bottom-up EFT for LFV

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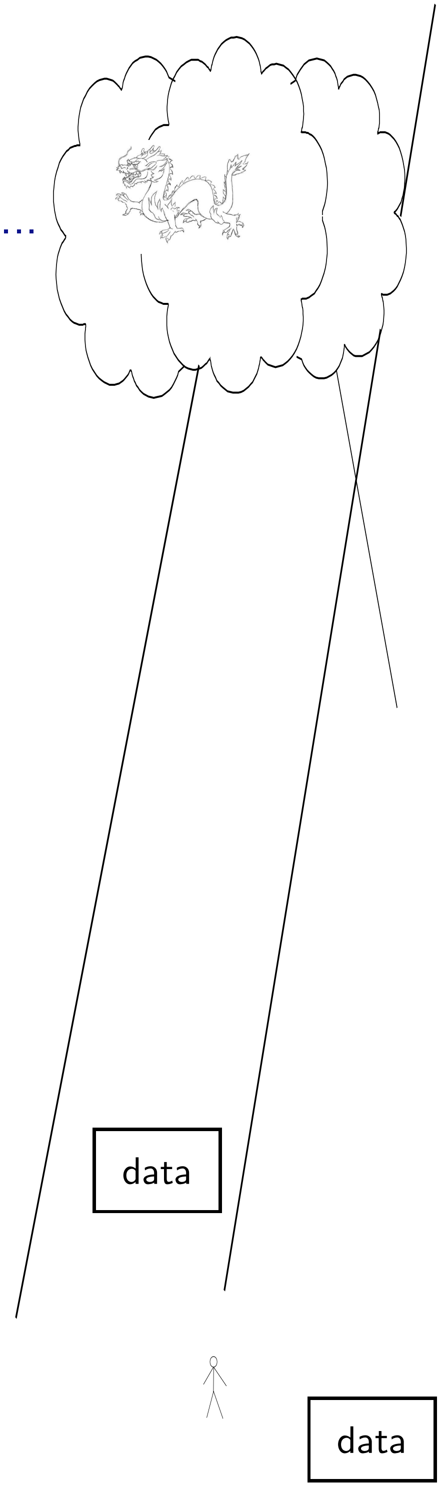


LFV is New Physics that *exists* (just that we don't see it yet)

$\Rightarrow$ what to do as a theorist?

...?use EFT as tool to learn about $\mathcal{L}_{NP}$ from data?

0. (beautiful intro by Kuno-sensei)

1. what we know = constraints on contact interactions

2. climbing the mountain = EFT: operators, RGEs,..

3. obtain @ $\Lambda_{\rm LFV}$: exptally allowed, excluded+ unconstrained regions .

caveats to reconstructing models from data

Q1: if want to learn about models— start from a model?

I don't know how to model-build,

$\Rightarrow$ been done: learned theorists can always build models just-beyond exptal bds.

Q2: I assume NP is heavy, and only SM dynamical below $\Lambda_{\rm LFV}$ —

but NP could be multiscale, involve q and leptons, and/or DM $=$ no hope to reconstruct just from lepton sector

$\Rightarrow$ sure; lets deal with complicated models later...

and NP could be light

$\Rightarrow$ Yes. Interesting alternative to EFT

What we know: categories of LFV data

$\boldsymbol{\Delta LF = 1, \Delta QF = 0}$
$\mu A \to eA$, $\tau \to 3l$, $h \to \tau^{\pm} l^{\mp}$... $(l \in \{e, \mu\})$

$\boldsymbol{\Delta LF = 2}$
$\mu\bar{e} \to e\bar{\mu}$, $\tau \to ee\bar{\mu}$...

$\boldsymbol{\Delta LF = \Delta QF = 1}$
$K \to \mu\bar{e}$

categories $\approx$ independent below $\boldsymbol{\Lambda_{\rm LFV}}$

- SM loops corrections to $\Delta LF = 2$ cannot give $\Delta LF = 1$ (LFV is at $\Lambda_{\rm LFV}$)
- $(\Delta LF = 1)^2 \to \Delta LF = 2$, but better exptal bds on $\Delta LF = 1$.
- $\Delta LF = \Delta QF = 1$ mixes with $\Delta LF = 1$ in SMEFT. But quark FCNC small, so effect $<$ "forseeable" exptal reach on $\Delta LF = 1$. (for $\Lambda_{\rm LFV} > 4$ TeV).

bounds/upcoming reach to $\Delta LF = 1, \Delta QF = 0$

some processes	current constraints on BR	future sensitivities
$\mu \to e\gamma$	$< 4.2 \times 10^{-13}$	6×10^{-14} (MEG)
$\mu \to e\bar{e}e$	$< 1.0 \times 10^{-12}$ (SINDRUM)	10^{-16} (202x, Mu3e)
$\mu A \to eA$	$<$ (6.1×10^{-13} Ti*), (SINDRUMII)	$10^{-(16\to?)}$ (Mu2e,COMET)
	$< 7 \times 10^{-13}$ Au, (SINDRUMII)	$10^{-(18\to?)}$ (PRISM/PRIME/ENIGMA)
$\tau \to \ell\gamma$	$< 3.3, 4.4 \times 10^{-8}$	few$\times 10^{-9}$ (Belle-II)
$\tau \to 3\ell$	$< 1.5 - 2.7 \times 10^{-8}$	few$\times 10^{-9}$ (Belle-II, LHCb?)
$\tau \to \ell\{\pi, \rho, \phi, ...\}$	$\lesssim$ few $\times 10^{-8}$	few$\times 10^{-9}$ (Belle-II)
$h \to \tau^\pm \ell^\mp$	$< 1.5, 2.2 \times 10^{-3}$ (ATLAS/CMS)	$< 2.4 \times 10^{-4}$ (ILC)
$h \to \mu^\pm e^\mp$	$< 6.1 \times 10^{-5}$ (ATLAS/CMS)	2.1×10^{-5} (ILC)
$Z \to e^\pm \mu^\mp$	$< 7.5 \times 10^{-7}$ (ATLAS)	
$Z \to l^\pm \tau^\mp$	$< \times 10^{-7}$ (ATLAS)	

$\mu A \to eA \equiv \mu$ in $1s$ state of nucleus A converts to e

* $(\mu Ti \to eTi)|_{PDB} < 4.1 \times 10^{-12}$, but there is conf. proc. $< 6.1 \times 10^{-13}$, see hepph/9909265 2204.06005

But to reconstruct $\mu \to e$ bottom-up, need all data?

$eg\ BR(\pi^0 \to e^\pm \mu^\mp) < 3.6 \times 10^{-10}$, or $BR(\Upsilon \to l_1 \bar{l}_2) \lesssim 10^{-6}$?

Ummm: μ decays weakly $\Leftrightarrow \tau_\mu \sim 10^{-6}$ sec.
vs $\tau_{\pi^0} \sim 10^{-16}$ sec (loop-suppressed QED), or $\tau_\Upsilon \sim 10^{-20}$ sec (tree QED/QCD)

Compare *weak* μ decays to *anomalous QED* π_0 decay
(write $\delta\mathcal{L} \sim \frac{1}{\Lambda^2_{\rm LFV}}(\bar{e}\mu)(\bar{q}q) + \frac{1}{\Lambda^2_{\rm LFV}}(\bar{e}\gamma\mu)(\bar{e}\gamma e)$) :

$$BR(\mu \to e\bar{e}e) = \frac{\Gamma(\mu \to e\bar{e}e)}{\Gamma(\mu \to e\bar{\nu}\nu)} \sim \left|\frac{m_\mu^2/\Lambda^2_{\rm LFV}}{m_\mu^2 G_F}\right|^2 \sim \frac{v^4}{\Lambda^4_{\rm LFV}} \lesssim 10^{-12} \Rightarrow \Lambda_{\rm LFV} \gtrsim 10^5 {\rm GeV}$$

$$BR(\pi_0 \to \bar{e}\mu) = \frac{\Gamma(\pi_0 \to \bar{e}\mu)}{\Gamma(\pi_0 \to \gamma\gamma)} \sim \left|\frac{m_\pi^2/\Lambda^2_{LFV}}{\alpha/4\pi}\right|^2 \sim \left(\sqrt{\frac{4\pi}{\alpha}}\frac{m_\pi}{\Lambda_{LFV}}\right)^4 \Rightarrow \Lambda_{\rm LFV} \gtrsim {\rm TeV}$$

... rare μ processes have exceptional *sensitivity*, because μ decay weak.
Other $\mu \to e$ processes constrain "orthogonal" operator coefficients, less well.

Climbing the mountain for $\mu \to e$: EFT

Renormalisation Group Eqns/matching/scheme-dep./...

(conceptually simple, technically involved)

Can't we do without RGEs, etc?

in discovery mode for LFV+electroweak loops are small...include later?

counterex: $\mu A \to eA$ in model giving tensor $2\sqrt{2}G_F C_T^{uu}(\overline{e}\sigma P_R\mu)(\overline{u}\sigma u)$ at weak scale

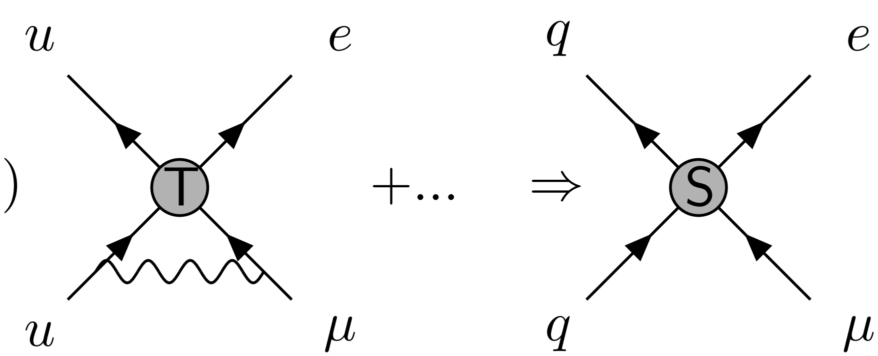
1: forget loops quark tensor matches to nucleon spin $\bar{N}\gamma\gamma_5 N$: $(N \in \{n,p\})$

$$\Rightarrow \boldsymbol{BR(\mu A \to eA) \approx BR_{SD} \approx \frac{1}{2}|C_T^{uu}|^2}$$

(CiriglianoDKuno) Hoferichter etal

2: include QED loops $m_W \to 2$ GeV:

$C_T^{uu}(\overline{u}\sigma u)(\overline{e}\sigma P_Y\mu)$



$$64\frac{\alpha_e}{4\pi}\log\frac{m_W}{m_\tau}\; C_T^{uu}\;(\overline{u}u)(\overline{e}P_Y\mu)$$

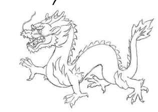
$$\Delta C_S^{uu}(m_\tau) \sim \frac{1}{7}C_T^{uu}(m_W)$$

Then, scalar ops have enhanced nuclear matrix elements, and are SpinIndep:

$$\boldsymbol{BR(\mu A \to eA) \approx BR_{SI} \sim Z^2|2C_T^{uu}|^2 \sim 10^3 BR_{SD}}$$

loops can change Lorentz structure/external legs $\Rightarrow$ different operator whose coefficient better constrained. Important for $\mu \to e$. (?not $\tau \to l$?)

need operators+bases for 3 EFTs?



$\Lambda_{NP} \gg$ TeV

$\{Z, W, \gamma, g, h, t, f\}$

$SU(3) \times SU(2) \times U(1)$

$m_W \sim m_h \sim m_t$

$\{\gamma, g, f\}$

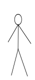
$QCD \times QED$

2 GeV$\sim m_c, m_b, m_\tau$

NB: $\frac{2\mathrm{GeV}}{m_\mu} \sim 20$

$\{n, p, \pi, \gamma, e, \mu\}$ QED $+ \chi PT$

data $(\mu \to e\gamma, \mu \to e\bar{e}e, \mu A \to eA)$



operators + RGEs: *everything to which data could be sensitive*

operator basis: below m_W, all gauge invariant operators with $\leq$ 4 legs$\approx$ 100 ops.
add to $\mathcal{L}_{SM}$ as $\delta\mathcal{L} = 2\sqrt{2}G_F C^{e\mu ee}_{V,LL}(\overline{e}\gamma\mu)(\overline{e}\gamma e) + ...$

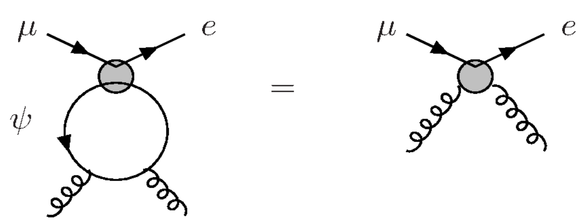
(not dim6: bottom-up perspective/ operator dim. not preserved in matching)

above m_W: dim 6 + selected dim 8 (guess by powercounting)

ArduDavidson

ex: $(\bar{e}\mu)G_{\alpha\beta}G^{\alpha\beta}$ is dim7 $< m_W$, dim8 in SMEFT. But

- dim6 heavy quark scalar ops $(\bar{e}\mu)(\bar{Q}Q)$ match to $(\bar{e}\mu)GG$ at m_Q (coef.$C_{QQ}/(m_Q\Lambda^2_{\rm LFV})$):



- gluons contribute most of the mass of the nucleon

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$$\langle N|m_N\overline{N}N|N\rangle = \sum_{q\in\{u,d,s\}}\langle N|m_q\overline{q}q|N\rangle \quad - \frac{\alpha_s}{8\pi}\beta_0\langle N|GG|N\rangle$$

$\Rightarrow$ dim7 $(\bar{e}\mu)GG$ contributes significantly to $\mu A \to eA$ via scalar $\mu \to e$ interactions with nucleons N.

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operators + RGEs: *everything to which data could be sensitive*

operator basis: below m_W, all gauge invariant operators with $\leq$ 4 legs$\approx$ 100 ops.
add to $\mathcal{L}_{SM}$ as $\delta\mathcal{L} = 2\sqrt{2}G_F C^{e\mu ee}_{V,LL}(\overline{e}\gamma\mu)(\overline{e}\gamma e) + ...$

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RGEs+matching: at "leading order"$\equiv$ largest contribution of each operator to each observable. (2GeV$\to m_W$:resum LL QCD, $\alpha_e \log$, some $\alpha_e^2 \log^2$, $\alpha_e^2 \log$)

why not just 1-loop RGEs?

- expand in loops, hierarchical Yukawas, $1/\Lambda^2_{\rm LFV}$,... largest effect maybe not 1-loop (ex: Barr-Zee)
- sometimes 1-loop vanishes...eg: 2-loop $\Delta a_\mu|_{EW} \simeq$ 1-loop $\Delta a_\mu|_{EW}$.
  Because 2-loop log-enhanced
  $=$ mixing vector ops to dipole in 2-loop RGEs.

What can one learn

in bottom-up EFT?

.

3 processes, many ops: if $\Delta QF = 0$, $\mu \to e$ occurs, will it contribute to $\mu \to e\gamma, \mu \to e\bar{e}e$ or $\mu A \to eA$?

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Probably yes: SM loops ensure almost every $\Delta QF = 0$, $\mu \to e$ interaction with ≤ 4 legs, contributes $\gtrsim \mathcal{O}(10^{-3})$ to amplitudes $\mu \to e\gamma$, $\mu \to e\bar{e}e$ and/or $\mu A \to eA$
(not $\bar{e}\mu G\widetilde{G}$, $\bar{e}\mu F\widetilde{F}$, $\bar{e}\gamma\mu F\partial F$...)

coefficient	$\mu \to e\gamma$	$\mu \to e\bar{e}e$	$\mu A \to eA$
$\lvert C_{D,X}\rvert$	1.12×10^{-8}	4.30×10^{-7}	2.35×10^{-7}
$\lvert C^{ee}_{V,XX}\rvert$	1.10×10^{-4}	7.80×10^{-7}	1.86×10^{-5}
$\lvert C^{ee}_{V,XY}\rvert$	2.55×10^{-4}	9.34×10^{-7}	3.77×10^{-5}
$\lvert C^{ee}_{S,XX}\rvert$	1.73×10^{-4}	2.8×10^{-6}	(3.64×10^{-3})
$\lvert C^{\mu\mu}_{V,XX}\rvert$	1.10×10^{-4}	5.60×10^{-5}	1.85×10^{-5}
$\lvert C^{\mu\mu}_{V,XY}\rvert$	2.56×10^{-4}	1.12×10^{-4}	3.77×10^{-5}
$\lvert C^{\mu\mu}_{S,XX}\rvert$	8.24×10^{-7}	(1.58×10^{-5})	(1.73×10^{-5})
$\lvert C^{\tau\tau}_{V,XX}\rvert$	3.80×10^{-4}	1.95×10^{-4}	1.24×10^{-5}
$\lvert C^{\tau\tau}_{V,XY}\rvert$	4.40×10^{-4}	1.91×10^{-4}	1.25×10^{-5}
$\lvert C^{\tau\tau}_{S,XX}\rvert$	5.33×10^{-6}	1.02×10^{-4}	1.12×10^{-4}
$\lvert C^{\tau\tau}_{S,XY}\rvert$	—	—	—
$\lvert C^{\tau\tau}_{T,XX}\rvert$	1.10×10^{-8}	(4.20×10^{-7})	(2.30×10^{-7})

sensitivities/1-at-a-time bds for $\delta\mathcal{L} = 2\sqrt{2}G_F C_i \mathcal{O}_i$; if model gives smaller coefficients, it is consistent with data. If it generates larger coefficients, need to arrange a cancellation...

$\Leftrightarrow$ modulo cancellations, probably find $\mu \leftrightarrow e$

$[\mu \to \tau] \times [\tau \to e] = [\mu \to e] \Rightarrow$?

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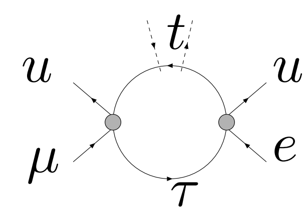
recall exptal reach: $\mathrm{BR}(\mu \to e) \to 10^{-(18\to 20)} \sim [\mathrm{BR}(\tau \to l) \to 10^{-9}]^2$
? learn about $\tau \to l$ from $\mu \to e$?

1. if model has $(\mu \to \tau)$,$(\tau \to e)$, then no conserved flavour, so "expect" $\mu \to e$

2. can one calculate anything model-independent? In SMEFT, $(\text{dim6})^2 \to$ dim8,
   eg $\overline{\ell} e \varepsilon \overline{q} u \times (\overline{\ell}\gamma\ell)(\overline{q}\gamma q) \to \overline{\ell} e \varepsilon \overline{q} u H^\dagger H$

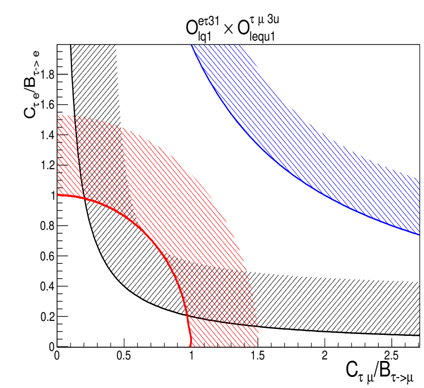
$$\frac{\Delta\ ^{(8)}C^{e\mu uu}}{\Lambda^4_{\rm LFV}} \simeq \frac{\{y_t^2, g^2\}}{16\pi^2}\frac{C^{e\tau ut}_{LQ}}{\Lambda^2_{\rm LFV}}\frac{C^{\tau\mu tu}_{LEQU}}{\Lambda^2_{\rm LFV}}$$



   so effective low-energy 4-fermion interaction $2\sqrt{2}G_F C_S$

$$\Delta\ ^{(6)}C^{e\mu uu}_S \propto \frac{v^4}{16\pi^2\Lambda^4_{\rm LFV}} C^{e\tau ut} C^{\tau\mu tu}$$

3. find eg, $\mu A \to eA$ sensitivity complementary
   to $B^- \to \{e, \mu\}\nu$ decays for some operators:



But 3 processes, ∼100 operators ⇒ zoo of flat directions?

DKunoYamanaka

Count constraints: (write $\delta\mathcal{L} = C^{flavour}_{Lorentz,XY}/v^n \ \mathcal{O}^{flav}_{Lorentz,XY} \ , X, Y \in \{L, R\}$)

$\boldsymbol{\mu \to e\gamma}$ **:** $BR(\mu \to e\gamma) = 384\pi^2(|C_{D,L}|^2 + |C_{D,R}|^2)$ ⇒ **2 constraints**

$\boldsymbol{\mu \to e\bar{e}e}$ **:** (e relativistic ≈ chiral, neglect interference between e_L, e_R)

$$BR = \frac{|C_{S,LL}|^2}{8} + 2|C_{V,RR} + 4eC_{D,L}|^2 + (64 \ln \frac{m_\mu}{m_e} - 136)|eC_{D,L}|^2 + |C_{V,RL} + 4eC_{D,L}|^2 + \{L \leftrightarrow R\}$$

⇒ **6 more constraints**

$\boldsymbol{\mu A \to eA}$ **:**(S^N_A,V^N_A=integral over nucleus A of N distribution×lepton wavefns, **different** for diff. A)

$$BR_{SI} \sim Z^2|V^p_A\tilde{C}^p_{V,L} + S^p_A\tilde{C}^p_{S,R} + V^n_A\tilde{C}^n_{V,L} + S^b_A\tilde{C}^n_{S,R} + D_AC_{D,R}|^2 + |L \leftrightarrow R|^2$$

$$BR_{SD} \sim |\tilde{C}^N_A + 2\tilde{C}^N_T|^2$$

SI bds on Au, Ti, (+ SD on ?Ti, Au?) ⇒ **$4 + 2$ more constraints**

future: improved theory, 3SI+2SD targets ⇒ **6+4** constraints

is 12-20 constraints on ~ 100 operators a problem?

many operators+few constraints=using inconvenient basis

Have 6 (+6) constraints on e_L (e_R) operator coefficients. Focus on e_L.
Want to change basis to *scale -dependent* **basis of constrained 6-d subspace.**

1. $\mu \to e\gamma$ measures $C_{D,R}(m_\mu)$
Have RGEs for coefficients (arranged in row vector)

$$\mu \frac{\partial}{\partial \mu} \vec{C}(\mu) = \vec{C}(\mu) \mathbf{\Gamma}(\mu, g_s(\mu), ...) \quad \Rightarrow \quad \vec{C}(m_\mu) = \vec{C}(m_W) \boldsymbol{G}(m_\mu, m_W)$$

solved as scale-ordered exponential (resummed QCD, $\alpha \log$, some $\alpha^2 \log^2$, $\alpha^2 \log$)
$\Rightarrow$ define scale-dep $\vec{v}_{\mu\to e\gamma}(\Lambda)$, column of **G** such that: $C_{DR}(m_\mu) = \vec{C}(\Lambda) \cdot \vec{v}_{\mu\to e\gamma}(\Lambda)$

$\boldsymbol{\vec{v}_{\mu\to e\gamma}(\Lambda)}$ **is scale-dep basis vector for constrainable subspace**

2-6. repeat for other independent constraints. So obtain scale-dep basis vectors for the subspace, defined from the observables.
The "flat directions" (experimentally inaccessible) are orthogonal, and therefore irrelevant.

Basis should span the finite-eigenvalue subspace of the correlation matrix.

what to do with this basis?

check a few things

1. Do the basis vectors stay orthogonal? $=$*Do* $\mu \to e\gamma, \mu \to e\bar{e}e$ *and* $\mu A \to eA$ *give complementary information about NP models?*

   (a) Yes, to $\mathcal{O}(10^{-3})$ in running 2 GeV$\to m_W$

   (b) But changing EFTs can give overlaps (diff. low-E operators can match to same high-E operator $\leftrightarrow$ measure same thing)

   ex1: at m_W, all low-E vector 4f operators match to penguins $C^{e\mu}_{HE}, C^{e\mu}_{HL}$.

   ex2: in matching at 2 GeV:

   $\langle p|\bar{u}u|p\rangle = \langle n|\bar{d}d|n\rangle$ (isospin ?)

   but also: $\langle p|\bar{d}d|p\rangle \simeq \langle p|\bar{u}u|p\rangle \simeq \langle n|\bar{d}d|n\rangle \simeq \langle n|\bar{u}u|n\rangle$

2. the basis vectors change length...by $\mathcal{O}(1)$ factors, so ok

   eg importance of dipole for $\mu \to e\bar{e}e$ grows with scale

Wanted to use EFT to take exptal info to models... so:

1. *(match to models, and explore what we can learn)*
   (not need to run RGEs at each point in model space)
   are some regions of 6-d space inaccessible to some models?

2. make plots of the excluded region in 6-d space ?
   ⇔ illustrate the reach and complementarity of experiments

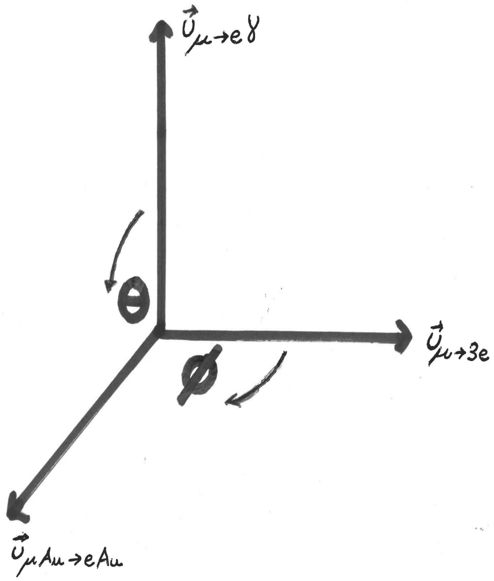
Plot complementarity+reach of $\mu \to e\gamma, \mu \to e\bar{e}e$ and $\mu A \to eA$

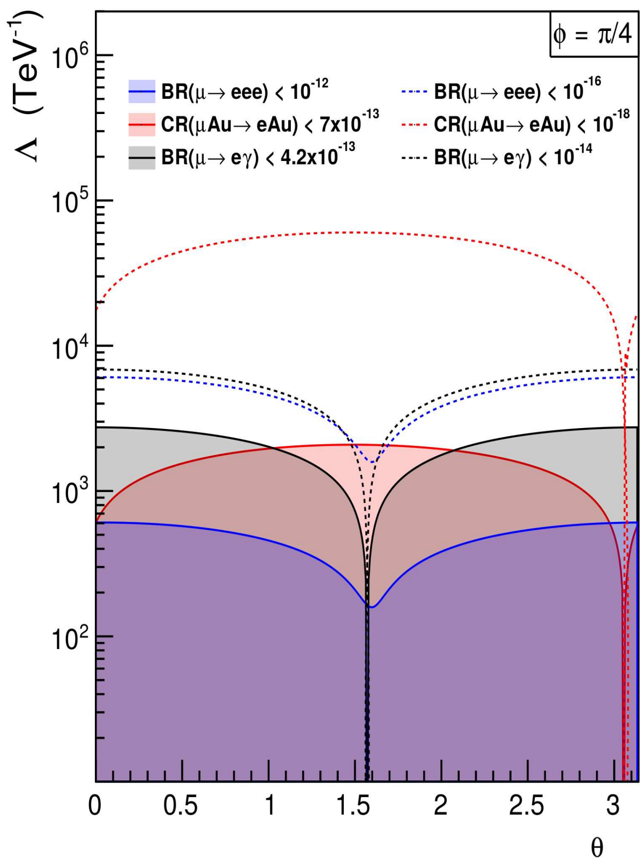
(in theoretically self-consistent EFT, including LO loops, cancellations...)

Restrict to 3-d space of coefficients of $\vec{v}_{\mu\to e_L\gamma}, \vec{v}_{\mu\to 3e_L}, \vec{v}_{\mu Au\to e_L Au} (= z, x, y)$.
Current *allowed* region an ellipse around origin... write instead:

$$\vec{C}\cdot\vec{v}_{\mu\to e_L\gamma} \equiv \frac{v^2\cos\theta}{\Lambda^2_{\rm LFV}}$$

$\Rightarrow \Lambda_{\rm LFV} \to \infty$ allowed





see 2204.00564

Plot complementarity+reach of $\mu \to e\gamma, \mu \to e\bar{e}e$ and $\mu A \to eA$

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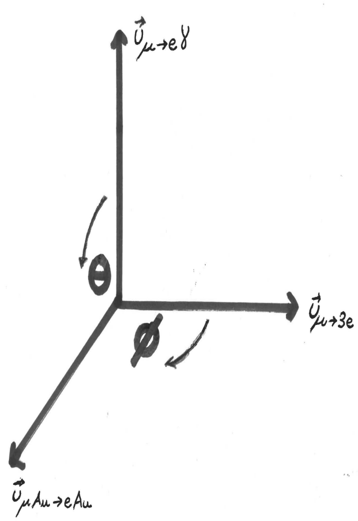
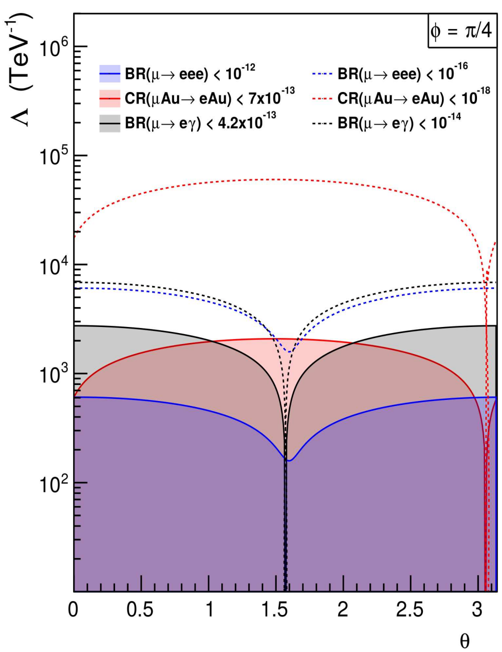
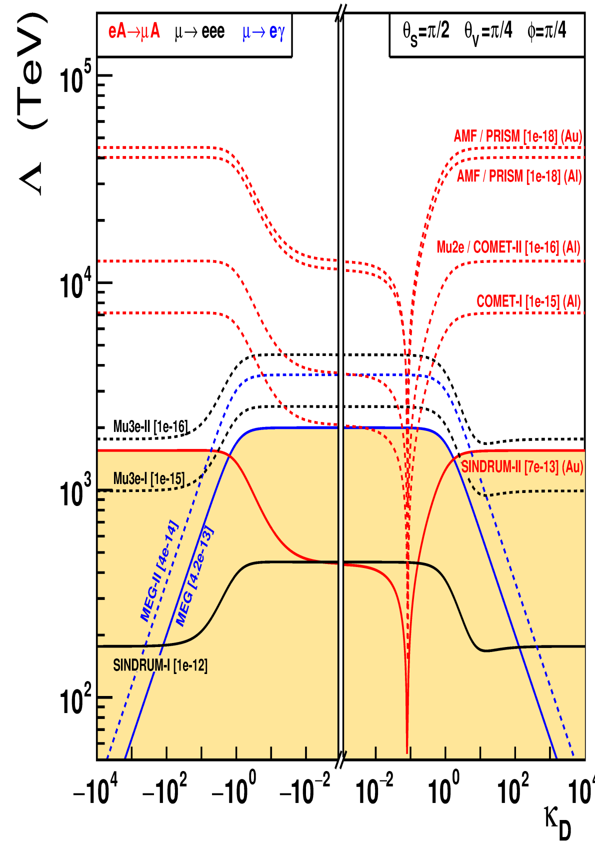
Define $\kappa_D = {\rm cotg}(\theta_D - \pi/2)$







see 2204.00564

Summary

$\mu \to e\gamma, \mu \to e\bar{e}e$ and $\mu A \to eA$ have exceptional sensitivity ($\Lambda_{\rm LFV} \lesssim 10^2 \to 10^3$ now, $\Lambda_{\rm LFV} \lesssim 10^3 \to 10^4$ upcoming), to only a few operators at low energy, so:

interesting to include RGEs at leading order, because ensure that almost every $\mu \to e$ operator (in chiral basis) with ≤ 4 legs contributes at $\gtrsim \mathcal{O}(10^{-3})$ to $\mu \to e\gamma$ and/or $\mu \to e\bar{e}e$ and/or $\mu A \to eA$
(possible exceptions :$\bar{e}\mu G\widetilde{G}$, $\bar{e}\mu F\widetilde{F}$, $\bar{e}\gamma\mu F\partial F$...)
Can even have interesting sensitivity to products of some $(\mu \to \tau) \times (\tau \to e)$ interactions!

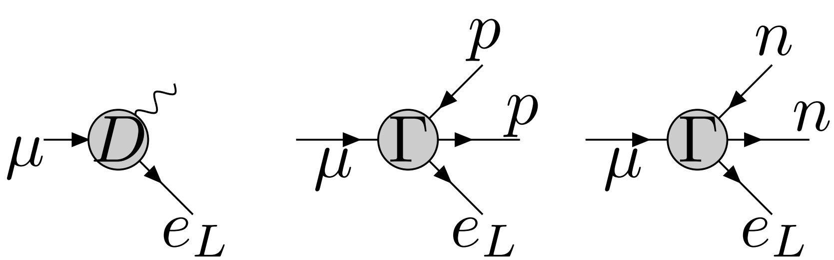
But most directions in coefficient space are untestable ("flat": *(not an EFT-problem, its a consequence of searching for NP under the lamppost, affects model studies in same way.)*
Can circumvent this by changing operator basis: a convenient basis for comparing models to $\mu \to e$ flavour-changing observables can be constructed from the observables.

BackUp

$\mu A \to eA$: most sensitive process, expt + th

- μ^- captured by Al nucleus, tumbles down to $1s$. $(r \sim Z\alpha/m_\mu \gtrsim r_{Al})$
- in SM: muon "capture" $\mu + p \to \nu + n$, or decay-in-orbit
- LFV:μ interacts with $\vec{E}$, nucleons (via $\widetilde{C}^N_{\Gamma,X}(\bar{e}\Gamma P_X N)(\bar{N}\Gamma N)$), converts to e

$(E_e \approx m_\mu$ so $e_L/e_R)$



$$\Gamma = \{I, \gamma_5, \gamma^\alpha, \gamma^\alpha\gamma_5, \sigma\}$$
$$\Gamma = \{S, P, V\ ,\ A\ ,\ T\}$$

$\approx$ WIMP scattering on nuclei

1) "Spin Independent" rate $\propto A^2$ (amplitude $\propto \sum_N \propto A$)

KitanoKoikeOkada

$$BR_{SI} \sim \quad Z^2|\sum ...\tilde{C}_{SI}|^2 \quad , \qquad \tilde{C}_{SI} \in \{\tilde{C}^p_V, \tilde{C}^p_S, \tilde{C}^n_V, \tilde{C}^n_S, C_D\}$$

2) "Spin Dependent" rate $\sim \Gamma_{SI}/A^2$ (sum over $N \propto$ spin of only unpaired nucleon)

$$BR_{SD} \sim ...|\tilde{C}^N_A + 2\tilde{C}^N_T|^2$$

CiriglianoDavidsonKuno
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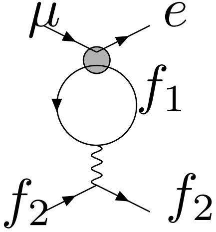
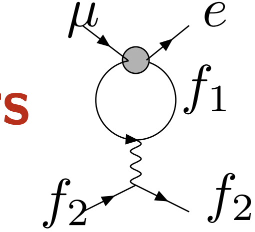
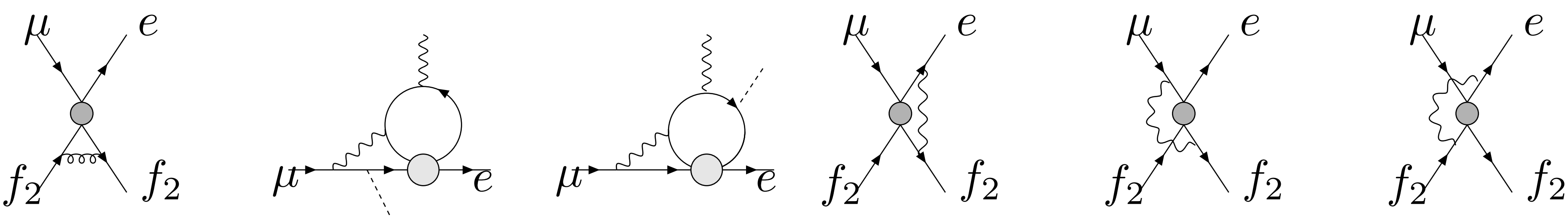


Including SM loop corrections to operators

ex: 1-loop QED + QCD (+2-loop QED V→D)





$$\mu\frac{\partial}{\partial\mu}\vec{C} = \frac{\alpha_s}{4\pi}\vec{C}\boldsymbol{\Gamma}^s + \frac{\alpha_{em}}{4\pi}\vec{C}\boldsymbol{\Gamma}$$

solve (analytically/numerically):

$$\vec{C}(m_\mu) = \vec{C}(\Lambda_{\rm LFV})\boldsymbol{G} \quad , \quad \boldsymbol{G} = \text{fn of SM parameters}, \log(\Lambda_{\rm LFV}/\Lambda_{exp})$$

For ex: $BR(\mu\to e\gamma) = 384\pi^2(|C_{D,L}|^2 + |C_{D,R}|^2) < 4.1\times 10^{-13} \Rightarrow C_{D,X} \lesssim 10^{-8}$

$$C_{D,X}(m_\mu) = C_{D,X}(m_W)\left(1 - 16\frac{\alpha_e}{4\pi}\ln\frac{m_W}{m_\mu}\right) - \frac{\alpha_e}{4\pi e}\left(C^{\mu\mu}_{S,XX} - 8\frac{m_\tau}{m_\mu}C^{\tau\tau}_{T,XX} + C_{2loop}\right)\ln\frac{m_W}{m_\mu}$$

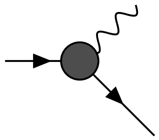
$$+16\frac{\alpha_e^2}{2e(4\pi)^2}\left(\frac{m_\tau}{m_\mu}C^{\tau\tau}_{S,XX}\right)\ln^2\frac{m_W}{m_\mu} - 8\lambda^{a_T} f_{TD}\frac{\alpha_e}{4\pi e}\left(\frac{2m_c}{m_\mu}C^{cc}_{T,XX} - \frac{m_s}{m_\mu}C^{ss}_{T,XX} - \frac{m_b}{m_\mu}C^{bb}_{T,XX}\right)\ln\frac{m}{2}$$

$$+16\frac{\alpha_e^2}{3e(4\pi)^2}\left(\sum_{u,c}4\frac{m_q}{m_\mu}C^{qq}_{S,XX} + \sum_{d,s,b}\frac{m_q}{m_\mu}C^{qq}_{S,XX}\right)\ln^2\frac{m_W}{2\text{GeV}}$$

$C^\zeta_{Lor}(m_W)$ on right. $\lambda = \alpha_s(m_W)/\alpha_s(2\text{GeV}) \simeq 0.44$, $f_{TS} \simeq 1.45$, $a_S = 12/23$, $a_T = -4/23$.

loop sensitivity of $\tau \to \ell\gamma$

Two dipole operators contribute to $\tau \to e\gamma$:



$$\delta\mathcal{L}_{teg} = -\frac{4G_F}{\sqrt{2}} m_\tau \left(C_{D,L}\overline{\tau_R}\sigma^{\alpha\beta}e_L F_{\alpha\beta} + C_{D,R}\overline{\tau_L}\sigma^{\alpha\beta}e_R F_{\alpha\beta}\right)$$

$$\widetilde{BR}(\tau \to e\gamma) = 384\pi^2(|C_{D,L}|^2 + |C_{D,R}|^2) < 1.9 \times 10^{-7}$$

$$\Rightarrow |C_{D,X}| \lesssim 7 \times 10^{-6}$$

Babar

How big does one expect $C_{D,X}$ to be? Suppose operator coefficient

			$n = 1$	$n = 2$
$C_{D,X}\frac{m_\tau}{v^2} \sim \frac{em_\tau}{(16\pi^2)^n\Lambda^2}$	$\Rightarrow$	probes	$\Lambda \lesssim 3$ TeV	0.2
$C_{D,X}\frac{m_\tau}{v^2} \sim \frac{ev}{(16\pi^2)^n\Lambda^2}$	$\Rightarrow$	probes	$\Lambda \lesssim 30$ TeV	2 TeV

$\Rightarrow$ NP contributing in loops to $\tau \to \ell$ maybe LHC-accessible?

(? can we also forget loops for $\mu \leftrightarrow e$?
... we are in discovery mode; loops are for precision physics, no?)

$BR(\tau \to 3\ell) < 2 \times 10^{-8}$

τ has many weak decays : $\to \begin{cases} \nu u \bar{d} \times 3(col.) \\ \nu \mu \bar{\nu} \\ \nu e \bar{\nu} \end{cases} \quad \Rightarrow BR(\tau \to e\bar{\nu}\nu) \simeq 0.175$

to estimate $\Lambda_{\rm LFV}$ for $\Gamma(\tau \to 3\ell)$ define

$$\widetilde{BR}(\tau \to 3\ell) \equiv \frac{\Gamma(\tau \to 3\ell)}{\Gamma(\tau \to e\bar{\nu}\nu)} \sim \frac{v^4}{\Lambda^4_{\rm LFV}} \lesssim 10^{-7} \Rightarrow \Lambda_{LFV} \sim 50v \simeq 10 \text{ TeV}$$

vs $\Lambda_{\rm LFV} \sim 200$ TeV from $BR(\mu \to e\bar{e}e) < 10^{-12}$.

operator list:Kuno-Okada, +CiriglianoKitanoOTuzon +BowmanChengLiMatis

Operator basis $m_\tau \to m_W$: ~ 90 operators

Add QCD×QED-invar operators, representing all 3,4 point interactions of μ with e and *flavour-diagonal* combination of γ, g, u, d, s, c, b. $Y \in L, R$.

$m_\mu(\overline{e}\sigma^{\alpha\beta}P_Y\mu)F_{\alpha\beta}$ *dim* 5

$(\overline{e}\gamma^\alpha P_Y\mu)(\overline{e}\gamma_\alpha P_Y e)$ $(\overline{e}\gamma^\alpha P_Y\mu)(\overline{e}\gamma_\alpha P_X e)$

$(\overline{e}P_Y\mu)(\overline{e}P_Y e)$ *dim* 6

$(\overline{e}\gamma^\alpha P_Y\mu)(\overline{\mu}\gamma_\alpha P_X\mu)$ $(\overline{e}\gamma^\alpha P_Y\mu)(\overline{\mu}\gamma_\alpha P_X\mu)$

$(\overline{e}P_Y\mu)(\overline{\mu}P_Y\mu)$

$(\overline{e}\gamma^\alpha P_Y\mu)(\overline{f}\gamma_\alpha P_Y f)$ $(\overline{e}\gamma^\alpha P_Y\mu)(\overline{f}\gamma_\alpha P_X f)$

$(\overline{e}P_Y\mu)(\overline{f}P_Y f)$ $(\overline{e}P_Y\mu)(\overline{f}P_X f)$ $f \in \{u, d, s, c, b, \tau\}$

$(\overline{e}\sigma P_Y\mu)(\overline{f}\sigma P_Y f)$

$\frac{1}{m_t}(\overline{e}P_Y\mu)G_{\alpha\beta}G^{\alpha\beta}$ $\frac{1}{m_t}(\overline{e}P_Y\mu)G_{\alpha\beta}\widetilde{G}^{\alpha\beta}$ *dim* 7

$\frac{1}{m_t}(\overline{e}P_Y\mu)F_{\alpha\beta}F^{\alpha\beta}$ $\frac{1}{m_t}(\overline{e}P_Y\mu)F_{\alpha\beta}\widetilde{F}^{\alpha\beta}$ **...zzz...but ~ 90 coeffs!**

$(P_X, P_Y = (1 \pm \gamma_5)/2)$, all operators with coeff $-2\sqrt{2}G_F C$.

operators at exptal scale

Kuno Okada

There are dipoles of 2 chiralities

$D \qquad \overline{e}\sigma^{\alpha\beta}P_L\mu F_{\alpha\beta} \qquad \overline{e}\sigma^{\alpha\beta}P_R\mu F_{\alpha\beta}$

which also contribute in $\mu\to e\gamma$, $\mu\to e\bar{e}e$.

Six 4-fermions for $\mu\to e\bar{e}e$, $Y, X \in \{L, R\}, Y \neq X$

$V \qquad (\overline{e}\gamma^\alpha P_Y\mu)(\overline{e}\gamma_\alpha P_Y e) \qquad (\overline{e}\gamma^\alpha P_Y\mu)(\overline{e}\gamma_\alpha P_X e)$

$S \qquad (\overline{e}P_Y\mu)(\overline{e}P_Y e)$

For $\mu A\to eA$, interactions with nucleons $N \in \{n, p\}$ parametrised by :

$S,\ V \qquad \overline{e}P_X\mu\overline{N}N \qquad \overline{e}\gamma^\alpha P_X\mu\overline{N}\gamma_\alpha N \qquad X \in \{L, R\}$

$A,\ T \qquad \overline{e}\gamma^\alpha P_X\mu\overline{N}\gamma_\alpha\gamma_5 N \qquad \overline{e}\sigma^{\alpha\beta}P_X\mu\overline{N}\sigma_{\alpha\beta}N$

$P,\ Der \qquad \overline{e}P_X\mu\overline{N}\gamma_5 N \qquad \overline{e}\gamma^\alpha P_X\mu(\overline{N}i\overleftrightarrow{\partial}_\alpha\gamma_5 N)$

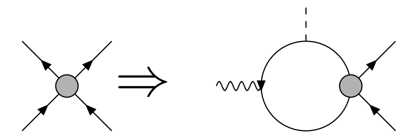
Matching in χPT gives Derivative. But absorb in matching into $G_O^{N,q}$= quark matrix elements in nucleons.

chiral basis for the lepton current (relativistic e), but not for the non-rel. nucleons.

"Accidental cancellations" and "naturalness" in EFT

("accidental" cancellations occur; in 1-loop RGEs give, for coeff.s at $\sim m_W$ of $\mathcal{O}_{D,L} = m_\mu(\overline{e}\sigma \cdot F P_L \mu)$, $\mathcal{O}^{\tau\tau}_{T,LL} = (\overline{e}\sigma P_L \mu)(\overline{\tau}\sigma P_L \tau)$:

$$BR(\mu \to e\gamma) \approx |0.938 C_{D,L} + 0.981 C^{\tau\tau}_{T,LL} + ...|^2$$)



If imagine that NP knows about all the SM parameters, but not about the scale at which you do expts, could argue that RG-stable cancellations in EFT can be "natural".
(caveat: NP does know about all the mass scales in the theory, which often determine the scales in the logs...)

So if resum RGs, cancellations among coeff.s with same anom dim are ok?
If not resum, can allow cancellations among all coeff.s who multiply same log?

Interest of this argument, is that forbidding "unnatural" calcellations transforms a single exptal bound into many bounds...but unnatural cancellations occur, see green parenthese: dipole is tree, tensor is log-enhanced loop.

Quantifying which targets give independent information (on nucleons)

1. neglect Dipole (better sensitivity of $\mu \to e\gamma$ (MEGII) and $\mu \to e\bar{e}e$(Mu3e).
   remain to determine: $\vec{C} \equiv (\widetilde{C}^{pp}_{VR}, \widetilde{C}^{pp}_{SL}, \widetilde{C}^{nn}_{VR}, \widetilde{C}^{nn}_{SL})$

2. recall that
   $$BR_{SI}(A\mu \to Ae) \propto \left|\vec{C} \cdot \vec{v}_A\right|^2$$
   where target vector for nucleus A
   $$\vec{v}_A \equiv \left(V_A^{(p)}, S_A^{(p)}, V_A^{(n)}, S_A^{(n)}\right)$$

3. So first experimental search (eg on Aluminium) probes projection of $\vec{C}$ of $\vec{v}_{Al}$
   ... next target needs to have component $\perp$ to Aluminium!
   $\Leftrightarrow$ plot misalignment angle θ between target vectors

4. how big does θ need to be?
   overlap integrals have theory uncertainty: $\Delta\theta \left\{ \begin{array}{cc} \text{nuclear} & \sim 5\%(KKO) \\ NLO\ \chi\text{PT} & \sim 10\%(?) \end{array} \right.$
   Both vectors uncertain by $\Delta\theta$; need misaligned by $2\Delta\theta \approx 10 \to 20\%$

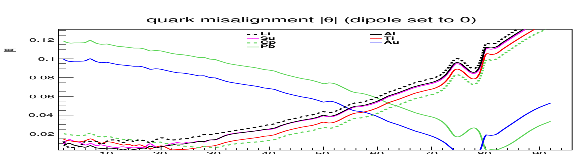
But what happens when match nucleons to quarks?

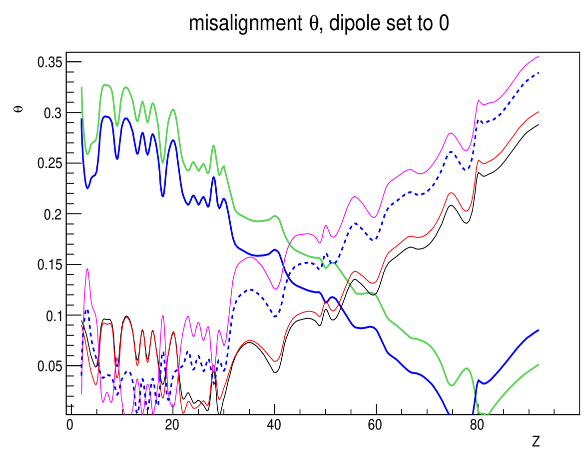
By measuring $\mu A \to eA$ on different targets, could determine coefficients of LFV ops with vector and scalar currents of n or p.

Match to quarks: $(\Gamma_O \in \{I, \gamma_5, \gamma^\alpha, \gamma^\beta\gamma_5, \sigma^{\alpha\beta}\})$

$$\begin{aligned} \langle N(P_f)|\bar{q}(x)\Gamma_O q(x)|N(P_i)\rangle &= G_O^{N,q}\langle N|\bar{N}(x)\Gamma_O N(x)|N\rangle \\ &= G_O^{N,q}\overline{u_N}(P_f)\Gamma_O u_N(P_i)e^{-i(P_f-P_i)x} \end{aligned}$$

But for scalar ops, $G_S^{p,u} = G_S^{n,d} \simeq G_S^{p,d} \simeq G_S^{n,u}$
so need great precision to differentiate LFV ops with scalar currents of u or d :(

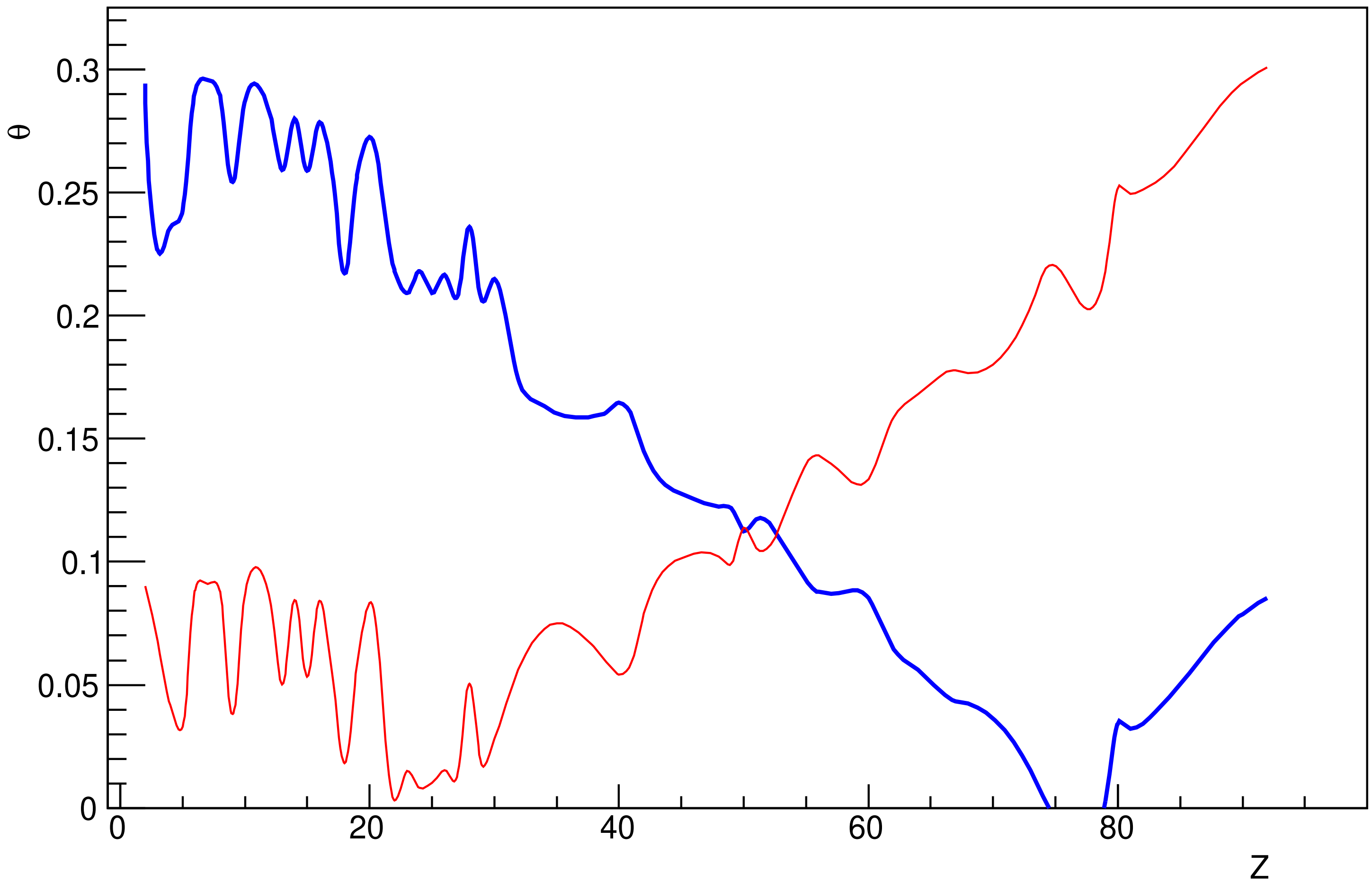




Current data+ theory uncertainty $\sim 10\%$: want $\Delta\theta > 0.2$

$$BR(\mu Au \to eAu) \leq 7 \times 10^{-13} \qquad (Au : Z = 79)$$

$$BR(\mu Ti \to eTi) \leq 4.3 \times 10^{-12} \qquad (Ti : Z = 22)$$



$\vec{v}_A = (V_A^{(p)}, S_A^{(p)}, V_A^{(n)}, S_A^{(n)})$, and $BR \propto |\vec{v}_A \cdot \vec{C}|^2$

$\vec{v}_{Au} \cdot \vec{v}_Z \equiv |\vec{v}_{Au}||\vec{v}_Z| \cos\theta$ ...plot θ on vertical axis

In the future...with a 5% theory uncertainty:

First target of Mu2e, COMET: Aluminium (Z=13, A=27)

$$\hat{v}_{Al} \approx \frac{1}{2}(1,1,1,1)$$

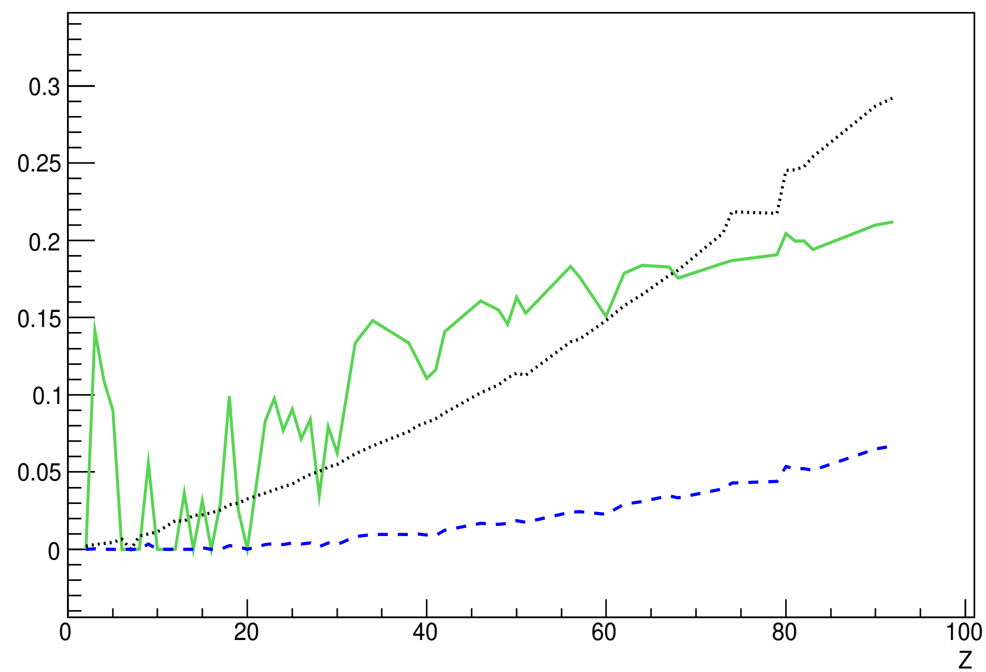
(recall $\tilde{C}_V^{pp}, \tilde{C}_S^{pp}, \tilde{C}_V^{nn}, \tilde{C}_S^{nn}$)

basis of three other "directions":

$$\hat{v}_{np} \equiv \frac{1}{2}(-1,-1,1,1)$$

$$\hat{v}_{VS} \equiv \frac{1}{2}(1,-1,1,-1)$$

$$\hat{v}_{IsoSV} \equiv \frac{1}{2}(-1,1,1,-1)$$

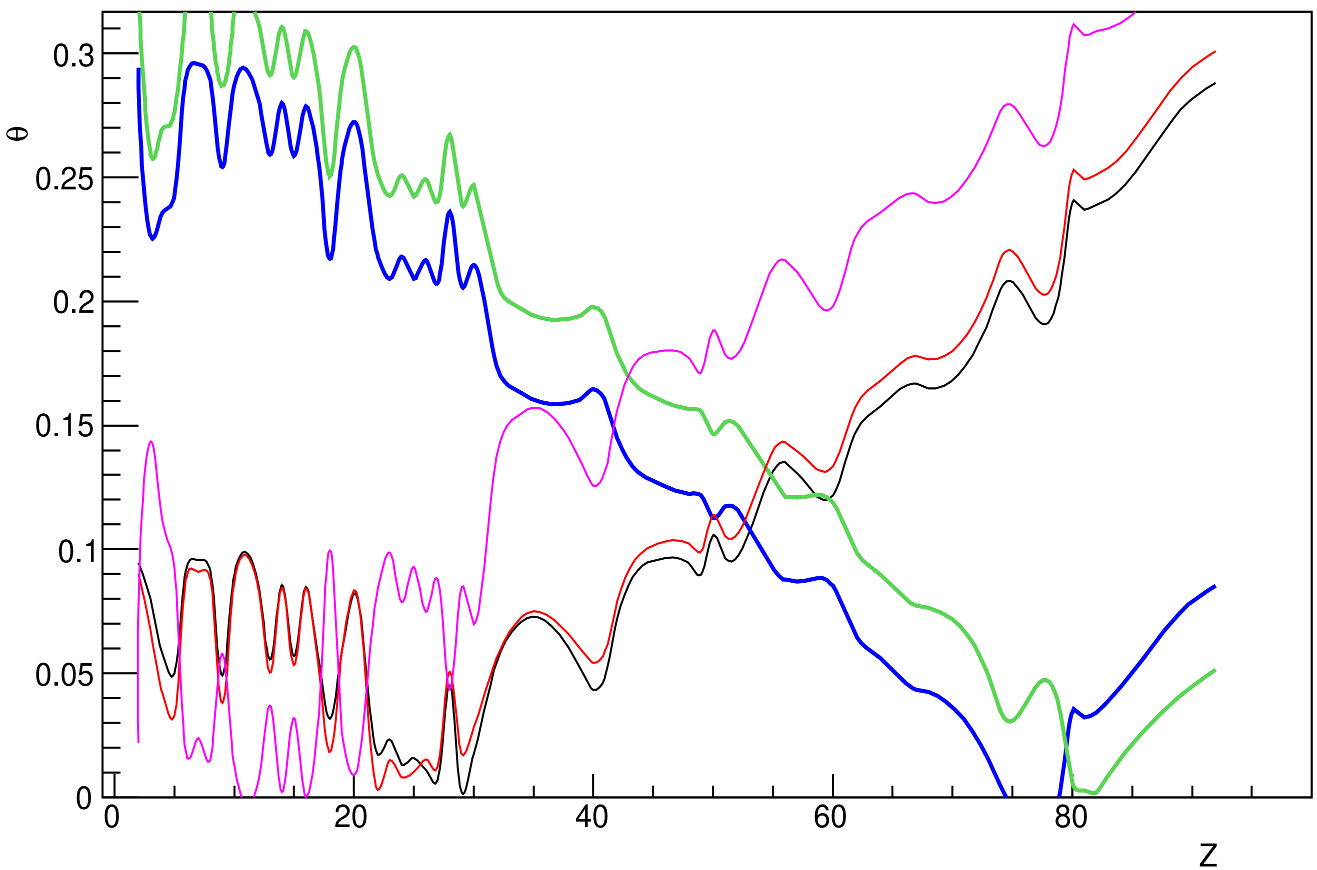


probe 3 combinations of SI coeffs

All current data...

$$BR(\mu Au \to eAu) \leq 7 \times 10^{-13} \qquad (Au : Z = 79)$$

$$BR(\mu Ti \to eTi) \leq 4.3 \times 10^{-12} \qquad (Ti : Z = 22)$$



$$BR(\mu Pb \to ePb) \leq 4.6 \times 10^{-11}$$

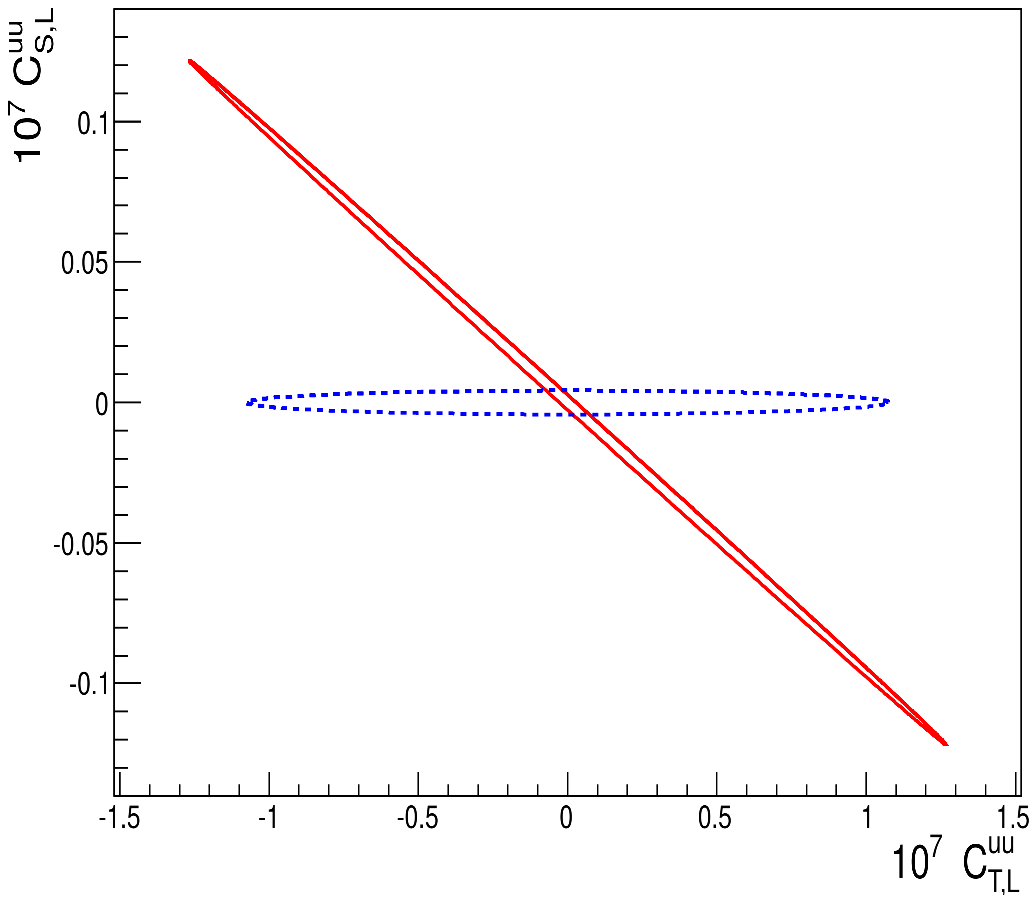
$BR(\mu S \to eS) \leq 7 \times 10^{-11}$ S = Sulpher, Z = 16

$BR(\mu Cu \to eCu) \leq 1.6 \times 10^{-8}$ Cu = Copper, Z = 29

sensitivity *vs* constraint

Suppose that $BR(\mu Al \to eAl) \lesssim 10^{-14}$, and :

$$\delta\mathcal{L}(m_W) = C_T^{uu}(\overline{e}\sigma P_Y\mu)(\overline{u}\sigma u) + C_S^{uu}(\overline{e}P_Y\mu)(\overline{u}u)$$



C_T^{uu}, C_S^{uu} constrained to live inside blue (red) ellipse at exptal scale (at m_W): sensitivity to C_S^{uu} = cut ellipse @ $C_T^{uu} = 0$; constraint = live in projection of ellipse onto C_S^{uu} axis.