Theoretical motivations to study the Σ observable (with comments)

Vladimir Vechernin

St.Petersburg State University

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Connection with two-particle correlation function C_2

For this traditional n-n FB correlation coefficient b_{nn} we have for small observation windows (see e.g.V.V.,Nucl.Phys.A939(2015)21):

$$b_{nn} = \frac{\langle n_F \rangle \langle n_B \rangle}{D_{n_F}} \frac{\mathrm{cov}(n_F, n_B)}{\langle n_F \rangle \langle n_B \rangle} \to \frac{\langle n_F \rangle \langle n_B \rangle}{D_{n_F}} C_2(\eta_F, \eta_B)$$

$$C_2(\eta_1, \eta_2) \equiv \frac{\rho_2(\eta_1, \eta_2)}{\rho(\eta_1)\rho(\eta_2)} - 1 \ , \qquad (1)$$

where

$$\rho(\eta) = \frac{dN_{ch}}{d\eta} \ , \qquad \rho_2(\eta_1, \eta_2) = \frac{d^2 N_{ch}}{d\eta_1 \, d\eta_2} \qquad (2)$$

This enables to extract from experimental data the absolute value of the $C_2(\eta_1, \eta_2)$ without any commonly used event mixing procedure, including the cases when a translation invariance in rapidity is absent.

C_2 through multiplicities in two small windows

For two small windows $\delta\eta_1$ and $\delta\eta_2$ around η_1 and η_2 we have

$$\rho(\eta) = \frac{\langle n \rangle}{\delta\eta} , \qquad \rho_2(\eta_1, \eta_2) = \frac{\langle n_1 n_2 \rangle}{\delta\eta_1\, \delta\eta_2} , \tag{3}$$

$$C_2(\eta_1, \eta_2) = \frac{\langle n_1 n_2 \rangle}{\langle n_1 \rangle \langle n_2 \rangle} - 1 , \tag{4}$$

where n_1 and n_2 are the event multiplisities in these windows $\delta\eta_1$ and $\delta\eta_2$. Note that when $\eta_1 = \eta_2 = \eta$, $\eta_{sep} = 0$, we have to use

$$\rho_2(\eta, \eta) = \frac{\langle n(n-1) \rangle}{\delta\eta^2} , \qquad C_2(0) = \frac{\langle n(n-1) \rangle}{\langle n \rangle^2} - 1 = \frac{\omega_n - 1}{\langle n \rangle} , \tag{5}$$

where n is the number of particles in small window $\delta\eta$ around the point η. (see e.g. [C.Pruneau,S.Gavin,S.Voloshin,Phys.Rev.C66(2002)044904] or [V.V.,Nucl.Phys.A939(2015)21]).

Strongly intensive observable $\Sigma(n_F, n_B)$

We define the strongly intensive observable $\Sigma(n_F, n_B)$ between multiplicities in forward (n_F) and backward (n_B) windows in accordance with [*M.I.Gorenstein,M.Gazdzicki,Phys.Rev.C84(2011)014904*] as

$$\Sigma(n_F, n_B) \equiv \frac{1}{\langle n_F \rangle + \langle n_B \rangle}[\langle n_F \rangle \, \omega_{n_B} + \langle n_B \rangle \, \omega_{n_F} - 2\,\mathrm{cov}(n_F\, n_B)] \;, \qquad (6)$$

where

$$\mathrm{cov}(n_F, n_B) \equiv \langle n_F n_B \rangle - \langle n_F \rangle \langle n_B \rangle \;, \qquad (7)$$

and ω_{n_F} and ω_{n_B} are the corresponding scaled variances of the multiplicities:

$$\omega_n \equiv \frac{D_n}{\langle n \rangle} = \frac{\langle n^2 \rangle - \langle n \rangle^2}{\langle n \rangle} \;. \qquad (8)$$

[*E.V.Andronov,Theor.Math.Phys.185(2015)1383*]

$\Sigma(n_F, n_B)$ for symmetric reaction and symmetric windows

For symmetric reaction and symmetric observation windows $\delta\eta_F = \delta\eta_B = \delta\eta$:

$$\langle n_F \rangle = \langle n_B \rangle \equiv \langle n \rangle \ , \qquad \omega_{n_F} = \omega_{n_B} \equiv \omega_n \tag{9}$$

and

$$\Sigma(n_F, n_B) = \omega_n - \mathrm{cov}(n_F, n_B)/\langle n \rangle = \frac{\langle n^2 \rangle - \langle n_F n_B \rangle}{\langle n \rangle} =$$

$$= \frac{D_n - \mathrm{cov}(n_F, n_B)}{\langle n \rangle} = \frac{\mathrm{cov}(n_F, n_F) - \mathrm{cov}(n_F, n_B)}{\langle n \rangle} \ . \tag{10}$$

Connection with FBC coefficient b_{nn}:

$$\Sigma(n_F, n_B) = \omega_n \, (1 - b_{nn}) \tag{11}$$

$\Sigma(n_F, n_B)$ through two-particle correlation function C_2

$$\omega_n = D_n/\langle n\rangle = 1 + \langle n\rangle\, I_{FF}\ , \qquad \mathrm{cov}(n_F, n_B)/\langle n\rangle = \langle n\rangle\, I_{FB}\ , \tag{12}$$

$$\Sigma(n_F, n_B) = \omega_n - \mathrm{cov}(n_F, n_B)/\langle n\rangle = 1 + \langle n\rangle\,[I_{FF} - I_{FB}]\ , \tag{13}$$

where

$$I_{FF} = \frac{1}{\delta\eta_F^2}\int_{\delta\eta_F} d\eta_1 \int_{\delta\eta_F} d\eta_2\ C_2(\eta_1 - \eta_2) \quad \to C_2(0)$$

$$I_{FB} = \frac{1}{\delta\eta_F\,\delta\eta_B}\int_{\delta\eta_F} d\eta_1 \int_{\delta\eta_B} d\eta_2\ C_2(\eta_1 - \eta_2) \quad \to C_2(\eta_{sep})$$

The last limit is valid for the small windows: $\delta\eta_F{=}\delta\eta_B{=}\delta\eta \ll \eta_{corr}$, then

$$\boxed{\Sigma(n_F, n_B) = 1 + \langle n\rangle[C_2(0) - C_2(\eta_{sep})]}$$

$$\omega_n = 1 + \langle n\rangle\, C_2(0)\ , \tag{14}$$

For FBC coefficient b_{nn} we had in [V.V.,Nucl.Phys.A939(2015)21]:

$$b_{nn} = \frac{\langle n\rangle \mathrm{cov}(n_F, n_B)}{\omega_n} = \frac{\langle n\rangle\, I_{FB}}{1 + \langle n\rangle\, I_{FF}} \quad \to \frac{\langle n\rangle\, C_2(\eta_{sep})}{1 + \langle n\rangle\, C_2(0)} \approx \langle n\rangle\, C_2(\eta_{sep})$$

Σ in the model with independent identical strings

The fundamental characteristics of a string:
one- and two-particle rapidity distributions from a single string decay:

$$\lambda(y) = \mu_0, \qquad \lambda_2(y_1, y_2) = \lambda_2(y_1 - y_2) = \lambda_2(\Delta\eta)$$

$\Lambda(\Delta\eta)$ - two-particle correlation function of a string:

$$\Lambda(\eta_1, \eta_2) \equiv \frac{\lambda_2(\eta_1, \eta_2)}{\lambda(\eta_1)\lambda(\eta_2)} - 1 = \frac{\lambda_2(\Delta\eta)}{\mu_0^2} - 1 = \Lambda(\Delta\eta) \ .$$

$\delta\eta$ - the width of the observation windows (below we suppose $\delta\eta \ll \eta_{corr}$),
$\Delta\eta = \eta_{sep}$ - the distance between the observation windows.

$$\boxed{\Sigma(\Delta\eta) = 1 + \mu_0\delta\eta[\Lambda(0) - \Lambda(\Delta\eta)]}$$

Andronov E, Vechernin V 2019 Eur. Phys. J. A 55 14
Vechernin V, Andronov E 2019 Universe 5 15
Vechernin V 2018 Eur. Phys. J.: Web of Conf. 191 04011

The model with independent identical strings

[*M.A. Braun, C. Pajares, V.V.V., Phys. Lett. B* **493**, *54 (2000)*]

1) The number of strings, N, fluctuates event by event around some mean value, $\langle N \rangle$, with some scaled variance, $\omega_N = D_N / \langle N \rangle$.

Intensive observable does not depends on $\langle N \rangle$.
Strongly intensive observable does not depends on $\langle N \rangle$ and ω_N.

2) The fragmentation of each string contributes event-by-event to the forward and backward observation rapidity windows, $\delta\eta_F$, and $\delta\eta_B$, the μ_F and μ_B charged particles correspondingly, which fluctuate around some mean values, $\langle \mu_F \rangle$ and $\langle \mu_B \rangle$, with some scaled variances, $\omega_{\mu_F} = D_{\mu_F} / \langle \mu_F \rangle$ and $\omega_{\mu_B} = D_{\mu_B} / \langle \mu_B \rangle$.

The observation rapidity windows are separated by some rapidity interval:
$\eta_{sep} = \Delta\eta$ - the distance between the centers of the $\delta\eta_F$ and $\delta\eta_B$.

Clear that in this model (and the same for n_B):

$$\langle n_F \rangle = \langle \mu_F \rangle \langle N \rangle = \langle N \rangle \mu_0, \qquad \omega_{n_F} = \omega_{\mu_F} + \langle \mu_F \rangle \omega_N \ ,$$

Two-particle correlation function of a string

Along with the observed standard two-particle correlation function:

$$C_2(\eta_1, \eta_2) \equiv \frac{\rho_2(\eta_1, \eta_2)}{\rho(\eta_1)\rho(\eta_2)} - 1 \ , \tag{15}$$

where

$$\rho(\eta) = \frac{dN_{ch}}{d\eta} \ , \qquad \rho_2(\eta_1, \eta_2) = \frac{d^2 N_{ch}}{d\eta_1 \, d\eta_2} \tag{16}$$

one can introduce the string two-particle correlation function, $\Lambda(\eta_1, \eta_2)$, characterizing correlation between particles, produced from the one string:

$$\Lambda(\eta_1, \eta_2) \equiv \frac{\lambda_2(\eta_1, \eta_2)}{\lambda(\eta_1)\lambda(\eta_2)} - 1 \ . \tag{17}$$

The $\Lambda(\eta_1, \eta_2)$ haracterizes the string decay properties

($z - \eta$ correspondence)

*[X.Artru,Phys.Rept.**97**(1983)147, V.V.,arXiv:0812.0604]*

Connection between the two-particle correlation functions

In this model we have the following connection:

$$C_2(\eta_1, \eta_2) = \frac{\omega_N + \Lambda(\eta_1, \eta_2)}{\langle N \rangle}$$

[*V.V., Nucl.Phys.A939(2015)21*]. (Note that one often looses the constant part $\omega_N / \langle N \rangle$ of C_2, using di-hadron correlation approach.)

At midrapidities, implying uniform rapidity distribution:

$$\lambda(\eta) = \mu_0 = \frac{\langle \mu_F \rangle}{\delta y_F} = \frac{\langle \mu_B \rangle}{\delta y_B}, \qquad \rho(\eta) = \frac{dN_{ch}}{d\eta} = \rho_0 = \frac{\langle n_F \rangle}{\delta y_F} = \frac{\langle n_B \rangle}{\delta y_B} = \langle N \rangle \mu_0$$

and the correlation functions depends only on a difference of rapidities:

$$\eta_{sep} = \eta_1 - \eta_2 = \Delta\eta$$

We suppose that the string correlation function

$$\Lambda(\Delta\eta) \to 0, \text{ when } \Delta\eta \gg \eta_{corr},$$

where the η_{corr} is the correlation length.

$\Sigma(n_F, n_B)$ for small observation windows

For small observation windows, of a width $\delta\eta \ll \eta_{corr}$, we find [V.V.,Nucl.Phys.A939(2015)21]:

$$\omega_n = D_n/\langle n \rangle = 1 + \mu_0 \delta\eta \, [\Lambda(0) + \omega_N] \ , \tag{18}$$

$$\mathrm{cov}(n_F, n_B)/\langle n \rangle = \mu_0 \delta\eta \, [\Lambda(\Delta\eta) + \omega_N] \ , \tag{19}$$

$$\Sigma(n_F, n_B) = \omega_n - \mathrm{cov}(n_F, n_B)/\langle n \rangle = 1 + \mu_0 \delta\eta \, [\Lambda(0) - \Lambda(\Delta\eta)] \ , \tag{20}$$

where $\Delta\eta = \eta_F - \eta_B = \eta_{sep}$ is a distance between the centers of the forward an backward observation windows. For a single string we have

$$\omega_\mu = D_\mu/\langle \mu \rangle = 1 + \mu_0 \delta\eta \, \Lambda(0) \ , \tag{21}$$

$$\mathrm{cov}(\mu_F, \mu_B)/\langle \mu \rangle = \mu_0 \delta\eta \, \Lambda(\Delta\eta) \ , \tag{22}$$

$$\Sigma(\mu_F, \mu_B) = \omega_\mu - \mathrm{cov}(\mu_F, \mu_B)/\langle \mu \rangle = 1 + \mu_0 \delta\eta \, [\Lambda(0) - \Lambda(\Delta\eta)] \ , \tag{23}$$

So in $\Sigma(n_F, n_B)$ we have the cancelation of the contributions from the fluctuation of the number of strings, ω_N, and it becames strongly intensive:

$$\Sigma(n_F, n_B) = \Sigma(\mu_F, \mu_B)$$

Strongly intensive observable $\Sigma(\mu_F, \mu_B)$

In general case the strongly intensive variable for a single string is defined similarly to $\Sigma(n_F, n_B)$ by

$$\Sigma(\mu_F, \mu_B) \equiv \frac{1}{\langle\mu_F\rangle + \langle\mu_B\rangle}[\langle\mu_F\rangle\,\omega_{\mu_B} + \langle\mu_B\rangle\,\omega_{\mu_F} - 2\,\mathrm{cov}(\mu_F, \mu_B)]\ . \quad (24)$$

It depends only on properties of a single string.
So in the model with independent identical strings for symmetric reaction and small symmetric observation windows we found for $\Sigma(n_F, n_B)$:

$$\boxed{\Sigma(\eta_{sep}) = 1 + \mu_0\delta\eta[\Lambda(0) - \Lambda(\eta_{sep})]}$$

We see that really the $\Sigma(\eta_{sep})$ is strongly intensive quantity.
It does not depend on $\langle N\rangle$ and ω_N, whereas for the b_{nn} we have

$$b_{nn} = \frac{\langle n\rangle\, C_2(\Delta\eta)}{1 + \langle n\rangle\, C_2(0)} = \frac{\mu_0\delta\eta\,[\omega_N + \Lambda(\Delta\eta)]}{1 + \mu_0\delta\eta\,[\omega_N + \Lambda(0)]}$$

Properties of Σ in model with independent identical strings

$$\Sigma(\eta_{sep}) = 1 + \mu_0 \delta\eta [\Lambda(0) - \Lambda(\eta_{sep})]$$

The $\Sigma(0) = 1$ and increases with the gap between windows, η_{sep},
because the $\Lambda(\eta_{sep})$ decrease with η_{sep},
as the correlations in string go off with increase of η_{sep}.

The rate of the $\Sigma(\eta_{sep})$ growth with η_{sep} is proportional
to the width of the observation window $\delta\eta$ and μ_0 -
- the multiplicity produced from one string.

The model predicts saturation of the $\Sigma(\eta_{sep})$ on the level

$$\Sigma(\eta_{sep}) = 1 + \mu_0 \delta\eta \, \Lambda(0) = \omega_\mu$$

at large η_{sep}, as $\Lambda(\eta_{sep}) \to 0$ at the $\eta_{sep} \gg \eta_{corr}$,
where the η_{corr} is a string correlation length.

Properties of Σ in model with independent identical strings

- We see that in the model with identical strings the $\Sigma(\Delta\eta)$ is a really strongly intensive quantity. It does not depend nor on the mean number of strings $\langle N\rangle$, nor on their event-by-event fluctuations $\omega_N \equiv D_N/\langle N\rangle$.
It depends ONLY on string parameters: μ_0 and $\Lambda(\Delta\eta)$.

- The $\Sigma(0) = 1$ and increases with the distance between windows, $\Delta\eta$, as the $\Lambda(\Delta\eta)$ decrease to 0 with $\Delta\eta$, since the correlations between string segments go off with increase of $\Delta\eta$.

- The rate of the $\Sigma(\Delta\eta)$ growth with $\Delta\eta$ is proportional to the width of the observation window $\delta\eta$ and μ_0 - the multiplicity produced from one string.

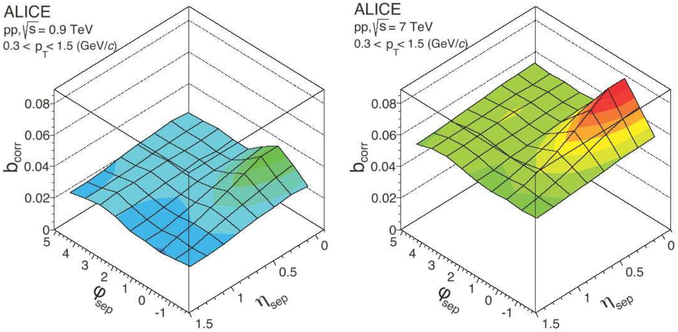
- The model predicts saturation of the $\Sigma(\Delta\eta)$ on the level

$$\Sigma(\Delta\eta) = 1 + \mu_0\delta\eta\,\Lambda(0) = \omega_\mu = D_\mu/\langle\mu\rangle$$

at large $\Delta\eta$, since $\Lambda(\Delta\eta) \to 0$ at the $\Delta\eta \gg \eta_{corr}$,
where the η_{corr} is a string correlation length.

The ALICE data on b_{nn} in pp

ALICE collab., JHEP 05(2015)097



$\Rightarrow \Lambda(\Delta\eta)$

V.V.,Nucl.Phys.A939(2015)21

The parametrization of the single correlation function

The parametrization for the pair correlation function $\Lambda(\eta,\phi)$ of a single string (reflecting the Schwinger mechanism of a string decay, was suggested in [V.V.,Nucl.Phys.A939(2015)21]:

$$\Lambda(\eta,\phi) = \Lambda_1 e^{-\frac{|\eta|}{\eta_1}} e^{-\frac{\varphi^2}{\varphi_1^2}} + \Lambda_2 \left(e^{-\frac{|\eta-\eta_0|}{\eta_2}} + e^{-\frac{|\eta+\eta_0|}{\eta_2}} \right) e^{-\frac{(|\varphi|-\pi)^2}{\varphi_2^2}} . \qquad (25)$$

This formula has the nearside peak, characterizing by parameters Λ_1, η_1 and φ_1, and the awayside ridge-like structure, characterizing by parameters Λ_2, η_2, η_0 and φ_2 (two wide overlapping hills shifted by $\pm\eta_0$ in rapidity, η_0 - the mean length of a string decay segment). We imply that in formula (25)

$$|\varphi| \le \pi \ . \qquad (26)$$

If $|\varphi| > \pi$, then we use the replacement $\varphi \to \varphi + 2\pi k$, so that (26) was fulfilled. With such completions the $\Lambda(\eta,\phi)$ meets the following properties

$$\Lambda(-\eta,\phi) = \Lambda(\eta,\phi) \ , \quad \Lambda(\eta;-\phi) = \Lambda(\eta,\phi) \ , \quad \Lambda(\eta,\phi+2\pi k) = \Lambda(\eta,\phi) \qquad (27)$$

Fitting the model parameters by FBC in small windows

$\Lambda(\eta_{sep}, \phi_{sep})$ was fitted by the ALICE b_{nn} pp data with FB windows of small acceptance, $\delta\eta = 0.2, \delta\phi = \pi/4$, separated in azimuth and rapidity *[ALICE collab.,JHEP 05(2015)097]*. It gives for the parameters:

	$\sqrt{s}$, TeV	0.9	2.76	7.0
LRC	$\mu_0\omega_N$	0.7	1.4	2.1
SRC	$\mu_0\Lambda_1$	1.5	1.9	2.3
	η_1	0.75	0.75	0.75
	ϕ_1	1.2	1.15	1.1
	$\mu_0\Lambda_2$	0.4	0.4	0.4
	η_2	2.0	2.0	2.0
	ϕ_2	1.7	1.7	1.7
	η_0	0.9	0.9	0.9

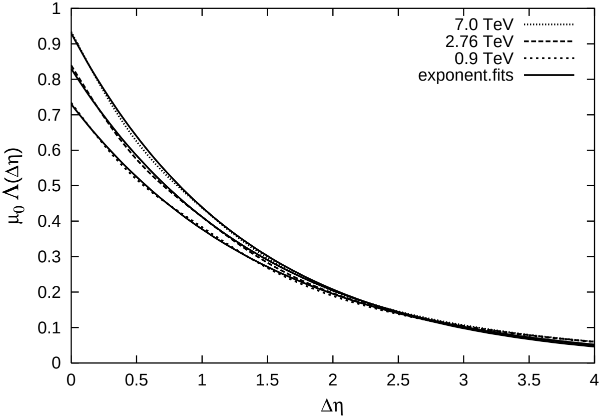
$\omega_N = \frac{\langle N^2\rangle - \langle N\rangle^2}{\langle N\rangle}$ is the e-by-e scaled variance of the number of strings,
μ_0 is the average rapidity density of the charged particles from one string,
$i=1$ corresponds to the nearside and $i=2$ to the awayside contributions,
η_0 is the mean length of a string decay segment.

[V.V.,Nucl.Phys.A939(2015)21]

The stringt correlation function $\Lambda(\Delta\eta)$

Then we find $\Lambda(\Delta\eta)$ integrating over azimuth:

$$\Lambda(\eta_{sep}) = \frac{1}{\pi} \int_0^{\pi} \Lambda(\eta_{sep}, \phi_{sep})\, d\phi_{sep} \ .$$



The string correlation function $\Lambda(\Delta\eta)$

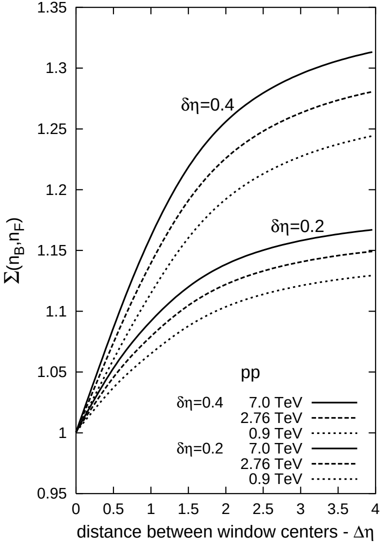
The obtained dependencies in this fugure for three initial energies are well approximated by the exponent:

$$\Lambda(\Delta\eta) = \Lambda_0 e^{-\frac{|\Delta\eta|}{\eta_{corr}}} , \quad (28)$$

with the parameters presented in the table:

$\sqrt{s}$, TeV	0.9	2.76	7.0
$\mu_0\Lambda_0$	0.73	0.83	0.93
η_{corr}	1.52	1.43	1.33

We see that the correlation length, η_{corr}, decreases with the increase of collision energy.
This can be interpreted as a signal of an increase with energy of the admixture of strings of a new type - the fused strings in pp collisions (see below).



Here it was used the $\Lambda(\Delta\eta, \Delta\phi)$, extracted in [V.Vechernin,Nucl.Phys.A939(2015)21] from the ALICE pp data on forward-backward correlations in windows of small acceptance, $\delta\eta = 0.2, \delta\phi = \pi/4$, separated in azimuth and rapidity [ALICE collab.,JHEP05(2015)097].

The string parameters occur dependent on initial energy (!?)

Hint at increasing the contribution of string clusters to $\Sigma(n_F, n_B)$ with collision energy in pp collisions

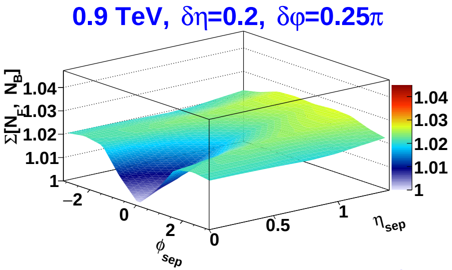
$\Sigma(n_F, n_B)$ in windows separated in azimuth and rapidity

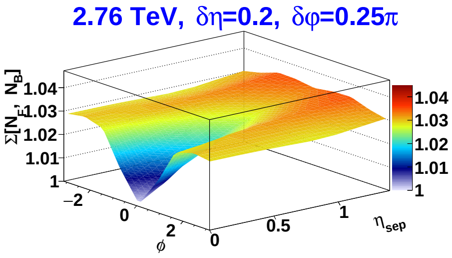
For small observation windows:

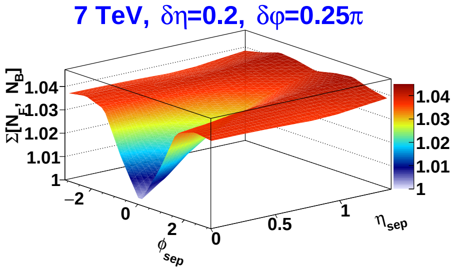
$$\Sigma(\Delta\eta, \Delta\phi) = 1 + \frac{\delta\eta\,\delta\phi}{2\pi}\mu_0\,[\Lambda(0,0) - \Lambda(\Delta\eta, \Delta\phi)]$$

$$\Delta\eta \equiv \eta_{sep}, \qquad \Delta\phi \equiv \phi_{sep}$$

Σ for $\delta\eta\ \delta\phi$ windows separated in azimuth and rapidity







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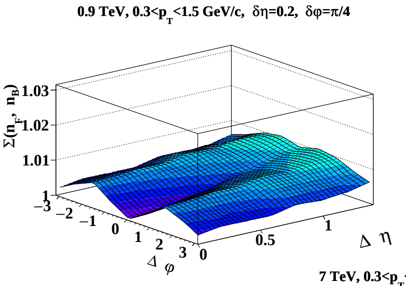
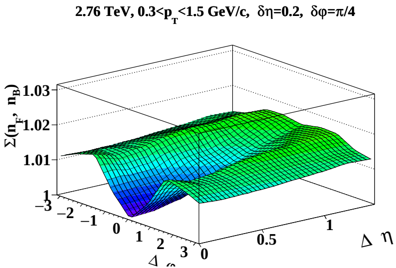
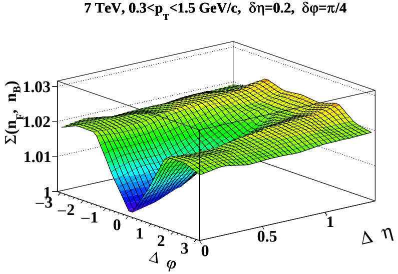
Σ for $\delta\eta$ $\delta\phi$ windows - PYTHIA







Andronov E, Vechernin V 2019 Eur.Phys.J.A55 14 (The cover journal image)

$\Sigma(n_F, n_B)$ with charges

$$\Sigma(n_F, n_B) = \Sigma(n_F^+, n_B^+) + \Sigma(n_F^-, n_B^+) - \Sigma(n_F^+, n_F^-) \ .$$

Andronov E, Vechernin V 2019 Eur. Phys. J. A 55 14
Vechernin V, Andronov E 2019 Universe 5 15:

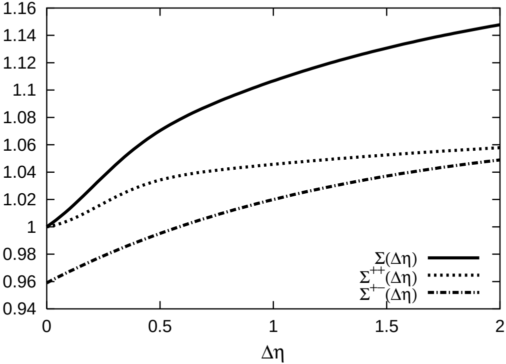
$$\Sigma(n_F^+, n_B^+) = 1 + \mu_0 \delta\eta [\Lambda^{same}(0) - \Lambda^{same}(\Delta\eta)]/2 \ ,$$
$$\Sigma(n_F^+, n_B^-) = 1 + \mu_0 \delta\eta [\Lambda^{same}(0) - \Lambda^{opp}(\Delta\eta)]/2 \ ,$$
$$\Sigma(n_F^+, n_F^-) = 1 + \mu_0 \delta\eta [\Lambda^{same}(0) - \Lambda^{opp}(0)]/2 \ .$$

The $\Lambda^{same}(\Delta\eta)$ and $\Lambda^{opp}(\Delta\eta)$ was extracted using the connections:

$$\Lambda(\Delta\eta) = \frac{\Lambda^{opp}(\Delta\eta) + \Lambda^{same}(\Delta\eta)}{2}, \qquad B(\Delta\eta) = \mu_0 \frac{\Lambda^{opp}(\Delta\eta) - \Lambda^{same}(\Delta\eta)}{2}.$$

The Balance Function $B(\Delta\eta)$ was taken from the paper [ALICE collab.,Eur.Phys.J. C76(2016)86]. and the $\Lambda(\Delta\eta)$ was already extracted in [V.Vechernin,Nucl.Phys.A939(2015)21] from the ALICE pp data on forward-backward correlations [ALICE collab.,JHEP05(2015)097]

$\Sigma(n_F, n_B)$ with charges



Note that similarly to the $\Sigma(n_F, n_B)$ we have

$$\Sigma(n_F^+, n_B^+) \to 1 \quad at \quad \Delta\eta \to 0 \ ,$$

but

$$\Sigma(n_F^+, n_B^-) \to \Sigma(n_F^+, n_F^-) = 1 + \mu_0 \delta\eta [\Lambda^{same}(0) - \Lambda^{opp}(0)]/2 = 0.96 < 1 \ .$$

$\Sigma(n_F, n_B)$ in the model with string clusters formation

In the model with string clusters formation
by a string fusion on transverse grid it was shown
[*S.N. Belokurova, V.V. Vechernin, Theor.Math.Phys. 200(2019)1094*]:

$$\Sigma(n_F, n_B) = \sum_{k=1}^{\infty} \alpha_k \, \Sigma_k(\mu_F, \mu_B) \; , \qquad \alpha_k = \frac{\langle n^{(k)} \rangle}{\langle n \rangle} \; ,$$

where k is a number of stings, which formed a given string cluster and $\langle n^{(k)} \rangle$ is a mean number of particles produced from all such clusters, $\sum \alpha_k = 1$.
Here $\Sigma_k(\mu_F, \mu_B)$ is the variable Σ for the cluster, formed by k strings:

$$\Sigma_k(\mu_F, \mu_B) = \Sigma_k(\Delta\eta) = 1 + \mu_0^{(k)} \delta\eta [\Lambda_k(0) - \Lambda_k(\Delta\eta)] \; ,$$

where $\mu_0^{(k)}$ and $\Lambda_k(\Delta\eta)$ are the corresponding parameters of the string cluster.

$\Sigma(n_F, n_B)$ in the model with string fusion

The similar result was obtained in the model with two types of string in [*E.V.Andronov, Theor.Math.Phys.185(2015)1383*] for the long-range part of $\Sigma(n_F, n_B)$, when at $\Delta\eta \gg \eta_{corr}$ we have $\Sigma_k(\mu_F, \mu_B) = \omega_\mu^{(k)}$, k =1,2 :

$$\Sigma(n_F, n_B)|_{\Delta\eta >> \eta_{corr}} = \frac{\langle n\rangle_1 \omega_\mu^{(1)} + \langle n\rangle_2 \omega_\mu^{(2)}}{\langle n\rangle}$$

If we suppose the formation of string clusters in central pp collisions at high energy (and in AA collisions) with some new characteristics, due to e.g. string fusion processes, then for a source with k fused strings

$$\Sigma_k(\Delta\eta) = 1 + \mu_0^{(k)} \delta\eta [\Lambda_k(0) - \Lambda_k(\Delta\eta)]$$

For these fused strings we expect, basing on the string decay picture [*V.V.*, Baldin ISHEPP XIX v.1(2008)276; arXiv:0812.0604]:

1) larger multiplicity from one string, $\mu_0^{(k)} > \mu_0$,

2) smaller correlation length, $\eta_{corr}^{(k)} < \eta_{corr}$.

$\Sigma(n_F, n_B)$ in the model with string fusion

This corresponds to the analysis of the net-charge fluctuations in the framework of the string model for pp and AA collisions *[A.Titov, V.V., PoS(Baldin ISHEPP XXI)047(2012)]*.

$$\Sigma_k(\Delta\eta) = 1 + \mu_0^{(k)} \delta\eta [\Lambda_k(0) - \Lambda_k(\Delta\eta)]$$

Both factors lead to the steeper increase of $\Sigma_k(\Delta\eta)$ with $\Delta\eta$ in the case of AA collisions, compared to pp.

In reality - a mixture of fused and single strings:

$$\Sigma(n_F, n_B) = \sum_{k=1}^{\infty} \alpha_k \, \Sigma_k(\mu_F, \mu_B) \, , \qquad \alpha_k = \frac{\langle n^{(k)} \rangle}{\langle n \rangle} \, ,$$

Unfortunately in this case through the weighting factors $\alpha_k = \langle n^{(k)} \rangle / \langle n \rangle$ the observable $\Sigma(n_F, n_B)$ becomes dependent on collision conditions and, strictly speaking, can not be considered any more as strongly intensive.

MC calculations of $\Sigma(n_F, n_B)$ in the model with string clusters formation

- V.V. Vechernin, S.N. Belokurova, in Proceedings of the 5th International Conference on Particle Physics and Astrophysics (ICPPA-20), 5-9 October 2020, Moscow (online), Journal of Physics: Conference Series (in press).

- S. Belokurova, in Proceedings of the LXX International conference NUCLEUS - 2020, 11-17 October 2020, St.Petersburg (online), Physics of Particles and Nuclei (in press); arXiv:2011.10434 [hep-ph].

- Modelling the initial string distribution in the impact parameter plane of pp collisions for different initial energies to take into account string fusion processes. [*V. Vechernin, I. Lakomov. Proceedings of Science (Baldin ISHEPP XXI) (2013) 072.*]

- Monte Carlo simulation of the weighting factors α_k as a function of centrality and initial energy of pp collision.

- Calculation the $\Sigma(n_F, n_B)$ for different centralities of pp collision at few LHC energies, using the expessions for $\mu_0^{(k)}$ and $\Lambda_k(\Delta\eta) = \Lambda_0^{(k)} exp[-|\Delta\eta|/\eta_{corr}^{(k)}]$.

$\Sigma(n_F, n_B)$ in the model with string fusion

$$\mu_0^{(k)} = \mu_0^{(1)}\sqrt{k}\ , \qquad \Lambda_0^{(k)} = \Lambda_0^{(1)} = const\ , \qquad \eta_{corr}^{(k)} = \eta_{corr}^{(1)}/\sqrt{k}\ ,$$

which is instructive to compare with

$$\mu_0^{(k)} = \mu_0^{(1)}\,k\ , \qquad \Lambda_0^{(k)} = \Lambda_0^{(1)}/k\ , \qquad \eta_{corr}^{(k)} = \eta_{corr}^{(1)} = const\ .$$

for the case without string fusion in the given transverse cell.

(In last case $\Sigma(n_F, n_B) = \Sigma_1(\mu_F, \mu_B)$ and does not depends on α_k.)

The values of the parameters $\Lambda_0^{(1)} = 0.8$ and $\eta_{corr}^{(1)} = 2.7$ were chosen so that to obtain a correspondence with the values of the $\Sigma(n_F,\ n_B)$ obtained in [Vechernin V 2018 Eur.Phys.J.:Web of Conf. 191 04011].

Note that in that paper the $\Sigma(n_F,\ n_B)$ was calculated on the base of the string pair correlation function, $\Lambda(\Delta\eta)$, extracted in [V.Vechernin, Nucl.Phys.A939(2015)21] from the ALICE data on the FB correlations [ALICE collab.,JHEP05(2015)097] in the approx. of IDENTICAL strings.

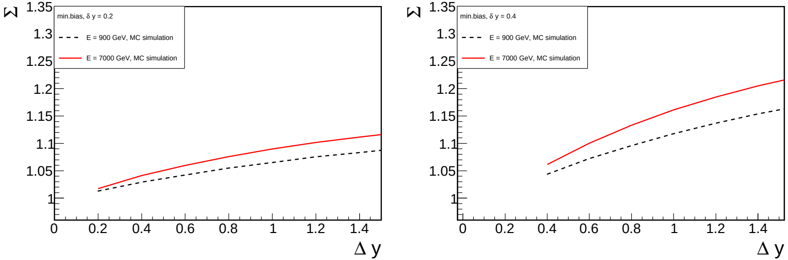


Figure: The strongly intensive observable $\Sigma(n_F, n_B)$ for pp collisions as a function of the rapidity distance $\Delta\eta = \Delta y$ between the centers of the FB observation windows, for two widths of windows: $\delta\eta$=0.2 (left panel) and $\delta\eta$=0.4 (right panel), and for two initial energies: 0.9 TeV (dashed lines) and 7 TeV (solid lines), calculated for particles with transverse momenta in the interval 0.3-1.5 GeV/c, as in the experimental analysis in [ALICE collab.,JHEP05(2015)097].

The increase of the $\Sigma(n_F, n_B)$ in pp collisions with energy is caused by the increasing contribution of string fusion processes and the formation of string clusters with new properties.

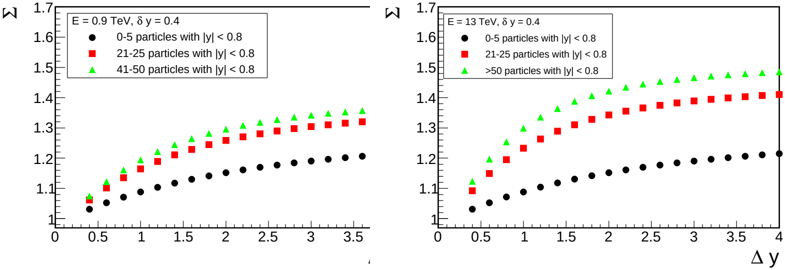


Figure: The strongly intensive variable $\Sigma(n_F,\ n_B)$ at different centralities as a function of the rapidity distance between the observation windows $\Delta\eta = \Delta y$ for *pp* collisions at energies 900 and 13000 GeV for the width of the observation windows $\delta y = 0.4$.

The increase of the $\Sigma(n_F, n_B)$ in pp collisions with the collision centrality is also caused by the increasing contribution of string fusion processes and the formation of string clusters with new properties.

$\Sigma(n_F, n_B)$ in the model with string clusters formation

In the model with string clusters formation the observable $\Sigma(n_F, n_B)$ loses the strongly intensive property, as it becomes equal to the weighted average of its values for different string clusters with the weights depending on collision conditions (the initial energy and centrality).

Nevertheless it can be used for the extraction of the information on the properties of the string clusters (the multiplicity density $\mu_0^{(k)}$ and the pair correlation function $\Lambda_k(\Delta\eta)$ of paricles, produced from a cluster decay).

The increase of the $\Sigma(n_F, n_B)$ in pp collisions with energy can be explained by the increasing role of string fusion processes and the formation of string clusters with new properties.

The increase of the $\Sigma(n_F, n_B)$ with the collision centrality also can be explained by the string fusion processes in the framework of the same approach.

Backup slides

Backup slides

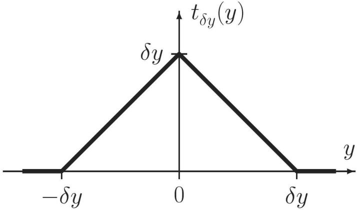
Integrals Over Rapidity and Azimuth Windows - 1

For symmetric rapidity windows, $\delta y_B = \delta y_F = \delta y$, whose centers are separated by $y_{FB} = y_F - y_B$, one has:

$$\int_{\delta y_F} dy_1 \int_{\delta y_B} dy_2\, f(|y_1 - y_2|) = \int_{-\delta y}^{\delta y} dy\, f(|y_{FB} + y|)\, t_{\delta y}(y) \ , \tag{29}$$

where $t_{\delta y}(y)$ is the "triangular" weight function:

$$t_{\delta y}(y) = [\theta(-y)(\delta y + y) + \theta(y)(\delta y - y)]\, \theta(\delta y - |y|) \ . \tag{30}$$



The phase space "triangular" weight function,
arising at integration over non-periodic FB windows

Integrals Over Rapidity and Azimuth Windows - 2

Formula (29) is valid for any distance between the centers of windows, in particular for coinciding windows, for which $y_{FB} = 0$. In this case, we have:

$$\int_{\delta y_F} dy_1 \int_{\delta y_F} dy_2 \, f(|y_1 - y_2|) = \int_{-\delta y}^{\delta y} dy \, f(|y|) \, t_{\delta y}(y) = 2 \int_0^{\delta y} dy \, f(|y|)(\delta y - y) \, .$$

The same general formula can be used for the integration over azimuthal windows:

$$\int_{\delta \varphi_F} d\varphi_1 \int_{\delta \varphi_B} d\varphi_2 \, f(|\varphi_1 - \varphi_2|) = \int_{-\delta\varphi}^{\delta\varphi} d\varphi \, f(|\varphi_{FB} + \phi|) \, t_{\delta\varphi}(\phi) \, . \qquad (31)$$

In this case the function $f(|\phi|)$ is periodic in ϕ such that $f(|\phi|) = f(|\phi + 2\pi k|)$. This implies that windows of full azimuthal acceptance, i.e. the full 2π range, allow for (31) to be simplified to the following:

$$\int_{-2\pi}^{2\pi} d\varphi \, f(|\varphi_{FB} + \phi|) \, t_{2\pi}(\phi) = 4\pi \int_0^{\pi} d\varphi \, f(|\phi|) \, . \qquad (32)$$