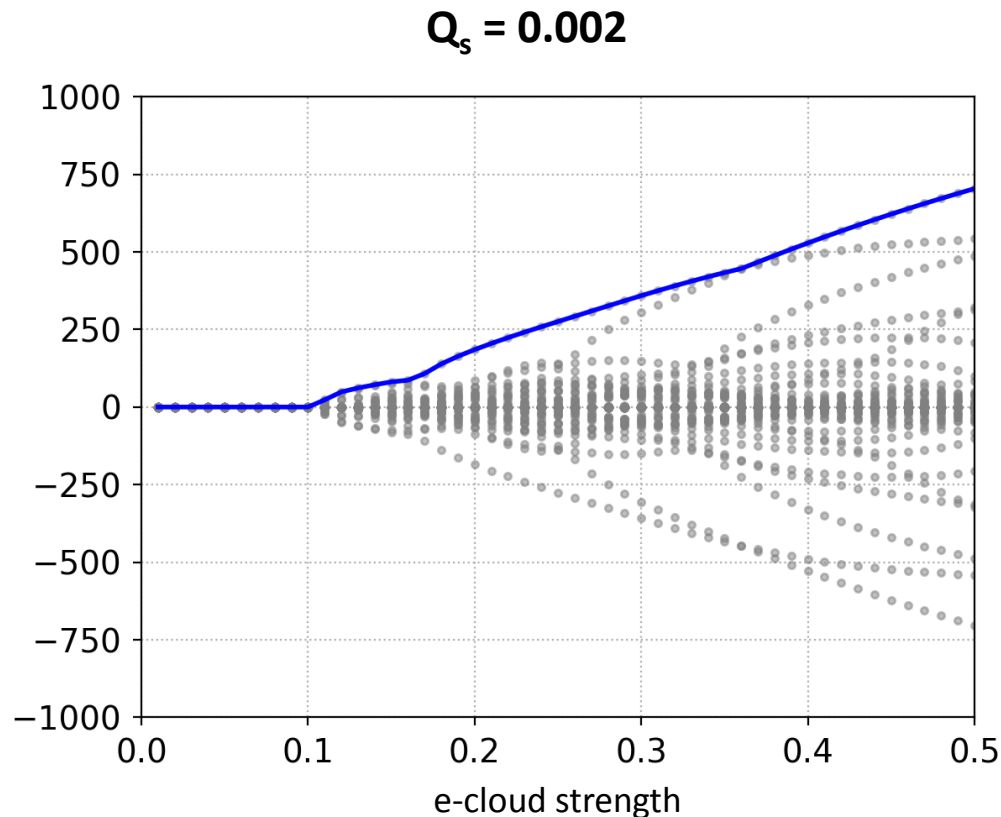
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- When  $Q_s$  decreases the synchrotron sidebands move closer together
  - A smaller e-cloud strength is sufficient to trigger instabilities
  - This is visible also on the imaginary part of the eigenvalues

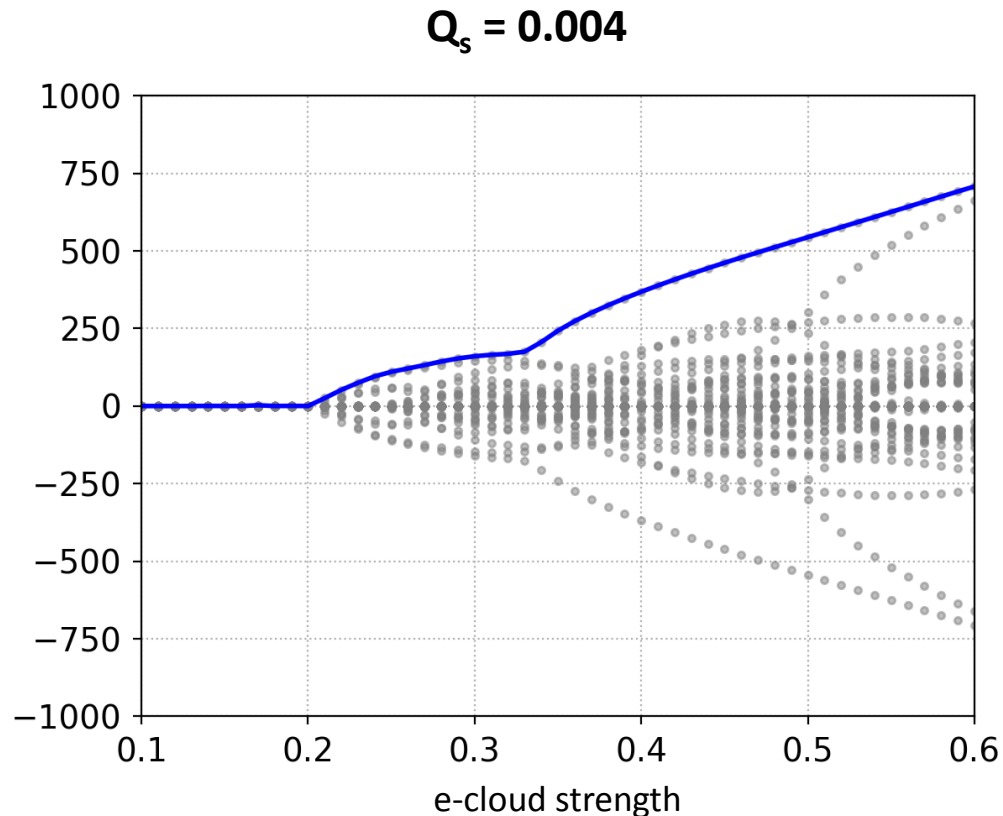


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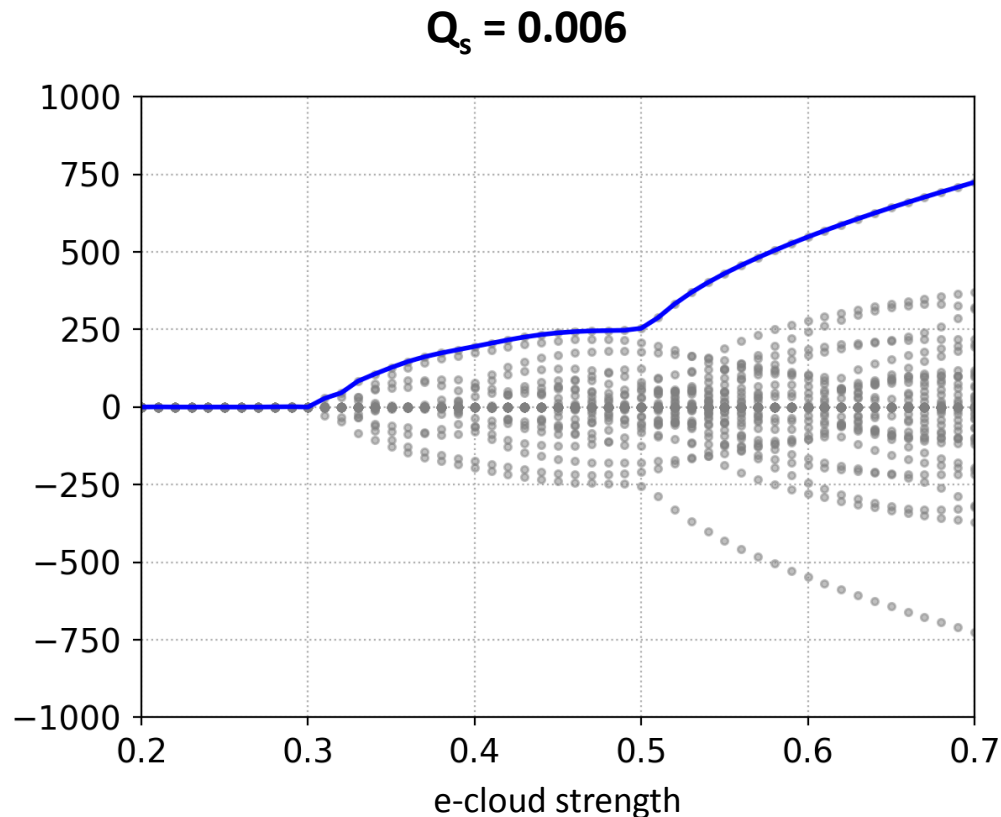
Note the sliding x-axis!



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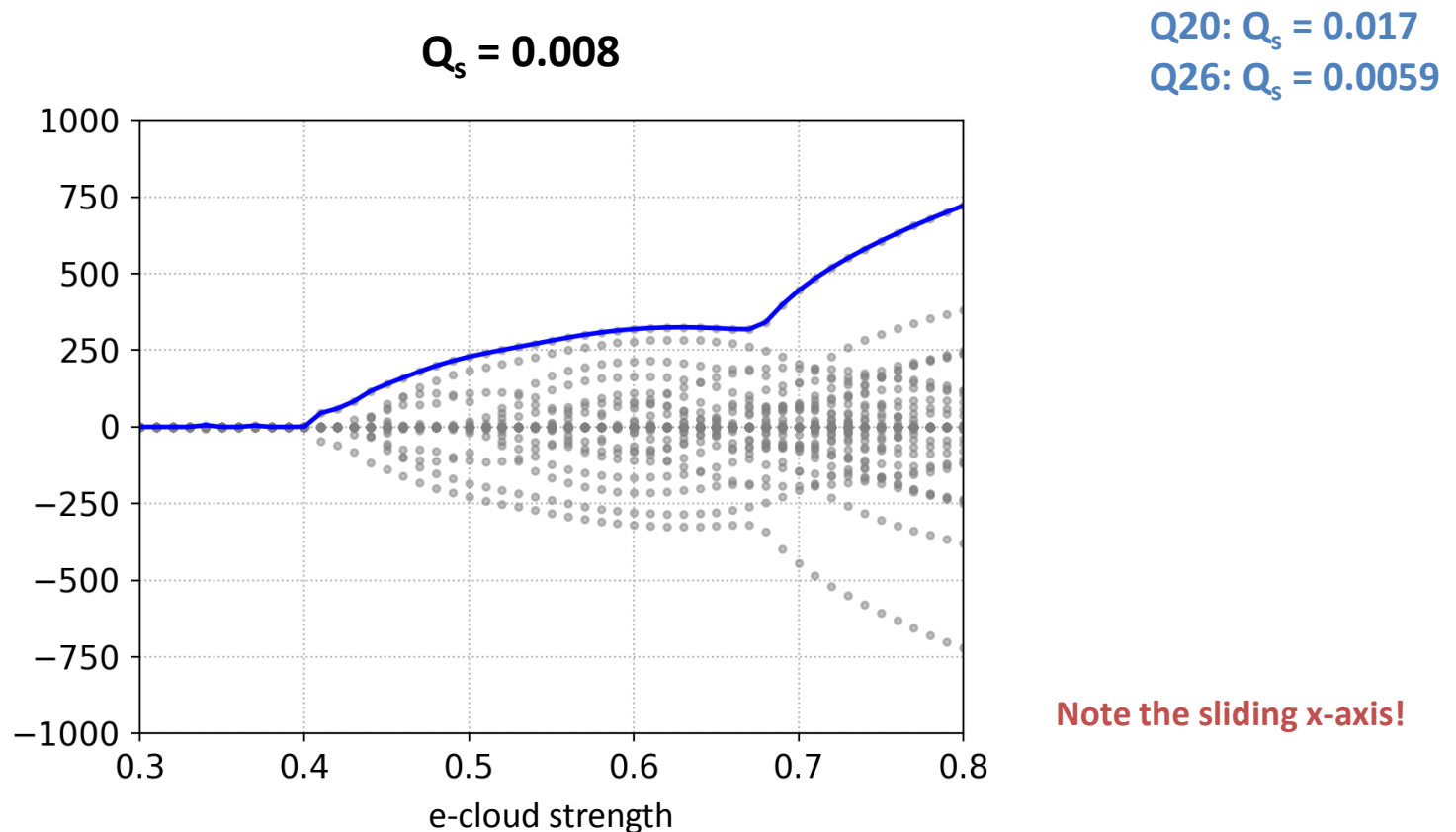
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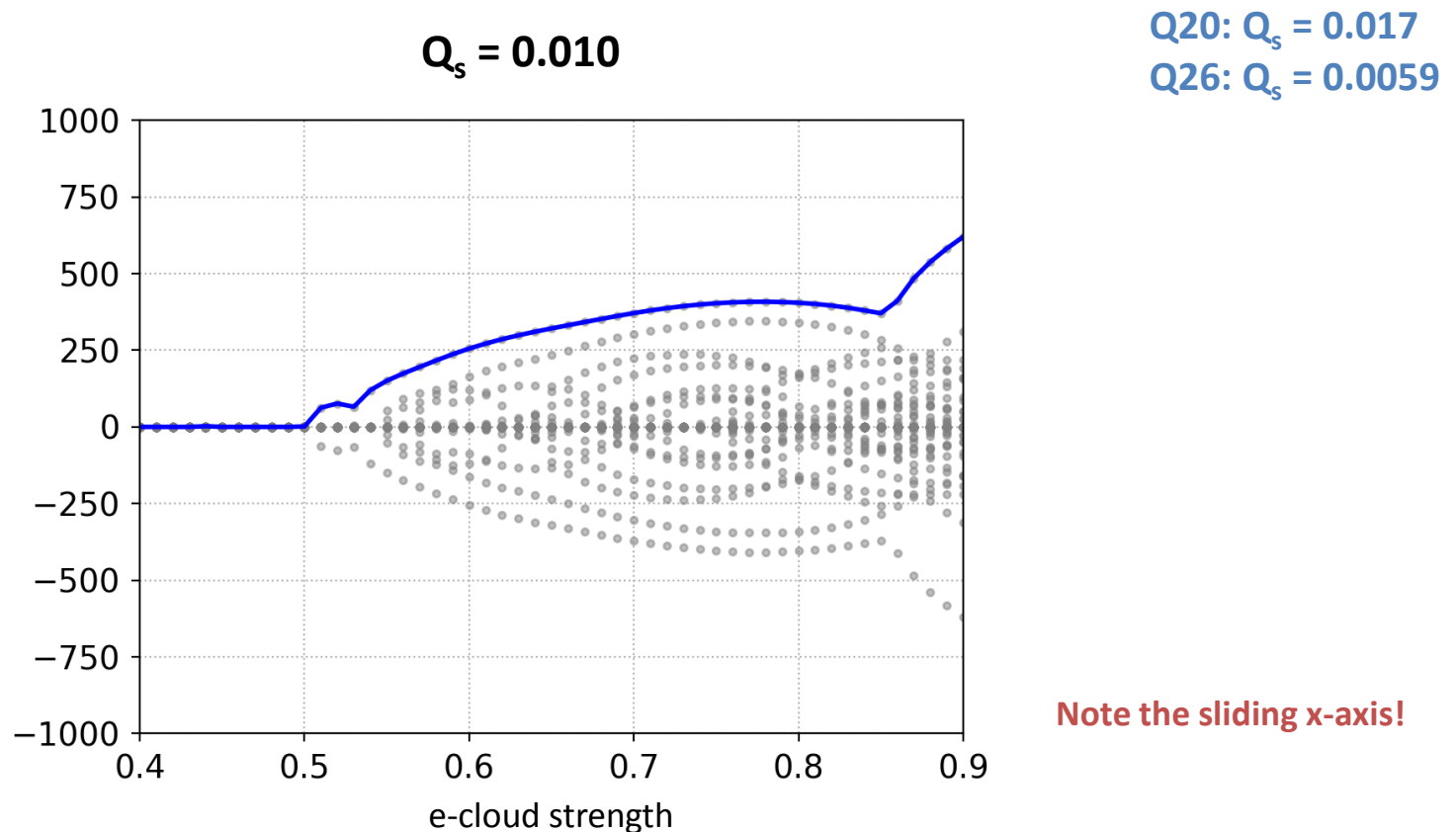
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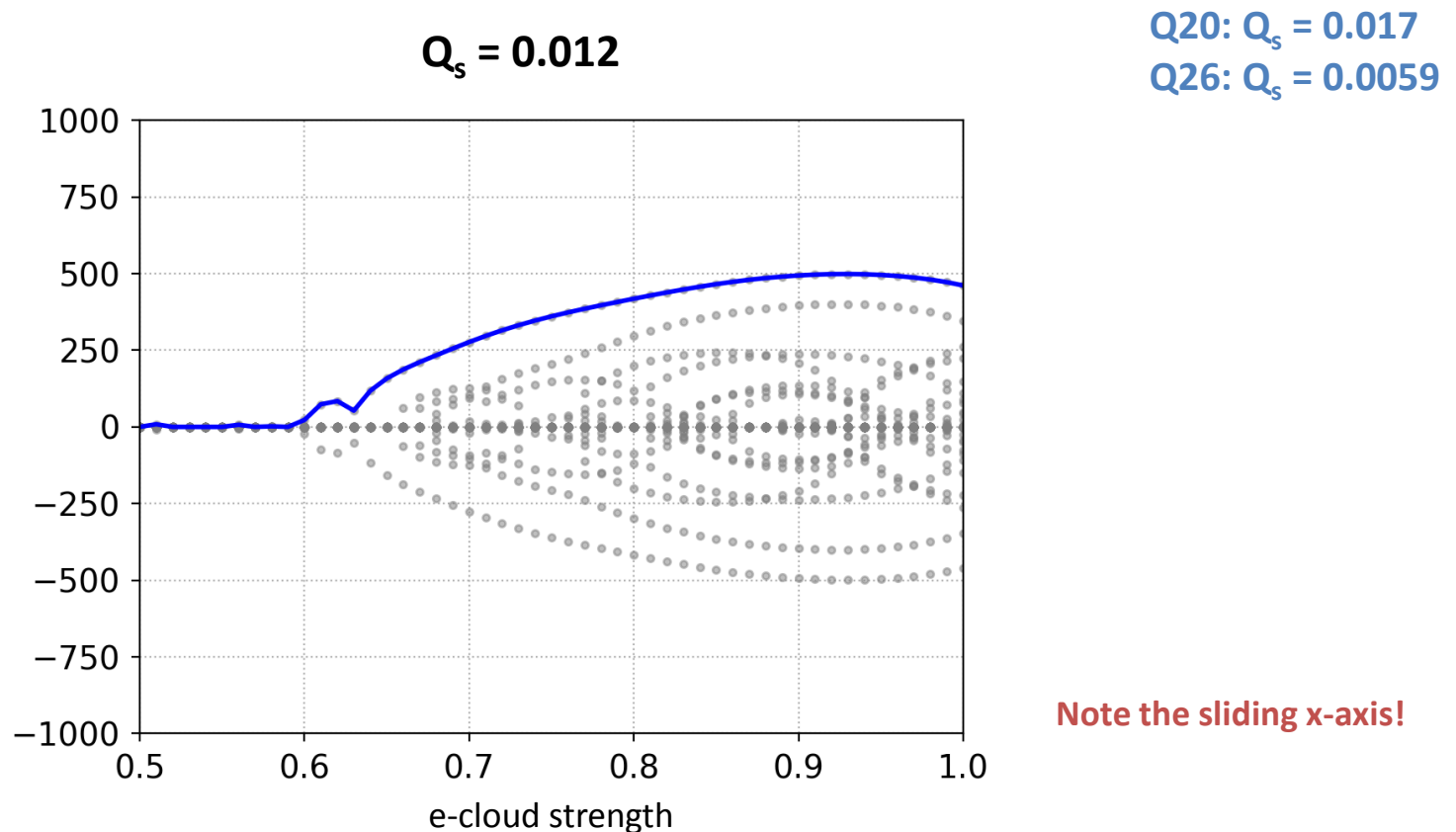
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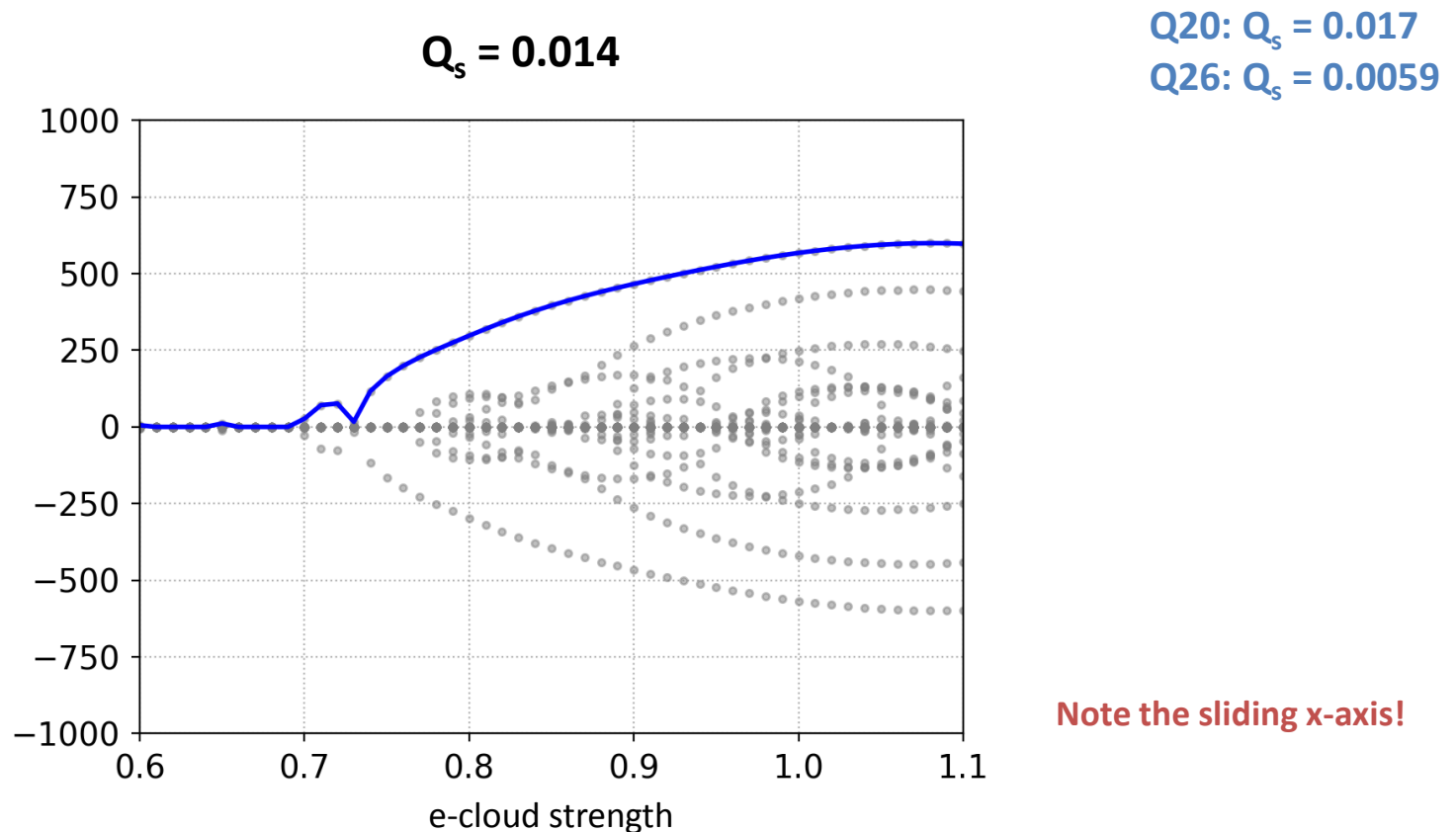
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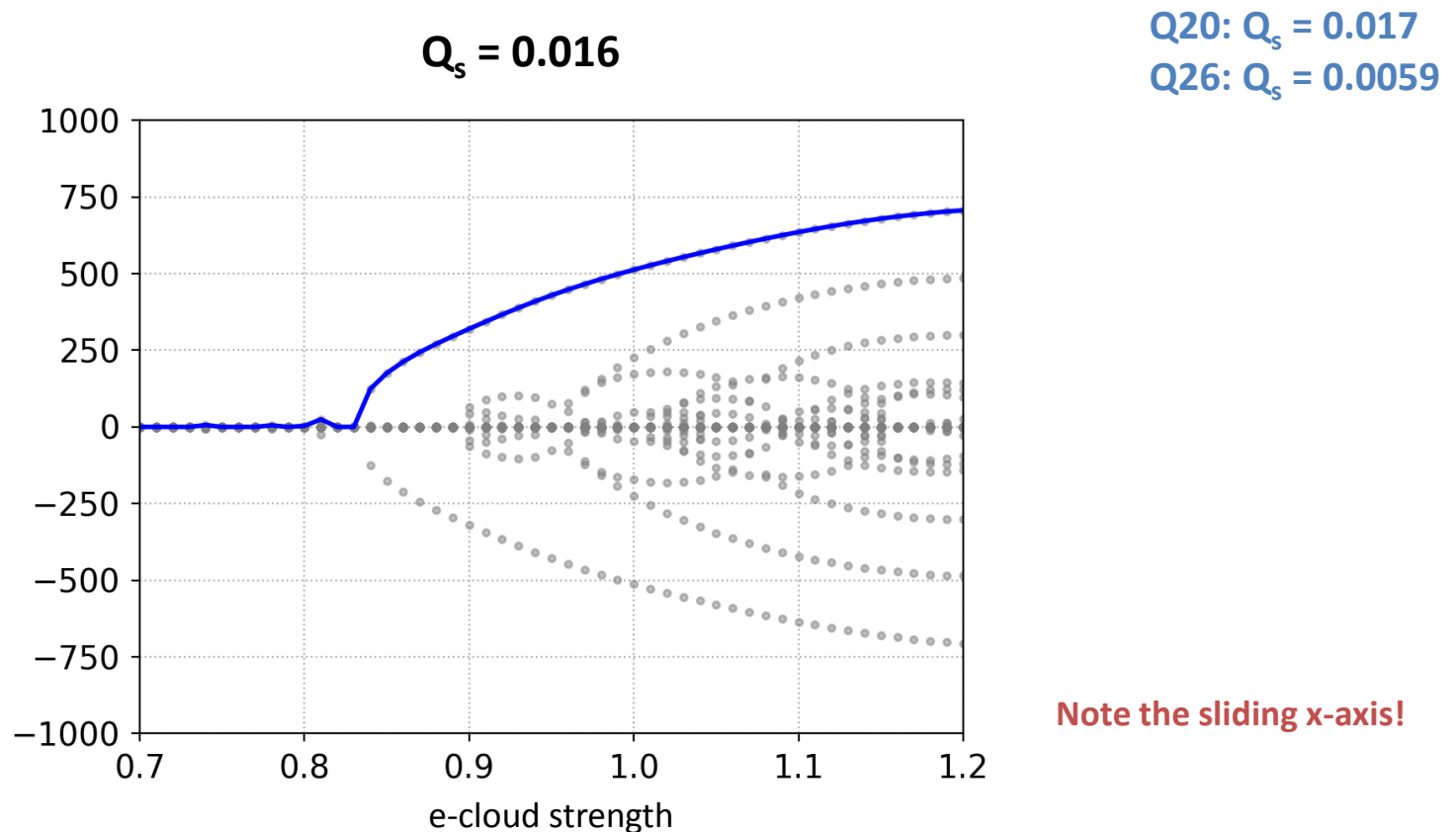
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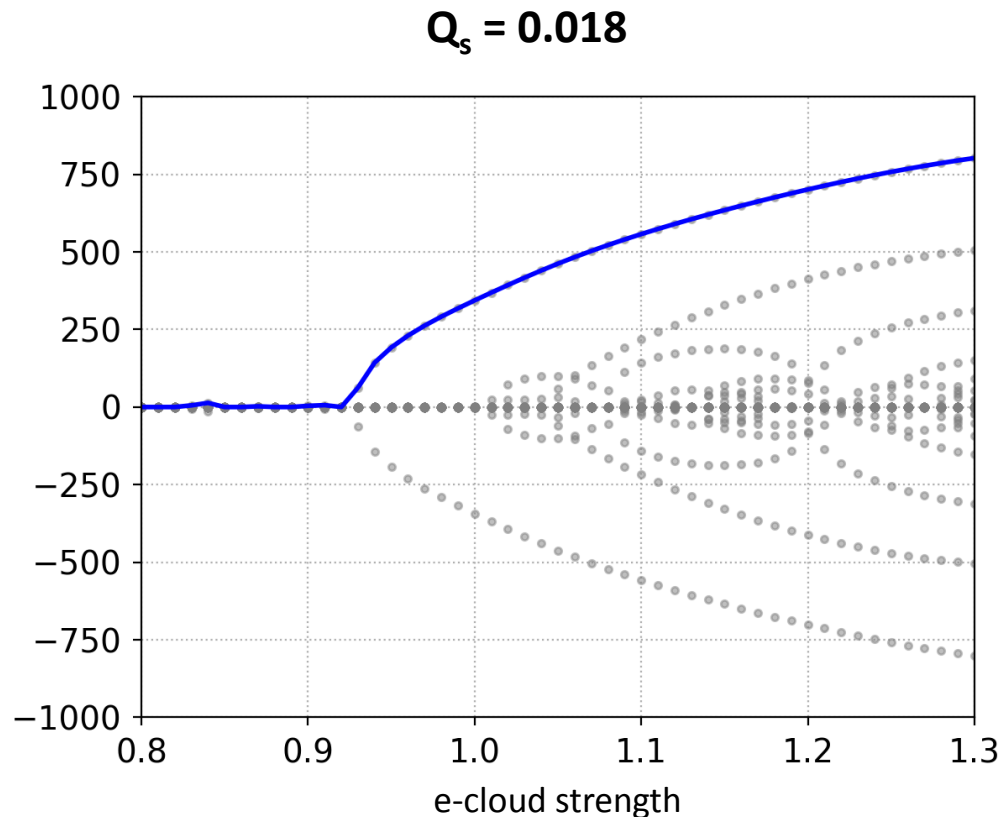
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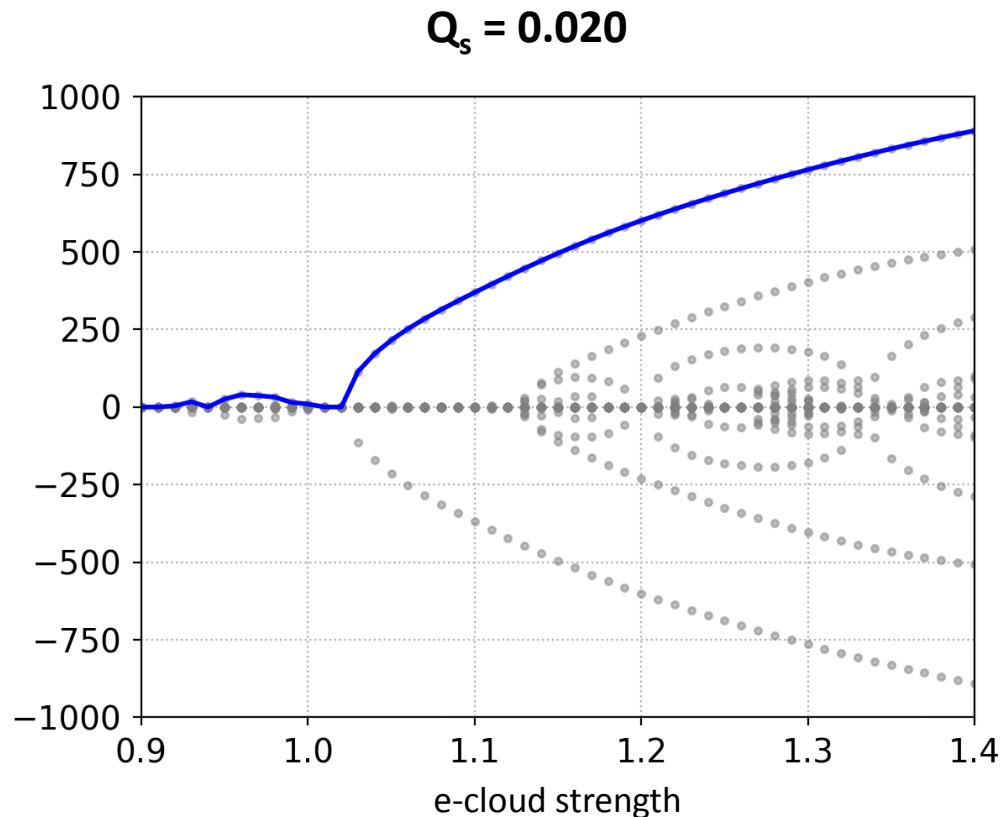
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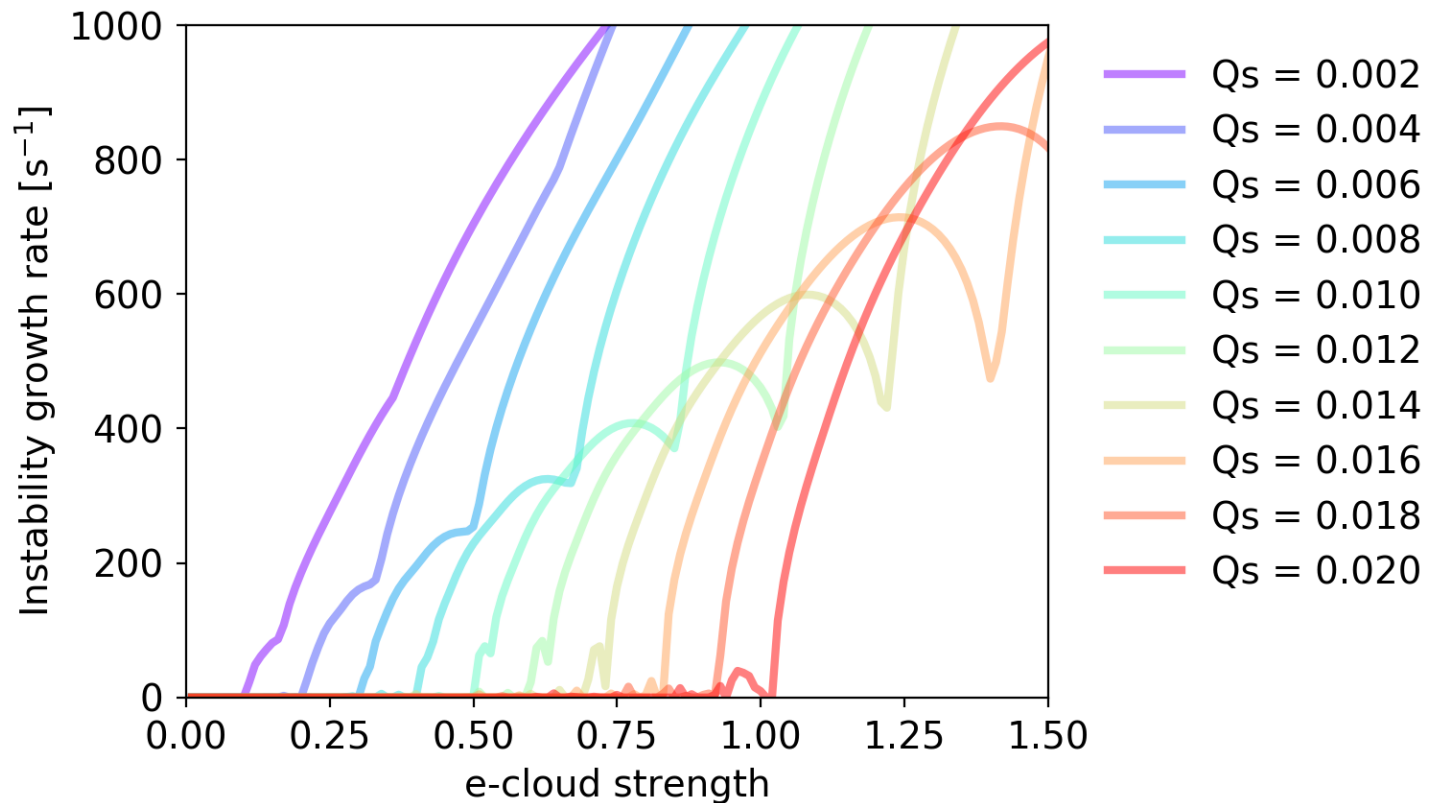
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Note the sliding x-axis!



# Instability growth rate vs $Q_s$

The instability growth rates as a function of the e-cloud strength for different  $Q_s$  with bunch intensity  $N_b = 2 \times 10^{11}$



# Summary and conclusions

The linearized model using the Vlasov method was applied to instabilities driven by e-cloud in the SPS dipoles at injection

- Instabilities are found to be triggered by transverse mode coupling (TMCI)
- The effect of the bunch intensity was studied
  - Increasing the bunch intensity leads to lower instability thresholds
  - Results are qualitatively in agreement with past PIC studies
- The effect of the synchrotron tune was studied
  - Decreasing the synchrotron tune leads to lower instability thresholds
  - This is due to the synchrotron sidebands becoming closer, thus facilitating the coupling between modes