CPV sources during an EWPT

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Based on 2206.01120 and 2209.xxxxx

Outline:

1. **Two sources and their origins and consequences**
2. **Non existence of the VIA source**
3. **How to explain numerical results that seem to qualitative agree with the VIA source**
4. **Derivation of resonantly enhanced flavour source**

2 methods: SC

Single flavour

$$\left(i\gamma^\mu\partial_\mu - m(z)e^{i\theta(z)}\right)\Psi = 0$$

Assume WKB solutions, it is straight forward to derive

$$F = -\frac{(m^2)'}{\omega} + s_{CP}\frac{s(m\theta')'}{2\omega^2}$$

Can also derive from first principles in Wigner space

2 methods: VIA

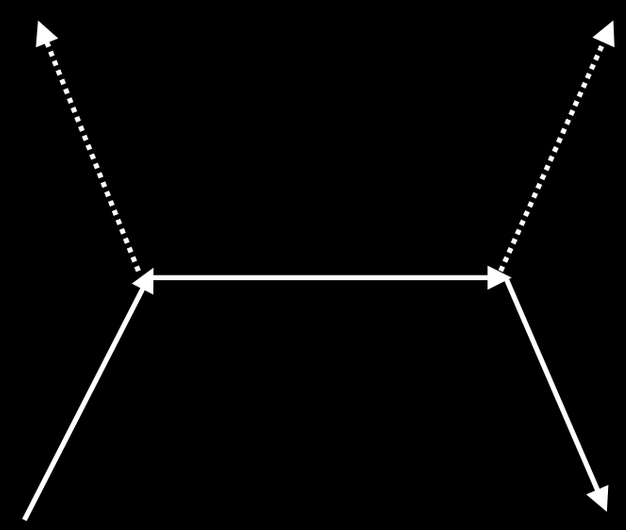
$$Eq_1 = \Delta^{-1}(x)G(x,y) = \Delta^{-1}(x)\Big(G_0(x) + G_0(x,w) \odot \Sigma(w,z)G(z,y)\Big)$$

$$Eq_2 = G(x,y)\Delta^{-1}(y) = \Big(G_0(x) + G(x,w) \odot \Sigma(w,z)G_0(z,y)\Big)\,\Delta^{-1}(y)$$

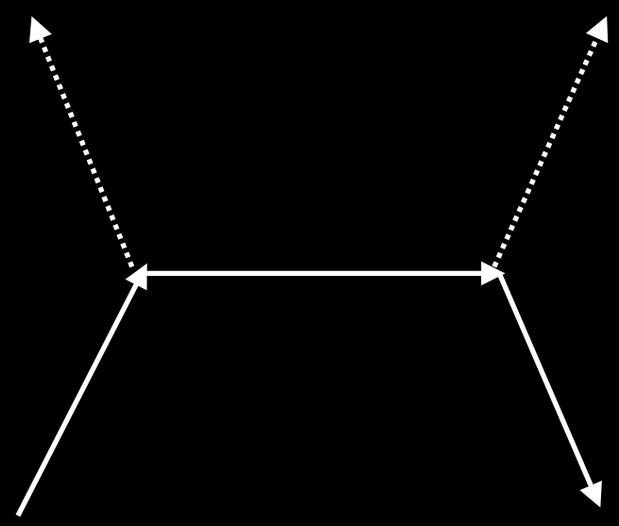
$$\lim_{x\to y}(Eq_1 - Eq_2)$$

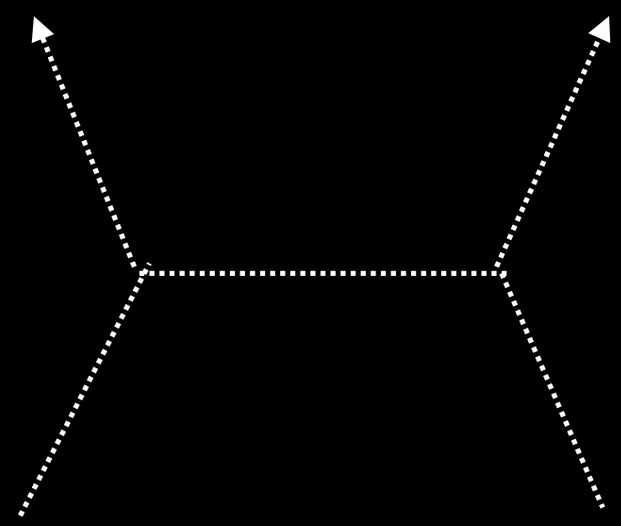
$$\Sigma^{<}(x,y) = f(x)S^{<}(x,y)g^{*}(y)$$

Use flavour basis
Treat vev as perturbation



2 methods: VIA



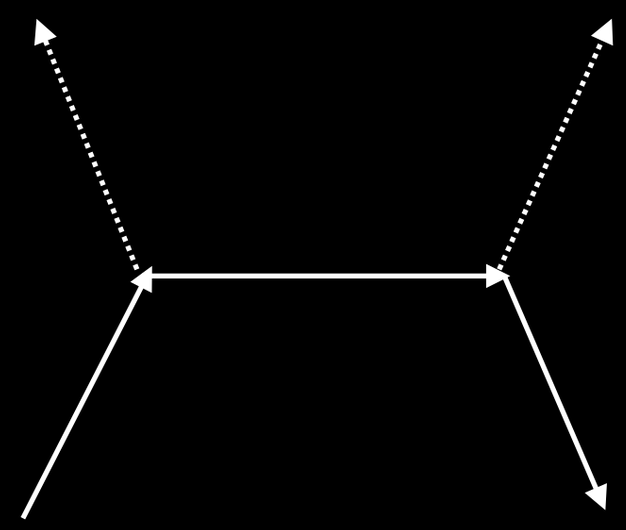


$$S \sim \frac{v_w y m_1 m_2 \sin\phi (v_1 v_2' - v_1' v_2)\Gamma}{((m_1^2 - m_2^2) + \Gamma^2)}$$

$$S \sim \frac{v_w |A| |\mu| \sin\phi (v_1 v_2' - v_1' v_2)\Gamma}{((m_1^2 - m_2^2) + \Gamma^2)}$$

$$\Gamma \sim \frac{1}{T^2} \frac{v^2 \Gamma m_1 m_2}{m_1^2 - m_2^2 + \Gamma^2}$$

Chiral source



Motivation $m_L \neq m_R \rightarrow S_L \neq S_R$

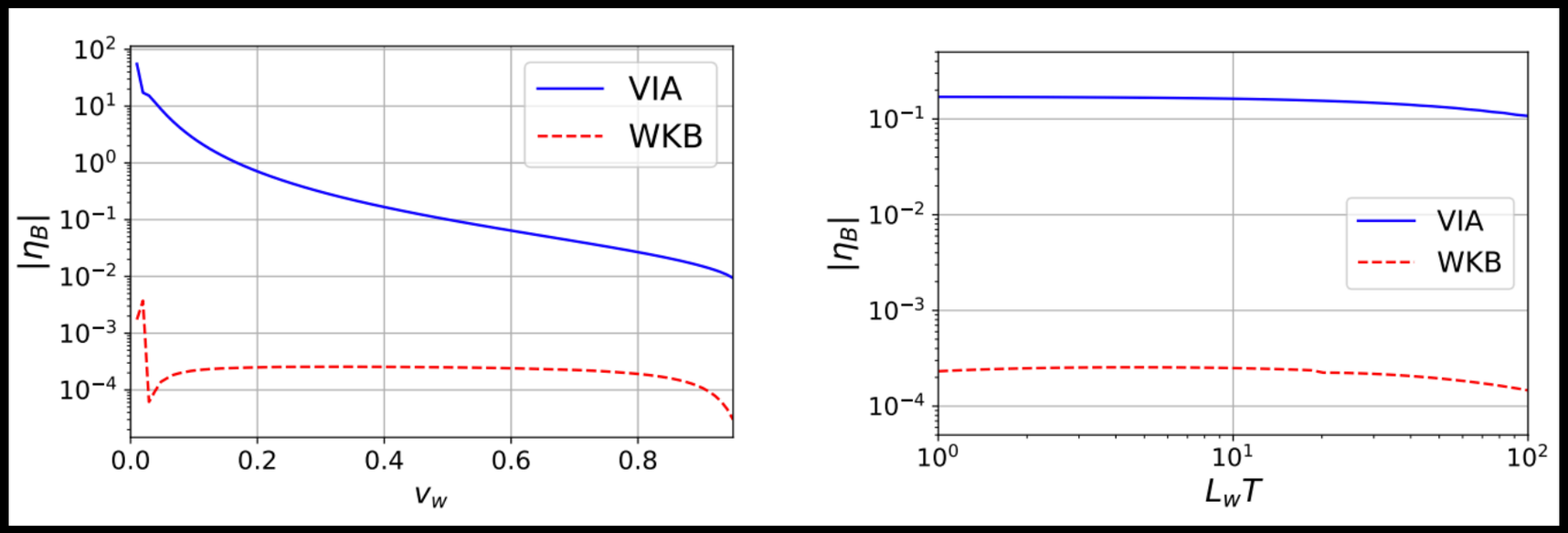
This means it is not necessarily true that

$$S_L = -S_R \rightarrow S_L = S_R = 0$$

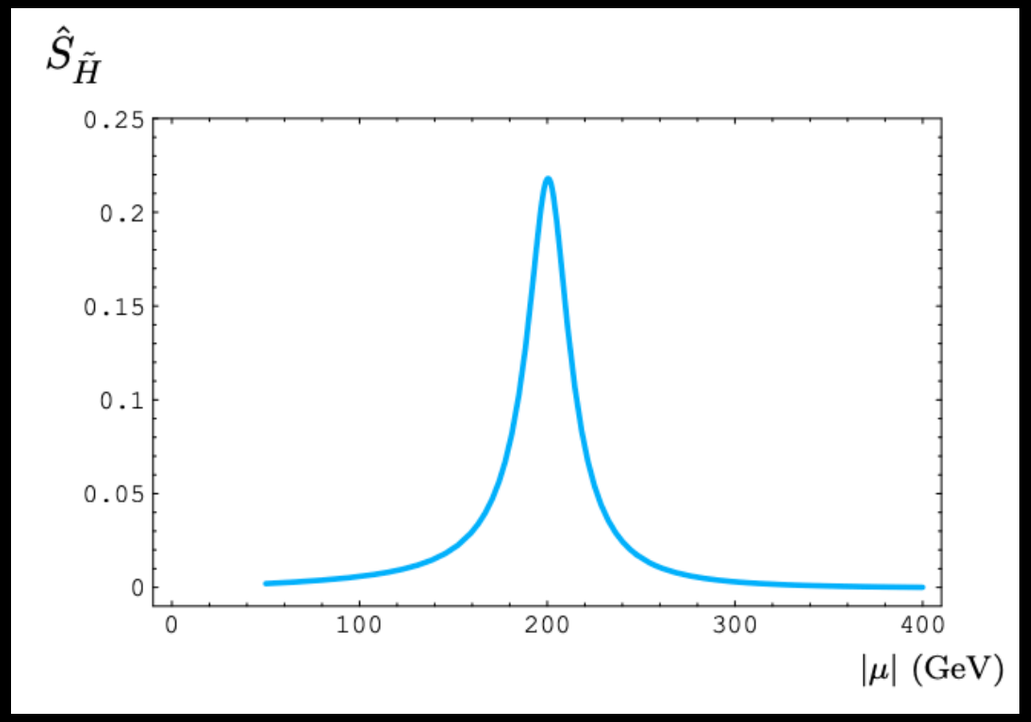
And perhaps when the masses differ, chirality acts like flavour since

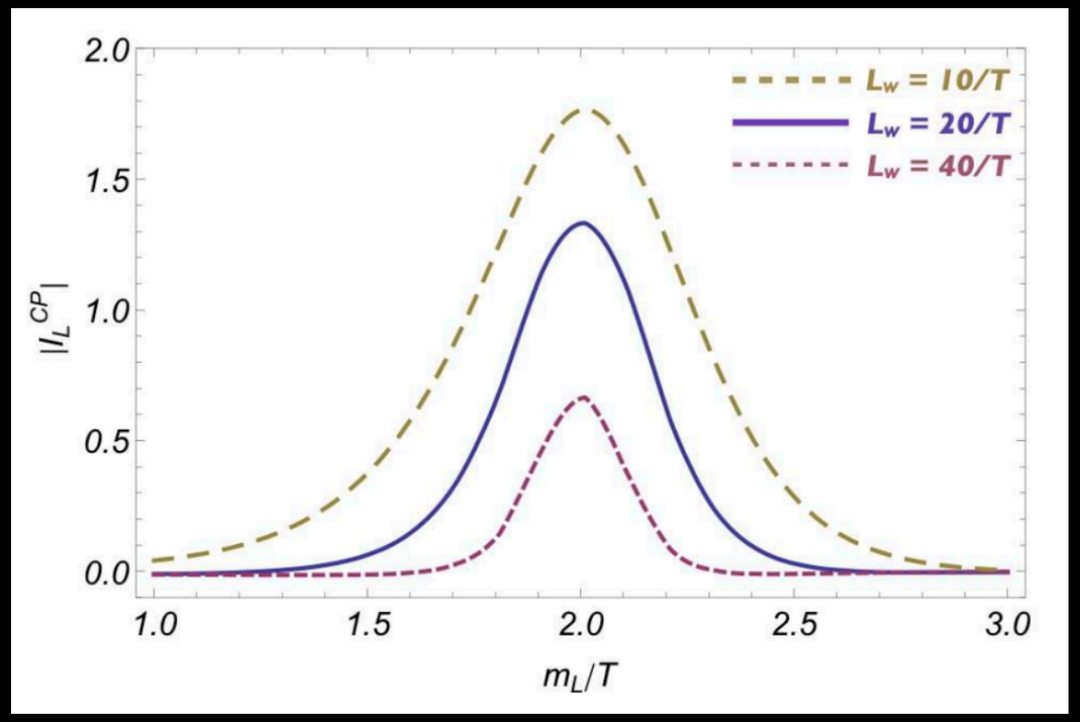
$$[m, S] \neq 0, \quad [m, \gamma^x] \neq 0$$

Relaxation term contains a divergence allegedly removed by normal ordering (this was critiqued in 2108.08336, Kainulainen)



Cline, Laurent 2108.04249





Cirigliano, Lee, Ramsey-Musolf 0412354

Cirigliano, Lee, Tulin 1106.0747

First principles method

Dyson Schwinger equation

$$G^x = G_0^x + G^x \odot \Sigma \odot G_0^x$$

Use equations of equations of motion

$$\Delta^{-1} G_0^x = \delta \quad \rightarrow \quad \Delta^{-1} G^x = \delta + G^x \odot \Sigma$$

- **Transform from** $(x, y) \rightarrow (k, X)$
- **take the hermitian and anti hermitian parts**

For scalars

$$\left(k^2 - \frac{1}{4}\partial^2\right) G^{++} = 1 + \frac{1}{2} e^{-i\diamond} \left(\left\{ M^2 + \Sigma^{++} - \Sigma^h, G^{++} \right\} - \Sigma^{-+} G^{+-} - G^{-+} \Sigma^{+-} \right)$$

$$\diamond\,(A, B) = \frac{1}{2}\left(\partial A \partial_k B - \partial B \partial_k A\right)$$

Solution gives form of propagator

$$2ik \cdot \partial G^{\pm\mp} = e^{-i\diamond} \left(\left[M^2, S^{\pm,\mp}\right] + \left[\Sigma^{\pm\mp}, G^h\right] + \frac{1}{2}\left(\left\{\Sigma^{+-}, G^{-+}\right\} - \left\{\Sigma^{-+}, G^{+-}\right\} \right) \right)$$

Solution gives kinetic equation as

$$i\partial_\mu \int \frac{d^4k}{(2\pi)^4} k^\mu (G^{+-} + G^{-+}) = -\,\partial_\mu J^\mu \to \partial_t n - D\nabla^2 n$$

How to do a vev insertion consistently:

Take propagator that solves the constraint equation and expand

$$G^{ab}_{IJ} = G^{ab}_{(0),IJ} + G^{ab}_{(1),IJ} + G^{ab}_{(2),IJ} + \cdots$$

$$G^{ab}_{(1),IJ} = \sum_{c} c G^{ac}_{(0),II} (\delta M^2)_{IJ} G^{cb}_{(0),JJ}$$

$$G^{ab}_{(2),IJ} = \sum_{cd} cd G^{ac}_{0,II} (\delta M^2)_{IJ} G^{cd}_{(0),JJ} (\delta M^2)_{JI} G^{db}_{(0),II}$$

Zeroth order propogator

$$G^{>,<}_{0,IJ} = g^{>,<}_{II}\rho_{(0),I}\delta_{IJ}$$ **Where** $$\rho_{(0),I} = \frac{\gamma_I}{(k^2 - m_I^2)^2 - \gamma_I^2/4}$$

Where γ_I is a thermal width

To derive self energies, assume the thermal corrections arise from equilibrium physics and are flavor diagonal

$$\Pi^A_{IJ} = \gamma_I\delta_{IJ} \rightarrow \Pi^{<,>}_{IJ} = g^{<,>}_{II}\gamma_I\delta_{IJ}$$

Vev expand the FULL KB equation

$$2ik \cdot \partial(G^{>} + G^{<})^{(2)} =$$

$$\left[\delta M^2, (G^{>}_{(1)} + G^{<}_{(1)})\right] + \left[M_d^2, (G^{>}_{(2)} + G^{<}_{(2)})\right] + \left[\Pi^{>} + \Pi^{<}, G^{h}_{(2)}\right] + \left(\{\Pi^{>}, G^{<}_{(2)}\} - \{\Pi^{<}, G^{>}_{(2)}\}\right)$$

$$G^{ab}_{(1),IJ} = \sum_{c} c G^{ac}_{(0),II} (\delta M^2)_{IJ} G^{cb}_{(0),JJ}$$

$$G^{ab}_{(2),IJ} = \sum_{cd} cd G^{ac}_{0,II} (\delta M^2)_{IJ} G^{cd}_{(0),JJ} (\delta M^2)_{JI} G^{db}_{(0),II}$$

How to turn this into the usual source

First reverse the Wigner transform

$$\left(\{\Pi^{>}, G^{<}_{(2)}\} - \{\Pi^{<}, G^{>}_{(2)}\}\right) = -m^2_{LR} m^2_{RL}\left(\left\{G^{>}_{RR}, G^{<}_{LL}\right\} - \left\{G^{<}_{RR}, G^{>}_{LL}\right\}\right)$$

$$\to -2\int d^4y \mathrm{Re}\left[m^2_{LR}(x) G^{<}_{RR}(x,y) m^2_{RL}(y) G^{>}_{LL}(y,x)\right.$$

$$\left. - m^2_{RL}(x) G^{>}_{RR}(x,y) m^2_{LR}(y) G^{<}_{LL}(y,x)\right]$$

The part which contributes to the source has the form

$$\sim -2\int d^4y \left(\mathrm{Im}[g(x,y) - g(y,x)] G^{<}_{RR}(x,y) G^{>}_{LL}(y,x) - G^{>}_{RR}(x,y) G^{<}_{LL}(y,x)\right)$$

Where $g(x,y) = m^2_{LR}(x) m^2_{RL}(y)$

Example: $m^2_{LR}(x) = b v_1(x) + c v_2(x) e^{i\phi}$, $m_{RL}(x) = b v_1(x) + c v_2(x) e^{-i\phi}$

$$\mathrm{Im}[g(x,y) - g(y,x)] \sim (x-y)\sin\phi (v_1(x) v_2'(x) - v_1'(x) v_2(x))$$

We end up with exactly the VIA source!

Calculating all relevant sources

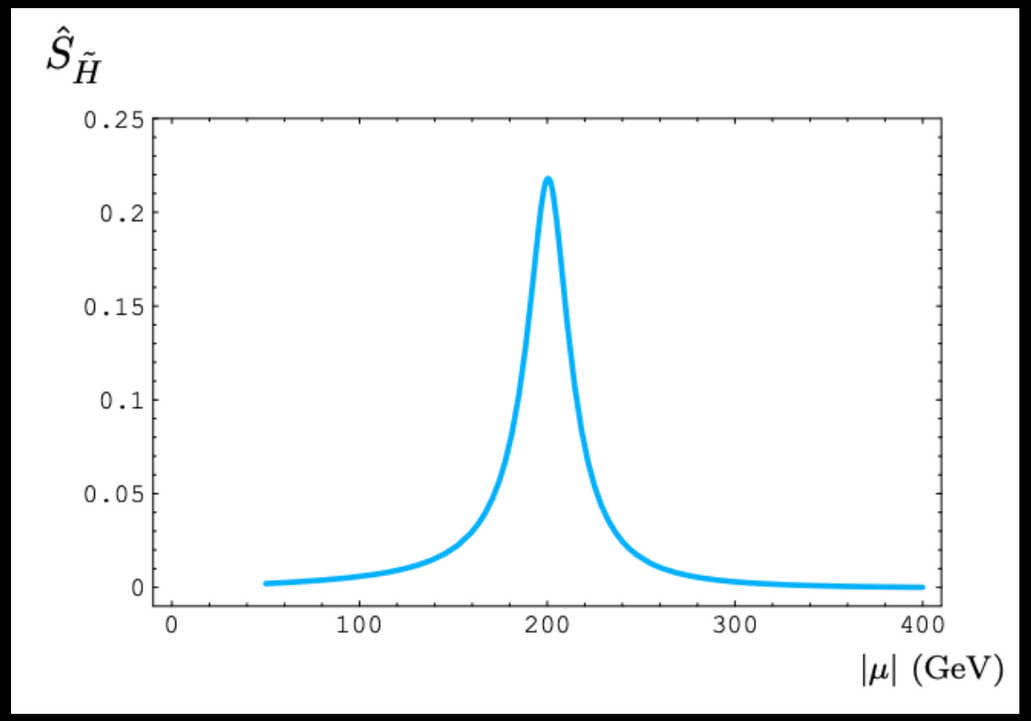
$$\left[\delta M^2, (G^{>}_{(1)} + G^{<}_{(1)})\right] = 2m^2_{LR}m^2_{RL}\rho_L\rho_R(n_L - n_R)$$

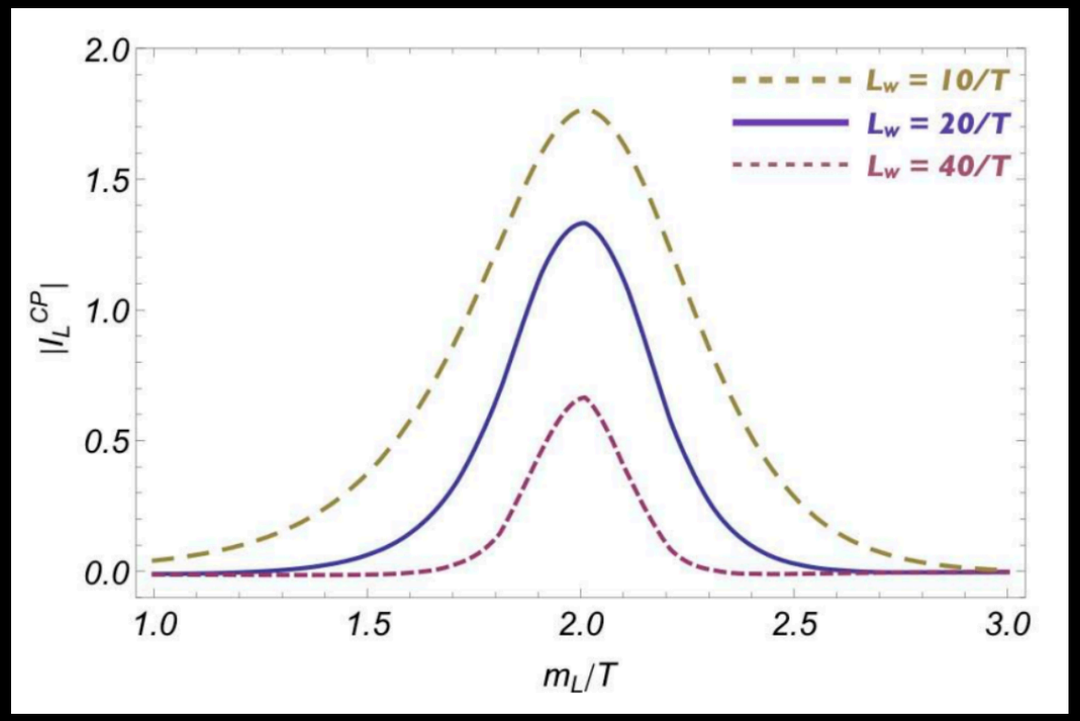
$$\left(\{\Pi^{>}, G^{<}_{(2)}\} - \{\Pi^{<}, G^{>}_{(2)}\}\right) = -\,2m^2_{LR}m^2_{RL}\rho_L\rho_R(n_L - n_R)$$

Cancels exactly!

Leading order VIA source does not exist!

So what the heck is this?





How to find resonantly enhanced sources

Let's ignore thermal corrections and write a simplified KB equation

$$\left[\not{k} + \frac{i}{2}\not{\partial} - M^H e^{-\frac{i}{2}\partial\cdot\partial_k} - i\gamma^5 M^A e^{-\frac{i}{2}\partial\cdot\partial_k}\right] S^< = 0$$

Perform a helicity decomposition

$$iS_s^< = -P_s\left[s\gamma^3\gamma^5 g_0^s - s\gamma^3 g_3^s + g_1^s - i\gamma^5 g_2^s\right]$$

Taking the trace with respect to $\frac{1}{2}\left\{1, s\gamma^3\gamma^5 - is\gamma^3, -\gamma^5\right\}$ **we have**

$$\begin{aligned}
2\mathrm{i}\hat{k}^0 g_0^s - 2\mathrm{i}s\hat{k}^z g_3^s - 2\mathrm{i}M^{\mathrm{H}}\mathrm{e}^{-\frac{\mathrm{i}}{2}\overleftarrow{\partial}\cdot\overrightarrow{\partial}_k} g_1^s - 2\mathrm{i}M^{\mathrm{A}}\mathrm{e}^{-\frac{\mathrm{i}}{2}\overleftarrow{\partial}\cdot\overrightarrow{\partial}_k} g_2^s &= 0\,,\\
2\mathrm{i}\hat{k}^0 g_1^s - 2s\hat{k}^z g_2^s - 2\mathrm{i}M^{\mathrm{H}}\mathrm{e}^{-\frac{\mathrm{i}}{2}\overleftarrow{\partial}\cdot\overrightarrow{\partial}_k} g_0^s + 2M^{\mathrm{A}}\mathrm{e}^{-\frac{\mathrm{i}}{2}\overleftarrow{\partial}\cdot\overrightarrow{\partial}_k} g_3^s &= 0\,,\\
2\mathrm{i}\hat{k}^0 g_2^s + 2s\hat{k}^z g_1^s - 2M^{H}\mathrm{e}^{-\frac{\mathrm{i}}{2}\overleftarrow{\partial}\cdot\overrightarrow{\partial}_k} g_3^s - 2\mathrm{i}M^{\mathrm{A}}\mathrm{e}^{-\frac{\mathrm{i}}{2}\overleftarrow{\partial}\cdot\overrightarrow{\partial}_k} g_0^s &= 0\,,\\
2\mathrm{i}\hat{k}^0 g_3^s - 2\mathrm{i}s\hat{k}^z g_0^s + 2M^{\mathrm{H}}\mathrm{e}^{-\frac{\mathrm{i}}{2}\overleftarrow{\partial}\cdot\overrightarrow{\partial}_k} g_2^s - 2M^{\mathrm{A}}\mathrm{e}^{-\frac{\mathrm{i}}{2}\overleftarrow{\partial}\cdot\overrightarrow{\partial}_k} g_1^s &= 0\,.
\end{aligned}$$

How to find resonantly enhanced sources

Define $g^s_{L,R} = g^s_0 \mp g^s_3$, solving the set of four equations and expanding to second order in gradients we find

$$
\begin{aligned}
& k^z \frac{\partial}{\partial z} g^s_{\mathrm{L}} + \underbrace{\frac{\mathrm{i}}{2}\left[MM^\dagger, g^s_{\mathrm{L}}\right]}_{\text{mixing term}} \\
& \underbrace{-\frac{1}{4}\left\{\left(MM^\dagger\right)', \partial_{k^z} g^s_{\mathrm{L}}\right\}}_{\text{classical force}} \underbrace{-\frac{1}{4k^z}\left(M' g^s_{\mathrm{R}} M^\dagger + M g^s_{\mathrm{R}} M'^\dagger\right) + \frac{1}{4k^z}\left(M' M^\dagger g^s_{\mathrm{L}} + g^s_{\mathrm{L}} M M'^\dagger\right)}_{\text{gradient-mixing terms}} \\
& \underbrace{+\frac{\mathrm{i}}{8}\left(M'' M^\dagger \partial_{k^3} \frac{g^s_{\mathrm{L}}}{k^z} - \partial_{k^z} \frac{g^s_{\mathrm{L}}}{k^z} M M''^\dagger\right) - \frac{\mathrm{i}}{8}\left(M'' \partial_{k^z} \frac{g^s_{\mathrm{R}}}{k^z} M^\dagger - M \partial_{k^3} \frac{g^s_{\mathrm{R}}}{k^z} M''^\dagger\right)}_{\text{semiclassical force}} \\
& -\frac{\mathrm{i}}{16}\left[\left(MM^\dagger\right)'', \partial^2_{k^z} g^s_{\mathrm{L}}\right] + \frac{\mathrm{i}}{8k^z}\left[M' M'^\dagger, \partial_{k^z} g^s_{\mathrm{L}}\right] = 0\,.
\end{aligned}
$$

But where is the resonant source?

How to find resonantly enhanced sources

Solve iteratively in powers of gradients (suppress spin indices)

$$g_{L/R,ij} = g^{(0)}_{L/R,ij} + g^{(1)}_{L/R,ij} + g^{(2)}_{L/R,ij} + \cdots$$

To leading order in gradients our differential equation is just

$$\frac{i}{2}\left[MM^\dagger, g_{L/R}\right] = 0$$

$$M = \begin{bmatrix} m_1 & \delta m_b(z) \\ \delta m_a(z) & m_2 \end{bmatrix}$$

$$g^{(0)}_{R,12} = \frac{(m_2 \delta m_a^\dagger + \delta m_b m_1)(g_{R,11} - g_{R,22})}{m_1^2 - m_2^2}$$

Resonance comes from feed back of off diagonals onto diagonals!

How to find resonantly enhanced sources

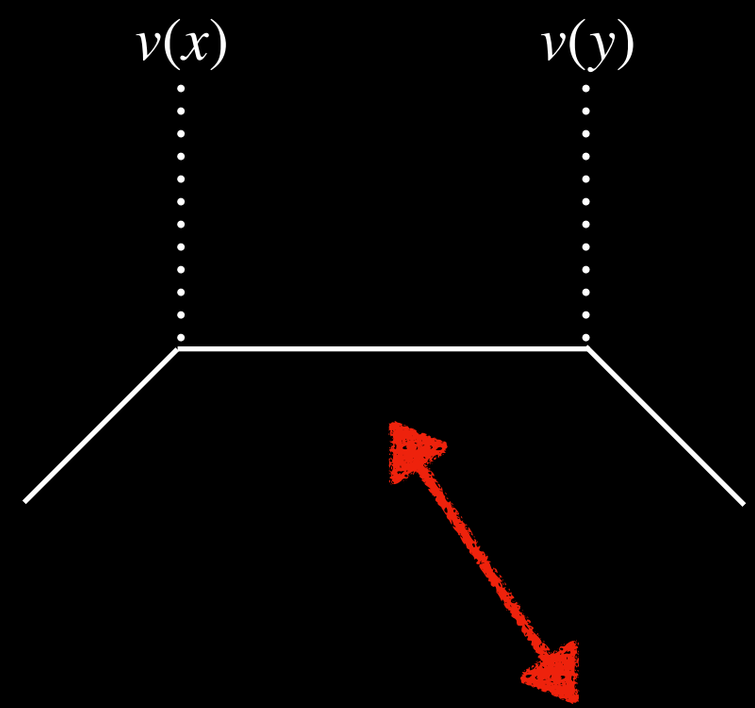
It is straightforward to iteratively solve at each order of gradients

$$(\partial_z j_5^z)_{11} \sim \frac{2s\sin\phi m_1 m_2}{k_z(m_1^2 - m_2^2)}\left(2v_1' v_2' + v_1 v_2'' + v_1'' v_2\right)\frac{1}{2k_z^2}(1 - k_z\partial_{k_z})g_{3,11}$$

Where we have used $\delta m_a = v_1,\ \delta m_b = v_2 e^{i\phi}, j_5^z = -2(g_0^+ - g_0^-)$

Can we find it with a vev insertion approach?

Resonant mixing source



$$(k+\frac{i}{2}\partial\!\!\!/ - M_0^H) - \delta M_0^H e^{-\frac{i}{2}\partial\cdot\partial_p} S_0^< - i\gamma^5 \delta M_A e^{-\frac{i}{2}\partial\cdot\partial_p} S_0^< S^< - e^{-iD}(\delta\Sigma_0^H S_0^< + \delta\Sigma_0^< S_0^H) = 0$$

$$\delta\Sigma^x = (\delta M^H + i\gamma^5 \delta M^A) S_0^x (\delta M^H + i\gamma^5 \delta M^A)$$

Need 2 equations: $\mathrm{Tr}\left[\gamma^5 \mathscr{K} \,.\, \mathscr{B}\right], \quad \text{and } \mathrm{Tr}\left[\gamma^3\gamma^5 \mathscr{K} \,.\, \mathscr{B}\right]$**, just show relevant terms**

$$(j_5^z)_{1,1} \supset (\partial\delta_M)_{12} S^H_{22} (\partial\delta_M)_{21} \partial_k^2 S^<_{11}$$

$$(\partial\delta_M)_{12} S^H_{22} (\partial\delta_M)_{21} \partial_k^2 S^<_{11} \to (\partial\delta_M)_{12} \frac{k_\mu\gamma^\mu + M_1}{k^2 - M_1^2} (\partial\delta_M)_{21} \partial_k^2 \delta(k^2 - M_2^2)(k_\mu\gamma^\mu + M_2) \times \cdots$$

$$\to \frac{M_1 M_2}{M_1^2 - M_2^2} \times \cdots$$

Total is

$$\partial_z j^{5z}_{11} \supset \frac{1}{2}\left(\partial M_{12}\partial M_{21} - \partial M^\dagger_{12}\partial M^\dagger_{21}\right) \frac{M_1 M_2}{M_1^2 - M_2^2} \frac{(-1 + k_z\partial_{k_z}) f_1}{k_z^2}$$

Summary

1. Electroweak Baryogenesis is a key testable paradigm answering a fundamental question
2. Theoretical confusion has existed in the literature for 3 decades!
3. Using CTP formalism and D-S equations allows for a consistent calculation
4. Doing so shows the usual source does not exist
5. Need to see if new sources appear when thermal corrections appear at non-zero gradients
6. Need to apply to realistic models