The anomalous magnetic moment of the muon: present status

Marc Knecht

Centre de Physique Théorique

CNRS, Aix-Marseille Université, IPhU, Université de Toulon

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INTRODUCTION

On April 7, 2021, the FNAL-E989 experiment released the first result of a measurement of the anomalous magnetic moment of the muon a_μ, based on the Run-1 data collected during Spring 2018

$$a_{\mu^+}^{\mathrm{E989}} = 116\,592\,040(54) \cdot 10^{-11} \quad [0.46\,\mathrm{ppm}]$$

B. Abi et al. [Muon g-2 Coll.], PRL 126, 120801 (2021)

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It confirms the value obtained almost 20 years ago by the BNL-E821 experiment with a comparable precision

$$a_{\mu}^{\mathrm{E821}} = 116\,592\,089(63) \cdot 10^{-11} \quad [0.54\,\mathrm{ppm}]$$

G. W. Bennett et al. [Muon g-2 Coll.], PRD 73, 072003 (2006)

INTRODUCTION

The world-average value

$$a_\mu^{\mathrm{exp;WA}} \quad = \quad 116\,592\,061(41)\cdot 10^{-11} \;\; [0.35\,\mathrm{ppm}]$$

when compared to the SM predicted value

$$a_\mu^{\mathrm{SM}} \quad = \quad 116\,591\,810(43)\cdot 10^{-11} \;\; [0.35\,\mathrm{ppm}]$$

leads to a discrepancy at the level of 4.2σ

$$a_\mu^{\mathrm{exp;WA}} - a_\mu^{\mathrm{SM}} \quad = \quad 251(59)\cdot 10^{-11} \;\; [4.2\sigma]$$

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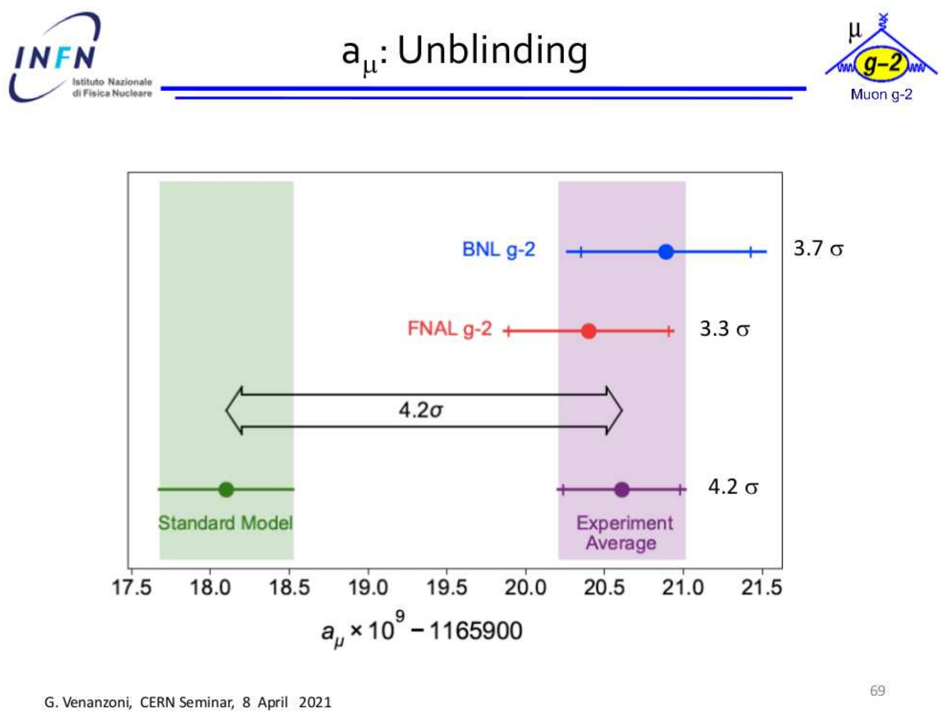

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- How is $a_\mu^{\rm exp}$ measured?
- How is $a_\mu^{\rm SM}$ computed to the required precision?
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- How is a_μ^{exp} measured?

$\longrightarrow$ will be addressed in this talk

- How is a_μ^{SM} computed to the required precision?

$\longrightarrow$ main topic of this talk

- If the discrepancy is real, what explains it?

$\longrightarrow$ hundreds of papers on arXiv since April 7, 2021

OUTLINE

- Brief overview of the main experimental aspects

- Theory aspects: a_μ in the SM

- Conclusion and a look into the (near) future

Experimental aspects

Recall: $\tau_\mu = 2.1969811(22) \cdot 10^{-6}\, s$

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Experiment	Years	Polarity	$a_\mu \times 10^{10}$	Precision [ppm]
CERN I	1961	μ^+	11 450 000(220 000)	4300
CERN II	1962-1968	μ^+	11 661 600(3100)	270
CERN III	1974-1976	μ^+	11 659 100(110)	10
CERN III	1975-1976	μ^-	11 659 360(120)	10
BNL E821	1997	μ^+	11 659 251(150)	13
BNL E821	1998	μ^+	11 659 191(59)	5
BNL E821	1999	μ^+	11 659 202(15)	1.3
BNL E821	2000	μ^+	11 659 204(9)	0.73
BNL E821	2001	μ^-	11 659 214(9)	0.72
FNAL E989	2021	μ^+	11 659 2040(54)	0.46
FNAL E989	2023?	μ^+	???	$\sim$ 0.14
[J-PARC E34	2027?	μ^+	???	$\sim$ 0.45]

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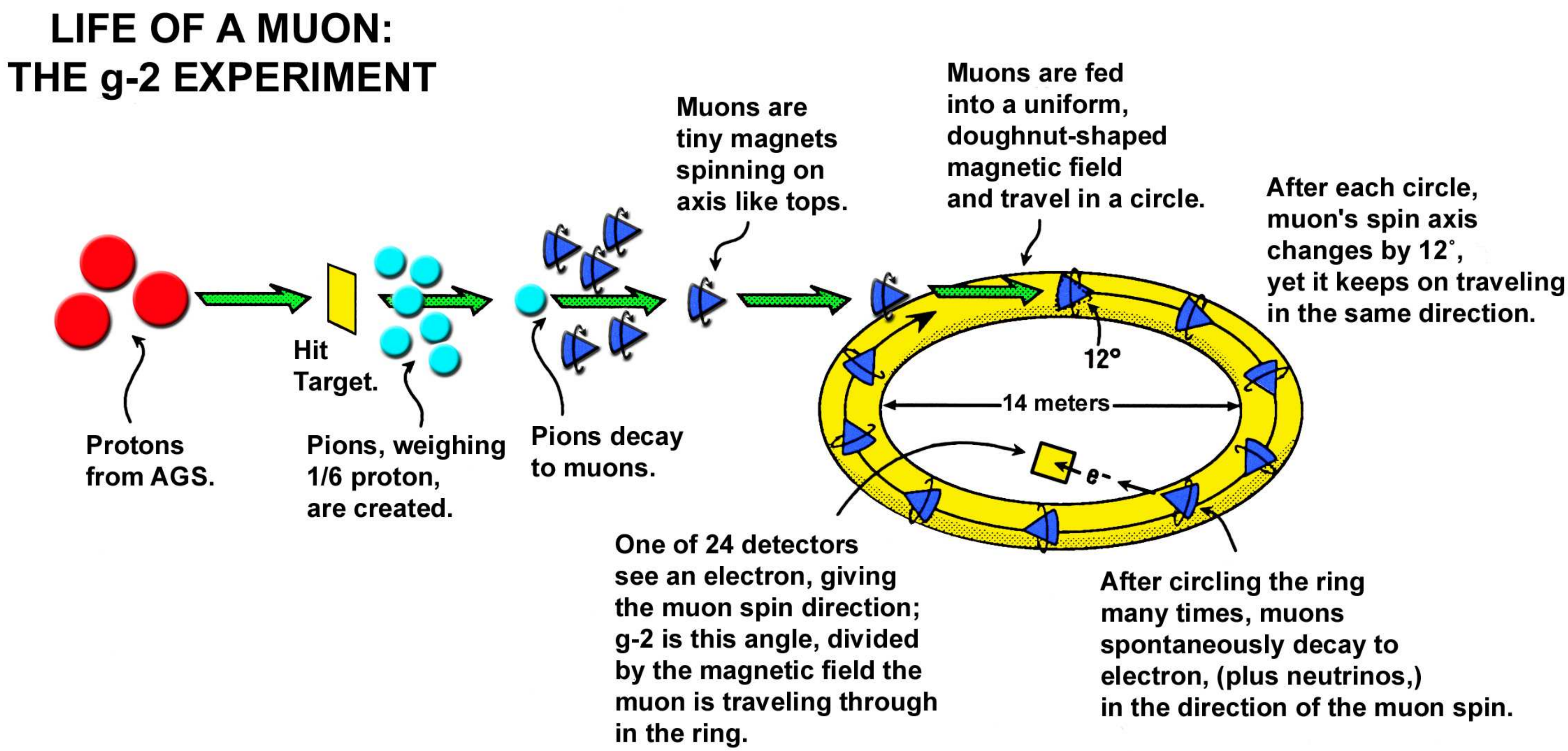

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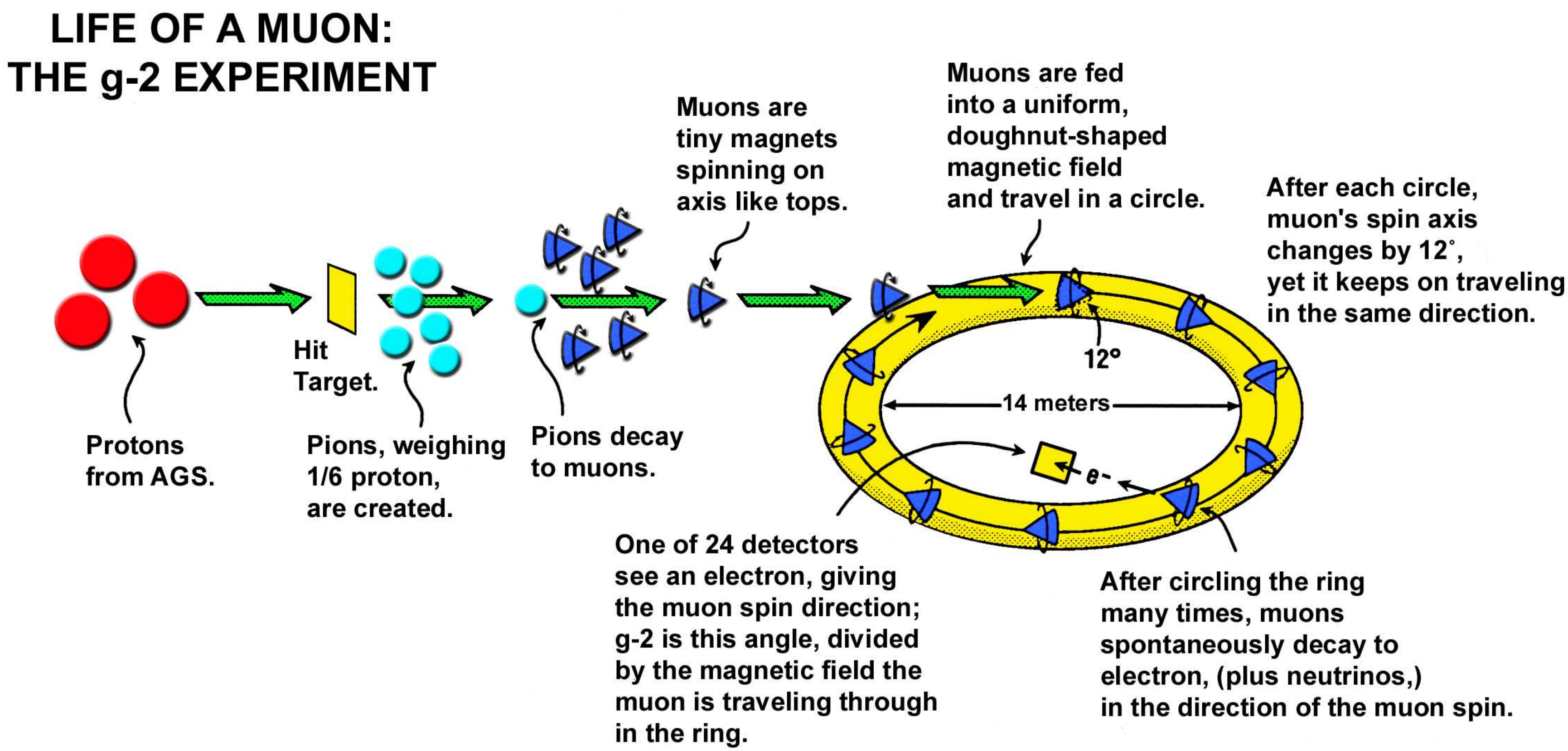

Two important features:

- the most energetic muons emitted in the decay of the pions are forwards polarized
- the most energetic positrons are emitted in the direction of the spin of the decaying μ^+

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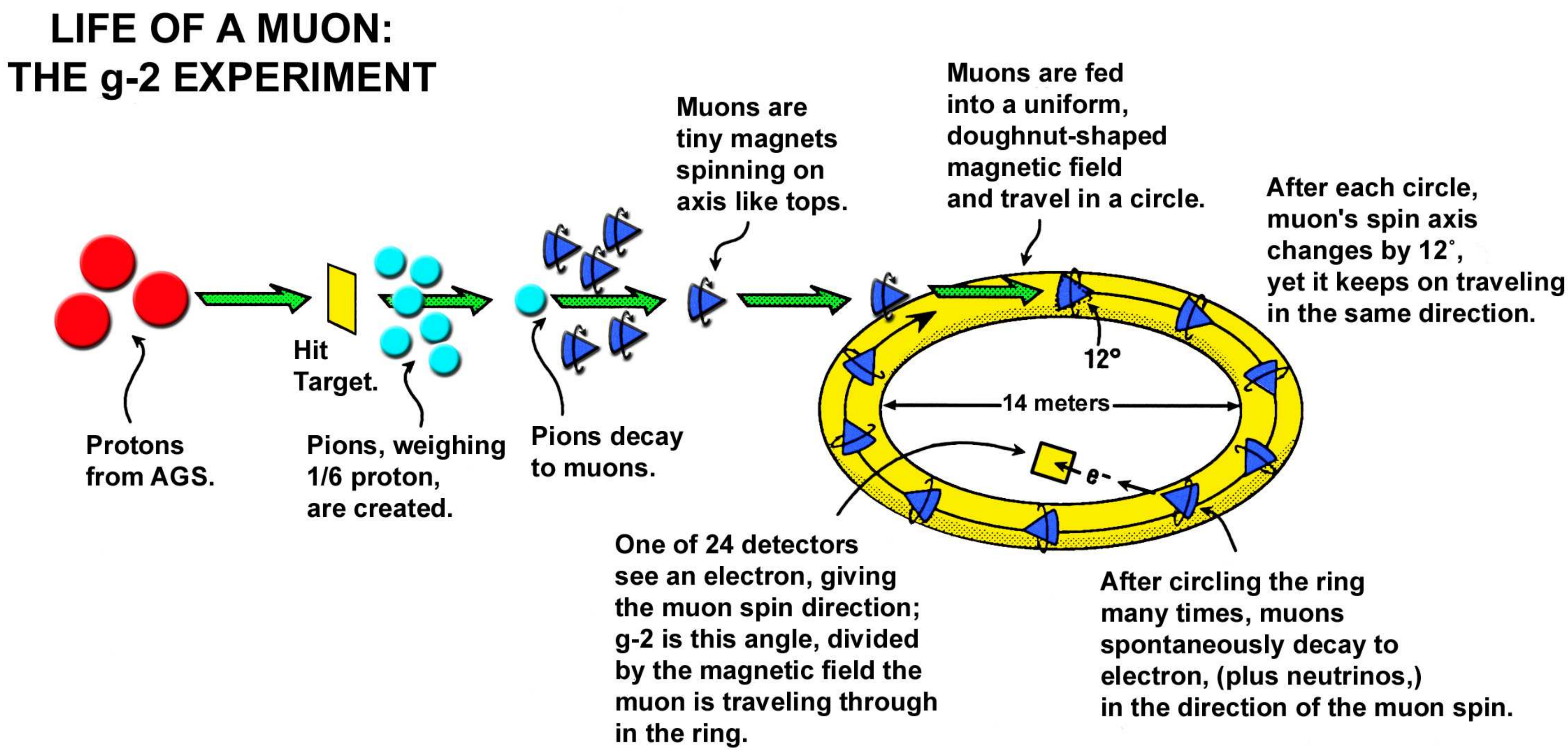


Two important features:

- the most energetic muons emitted in the decay of the pions are forwards polarized

- the most energetic positrons are emitted in the direction of the spin of the decaying μ^+

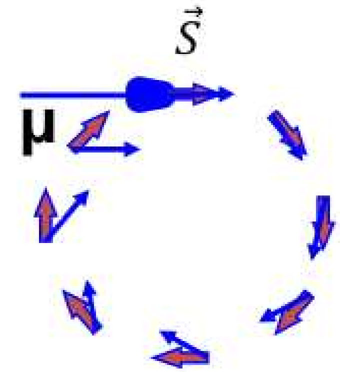
these are experimental facts, no need to assume SM valid!

Key ingredients

1) Polarized muons

~95% polarized for forward decay

2) Precession proportional to (g-2)



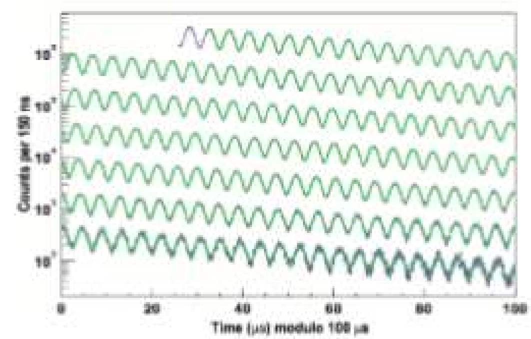
$$\omega_a = \omega_{spin} - \omega_{cyclotron} = \left(\frac{g-2}{2}\right)\frac{eB}{mc}$$

$a_\mu=(g-2)/2$

3) P_μ magic momentum = 3.09 GeV/c

$$\vec{\omega}_a = \frac{e}{mc}\left[a_\mu \vec{B} - \left(a_\mu - \frac{1}{\gamma^2 - 1}\right)\vec{\beta}\times\vec{E}\right]$$

4) Decay e^+ emitted preferably in spin direction of the muon



$$\mu^+ \rightarrow e^+ \nu_e \bar{\nu}_\mu$$

- muon storage ring experiment $[\vec{\beta}\cdot\vec{E} = \vec{\beta}\cdot\vec{B} = 0]$

$$\vec{\omega}_s - \vec{\omega}_c = -\frac{e}{m_\mu c}[a_\mu \vec{B} - (a_\mu - \frac{1}{\gamma^2 - 1})\vec{\beta}\times\vec{E}] - 2\,\vec{d}_\mu \cdot [\vec{\beta}\times\vec{B} + \vec{E}]$$

- $\gamma \sim 29.3$ [electrostatic focusing will not affect the spin]

Muon g-2 Coll., H. N. Brown et al., Phys. Rev. Lett. 86, 2227 (2001)

- $|d_\mu| < 1.9 \cdot 10^{-19}$ [95% CL] Muon g-2 Coll., G. W. Bennett et al., Phys. Rev. D 80 (2009)

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$$\vec{\omega}_s - \vec{\omega}_c = -\frac{e}{m_\mu c}[a_\mu \vec{B} - (a_\mu - \frac{1}{\gamma^2 - 1})\vec{\beta}\times\vec{E}] - 2\,\vec{d}_\mu \cdot [\vec{\beta}\times\vec{B} + \vec{E}]$$

- Still need to measure the magnetic field $\longrightarrow$ NMR probes $\longrightarrow \omega_{\rm p} = -g_{\rm p}\frac{e}{2m_{\rm p}}B$

$$a_\mu = \frac{g_{\rm p}}{2}\frac{\omega_a}{\omega_{\rm p}}\frac{m_\mu}{m_{\rm p}} = \frac{g_e}{2}\frac{\omega_a}{\omega_{\rm p}}\frac{m_\mu}{m_{\rm p}}\frac{\mu_{\rm p}}{\mu_e}$$

$$\frac{\Delta g_e}{g_e} = 0.26\text{ppt}, \quad \frac{\Delta(m_\mu/m_e)}{m_\mu/m_e} = 22\text{ppb}, \quad \frac{\Delta\mu_{\rm p}/\mu_e}{\mu_{\rm p}/\mu_e} = 3\text{ppb}, \quad \frac{\Delta\omega_{\rm p}}{\omega_{\rm p}} = 70\text{ppb}$$

Theory aspects

One wants to probe the response of a charged lepton to an external (and static) electromagnetic field

$$\langle \ell; p' | \mathcal{J}_\rho(0) | \ell; p \rangle \equiv \bar{\mathrm{u}}(p')\Gamma_\rho(p', p)\mathrm{u}(p)$$

$$= \bar{\mathrm{u}}(p')\left[F_1(k^2)\gamma_\rho + \frac{i}{2m_\ell}F_2(k^2)\sigma_{\rho\nu}k^\nu - F_3(k^2)\gamma_5\sigma_{\rho\nu}k^\nu + F_4(k^2)(k^2\gamma_\rho - 2m_\ell k_\rho)\gamma_5\right]\mathrm{u}(p)$$

(uses only the conservation of the electromanetic current $\mathcal{J}_\rho$, $k_\mu \equiv p'_\mu - p_\mu$)

$F_1(k^2) \rightarrow$ Dirac form factor , $F_1(0) = 1$

$F_2(k^2) \rightarrow$ Pauli form factor $\rightarrow F_2(0) = a_\ell$

$F_3(k^2) \rightarrow \not{P}, \not{T}$, electric dipole moment $\rightarrow F_3(0) = d_\ell / q_\ell$

$F_4(k^2) \rightarrow \not{P}$, anapole moment

$$G_E(k^2) = F_1(k^2) + \frac{k^2}{4m_\ell^2}F_2(k^2),\ G_M(k^2) = F_1(k^2) + F_2(k^2)$$

in the SM, $F_2(k^2)$, $F_3(k^2)$, $F_4(k^2)$ are only induced by loops $\longrightarrow$ calculable!

[tree-level contributions would correspond to terms with dim > 4]

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$$\boldsymbol{\mu_\ell} = g_\ell\left(\frac{q_\ell}{2m_\ell c}\right)\mathbf{S}, \quad \mathbf{S} = \hbar\,\frac{\boldsymbol{\sigma}}{2}$$

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The *anomalous* magnetic moment is induced at loop level

$$a_\ell \equiv \frac{g_\ell - g_\ell^{\rm Dirac}}{g_\ell^{\rm Dirac}} \ (\equiv F_2(0))$$

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a_ℓ probes all the degrees of freedom of the standard model, *and possibly beyond...*

Considering SM contributions only, one has, by order of importance

$$a_\mu = a_\mu^{\text{QED}} + a_\mu^{\text{had}} + a_\mu^{\text{weak}}$$

a_μ^{QED}: loops with only photons and leptons

a_μ^{had}: loops with photons and leptons and at least one quark loop dressed with gluons

a_μ^{weak}: loops with also contributions from the electroweak sector

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For a full and detailed account [up to June 15, 2020], see the White Paper

T. Aoyama et al., Phys. Rep. 887, 1 - 166 (2020)

Theory I: QED

QED contributions : loops with only photons and leptons

$\longrightarrow$ can be computed in perturbation theory:

$$a_\ell^{\text{QED}} = C_\ell^{(2)} \left(\frac{\alpha}{\pi}\right) + C_\ell^{(4)} \left(\frac{\alpha}{\pi}\right)^2 + C_\ell^{(6)} \left(\frac{\alpha}{\pi}\right)^3 + C_\ell^{(8)} \left(\frac{\alpha}{\pi}\right)^4 + C_\ell^{(10)} \left(\frac{\alpha}{\pi}\right)^5 + \ldots$$

$$C_\ell^{(2n)} = A_1^{(2n)} + A_2^{(2n)}(m_\ell/m_{\ell'}) + A_3^{(2n)}(m_\ell/m_{\ell'}, m_\ell/m_{\ell''})$$

$A_1^{(2n)} \longrightarrow$ mass-independent (universal) contributions (one-flavour QED)

$A_2^{(2n)}(m_\ell/m_{\ell'}),\ A_3^{(2n)}(m_\ell/m_{\ell'}, m_\ell/m_{\ell''}) \longrightarrow$

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– a_ℓ is finite (no renormalization needed) and dimensionless

– QED is decoupling

— Massive fermions with $m_{\ell'} \gg m_\ell$ contribute to a_ℓ through powers of $m_\ell^2/m_{\ell'}^2$, times logarithms (*)

— Light degrees of freedom with $m_{\ell'} \ll m_\ell$ give logarithmic contributions to a_ℓ, e.g. $\ln(m_\ell^2/m_{\ell'}^2)$ $\left(\pi^2 \ln \frac{m_\mu}{m_e} \sim 50\right)$

(*) also applies to BSM physics to the extent that it is decoupling (and preserves LFU)!

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$\longrightarrow$ for a_μ $A_2^{(2n)}(m_\ell/m_{\ell'})$ and $A_3^{(2n)}(m_\ell/m_{\ell'}, m_\ell/m_{\ell''})$ matter (for $m_{\ell'} = m_e$)

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Expressions for $A_1^{(2)}$, $A_1^{(4)}$, $A_2^{(4)}$, $A_1^{(6)}$, $A_2^{(6)}$, $A_3^{(6)}$ known analytically

J. Schwinger, Phys. Rev. 73, 416L (1948)

C. M. Sommerfield, Phys. Rev. 107, 328 (1957); Ann. Phys. 5, 26 (1958)

A. Petermann, Helv. Phys. Acta 30, 407 (1957)

H. Suura and E. Wichmann, Phys. Rev. 105, 1930 (1955)

A. Petermann, Phys. Rev. 105, 1931 (1955)

H. H. Elend, Phys. Lett. 20, 682 (1966); Err. Ibid. 21, 720 (1966)

M. Passera, Phys. Rev. D 75, 013002 (2007)

S. Laporta, E. Remiddi, Phys. Lett. B265, 182 (1991); B356, 390 (1995); B379, 283 (1996)

S. Laporta, Phys. Rev. D 47, 4793 (1993); Phys. Lett. B343, 421 (1995)

$\longrightarrow$ no uncertainties in $A_1^{(2)}$, $A_1^{(4)}$, $A_1^{(6)}$

$\longrightarrow$ precision of $A_2^{(4)}$, $A_2^{(6)}$, $A_3^{(6)}$ only limited by precision in $m_\ell/m_{\ell'}$

order $(\alpha/\pi)^4$: 891 diagrams

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$A_1^{(8)}$ has also been evaluated! (a_e)

S. Laporta, Phys. Lett. B 772, 232 (2017)

$$A_1^{(8)} = -1.912\,245\,764\,926\,445\,574\,152\,647\,167\,439\,830\,054\,060\,873\,390\,658\,725\,345\,171\,329\ldots$$

Good agreement with earlier numerical evaluations

$$A_1^{(8)} = -1.912\,98(84)$$

T. Aoyama et al., Phys. Rev. D 91, 033006 (2015)

Mass-dependent contributions (a_μ)

only a few diagrams are known analytically $\longrightarrow$ numerical evaluation

Automated generation of diagrams, systematic numerical evaluation of multi-dimensional integrals over Feynman-parameter space

T. Kinoshita and M. Nio, Phys. Rev. D 73, 053007 (2006); T. Aoyama et al., Phys. Rev. D 91, 033006 (2015)

$$A_2^{(8)}(m_e/m_\mu) = 9.161\,970\,703(373) \cdot 10^{-4} \quad A_2^{(8)}(m_e/m_\tau) = 7.429\,24(118) \cdot 10^{-6}$$

$$A_3^{(8)}(m_e/m_\mu, m_e/m_\tau) = 7.468\,7(28) \cdot 10^{-7}$$

$$A_2^{(8)}(m_\mu/m_e) = 132.685\,2(60) \quad A_2^{(8)}(m_\mu/m_\tau) = 0.042\,34(12)$$

$$A_3^{(8)}(m_\mu/m_e, m_\mu/m_\tau) = 0.062\,72(4)$$

Independent check of mass-dependent contributions

A. Kataev, Phys. Rev. D 86, 013019 (2012)

A. Kurz et al., Nucl. Phys. B 879, 1 (2014); Phys. Rev. D 92, 073019 (2015)

Agreement at the level of accuracy required by present and future experiments for a_μ

order $(\alpha/\pi)^5$: 12 672 diagrams...

6 classes, 32 gauge invariant subsets

Five of these subsets are known analytically

S. Laporta, Phys. Lett. B 328, 522 (1994)

J.-P. Aguilar, D. Greynat, E. de Rafael, Phys. Rev. D 77, 093010 (2008)

Complete numerical results have been published

T. Kinoshita and M. Nio, Phys. Rev. D 73, 053007 (2006); T. Aoyama et al., Phys. Rev. D 78, 053005 (2008); D 78, 113006 (2008); D 81, 053009 (2010); D 82, 113004 (2010); D 83, 053002 (2011); D 83, 053003 (2011); D 84, 053003 (2011); D 85, 033007 (2012); Phys. Rev. Lett. 109, 111807 (2012); Phys. Rev. Lett. 109, 111808 (2012)

No systematic cross-checks even for mass-dependent contributions

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An independent numerical evaluation of $A_1^{(10)}$ (a_e) is in progress

S. Volkov, Phys. Rev. D 98, 076018 (2018); arXiv:1905.08007; Phys.Rev.D 100, 096004 (2019)

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$\longrightarrow$ discrepancy [4.8σ] found in the contribution of graphs without fermion loops

$\longrightarrow$ semi-analytical evaluation by S. Laporta?

QED contributions : loops with only photons and leptons

$\longrightarrow$ can be computed in perturbation theory:

$$a_\ell^{\text{QED}} = C_\ell^{(2)}\left(\frac{\alpha}{\pi}\right) + C_\ell^{(4)}\left(\frac{\alpha}{\pi}\right)^2 + C_\ell^{(6)}\left(\frac{\alpha}{\pi}\right)^3 + C_\ell^{(8)}\left(\frac{\alpha}{\pi}\right)^4 + C_\ell^{(10)}\left(\frac{\alpha}{\pi}\right)^5$$

	$\ell = e$	$\ell = \mu$
$C_\ell^{(2)}$	0.5	0.5
$C_\ell^{(4)}$	$-0.328\,478\,444\,00\ldots$	$0.765\,857\,425(17)$
$C_\ell^{(6)}$	$1.181\,234\,017\ldots$	$24.050\,509\,96(32)$
$C_\ell^{(8)}$	$-1.911\,321\,390\ldots$	$130.878\,0(61)$
$C_\ell^{(10)}$	$6.733(159)$	$750.72(93)$

n	1	2	3	4	5
$(\alpha/\pi)^n$	$2.32\ldots\cdot 10^{-3}$	$5.39\ldots\cdot 10^{-6}$	$1.25\ldots\cdot 10^{-8}$	$2.91\ldots\cdot 10^{-11}$	$6.76\ldots\cdot 10^{-14}$

$$\Delta C_e^{(10)} \cdot (\alpha/\pi)^5 \sim 0.15 \cdot 10^{-13} \qquad \Delta a_e^{\text{exp}} = 2.8 \cdot 10^{-13}$$

$[\Delta C_e^{(8)} \cdot (\alpha/\pi)^4$ was $\sim 0.2 \cdot 10^{-13}$ before Laporta's calculation$]$

A few comments about the QED contributions

- Uncertainties on the coefficients $C_\mu^{(2n)}$ not relevant for a_μ at the present (and future) level of precision

$$\Delta C_\mu^{(4)} \cdot (\alpha/\pi)^2 \sim 0.9 \cdot 10^{-13} \qquad \Delta C_\mu^{(6)} \cdot (\alpha/\pi)^3 \sim 0.04 \cdot 10^{-13}$$
$$\Delta C_\mu^{(8)} \cdot (\alpha/\pi)^4 \sim 1.8 \cdot 10^{-13} \qquad \Delta C_\mu^{(10)} \cdot (\alpha/\pi)^5 \sim 0.7 \cdot 10^{-13} \qquad \Delta a_\mu^{\text{exp}} = 41 \cdot 10^{-11}$$

- Order $\mathcal{O}(\alpha^4)$ and even order $\mathcal{O}(\alpha^5)$ relevant for a_μ at the present (and future) level of precision

$$C_\mu^{(8)} \cdot (\alpha/\pi)^4 \sim 380 \cdot 10^{-11} \qquad C_\mu^{(10)} \cdot (\alpha/\pi)^5 \sim 5 \cdot 10^{-11}$$

- Drastic increase with n in the coefficients $C_\mu^{(2n)}$ [$\pi^2 \ln(m_\mu/m_e) \sim 50$!]
- Estimate of $\mathcal{O}(\alpha^6)$ contributions with these enhancement factors

$$\delta a_\mu \sim A_2^{(6)}(m_\mu/m_e;\mathrm{LxL}) \left[\frac{2}{3}\ln\frac{m_\mu}{m_e} - \frac{5}{9}\right]^3 \cdot 10 \left(\frac{\alpha}{\pi}\right)^6 \sim 0.54 \cdot 10^4 \cdot \left(\frac{\alpha}{\pi}\right)^6 \sim 0.08 \cdot 10^{-11}$$

- No sign of substantial contribution to a_μ from higher order QED

A few comments about the QED contributions

- $a_\mu^{\mathrm{QED}}(Cs19) = 116\,584\,718.931(7)_{\mathrm{mass}}(17)_{\alpha^4}(6)_{\alpha^5}(100)_{\alpha^6}(23)_{\alpha(Cs19)} \cdot 10^{-11}$

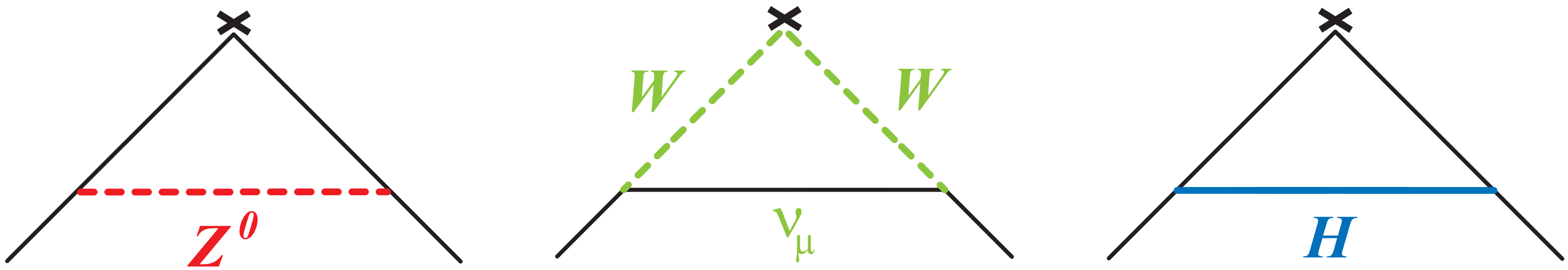
- $$a_\mu^{\mathrm{exp;WA}} - a_\mu^{\mathrm{QED}}(Cs19) = 7342(41) \cdot 10^{-11}$$

- QED provides more than 99.99% of the total value, without uncertainties at this level of experimental precision

- The missing part has to be provided by weak and strong interactions (or else, new physics...)

Theory II: weak interactions

- Weak contributions : W, Z,... loops



$$a_\mu^{\text{weak(1)}} = \frac{G_F}{\sqrt{2}}\frac{m_\mu^2}{8\pi^2}\left[\frac{5}{3}+\frac{1}{3}\left(1-4\sin^2\theta_W\right)^2+\mathcal{O}\left(\frac{m_\mu^2}{M_Z^2}\log\frac{M_Z^2}{m_\mu^2}\right)+\mathcal{O}\left(\frac{m_\mu^2}{M_H^2}\log\frac{M_H^2}{m_\mu^2}\right)\right]$$
$$= 194.8\cdot 10^{-11}$$

W.A. Bardeen, R. Gastmans and B.E. Lautrup, Nucl. Phys. B46, 315 (1972)

G. Altarelli, N. Cabbibo and L. Maiani, Phys. Lett. 40B, 415 (1972)

R. Jackiw and S. Weinberg, Phys. Rev. D 5, 2473 (1972)

I. Bars and M. Yoshimura, Phys. Rev. D 6, 374 (1972)

M. Fujikawa, B.W. Lee and A.I. Sanda, Phys. Rev. D 6, 2923 (1972)

Two-loop bosonic contributions

$$a_\mu^{\text{weak(2);b}} = \frac{G_F}{\sqrt{2}} \frac{m_\mu^2}{8\pi^2} \frac{\alpha}{\pi} \cdot \left[-5.96 \ln \frac{M_W^2}{m_\mu^2} + 0.19 \right] = \frac{G_F}{\sqrt{2}} \frac{m_\mu^2}{8\pi^2} \left(\frac{\alpha}{\pi} \right) \cdot (-79.3)$$

A. Czarnecki, B. Krause, W. J. Marciano, Phys. Rev. Lett. 76, 3267 (1996)

Two-loop fermionic contributions

A. Czarnecki, B. Krause, W. J. Marciano, Phys. Rev. D 52, R2619 (1995)
M. K., S. Peris, M. Perrottet, E. de Rafael, JHEP11, 003 (2002)
A. Czarnecki, W.J. Marciano, A. Vainshtein, Phys. Rev. D 67, 073006 (2003). Err.-ibid. D 73, 119901 (2006)

$$a_\mu^{\text{weak}} = (154 \pm 1) \cdot 10^{-11}$$
$$a_e^{\text{weak}} = (0.0297 \pm 0.0005) \cdot 10^{-12}$$

Updated a few years ago: $a_\mu^{\text{weak}} = (153.6 \pm 1.0) \cdot 10^{-11}$

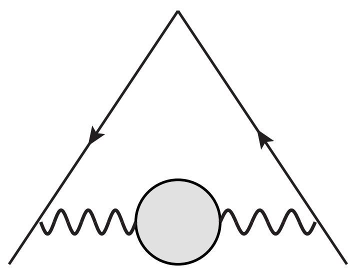
C. Gnendiger, D. Stöckinger, H. Stöckinger-Kim, Phys. Rev. D 88, 053005 (2013)

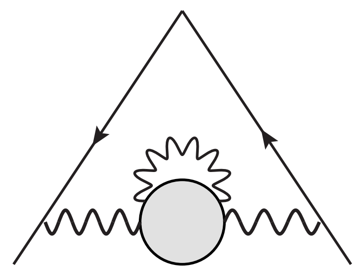
Numerical evaluation: $a_\mu^{\text{weak}} = (152.9 \pm 1.0) \cdot 10^{-11}$

T. Ishikawa, N. Nakazawa and Y. Yasui, Phys. Rev. D 99, 073004 (2019)

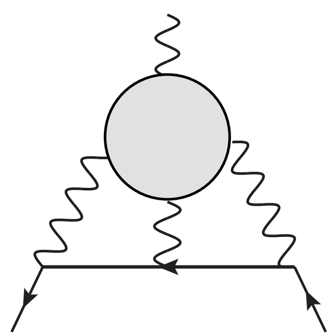
Theory III: strong interactions

- hadronic vacuum polarization





- (virtual) hadronic light-by-light (HLxL



$\longrightarrow$ non-perturbative regime of QCD

Hadronic vacuum polarization

- Occurs first at order $\mathcal{O}(\alpha^2)$

- Can be expressed as (optical theorem)

$$a_\ell^{\text{HVP-LO}} = \frac{1}{3}\left(\frac{\alpha}{\pi}\right)^2 \int_{4M_\pi^2}^{\infty} \frac{ds}{s} K(s) R^{\text{had}}(s) \quad K(s) = \int_0^1 dx \, \frac{x^2(1-x)}{x^2 + (1-x)\,\frac{s}{m_\ell^2}}$$

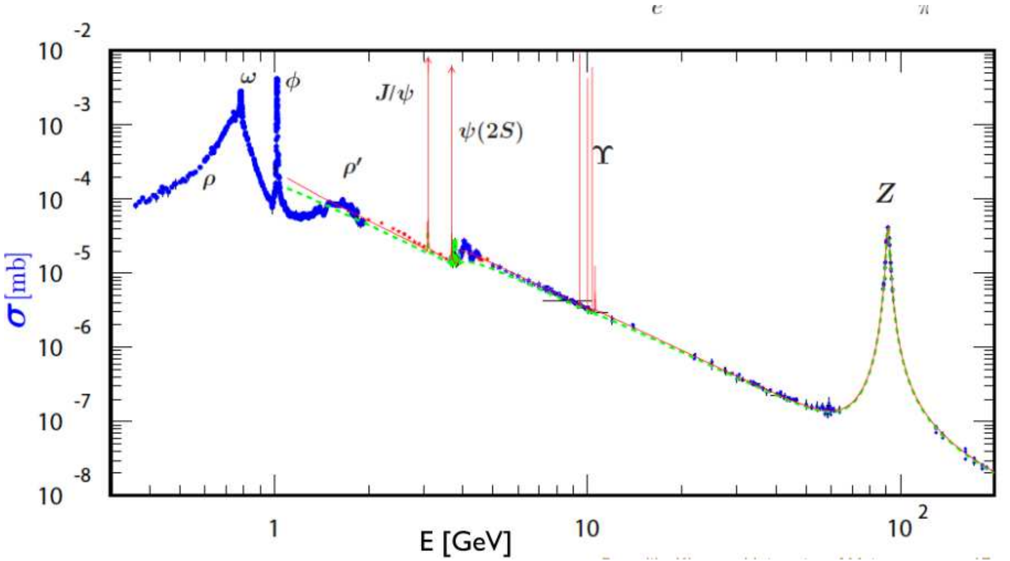
C. Bouchiat, L. Michel, J. Phys. Radium 22, 121 (1961)
L. Durand, Phys. Rev. 128, 441 (1962); Err.-ibid. 129, 2835 (1963)
M. Gourdin, E. de Rafael, Nucl. Phys. B 10, 667 (1969)

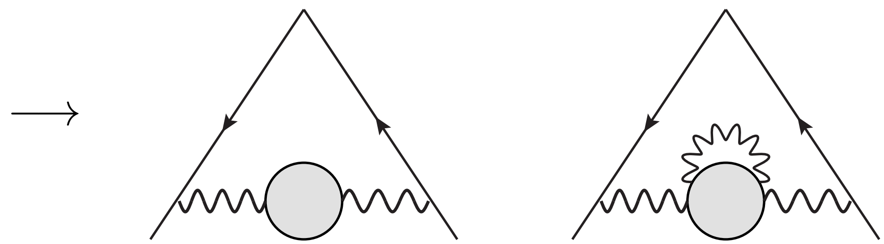
- $K(s) > 0$ and $R^{\text{had}}(s) > 0 \Longrightarrow a_\ell^{\text{HVP-LO}} > 0$

- $K(s) \sim m_\ell^2/(3s)$ as $s \to \infty \Longrightarrow$ the (non perturbative) low-energy region dominates

Hadronic vacuum polarization

- Can be evaluated using available experimental data





- Combination of ~ 39 exclusive channels

$\longrightarrow$ Scan experiments (e.g. @ VEPP)

$\longrightarrow$ ISR experiments (e.g. @ DAΦNE, B-factories, BEPC)

Hadronic vacuum polarization

$a_\mu^{\text{HVP-LO}} \cdot 10^{10}$, e^+e^-

$692.3(4.2)$	M. Davier et al., Eur. Phys. J. C 71, 1515 (2011)
$694.9(4.3)$	K. Hagiwara et al., J. Phys. G 38, 085003 (2011)
$690.75(4.72)$	F. Jegerlehner, R. Szafron, Eur. Phys. J. C 71, 1632 (2011)
$688.07(4.14)$	F. Jegerlehner, EPJ Web Conf. 166, 00022 (2018)
$693.1(3.4)$	M. Davier et al., Eur. Phys. J. C 77, 827 (2017)
$693.26(2.46)$	A. Keshavarzi et al., Phys. Rev. D 97, 114025 (2018)
$694.0(4.0)$	M. Davier et al., Eur. Phys. J. C 80, 341 (2020); Err. C 80, 410 (2020)
$692.78(2.42)$	A. Keshavarzi et al., Phys. Rev. D 101, 014029 (2020)

$a_\mu^{\text{HVP-NLO}} \cdot 10^{10}$, e^+e^-

$-9.84(7)$	K. Hagiwara et al., J. Phys. G 38, 085003 (2011)
$-9.93(7)$	F. Jegerlehner, EPJ Web Conf. 166, 00022 (2018)
$-9.82(4)$	A. Keshavarzi et al., Phys. Rev. D 97, 114025 (2018)
$-9.83(4)$	A. Keshavarzi et al., Phys. Rev. D 101, 014029 (2020)

$a_\mu^{\text{HVP-NNLO}} \cdot 10^{10}$, e^+e^-

$1.24(1)$	A. Kurz et al., Phys. Lett. B 734, 144 (2014)
$1.22(1)$	F. Jegerlehner, EPJ Web Conf. 166, 00022 (2018)

Hadronic vacuum polarization

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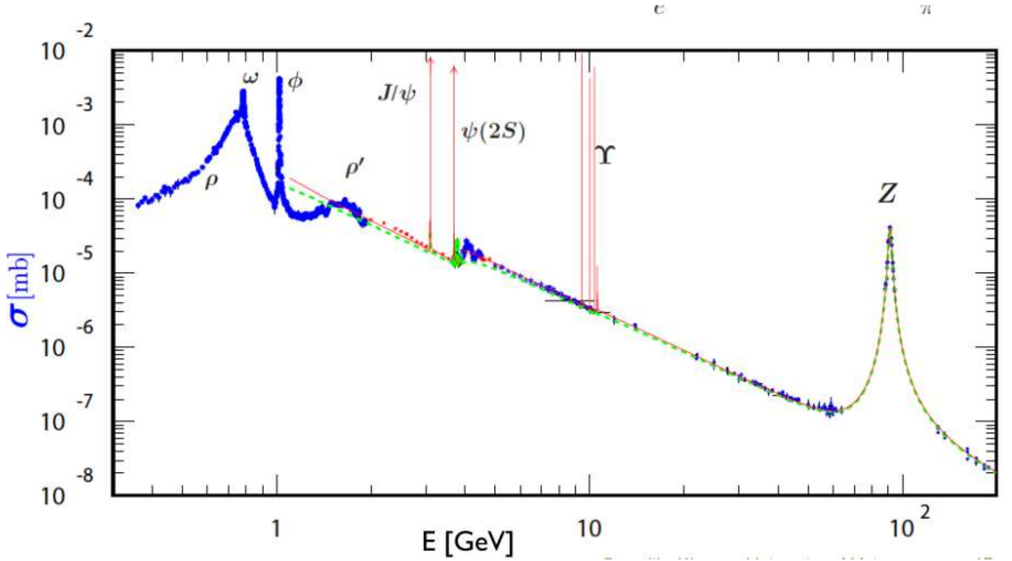
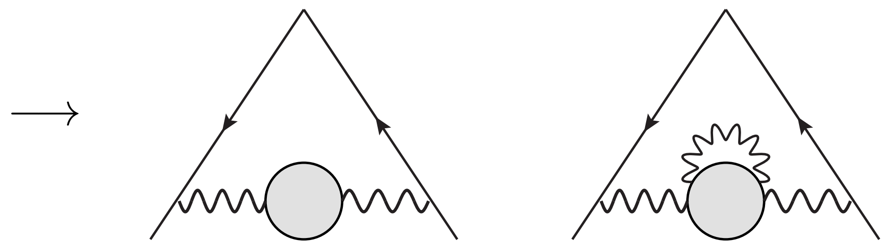




- Combination of ~ 39 exclusive channels

$\longrightarrow$ Scan experiments (e.g. @ VEPP)

$\longrightarrow$ ISR experiments (e.g. @ DAΦNE, B-factories, BEPC)

- More recently: lattice results (for the time being, stick to WP)

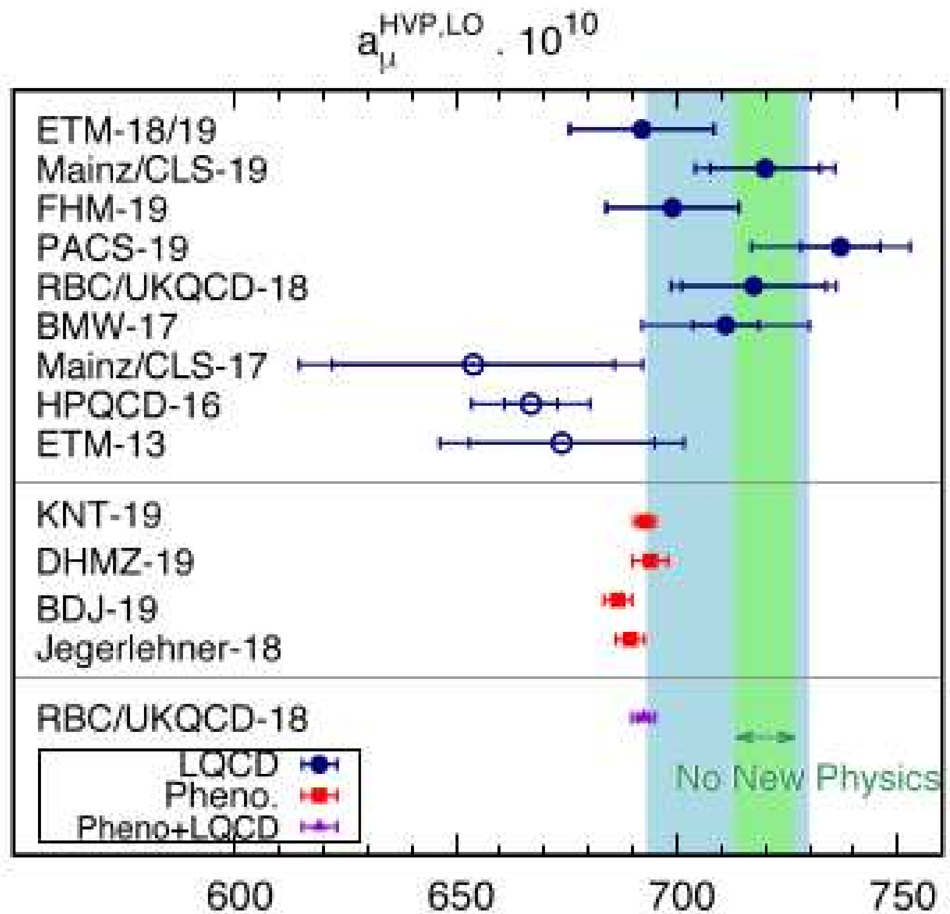
A. Gérardin *et al.*, Phys. Rev. D 100, 014510 (2019)
C. T. H. Davies *et al.*, arXiv:1902.04223 [hep-lat]
E. Shintani and Y. Kuramashi, arXiv:1902.00885 [hep-lat]
D. Giusti *et al.*, Phys. Rev. D 99, 114502 (2019)
T. Blum *et al.*, Phys. Rev. Lett. 121, 022003 (2018)
S. Borsanyi *et al.*, Phys. Rev. Lett. 121, 022002 (2018)
M. Della Morte *et al.*, JHEP 10, 020 (2017)

White Paper summary

- Data evaluation:

$$a_\mu^{\mathrm{HVP;LO}} = 6931(40)\cdot 10^{-11} \quad a_\mu^{\mathrm{HVP;NLO}} = -98.3(7)\cdot 10^{-11} \quad a_\mu^{\mathrm{HVP;LO}} = 12.4(1)\cdot 10^{-11}$$

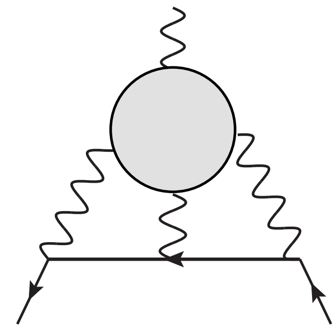
- Lattice WA: $a_\mu^{\mathrm{HVP;LO}} = 7043(150)\cdot 10^{-11}$



Hadronic light-by-light

- Occurs at order $\mathcal{O}(\alpha^3)$
- Not related, as a whole, to an experimental observable...

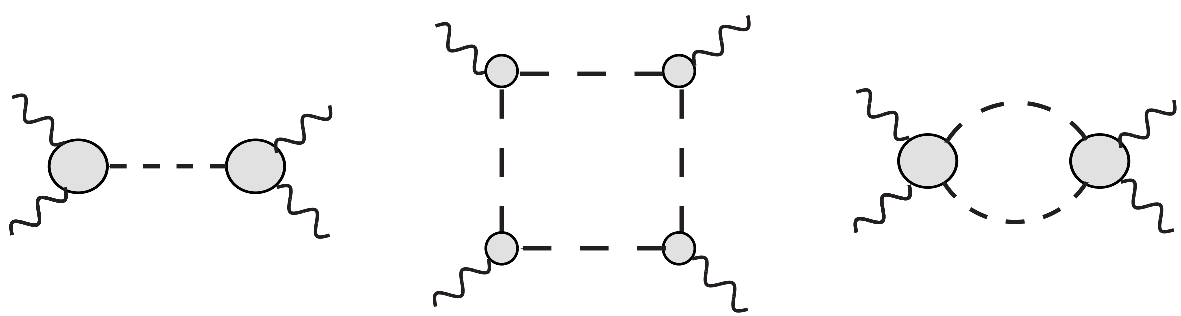
? $\longrightarrow$



- Involves the fourth-rank vacuum polarization tensor

$$\text{F.T.}\ \langle 0|T\{VVVV\}|0\rangle \longrightarrow \Pi_{\mu\nu\rho\sigma}(q_1, q_2, q_3, q_4) \quad q_1 + q_2 + q_3 + q_4 = 0$$

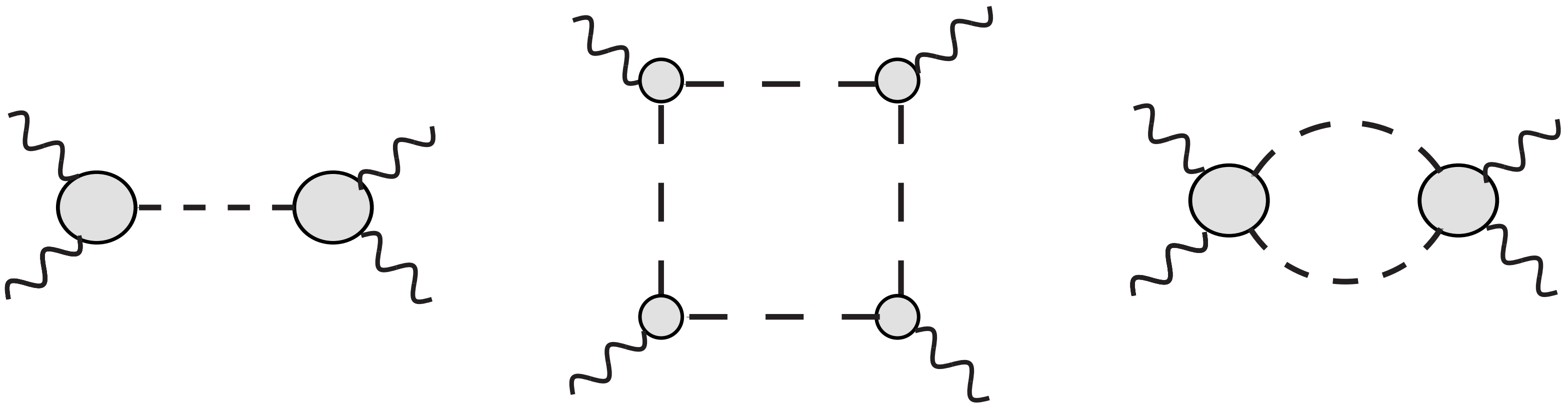
- Many individual contributions have been identified...



Hadronic light-by-light

- More recently: dispersive approaches

– $\Pi_{\mu\nu\rho\sigma} \longrightarrow$



$$\Pi = \Pi^{\pi^0,\eta,\eta' \text{ poles}} + \Pi^{\pi^\pm,K^\pm \text{ loops}} + \Pi^{\pi\pi} + \Pi^{\text{residual}}$$

G. Colangelo, M. Hoferichter, M. Procura, P. Stoffer, JHEP09, 091 (2014); JHEP09, 074 (2015)

Needs input from data (transition form factors,...)

G. Colangelo, M. Hoferichter, B. Kubis, M. Procura, P. Stoffer, Phys. Lett. B 738, 6 (2014)

A. Nyffeler, arXiv:1602.03398 [hep-ph]

Hadronic light-by-light

- Lattice QCD calculations

T. Blum et al., Phys. Rev. Lett. 124, 132002 (2020)

T. Blum et al., Phys. Rev. D 93, 014503 (2016); Phys. Rev. Lett 118, 022005 (2017)

N. Asmussen et al., arXiv:1609.08454 [hep-lat]; arXiv:1510.08384 [hep-lat]

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T. Blum et al., Phys. Rev. Lett. 124, 132002 (2020)
T. Blum et al., Phys. Rev. D 93, 014503 (2016); Phys. Rev. Lett 118, 022005 (2017)
N. Asmussen et al., arXiv:1609.08454 [hep-lat]; arXiv:1510.08384 [hep-lat]

White Paper summary

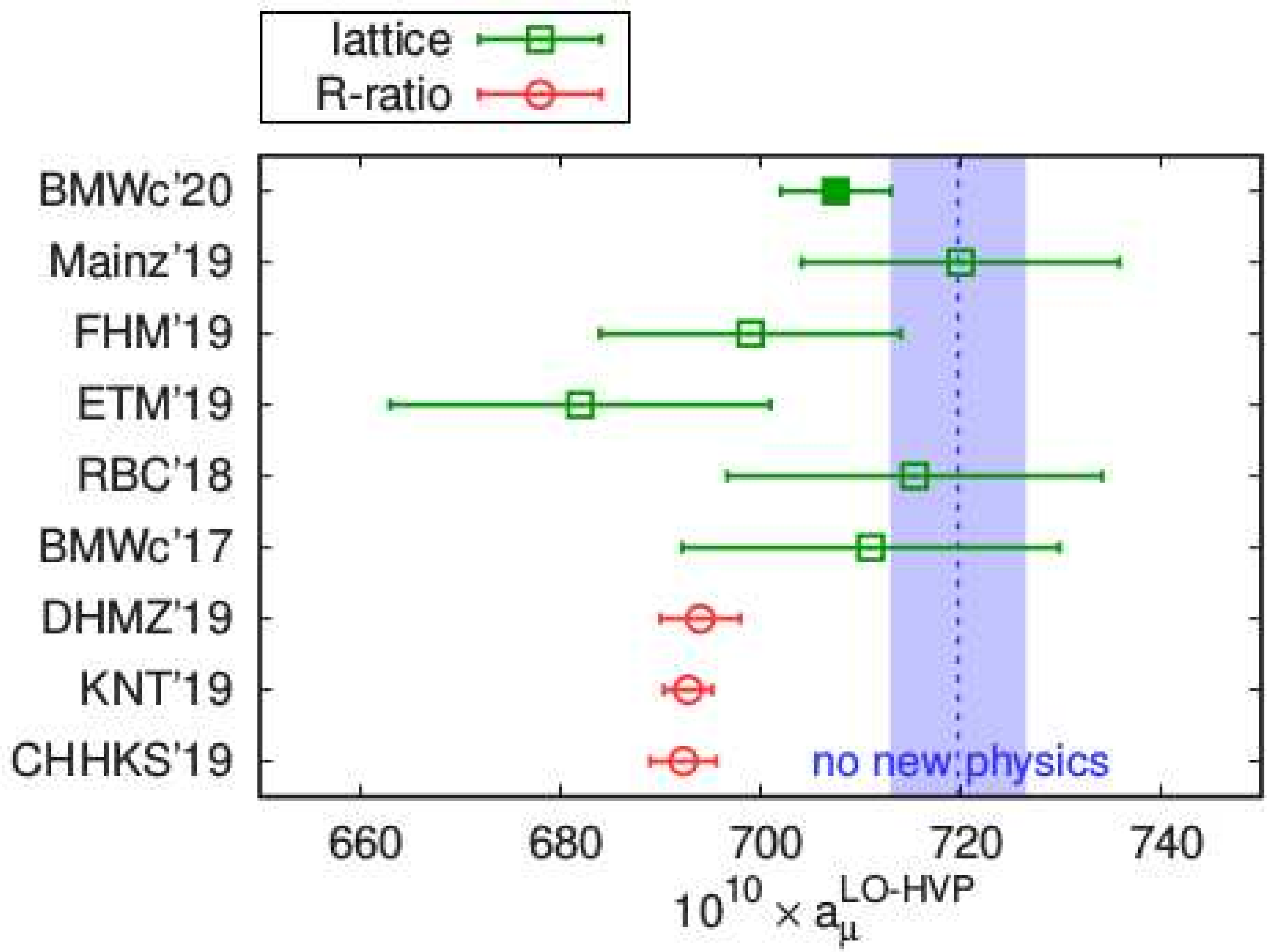
$$a_\mu^{\mathrm{HLxL}} = 92(19) \cdot 10^{-11}$$

Post-WP results

- New lattice QCD result for HVP with 0.8% accuracy

$$a_\mu^{\mathrm{HVP;LO}} = 7075(55) \cdot 10^{-11}$$

S. Borsanyi et al., Nature 593, 7857 (2021)



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Systematic effets (finite size, discretization,...) need to be scrutinized

M. Golterman, arXiv:2208.05560 [hep-ph]

Requires independent confirmation

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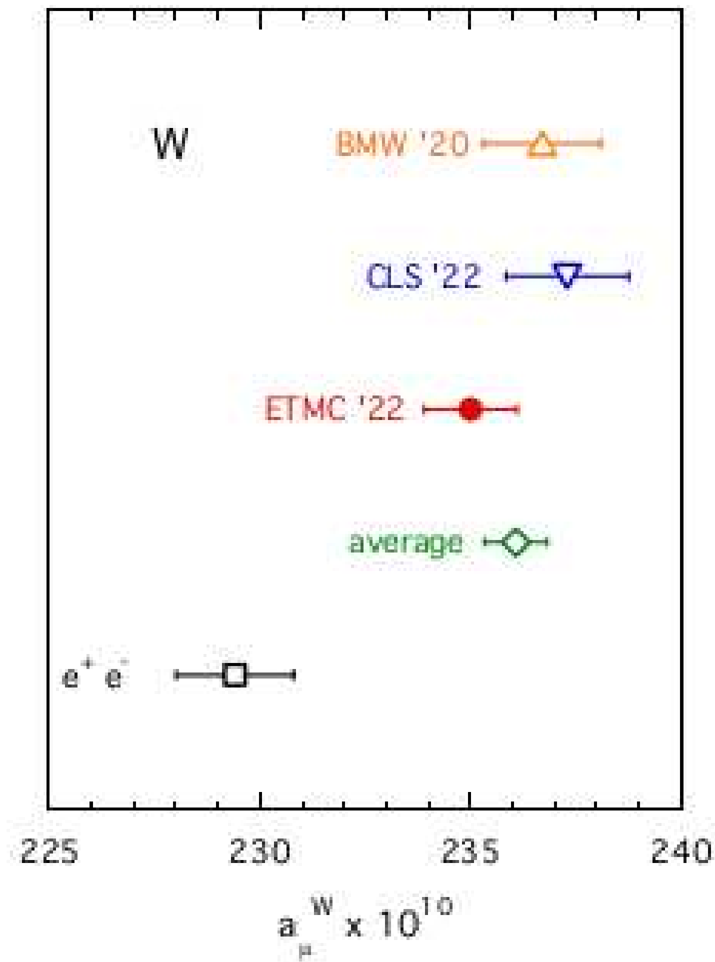
S. Borsanyi et al., Nature 593, 7857 (2021)

M. Cè et al., arXiv:2206.06582 [hep-lat]

C. Alexandrou et al., arXiv:2206.15084 [hep-lat]

a_μ^{W}: $0.4\mathrm{fm} \leq t_{\mathrm{E}} \leq 1.0\mathrm{fm}$

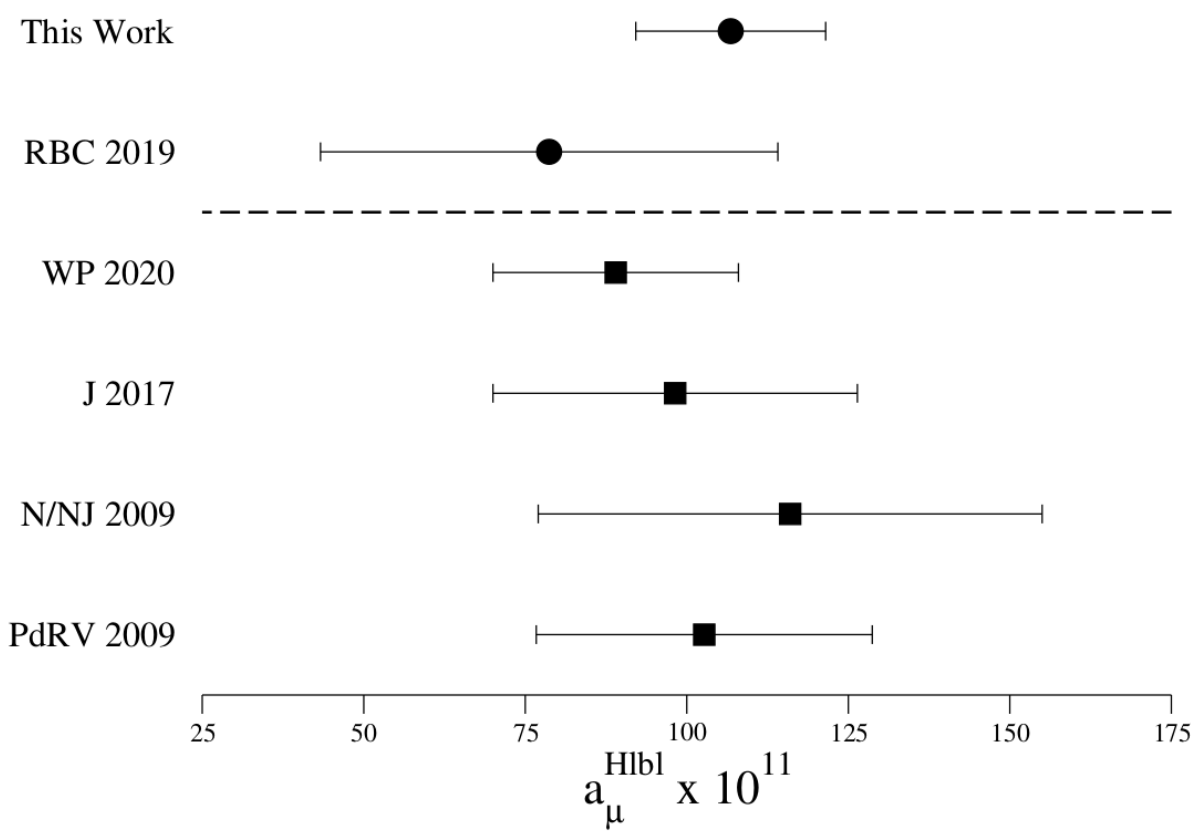


Post-WP results

- New lattice QCD result for HLxL at 15% accuracy

$$a_\mu^{\mathrm{HVP;LO}} = 107.4(11.3)(9.2) \cdot 10^{-11}$$

E.-H. Chao et al., Eur. Phys. J. C 81, 651 (2021)



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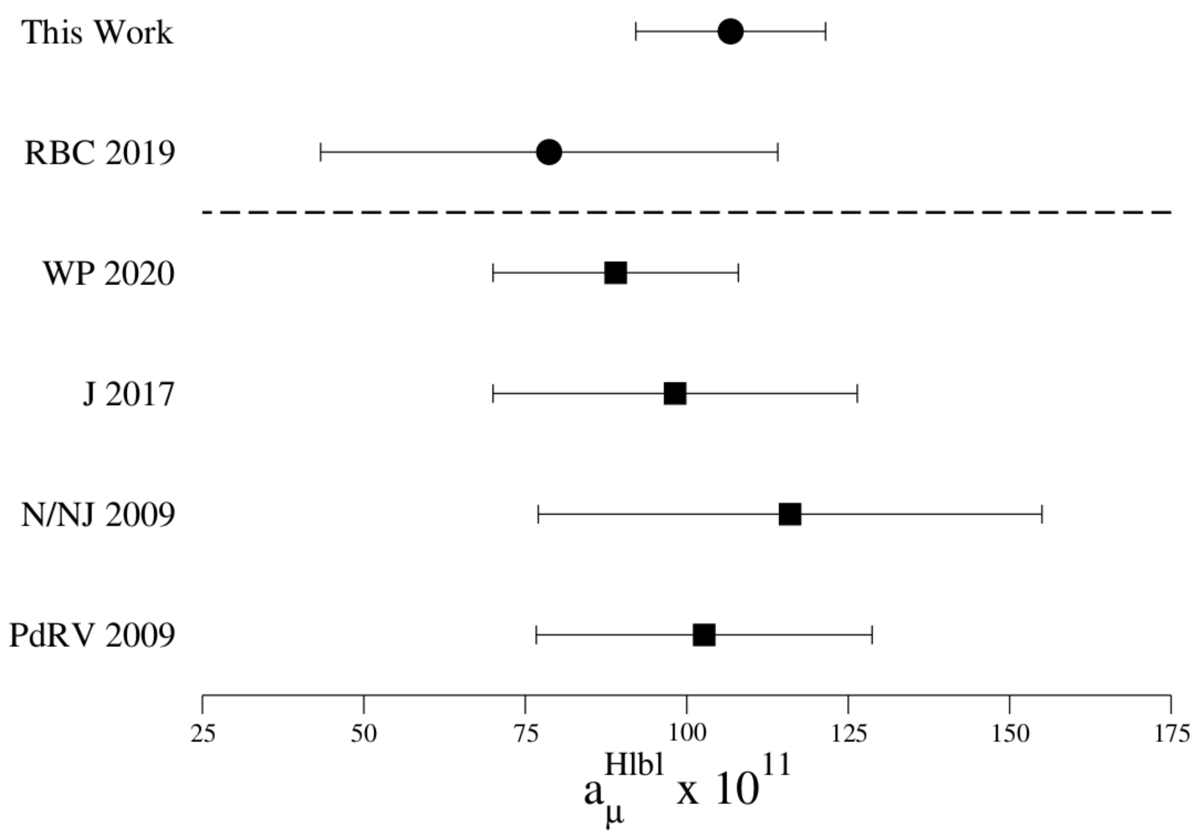

~10% accuracy goal seems within reach

Conclusion and outlook

- FNAL-E989 seems to work fine, BNL-E821 result confirmed

$$a_{\mu^+}^{\mathrm{E989}} = 116\,592\,040(54) \cdot 10^{-11} \quad [0.46\,\mathrm{ppm}]$$

B. Abi et al. [Muon g-2 Coll.], PRL 126, 120801 (2021)

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B. Abi et al. [Muon g-2 Coll.], PRL 126, 120801 (2021)

- Theory situation (as to June 2020) described in detail in the WP

$$a_{\mu}^{\text{SM}} = 116\,591\,810(43) \cdot 10^{-11} \quad [0.35\,\text{ppm}]$$

R. Aoyama et al., Phys. Rep. 887, 1 (2020)

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R. Aoyama et al., Phys. Rep. 887, 1 (2020)

- Discrepancy between the SM prediction and the world-average experimental value

$$a_\mu^{\mathrm{exp;WA}} - a_\mu^{\mathrm{SM}} = 251(59) \cdot 10^{-11} \quad [4.2\sigma]$$

- FNAL-E989 seems to work fine, BNL-E821 result confirmed

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- Discrepancy between the SM prediction and the world-average experimental value

$$a_\mu^{\rm exp;WA} - a_\mu^{\rm SM} = 251(59) \cdot 10^{-11} \;\; [4.2\sigma]$$

- No obvious explanation within the SM

$$a_\mu^{\text{exp;WA}} - a_\mu^{\text{SM}} \sim \begin{cases} a_\mu^{\text{QED}}(\alpha^4) \\ 60 \cdot a_\mu^{\text{QED}}(\alpha^5) \\ 5 \cdot a_\mu^{\text{weak(2)}} \\ 3 \cdot a_\mu^{\text{HLxL}} \end{cases}$$

• Could the problem lie in the determination of HVP using $e^+e^- \to \mathrm{hadron}$ data, as suggested by the latest result from lattice QCD?

$$a_\mu^{\mathrm{HVP;LO\ data}} = 6931(40) \cdot 10^{-11} \quad \mathrm{vs} \quad a_\mu^{\mathrm{HVP;LO\ BMWc}} = 7075(55) \cdot 10^{-11}$$

First (partial) cross-checks from other lattice collaborations available

$\longrightarrow$ so far BMWc result confirmed

No straightforward or convincing phenomenological explanation for the lattice vs. data discrepancy

- Possibility to measure HVP in the space-like region from μe scattering?

C. M. Carloni-Calame, M. Passera, L. Trentadue, G.Venanzoni, Phys. Lett. B 476, 325 (2015)

G. Abbiendi et al., Eur. Phys. J. C 77, 139 (2017)

- $a_\mu^{\mathrm{HVP}} - \frac{\alpha}{\pi} \int_0^1 dx (1-x) \Delta\alpha_{\mathrm{had}}(-\frac{x^2}{1-x} m_\mu^2)$

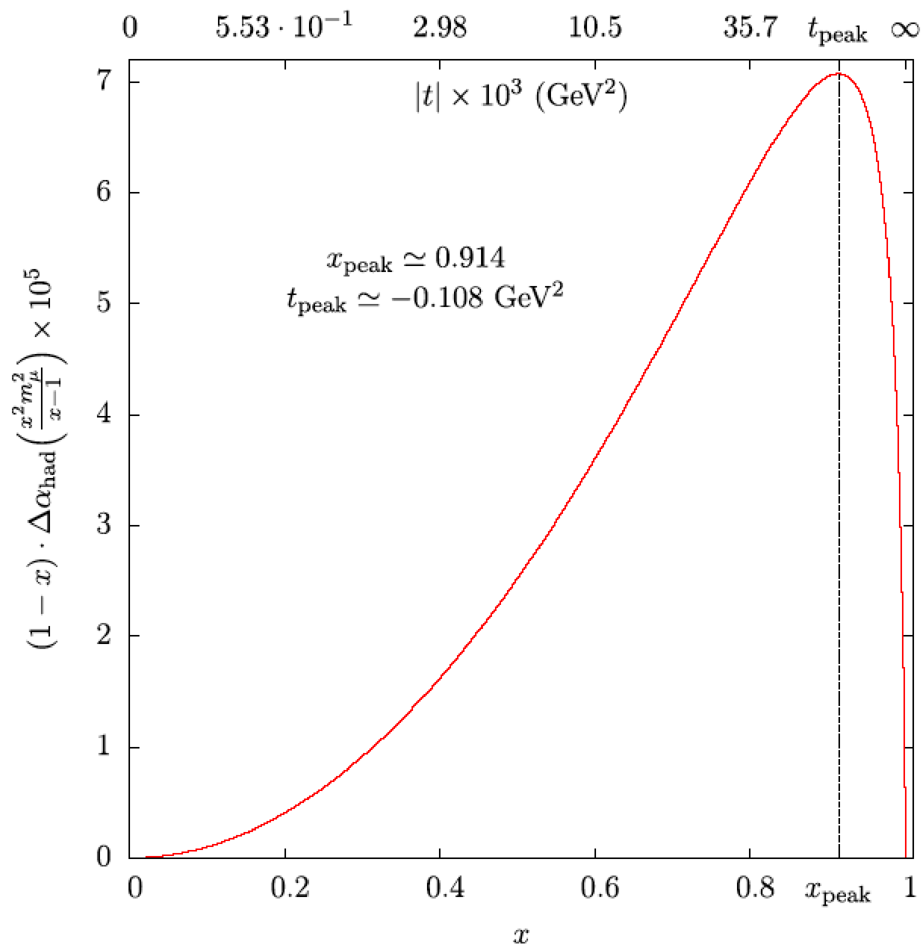
$t = \frac{x^2 m_\mu^2}{x-1} \quad 0 \le -t < +\infty \; 0 \le x < 1$

- a_μ^{HVP} given by the integral

- measurement of $\Delta\alpha_{\mathrm{had}}$ in the space-like region

- contribution at small t enhanced

- a 0.3% error can be achieved in 2y of data taking with 1.3x10^7 μ/s (CERN)



$\longrightarrow$ challenging (systematics)

$\longrightarrow$ inclusive measurement (more like lattice QCD)

$\longrightarrow$ MUonE coll. LoI CERN-LHCC-2017-009/CMS-TDR-014

$\longrightarrow$ test run (proof of concept, assessment of systematics,...) scheduled for the end of 2021

$\longrightarrow$ postponed

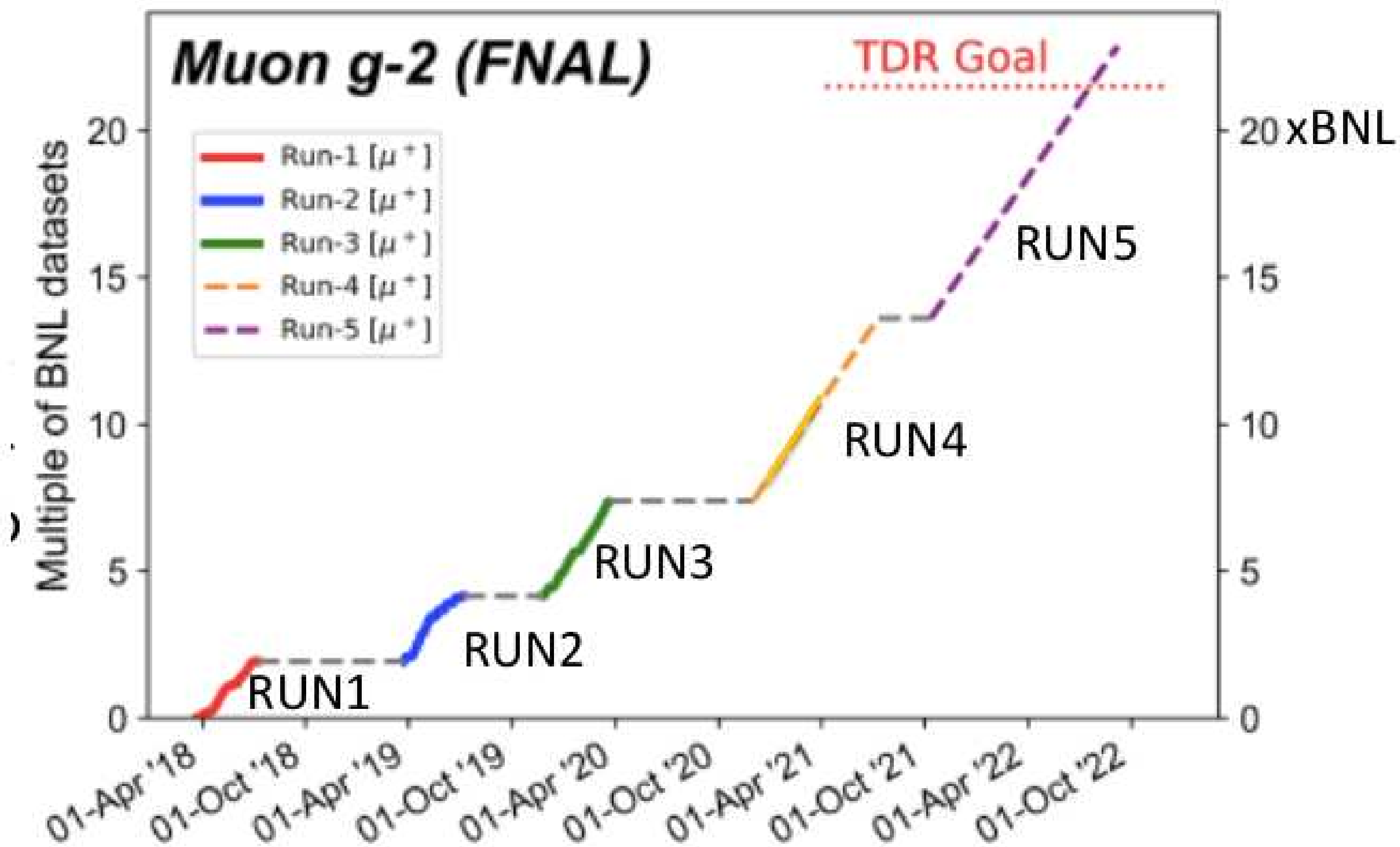
$\longrightarrow$ project starting in 202?, running time during LHC-Run3

- could the experiment be wrong?

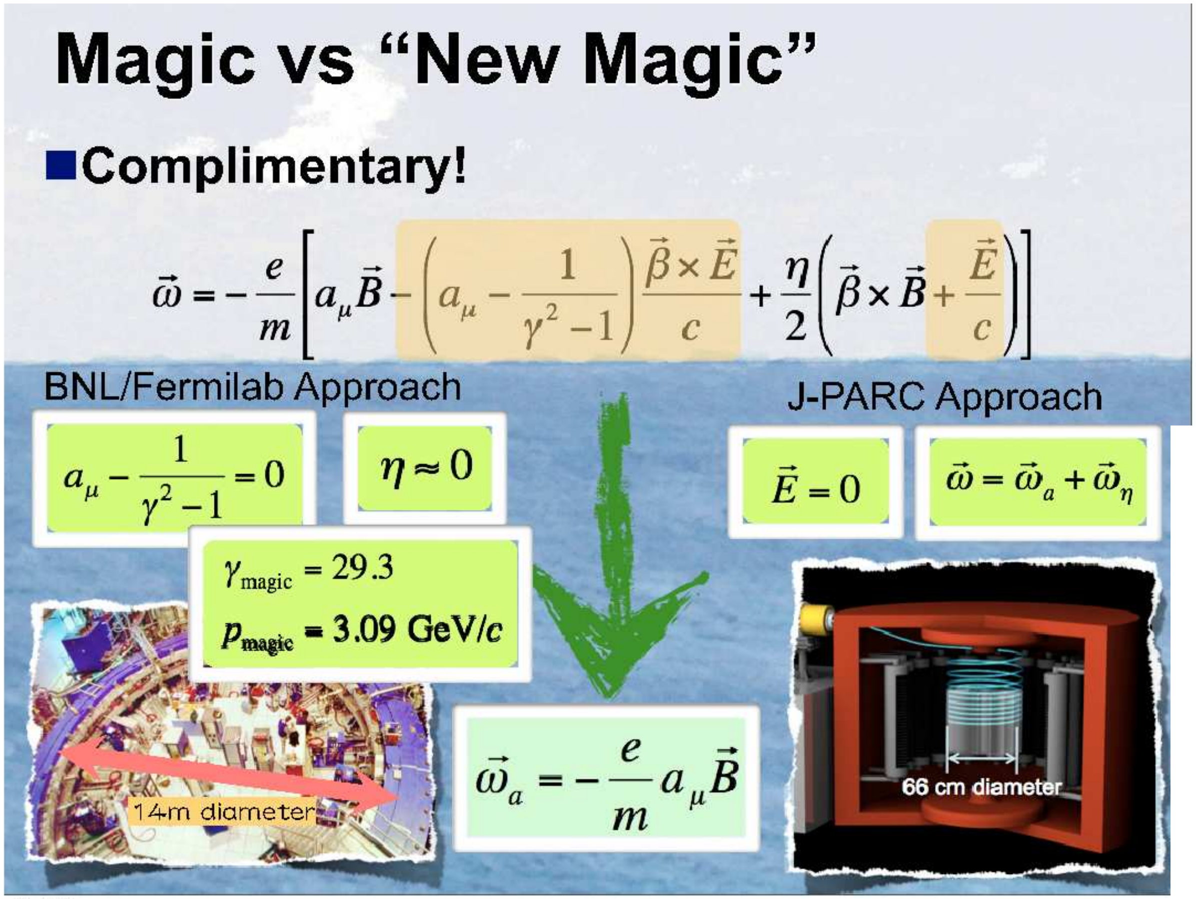
- could the experiment be wrong? We'll know more soon

⟶ only part of the data collected so far has been analysed

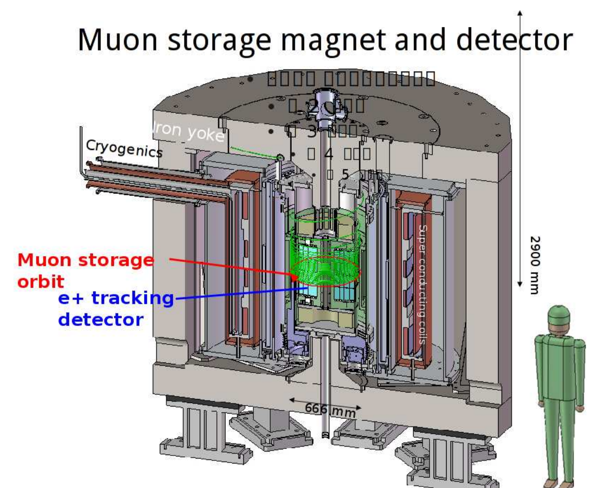
⟶ more habe been taken, to reach the accuracy goal of ~ 0.14ppm



- could the experiment be wrong? $\longrightarrow$ project to measure a_μ at J-PARC (E34)

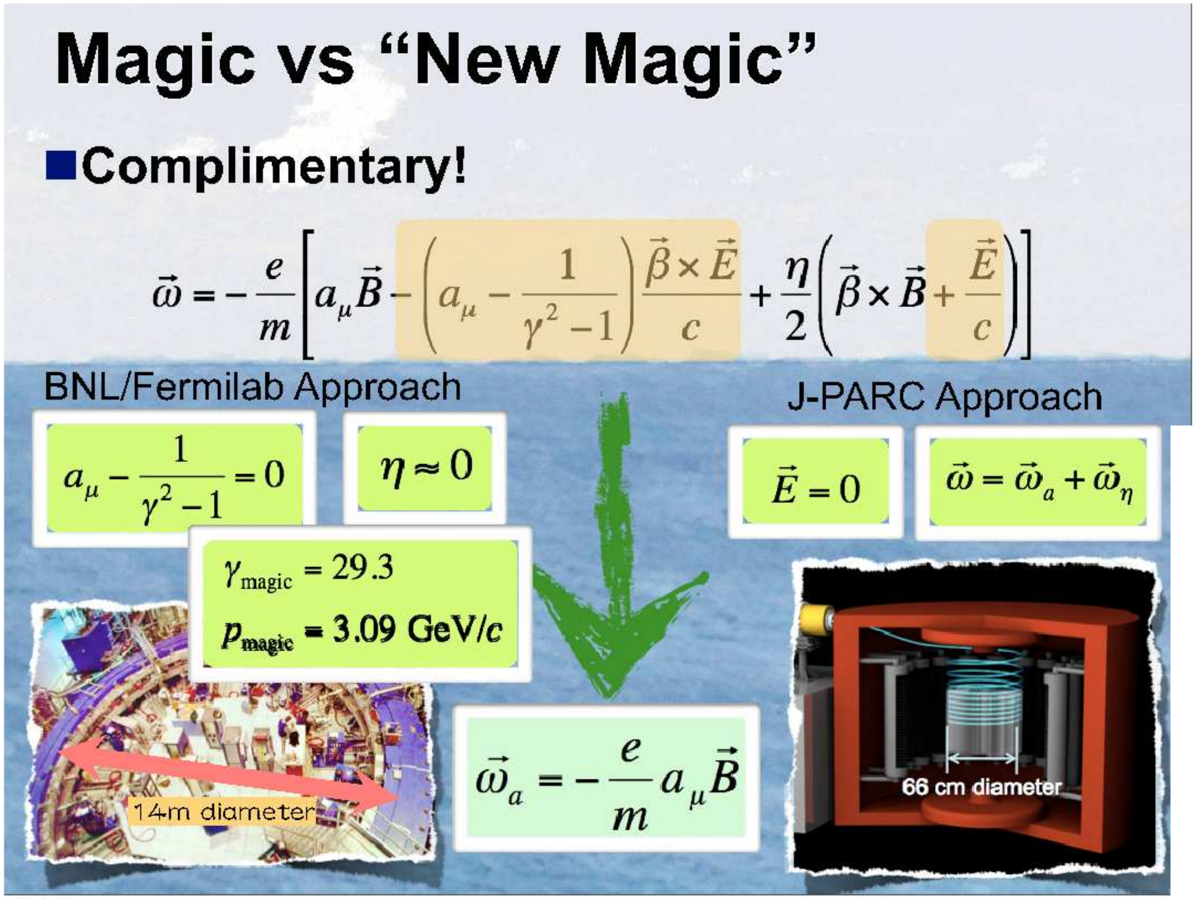


N. Saito, LPNHE Workshop Paris, May 2012

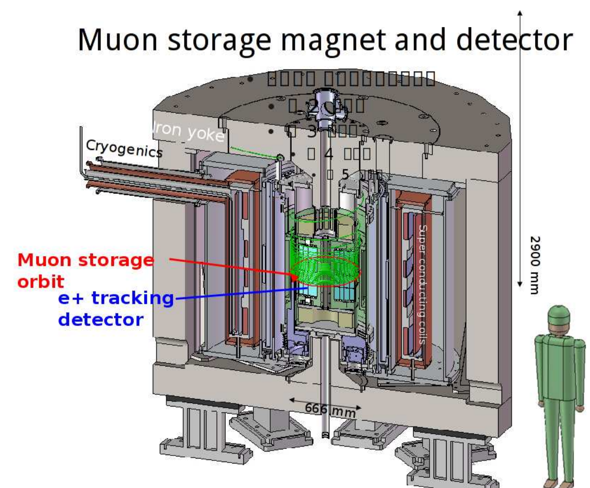


T. Mibe, LPNHE Workshop Paris, Dec. 2014

- could the experiment be wrong? $\longrightarrow$ project to measure a_μ at J-PARC (E34)



N. Saito, LPNHE Workshop Paris, May 2012



T. Mibe, LPNHE Workshop Paris, Dec. 2014

$\longrightarrow$ completely different set-up (uses slow muons)

$\longrightarrow$ never been tested before

$\longrightarrow$ data taking might start in 2025, accuracy goal 0.45ppm

A. Abe et al., PTEP 2019, 053C02 (2019)

- Testing the SM with a_e?

G. F. Giudice, P. Paradisi, M. Passera, JHEP 1211, 113 (2012)

$\longrightarrow$ a_e one of the most precisely measured observable in particle physics

D. Hanneke, S. Forgwell, G. Gabrielse, PRL 100, 120801 (2008)

$\longrightarrow$ accuracy goal: from 0.24ppb to 0.02ppb (vs. 0.14ppm for a_μ)

G. Gabrielse, S. E. Fayer, T. G. Myers, X. Fan, Atoms 7, 45 (2019)

$\longrightarrow$ need to determine the fine structure constant at the same level of accuracy! (at least)

Thanks for your attention!