Confronting GUTs with Proton Decay and Gravitational Waves

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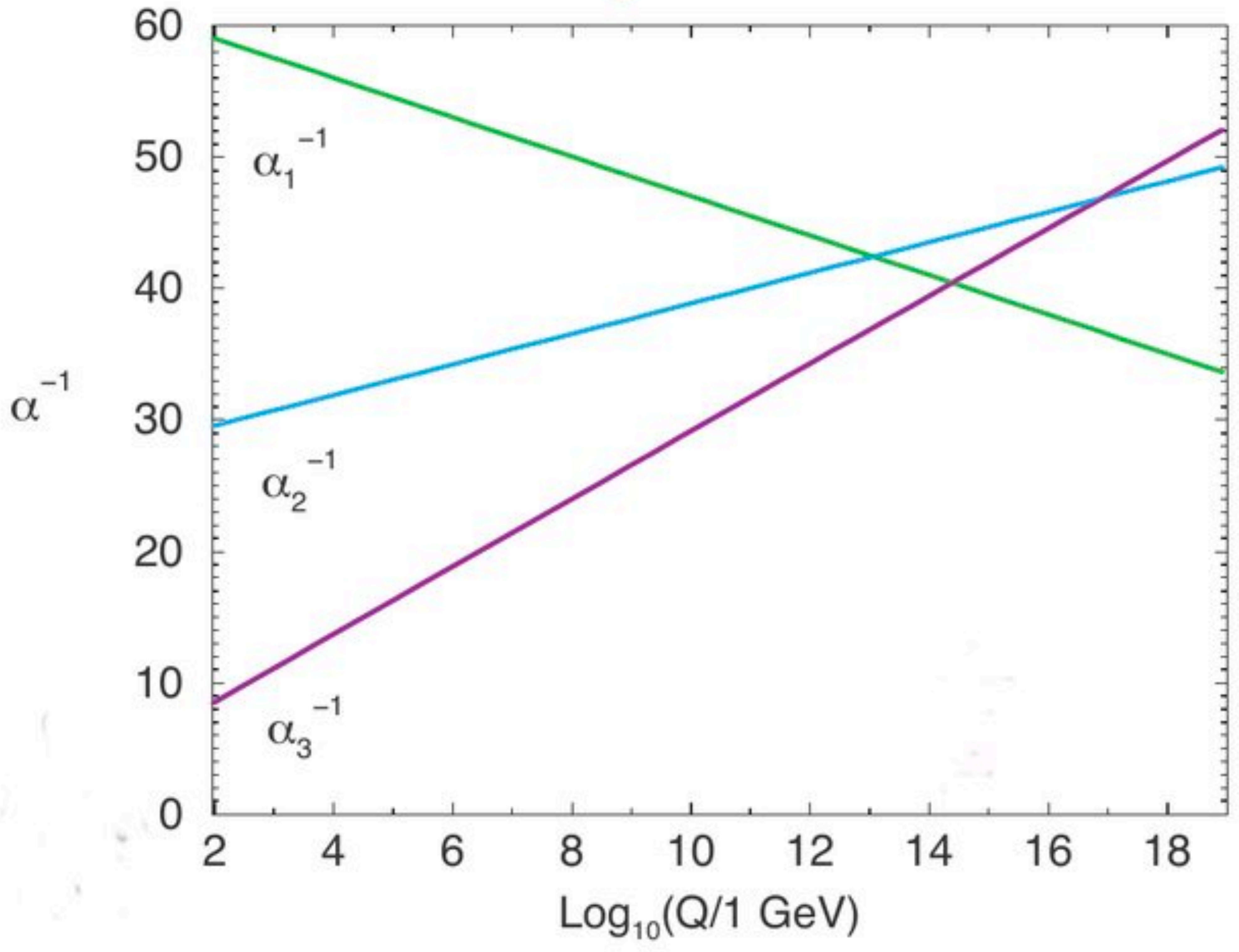
Based on 2005.13549, 2106.15634, 2209.00021

Motivation for Grand Unified Theories

- 19 **free and seemingly arbitrary** Standard Model parameters
- GUT unifies SM gauge interactions into a **single gauge group**
- SM fermionic multiplet $\Longrightarrow$ single GUT irrep $\Longrightarrow$ **reduces # parameters**
- Many GUTs predict non-zero **neutrino masses**, dark matter candidate etc



GUT Predictions - Proton Decay

- GUTs unify leptons and quarks into common multiplets.

- GUTs spontaneously broken to SM gauge group $\Longrightarrow$ heavy gauge boson integrated out $\Longrightarrow$ **baryon number violating process** e.g. proton decay

Weinberg, 1979

$$\frac{\epsilon_{\alpha\beta}}{\Lambda_1^2}\left[\left(\overline{u_R^c}\gamma^\mu Q_\alpha\right)\left(\overline{d_R^c}\gamma_\mu L_\beta\right)+\left(\overline{u_R^c}\gamma^\mu Q_\alpha\right)\left(\overline{e_R^c}\gamma_\mu Q_\beta\right)\right]$$
$$+\frac{\epsilon_{\alpha\beta}}{\Lambda_2^2}\left[\left(\overline{d_R^c}\gamma^\mu Q_\alpha\right)\left(\overline{u_R^c}\gamma_\mu L_\beta\right)+\left(\overline{d_R^c}\gamma^\mu Q_\alpha\right)\left(\overline{\nu_R^c}\gamma_\mu Q_\beta\right)\right]$$

$\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ invariant but BNV

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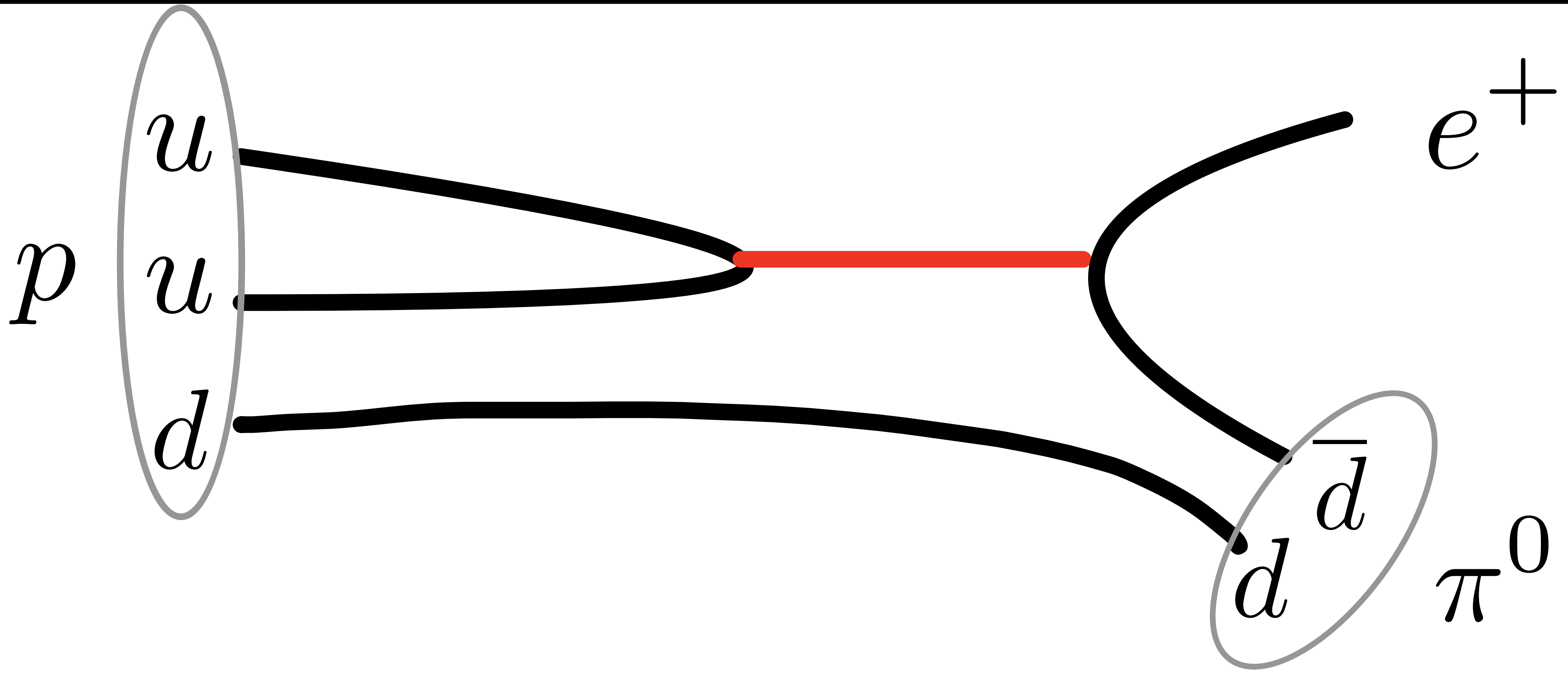
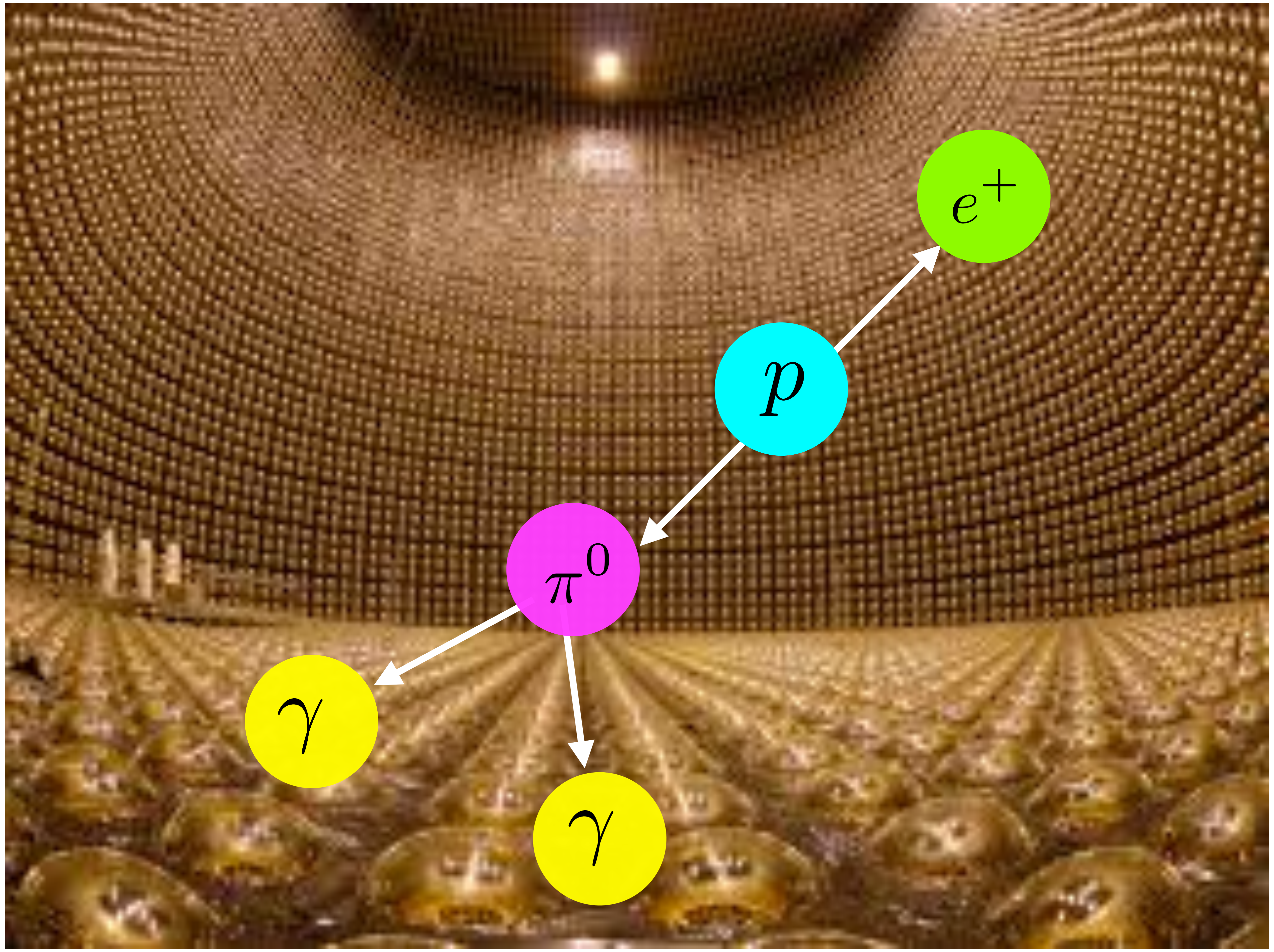


Golden Channel
In non-SUSY GUTs

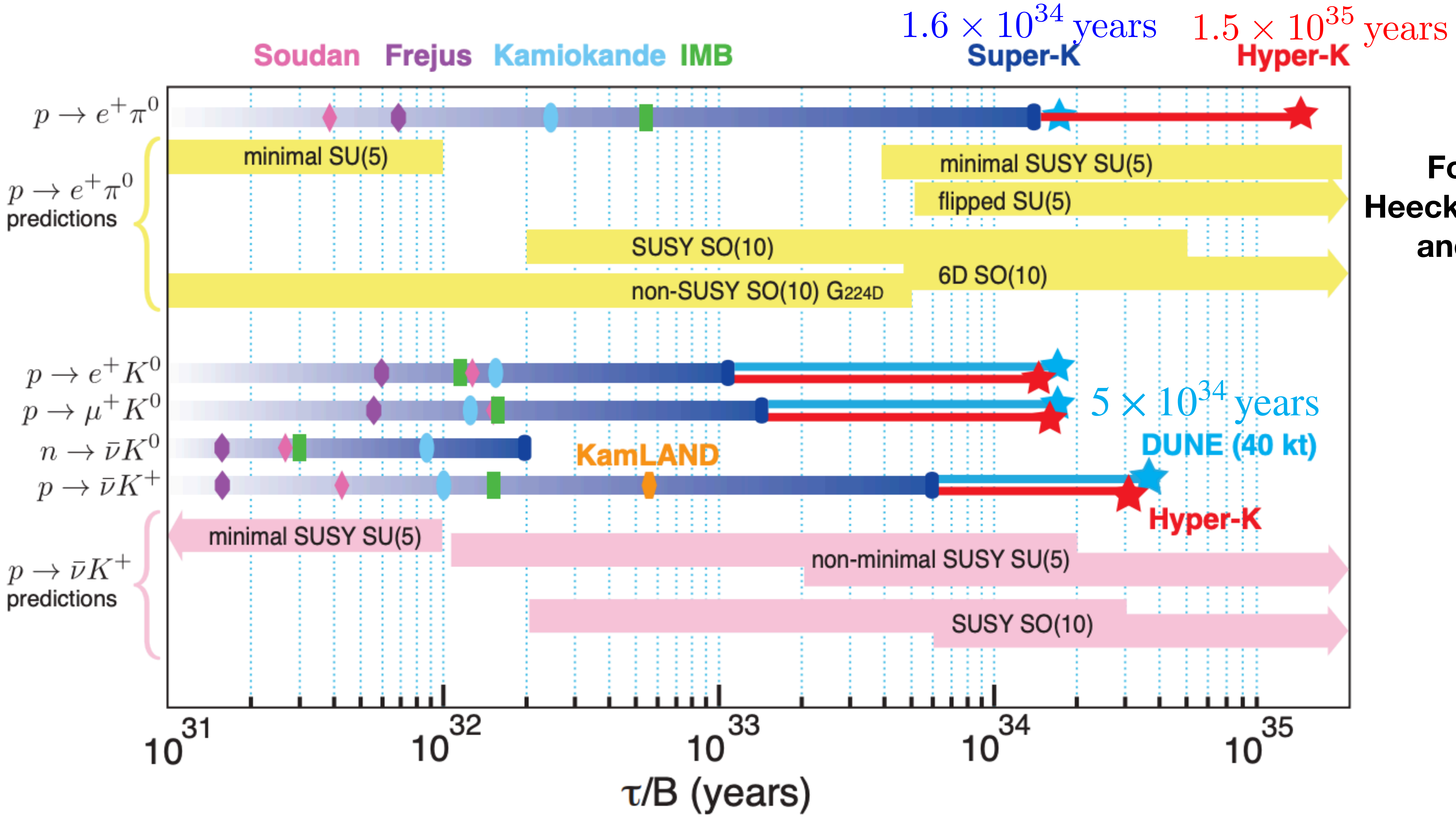
World Leading Limit on Proton Decay

Neutrino experiments are large vats of proton sitting around for a long time.

$$\tau_{\pi^0 e^+} > 1.6 \times 10^{34} \text{ years}$$ SK (1610.03597)



Future Limits from Neutrino Experiments



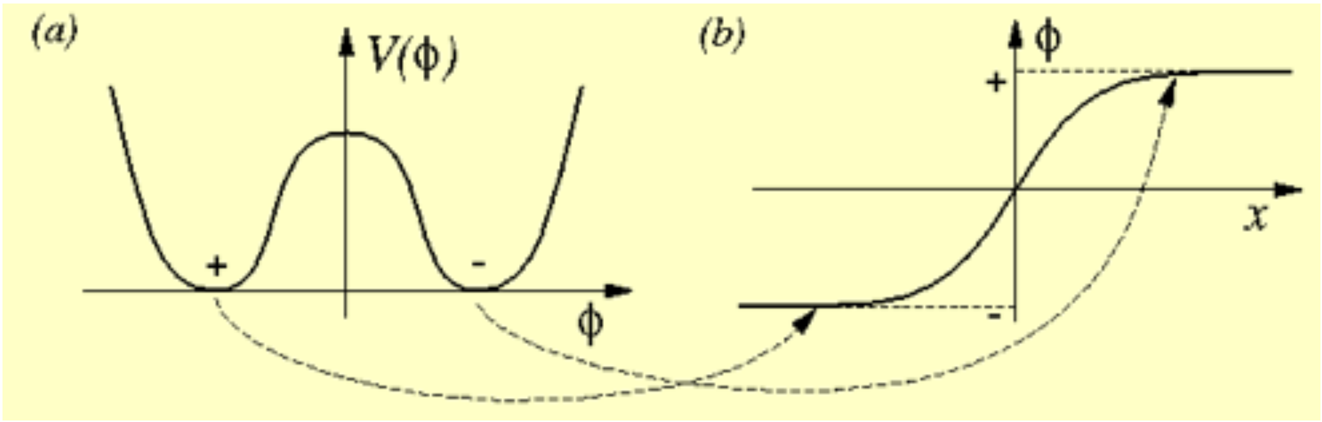
For review see
Heeck etal: 1910.07647
and B. Pal's talk

Water Cherenkov sensitive to $p \to e^+\pi^0$ LArTPC more sensitive to $p \to K^+\nu$

GUT Predictions - Topological Defects

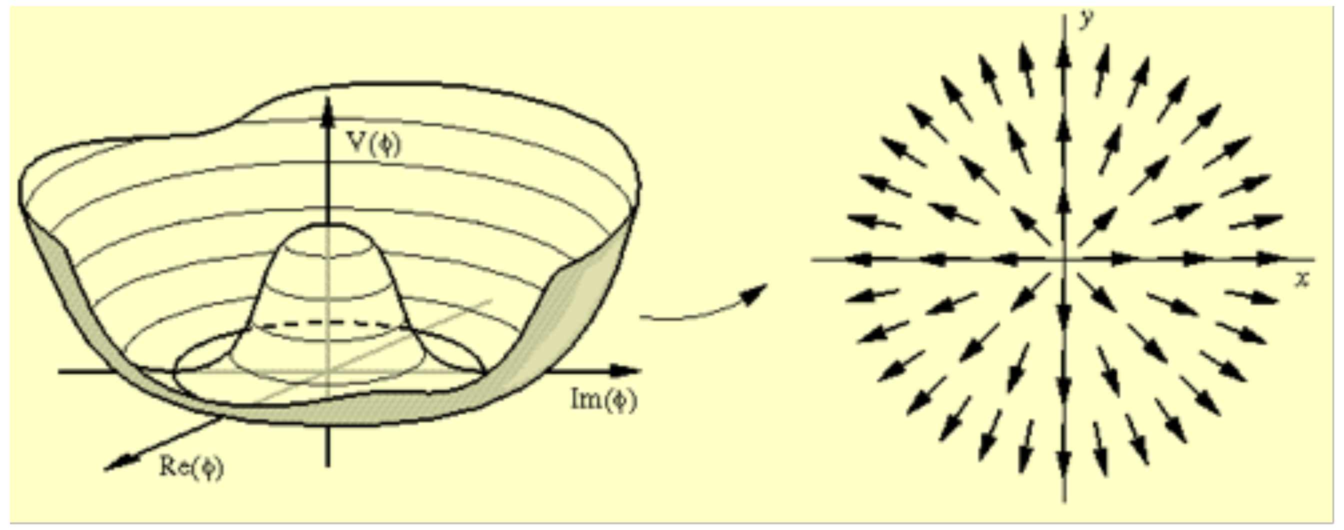
During SSB from $G_{GUT} \to \cdots \to G_{SM}$ topological defects may form.

Domain wall



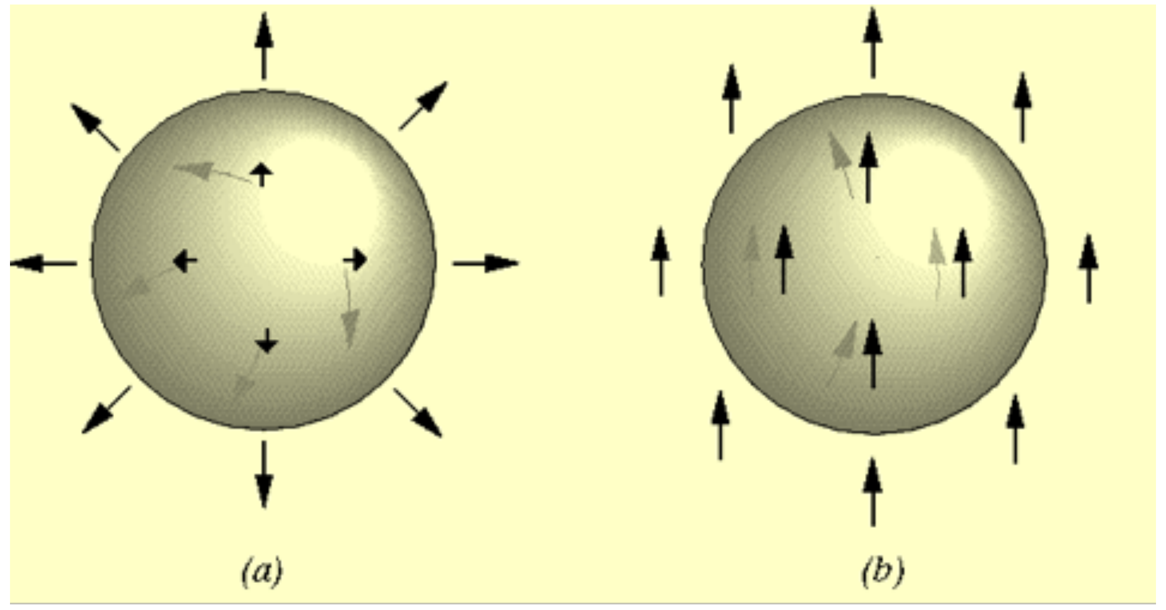
$\pi_0(\mathrm{G/H}) \neq 0$

Cosmic strings



$\pi_1(\mathrm{G/H}) \neq 0$

Monopoles



$\pi_2(\mathrm{G/H}) \neq 0$

cambridge cosmic structures

GUT Predictions - Topological Defects

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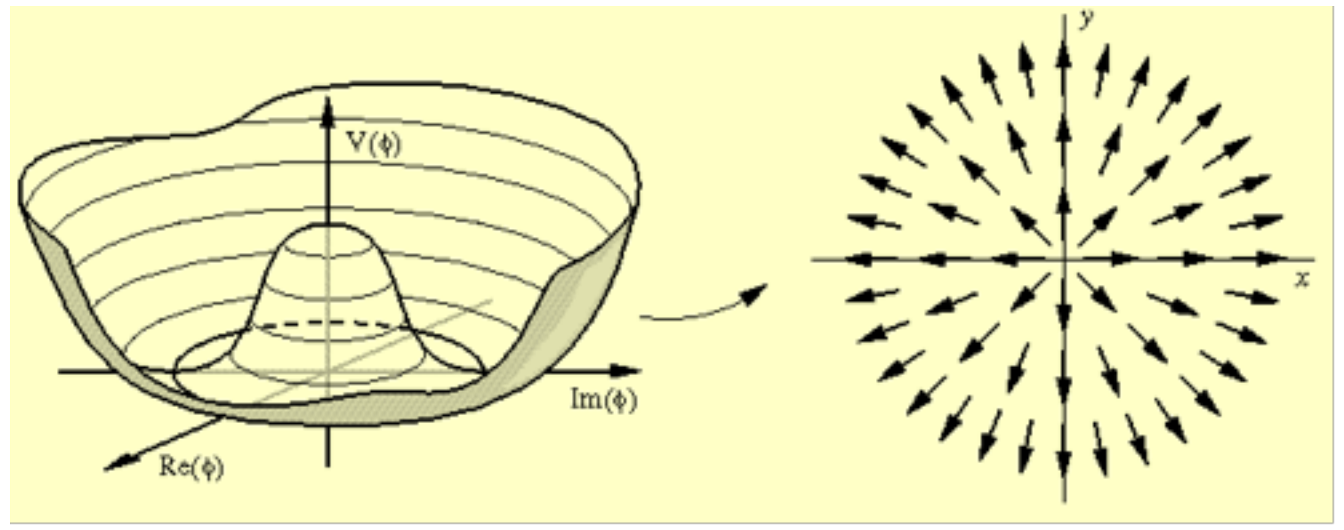
Domain w

Inflation to dilute

$\pi_0(\mathrm{G/H}) \neq 0$

Cosmic strings



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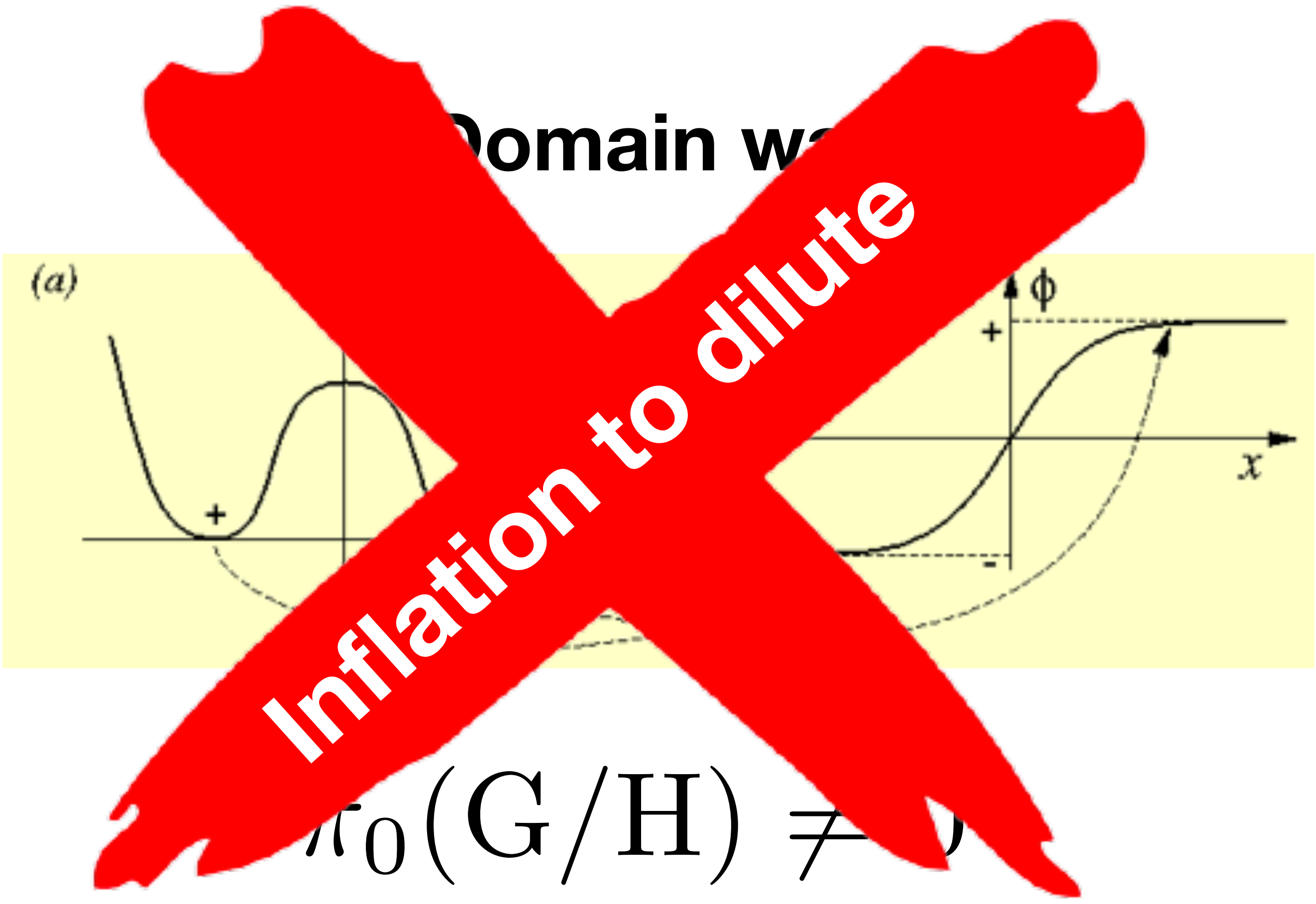

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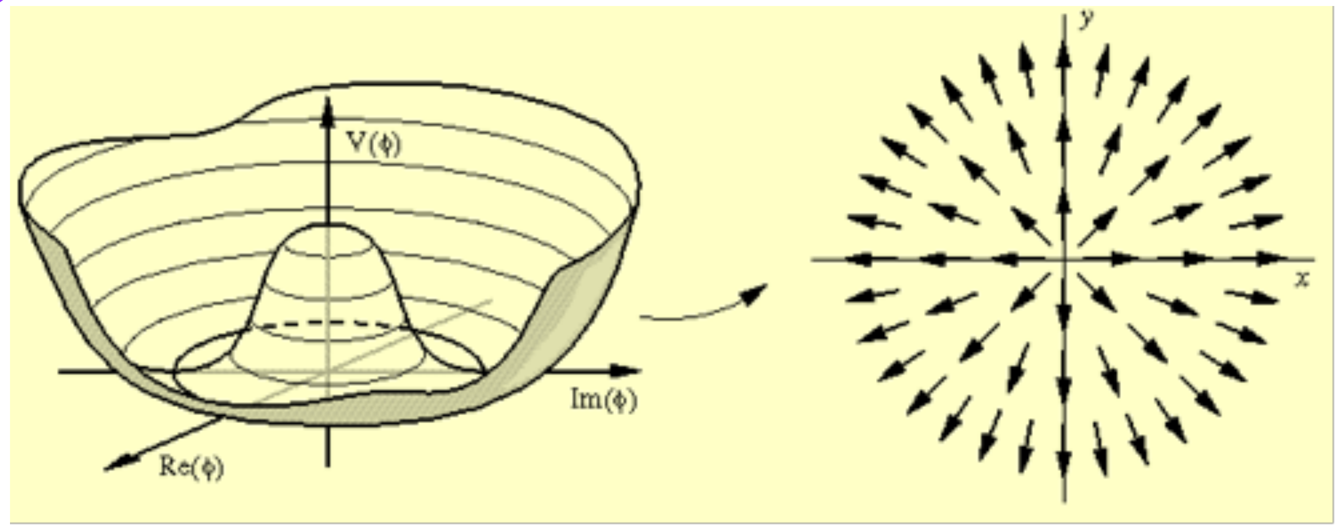

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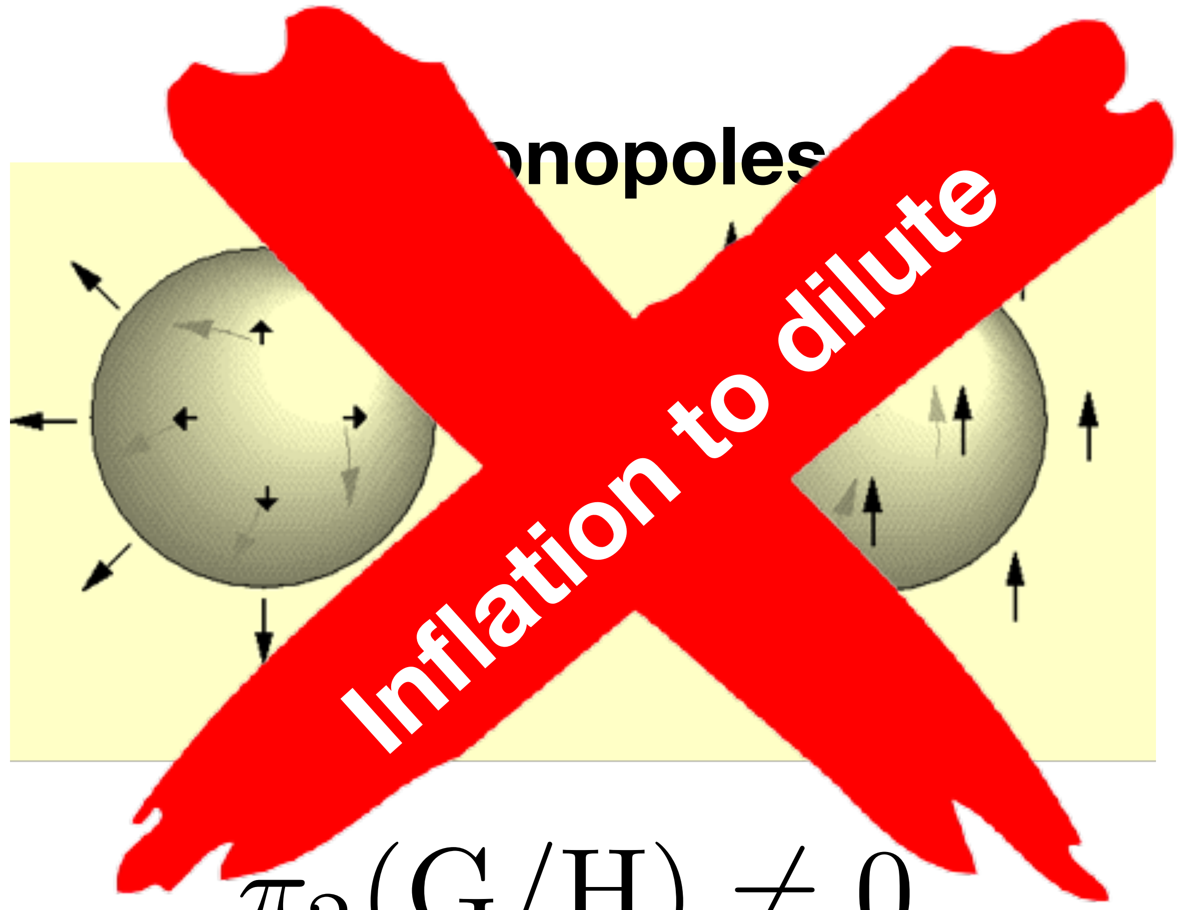


$\pi_2(\mathrm{G/H}) \neq 0$

<u>*cambridge cosmic structures*</u>

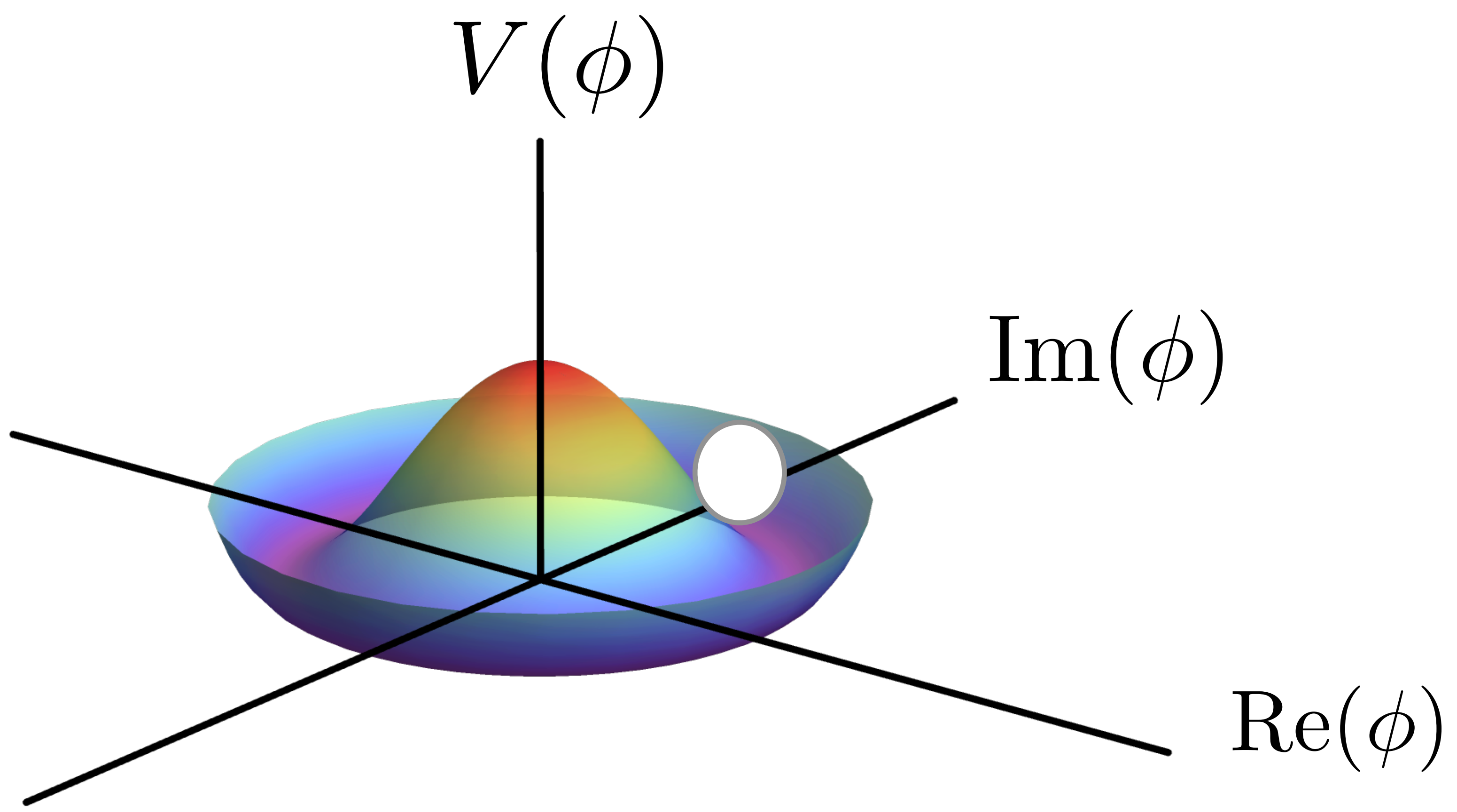
Cosmic strings induced via U(1) breaking are ubiquitously as GUT breaks to SM

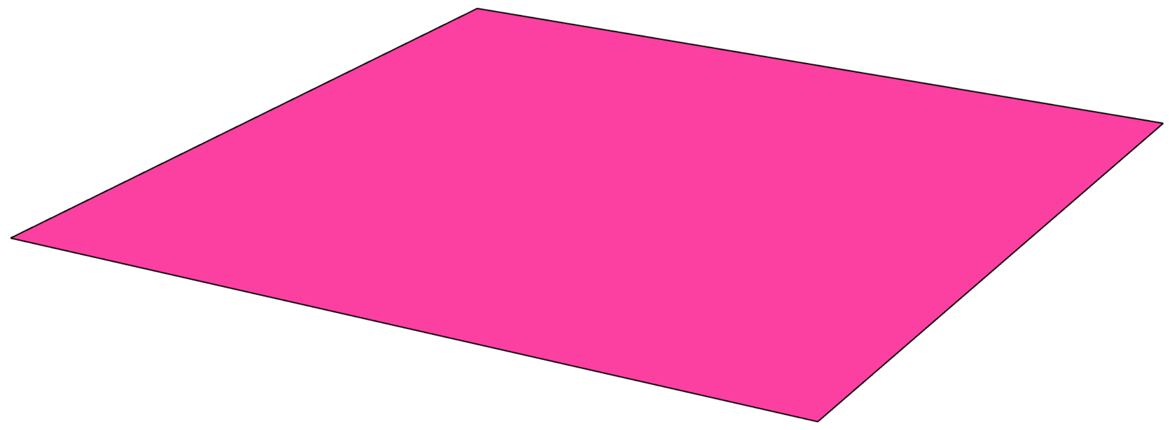
GUT Predictions - Topological Defects

Abelian Higgs Model

$$S_{U(1)} = \int d^4x \left[\partial_\mu \phi \partial^\mu \phi^* - V\left(|\phi|^2\right)\right]$$

$$V(\phi) = \frac{\lambda}{4}\left(|\phi|^2 - \eta^2\right)^2$$





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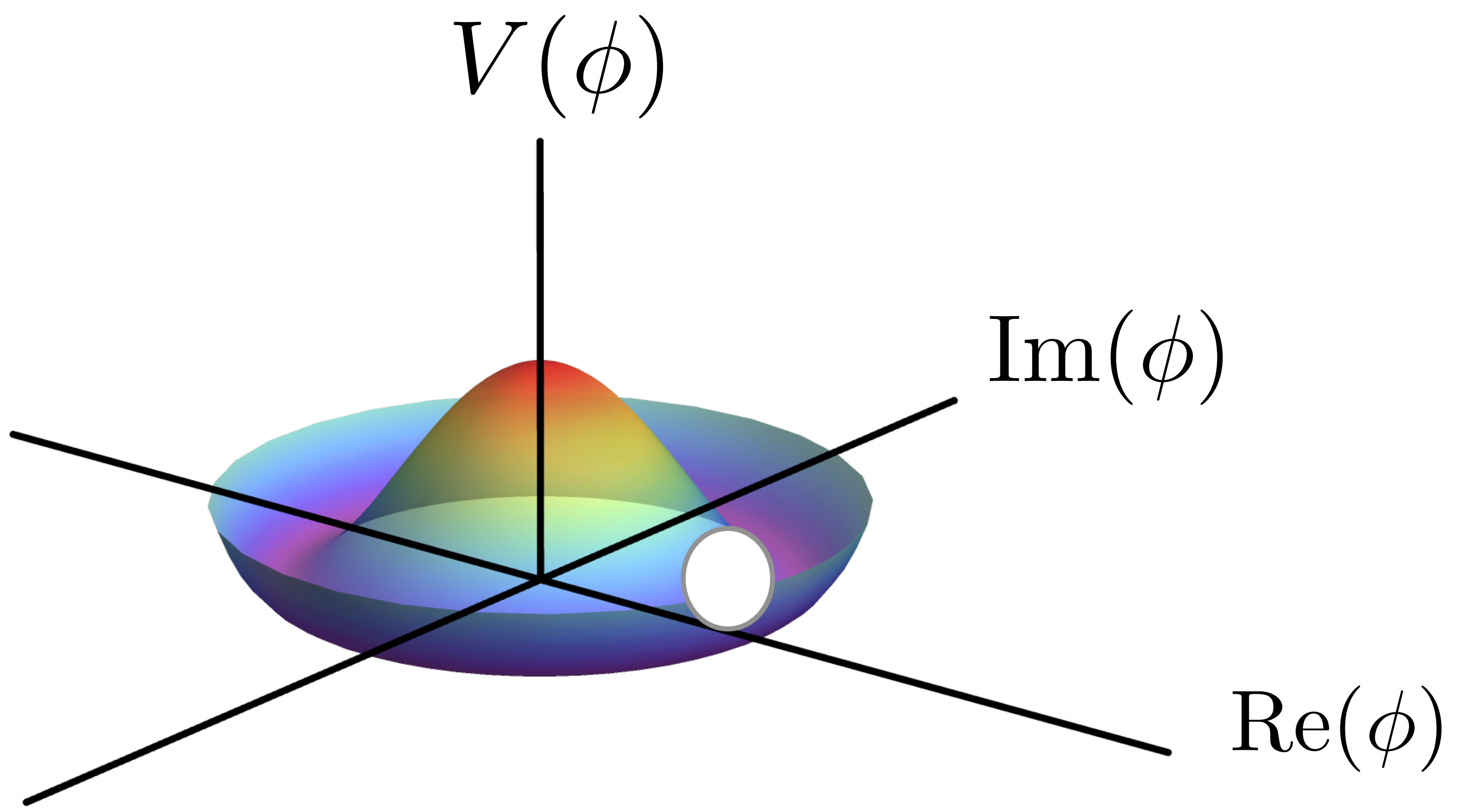

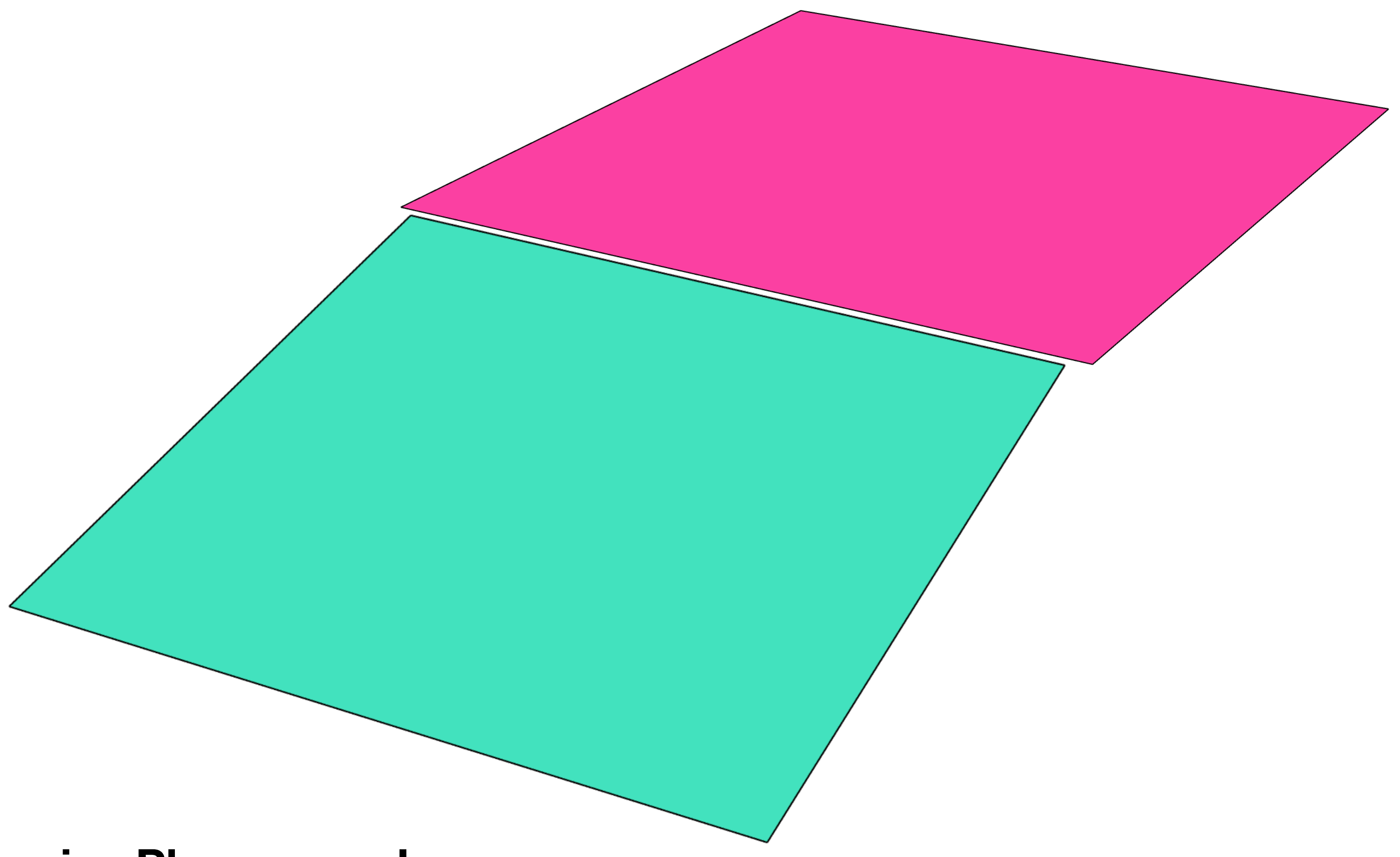

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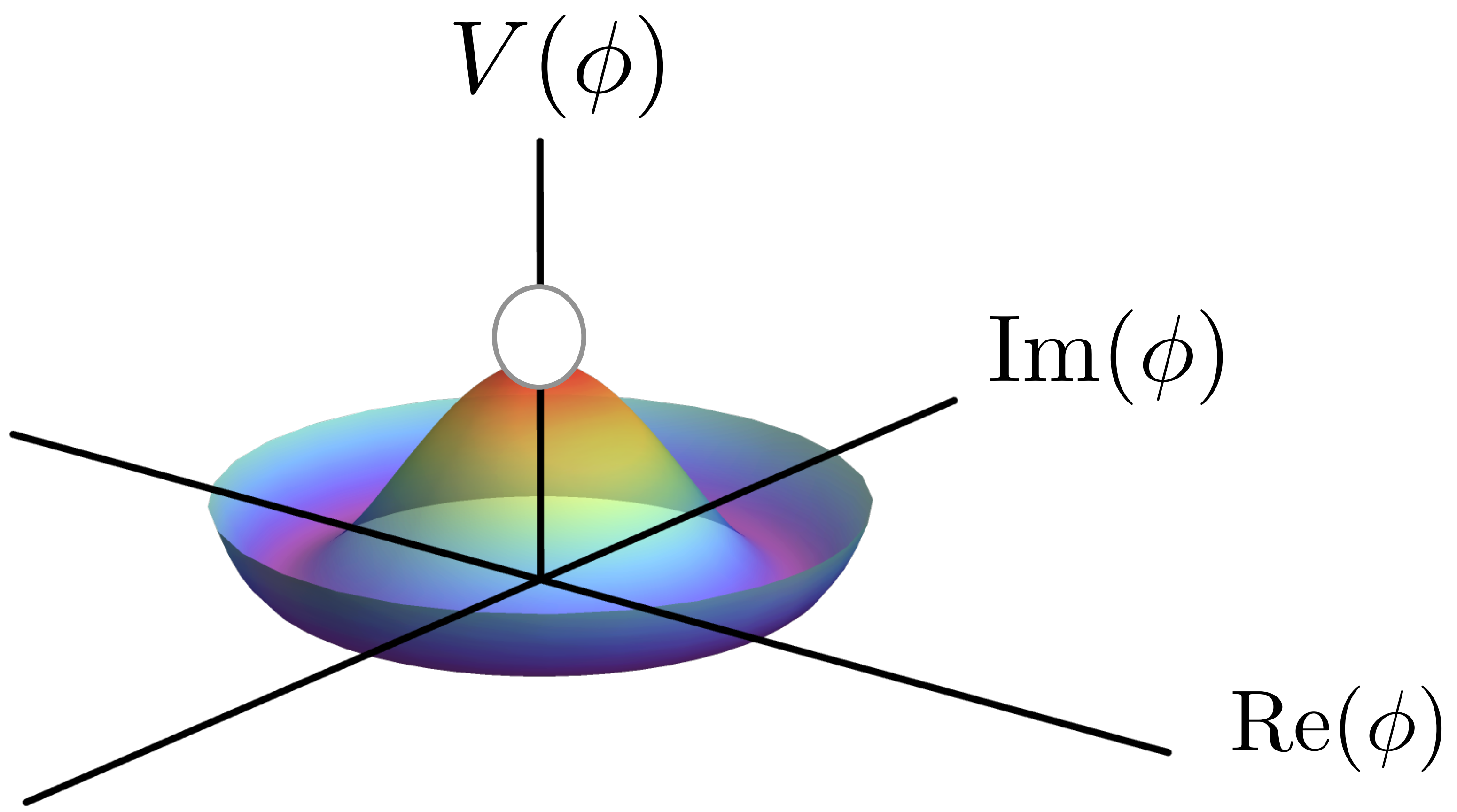

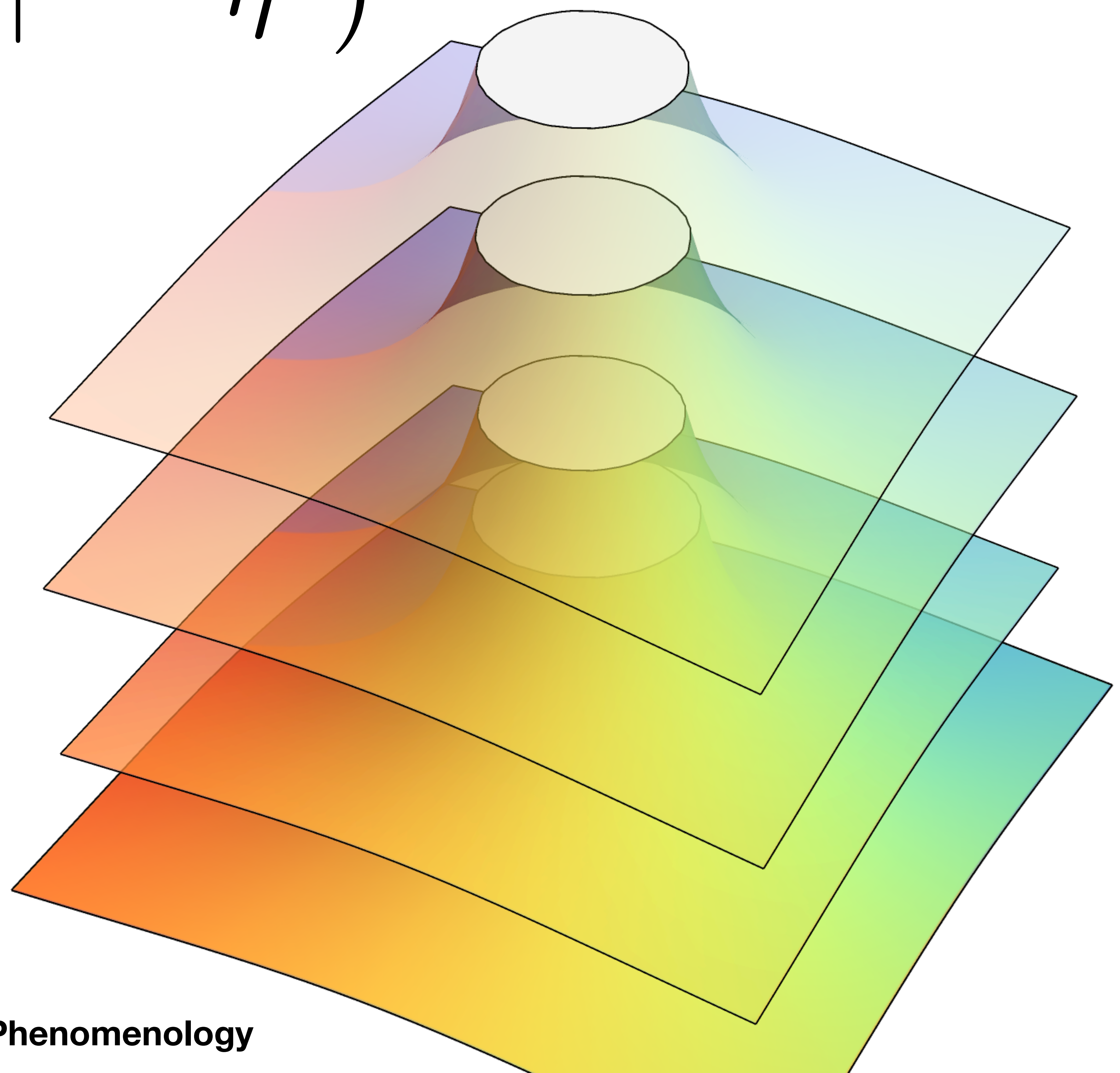

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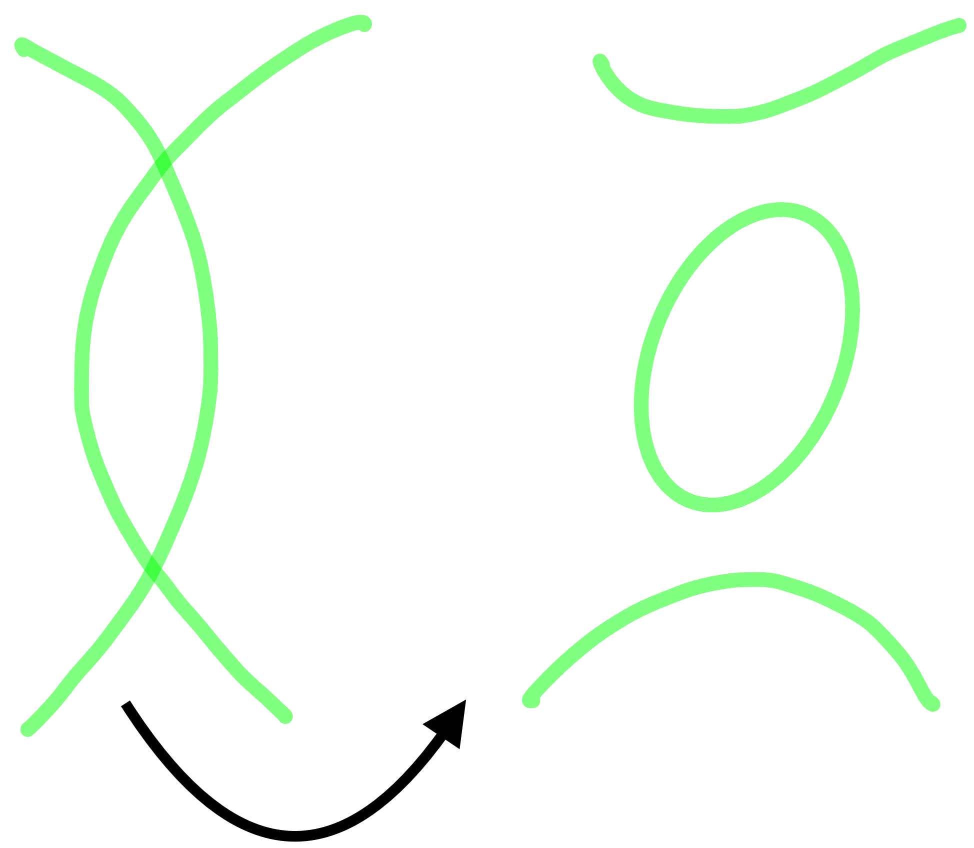
Kibble, & Nielsen, Ole



Strings intercommute $\implies$ closed loops
Oscillate and emit GWs

Scaling Solution →

$$\frac{\rho_{\rm string}}{\rho_{\rm tot}} \propto G\mu \sim \left(\frac{\langle\phi\rangle}{M_{\rm pl}}\right)^2$$

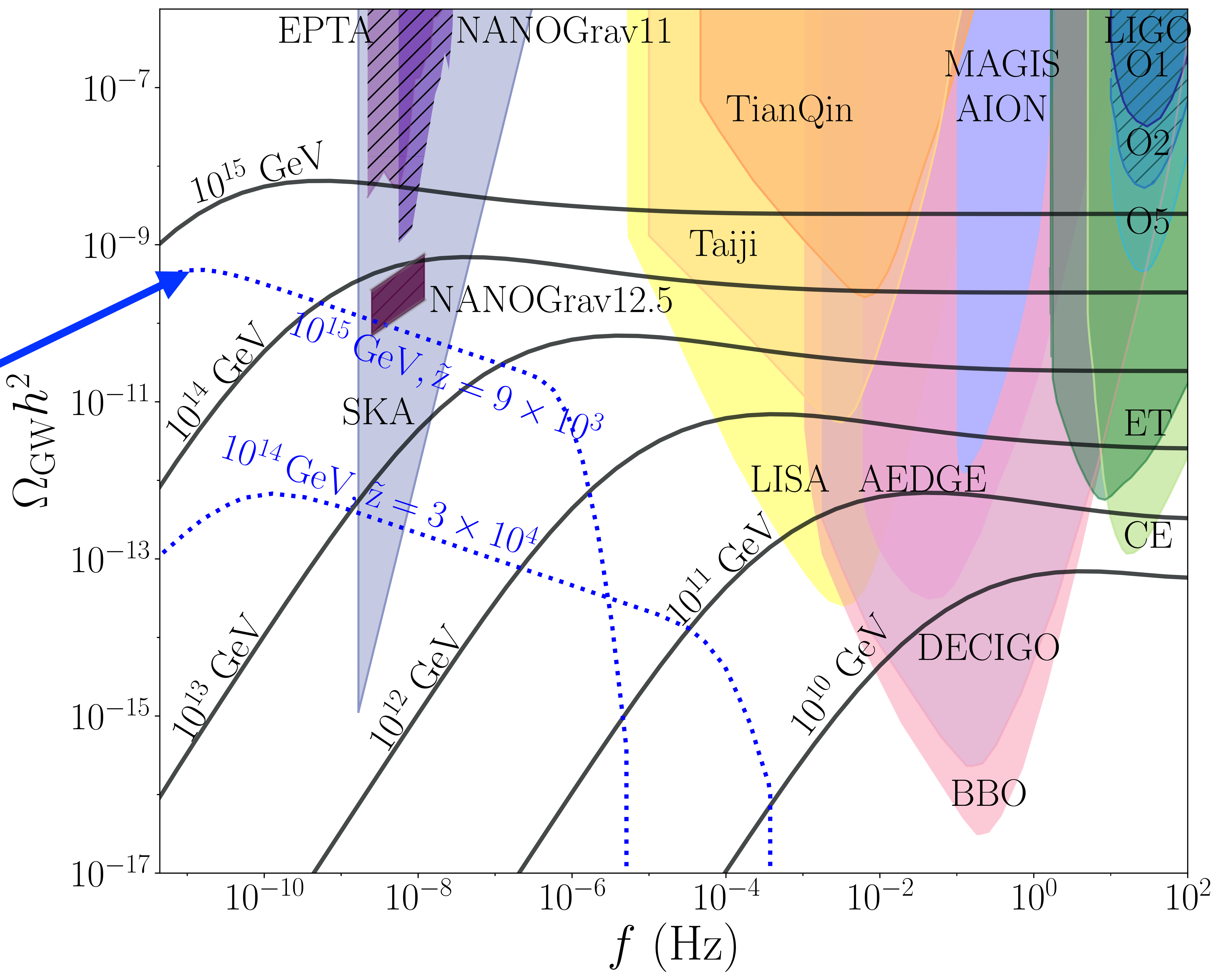
$$\mu \equiv \text{string parameter}$$

GUT Predictions - Cosmic Strings

- Inflation occurs **before** string formation → string network gives "scaling" solution
- Inflation occurs **after** string formation → string network diluted and **no GW signal**
- Inflation occurs **during** string formation → partly diluted string network → **GW spectrum broken power law behaviour** (Cui, Lewicki, Morrissey) *1912.08832*

Diluted by inflation

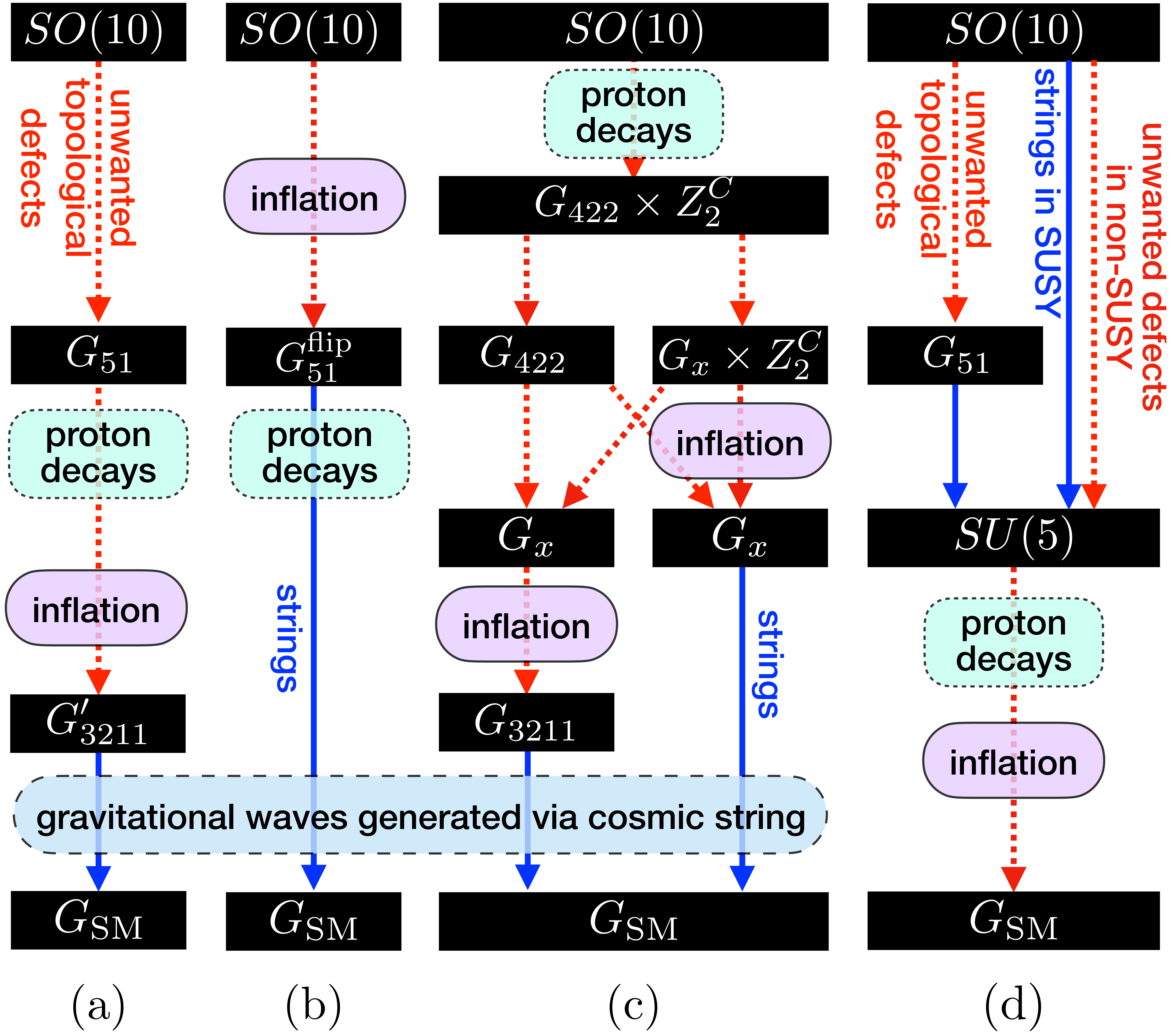
$$\Lambda_{\rm cs} \equiv \sqrt{\mu}$$



$$\Lambda_{\rm cs} = 10^{14}\,{\rm GeV}$$

$$G\mu = 0.7 \times 10^{-10}$$

Topological defects in non-supersymmetric $SO(10)$



Jeannerot et al classified all GUT "breaking chains" 0308134

- **2005.13549 King, Pascoli, JT, Zhou** use PD and GWs to examine viable non-SUSY SO(10) GUT breaking chains.

$$G_x = G_{3221} \text{ or } G_{421}$$

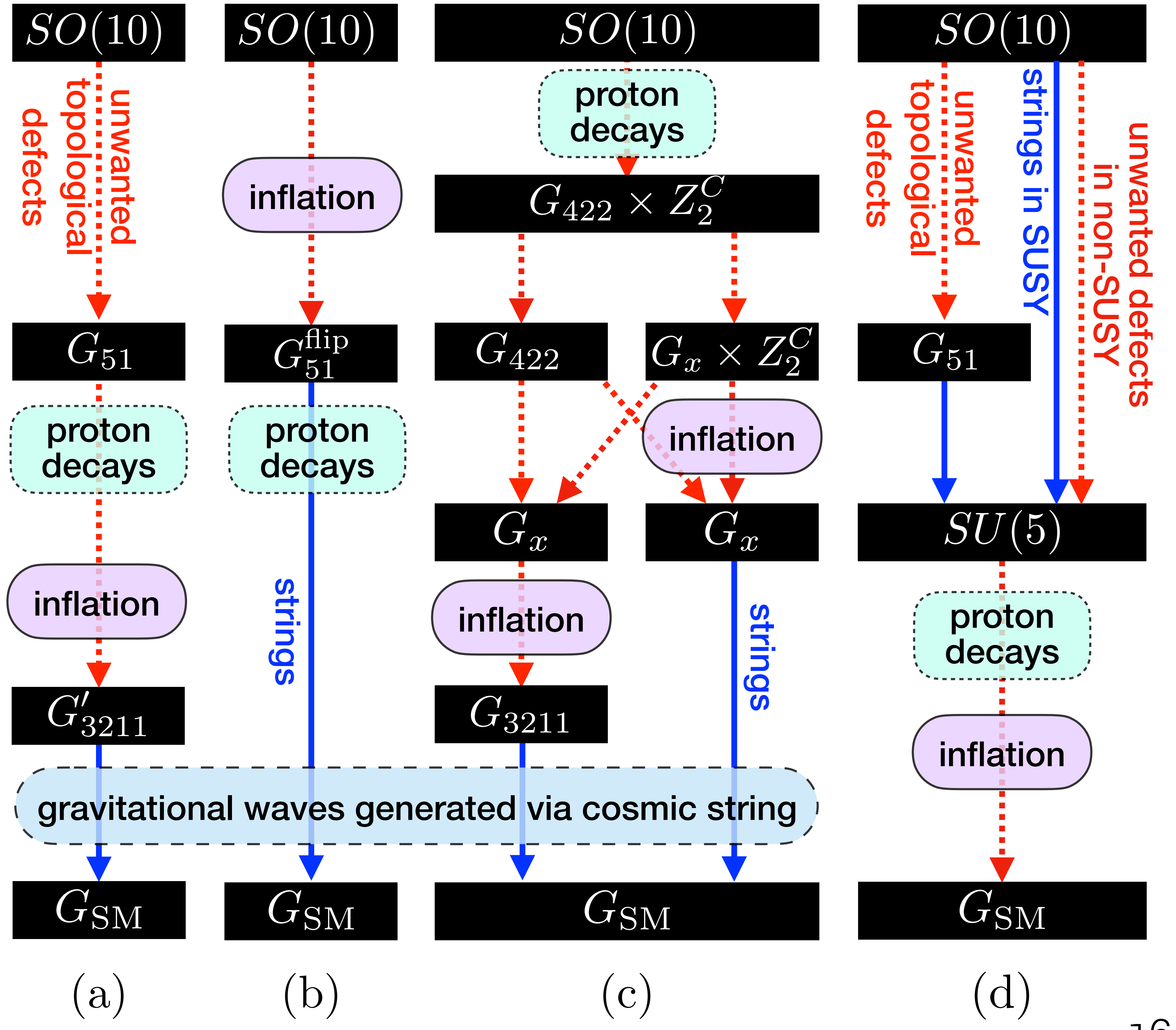
- Assume **inflation at highest scale to remove unwanted defects** and preserve cosmic strings

- GW & GUTs also explored by Buchmuller et al 1912.03695

Proton decay and GWs as complementary windows

- Type (a) via SU(5) x U(1): $\Lambda_{pd} > \Lambda_{cs}$
- Type (b) via flipped SU(5) x U(1): $\Lambda_{pd} \sim \Lambda_{cs}$
- Type (c) via SU(4) x SU(2)$_L$ x SU(2)$_R$: $\Lambda_{pd} > \Lambda_{cs}$
- Type (d) via SU(5) no GWs

Observables		Proton decays
		$p \to \pi^0 e^+$ observed $\Rightarrow$ non-SUSY contribution indicated
GWs	Observed	• types (a) and (c) favoured • types (b) and (d) excluded
	Marginal	• types (a) and (c) favoured • type (d) excluded • type (b) allowed if $p \to K^+ \bar{\nu}$ not observed and $\Lambda_{pd} \sim \Lambda_{cs}$



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Further study in 2106.15634
31 breaking chain all provide **unification & GW signal**

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GWs	Observed	• types (a) and (c) favoured • types (b) and (d) excluded
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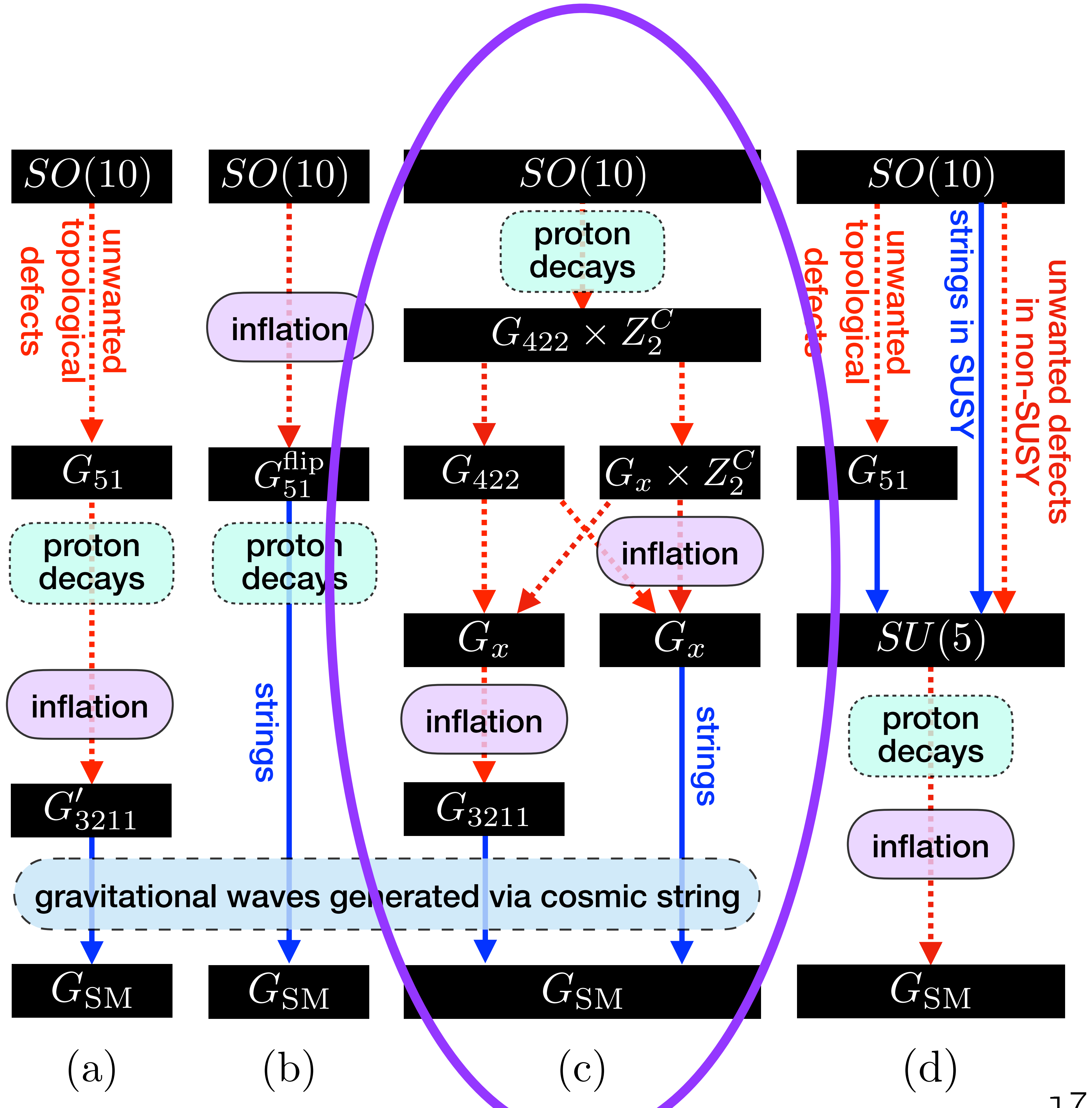

Proton decay and GWs as complementary windows

$SO(10)$	$\xrightarrow[\text{Higgs}]{\text{defect}}$	G_1	$\xrightarrow[\text{Higgs}]{\text{defect}}$	G_{SM}	Observable strings?
I1:	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{3221}	$\xrightarrow[\overline{\mathbf{126}}]{\text{s}}$		✓
I2:	$\xrightarrow[\mathbf{210}]{\text{m,s}}$	G^C_{3221}	$\xrightarrow[\overline{\mathbf{126}}]{\text{s,w}}$		✗
I3:	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{421}	$\xrightarrow[\overline{\mathbf{126}}]{\text{s}}$		✓
I4:	$\xrightarrow[\mathbf{210}]{\text{m}}$	G_{422}	$\xrightarrow[\overline{\mathbf{126}},\mathbf{45}]{\text{m}}$		✗
I5:	$\xrightarrow[\mathbf{54}]{\text{m,s}}$	G^C_{422}	$\xrightarrow[\overline{\mathbf{126}},\mathbf{45}]{\text{m,w}}$		✗
I6:	$\xrightarrow[\mathbf{210}]{\text{m}}$	G_{3211}	$\xrightarrow[\overline{\mathbf{126}}]{\text{s}}$		✓

$SO(10)$	$\xrightarrow[\text{Higgs}]{\text{defect}}$	G_2	$\xrightarrow[\text{Higgs}]{\text{defect}}$	G_1	$\xrightarrow[\text{Higgs}]{\text{defect}}$	G_{SM}	Observable strings?
II1:	$\xrightarrow[\mathbf{210}]{\text{m}}$	G_{422}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{3221}	$\xrightarrow[\overline{\mathbf{126}}]{\text{s}}$		✓
II2:	$\xrightarrow[\mathbf{54}]{\text{m,s}}$	G^C_{422}	$\xrightarrow[\mathbf{210}]{\text{m}}$	G^C_{3221}	$\xrightarrow[\overline{\mathbf{126}}]{\text{s,w}}$		✗
II3:	$\xrightarrow[\mathbf{54}]{\text{m,s}}$	G^C_{422}	$\xrightarrow[\mathbf{45}]{\text{m,w}}$	G_{3221}	$\xrightarrow[\overline{\mathbf{126}}]{\text{s}}$		✓
II4:	$\xrightarrow[\mathbf{210}]{\text{m,s}}$	G^C_{3221}	$\xrightarrow[\mathbf{45}]{\text{w}}$	G_{3221}	$\xrightarrow[\overline{\mathbf{126}}]{\text{s}}$		✓
II5:	$\xrightarrow[\mathbf{210}]{\text{m}}$	G_{422}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{421}	$\xrightarrow[\overline{\mathbf{126}}]{\text{s}}$		✓
II6:	$\xrightarrow[\mathbf{54}]{\text{m,s}}$	G^C_{422}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{421}	$\xrightarrow[\overline{\mathbf{126}}]{\text{s}}$		✓
II7:	$\xrightarrow[\mathbf{54}]{\text{m,s}}$	G^C_{422}	$\xrightarrow[\mathbf{210}]{\text{w}}$	G_{422}	$\xrightarrow[\overline{\mathbf{126}},\mathbf{45}]{\text{m}}$		✗
II8:	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{3221}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{3211}	$\xrightarrow[\overline{\mathbf{126}}]{\text{s}}$		✓
II9:	$\xrightarrow[\mathbf{210}]{\text{m,s}}$	G^C_{3221}	$\xrightarrow[\mathbf{45}]{\text{m,w}}$	G_{3211}	$\xrightarrow[\overline{\mathbf{126}}]{\text{s}}$		✓
II10:	$\xrightarrow[\mathbf{210}]{\text{m}}$	G_{422}	$\xrightarrow[\mathbf{210}]{\text{m}}$	G_{3211}	$\xrightarrow[\overline{\mathbf{126}}]{\text{s}}$		✓
II11:	$\xrightarrow[\mathbf{54}]{\text{m,s}}$	G^C_{422}	$\xrightarrow[\mathbf{210}]{\text{m,w}}$	G_{3211}	$\xrightarrow[\overline{\mathbf{126}}]{\text{s}}$		✓
II12:	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{421}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{3211}	$\xrightarrow[\overline{\mathbf{126}}]{\text{s}}$		✓

$SO(10)$	$\xrightarrow[\text{Higgs}]{\text{defect}}$	G_3	$\xrightarrow[\text{Higgs}]{\text{defect}}$	G_2	$\xrightarrow[\text{Higgs}]{\text{defect}}$	G_1	$\xrightarrow[\text{Higgs}]{\text{defect}}$	G_{SM}	Observable strings?
III1:	$\xrightarrow[\mathbf{54}]{\text{m,s}}$	G^C_{422}	$\xrightarrow[\mathbf{210}]{\text{w}}$	G_{422}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{421}	$\xrightarrow[\overline{\mathbf{126}}]{\text{s}}$		✓
III2:	$\xrightarrow[\mathbf{54}]{\text{m,s}}$	G^C_{422}	$\xrightarrow[\mathbf{210}]{\text{w}}$	G_{422}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{3221}	$\xrightarrow[\overline{\mathbf{126}}]{\text{s}}$		✓
III3:	$\xrightarrow[\mathbf{54}]{\text{m,s}}$	G^C_{422}	$\xrightarrow[\mathbf{210}]{\text{w}}$	G_{422}	$\xrightarrow[\mathbf{210}]{\text{m}}$	G_{3211}	$\xrightarrow[\overline{\mathbf{126}}]{\text{s}}$		✓
III4:	$\xrightarrow[\mathbf{54}]{\text{m,s}}$	G^C_{422}	$\xrightarrow[\mathbf{210}]{\text{m}}$	G^C_{3221}	$\xrightarrow[\mathbf{45}]{\text{w}}$	G_{3221}	$\xrightarrow[\overline{\mathbf{126}}]{\text{s}}$		✓
III5:	$\xrightarrow[\mathbf{54}]{\text{m,s}}$	G^C_{422}	$\xrightarrow[\mathbf{210}]{\text{m}}$	G^C_{3221}	$\xrightarrow[\mathbf{45}]{\text{m,w}}$	G_{3211}	$\xrightarrow[\overline{\mathbf{126}}]{\text{s}}$		✓
III6:	$\xrightarrow[\mathbf{54}]{\text{m,s}}$	G^C_{422}	$\xrightarrow[\mathbf{45}]{\text{m,w}}$	G_{3221}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{3211}	$\xrightarrow[\overline{\mathbf{126}}]{\text{s}}$		✓
III7:	$\xrightarrow[\mathbf{210}]{\text{m,s}}$	G^C_{3221}	$\xrightarrow[\mathbf{45}]{\text{w}}$	G_{3221}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{3211}	$\xrightarrow[\overline{\mathbf{126}}]{\text{s}}$		✓
III8:	$\xrightarrow[\mathbf{210}]{\text{m}}$	G_{422}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{3221}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{3211}	$\xrightarrow[\overline{\mathbf{126}}]{\text{s}}$		✓
III9:	$\xrightarrow[\mathbf{54}]{\text{m,s}}$	G^C_{422}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{421}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{3211}	$\xrightarrow[\overline{\mathbf{126}}]{\text{s}}$		✓
III10:	$\xrightarrow[\mathbf{210}]{\text{m}}$	G_{422}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{421}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{3211}	$\xrightarrow[\overline{\mathbf{126}}]{\text{s}}$		✓

$SO(10)$	$\xrightarrow[\text{Higgs}]{\text{defect}}$	G_4	$\xrightarrow[\text{Higgs}]{\text{defect}}$	G_3	$\xrightarrow[\text{Higgs}]{\text{defect}}$	G_2	$\xrightarrow[\text{Higgs}]{\text{defect}}$	G_1	$\xrightarrow[\text{Higgs}]{\text{defect}}$	G_{SM}	Observable strings?
IV1:	$\xrightarrow[\mathbf{54}]{\text{m,s}}$	G^C_{422}	$\xrightarrow[\mathbf{210}]{\text{m}}$	G^C_{3221}	$\xrightarrow[\mathbf{45}]{\text{w}}$	G_{3221}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{3211}	$\xrightarrow[\overline{\mathbf{126}}]{\text{s}}$		✓
IV2:	$\xrightarrow[\mathbf{54}]{\text{m,s}}$	G^C_{422}	$\xrightarrow[\mathbf{210}]{\text{w}}$	G_{422}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{3221}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{3211}	$\xrightarrow[\overline{\mathbf{126}}]{\text{s}}$		✓
IV3:	$\xrightarrow[\mathbf{54}]{\text{m,s}}$	G^C_{422}	$\xrightarrow[\mathbf{210}]{\text{w}}$	G_{422}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{421}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{3211}	$\xrightarrow[\overline{\mathbf{126}}]{\text{s}}$		✓

Proton decay and GWs as complementary windows

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I2:	$\xrightarrow[\mathbf{210}]{\text{m,s}}$	G_{3221}^C	$\xrightarrow[\mathbf{126}]{\text{s,w}}$		✗
I3:	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{421}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓
I4:	$\xrightarrow[\mathbf{210}]{\text{m}}$	G_{422}	$\xrightarrow[\mathbf{126},\mathbf{45}]{\text{m}}$		✗
I5:	$\xrightarrow[\mathbf{54}]{\text{m,s}}$	G_{422}^C	$\xrightarrow[\mathbf{126},\mathbf{45}]{\text{m,w}}$		✗
I6:	$\xrightarrow[\mathbf{210}]{\text{m}}$	G_{3211}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓

Monopole

cosmic strings

Domain walls

$SO(10)$	$\xrightarrow[\text{Higgs}]{\text{defect}}$	G_2	$\xrightarrow[\text{Higgs}]{\text{defect}}$	G_1	$\xrightarrow[\text{Higgs}]{\text{defect}}$	G_{SM}	Observable strings?
II1:	$\xrightarrow[\mathbf{210}]{\text{m}}$	G_{422}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{3221}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓
II2:	$\xrightarrow[\mathbf{54}]{\text{m,s}}$	G_{422}^C	$\xrightarrow[\mathbf{210}]{\text{m}}$	G_{3221}^C	$\xrightarrow[\mathbf{126}]{\text{s,w}}$		✗
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II5:	$\xrightarrow[\mathbf{210}]{\text{m}}$	G_{422}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{421}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓
II6:	$\xrightarrow[\mathbf{54}]{\text{m,s}}$	G_{422}^C	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{421}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓
II7:	$\xrightarrow[\mathbf{54}]{\text{m,s}}$	G_{422}^C	$\xrightarrow[\mathbf{210}]{\text{w}}$	G_{422}	$\xrightarrow[\mathbf{126},\mathbf{45}]{\text{m}}$		✗
II8:	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{3221}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{3211}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓
II9:	$\xrightarrow[\mathbf{210}]{\text{m,s}}$	G_{3221}^C	$\xrightarrow[\mathbf{45}]{\text{m,w}}$	G_{3211}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓
II10:	$\xrightarrow[\mathbf{210}]{\text{m}}$	G_{422}	$\xrightarrow[\mathbf{210}]{\text{m}}$	G_{3211}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓
II11:	$\xrightarrow[\mathbf{54}]{\text{m,s}}$	G_{422}^C	$\xrightarrow[\mathbf{210}]{\text{m,w}}$	G_{3211}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓
II12:	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{421}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{3211}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓

$SO(10)$	$\xrightarrow[\text{Higgs}]{\text{defect}}$	G_3	$\xrightarrow[\text{Higgs}]{\text{defect}}$	G_2	$\xrightarrow[\text{Higgs}]{\text{defect}}$	G_1	$\xrightarrow[\text{Higgs}]{\text{defect}}$	G_{SM}	Observable strings?
III1:	$\xrightarrow[\mathbf{54}]{\text{m,s}}$	G_{422}^C	$\xrightarrow[\mathbf{210}]{\text{w}}$	G_{422}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{421}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓
III2:	$\xrightarrow[\mathbf{54}]{\text{m,s}}$	G_{422}^C	$\xrightarrow[\mathbf{210}]{\text{w}}$	G_{422}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{3221}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓
III3:	$\xrightarrow[\mathbf{54}]{\text{m,s}}$	G_{422}^C	$\xrightarrow[\mathbf{210}]{\text{w}}$	G_{422}	$\xrightarrow[\mathbf{210}]{\text{m}}$	G_{3211}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓
III4:	$\xrightarrow[\mathbf{54}]{\text{m,s}}$	G_{422}^C	$\xrightarrow[\mathbf{210}]{\text{m}}$	G_{3221}^C	$\xrightarrow[\mathbf{45}]{\text{w}}$	G_{3221}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓
III5:	$\xrightarrow[\mathbf{54}]{\text{m,s}}$	G_{422}^C	$\xrightarrow[\mathbf{210}]{\text{m}}$	G_{3221}^C	$\xrightarrow[\mathbf{45}]{\text{m,w}}$	G_{3211}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓
III6:	$\xrightarrow[\mathbf{54}]{\text{m,s}}$	G_{422}^C	$\xrightarrow[\mathbf{45}]{\text{m,w}}$	G_{3221}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{3211}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓
III7:	$\xrightarrow[\mathbf{210}]{\text{m,s}}$	G_{3221}^C	$\xrightarrow[\mathbf{45}]{\text{w}}$	G_{3221}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{3211}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓
III8:	$\xrightarrow[\mathbf{210}]{\text{m}}$	G_{422}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{3221}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{3211}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓
III9:	$\xrightarrow[\mathbf{54}]{\text{m,s}}$	G_{422}^C	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{421}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{3211}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓
III10:	$\xrightarrow[\mathbf{210}]{\text{m}}$	G_{422}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{421}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{3211}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓

$SO(10)$	$\xrightarrow[\text{Higgs}]{\text{defect}}$	G_4	$\xrightarrow[\text{Higgs}]{\text{defect}}$	G_3	$\xrightarrow[\text{Higgs}]{\text{defect}}$	G_2	$\xrightarrow[\text{Higgs}]{\text{defect}}$	G_1	$\xrightarrow[\text{Higgs}]{\text{defect}}$	G_{SM}	Observable strings?
IV1:	$\xrightarrow[\mathbf{54}]{\text{m,s}}$	G_{422}^C	$\xrightarrow[\mathbf{210}]{\text{m}}$	G_{3221}^C	$\xrightarrow[\mathbf{45}]{\text{w}}$	G_{3221}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{3211}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓
IV2:	$\xrightarrow[\mathbf{54}]{\text{m,s}}$	G_{422}^C	$\xrightarrow[\mathbf{210}]{\text{w}}$	G_{422}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{3221}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{3211}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓
IV3:	$\xrightarrow[\mathbf{54}]{\text{m,s}}$	G_{422}^C	$\xrightarrow[\mathbf{210}]{\text{w}}$	G_{422}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{421}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{3211}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓

Proton decay and GWs as complementary windows

$SO(10)$	$\xrightarrow[\text{Higgs}]{\text{defect}}$	G_1	$\xrightarrow[\text{Higgs}]{\text{defect}}$	G_{SM}	Observable strings?
I1:	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{3221}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓
I2:	$\xrightarrow[\mathbf{210}]{\text{m,s}}$	G^C_{3221}	$\xrightarrow[\mathbf{126}]{\text{s,w}}$		✗
I3:	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{421}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓
I4:	$\xrightarrow[\mathbf{210}]{\text{m}}$	G_{422}	$\xrightarrow[\mathbf{126},\mathbf{45}]{\text{m}}$		✗
I5:	$\xrightarrow[\mathbf{54}]{\text{m,s}}$	G^C_{422}	$\xrightarrow[\mathbf{126},\mathbf{45}]{\text{m,w}}$		✗
I6:	$\xrightarrow[\mathbf{210}]{\text{m}}$	G_{3211}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓

SO(10) **Higgs multiplets**

$SO(10)$	$\xrightarrow[\text{Higgs}]{\text{defect}}$	G_2	$\xrightarrow[\text{Higgs}]{\text{defect}}$	G_1	$\xrightarrow[\text{Higgs}]{\text{defect}}$	G_{SM}	Observable strings?
II1:	$\xrightarrow[\mathbf{210}]{\text{m}}$	G_{422}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{3221}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓
II2:	$\xrightarrow[\mathbf{54}]{\text{m,s}}$	G^C_{422}	$\xrightarrow[\mathbf{210}]{\text{m}}$	G^C_{3221}	$\xrightarrow[\mathbf{126}]{\text{s,w}}$		✗
II3:	$\xrightarrow[\mathbf{54}]{\text{m,s}}$	G^C_{422}	$\xrightarrow[\mathbf{45}]{\text{m,w}}$	G_{3221}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓
II4:	$\xrightarrow[\mathbf{210}]{\text{m,s}}$	G^C_{3221}	$\xrightarrow[\mathbf{45}]{\text{w}}$	G_{3221}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓
II5:	$\xrightarrow[\mathbf{210}]{\text{m}}$	G_{422}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{421}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓
II6:	$\xrightarrow[\mathbf{54}]{\text{m,s}}$	G^C_{422}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{421}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓
II7:	$\xrightarrow[\mathbf{54}]{\text{m,s}}$	G^C_{422}	$\xrightarrow[\mathbf{210}]{\text{w}}$	G_{422}	$\xrightarrow[\mathbf{126},\mathbf{45}]{\text{m}}$		✗
II8:	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{3221}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{3211}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓
II9:	$\xrightarrow[\mathbf{210}]{\text{m,s}}$	G^C_{3221}	$\xrightarrow[\mathbf{45}]{\text{m,w}}$	G_{3211}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓
II10:	$\xrightarrow[\mathbf{210}]{\text{m}}$	G_{422}	$\xrightarrow[\mathbf{210}]{\text{m}}$	G_{3211}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓
II11:	$\xrightarrow[\mathbf{54}]{\text{m,s}}$	G^C_{422}	$\xrightarrow[\mathbf{210}]{\text{m,w}}$	G_{3211}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓
II12:	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{421}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{3211}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓

$SO(10)$	$\xrightarrow[\text{Higgs}]{\text{defect}}$	G_3	$\xrightarrow[\text{Higgs}]{\text{defect}}$	G_2	$\xrightarrow[\text{Higgs}]{\text{defect}}$	G_1	$\xrightarrow[\text{Higgs}]{\text{defect}}$	G_{SM}	Observable strings?
III1:	$\xrightarrow[\mathbf{54}]{\text{m,s}}$	G^C_{422}	$\xrightarrow[\mathbf{210}]{\text{w}}$	G_{422}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{421}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓
III2:	$\xrightarrow[\mathbf{54}]{\text{m,s}}$	G^C_{422}	$\xrightarrow[\mathbf{210}]{\text{w}}$	G_{422}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{3221}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓
III3:	$\xrightarrow[\mathbf{54}]{\text{m,s}}$	G^C_{422}	$\xrightarrow[\mathbf{210}]{\text{w}}$	G_{422}	$\xrightarrow[\mathbf{210}]{\text{m}}$	G_{3211}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓
III4:	$\xrightarrow[\mathbf{54}]{\text{m,s}}$	G^C_{422}	$\xrightarrow[\mathbf{210}]{\text{m}}$	G^C_{3221}	$\xrightarrow[\mathbf{45}]{\text{w}}$	G_{3221}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓
III5:	$\xrightarrow[\mathbf{54}]{\text{m,s}}$	G^C_{422}	$\xrightarrow[\mathbf{210}]{\text{m}}$	G^C_{3221}	$\xrightarrow[\mathbf{45}]{\text{m,w}}$	G_{3211}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓
III6:	$\xrightarrow[\mathbf{54}]{\text{m,s}}$	G^C_{422}	$\xrightarrow[\mathbf{45}]{\text{m,w}}$	G_{3221}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{3211}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓
III7:	$\xrightarrow[\mathbf{210}]{\text{m,s}}$	G^C_{3221}	$\xrightarrow[\mathbf{45}]{\text{w}}$	G_{3221}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{3211}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓
III8:	$\xrightarrow[\mathbf{210}]{\text{m}}$	G_{422}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{3221}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{3211}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓
III9:	$\xrightarrow[\mathbf{54}]{\text{m,s}}$	G^C_{422}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{421}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{3211}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓
III10:	$\xrightarrow[\mathbf{210}]{\text{m}}$	G_{422}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{421}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{3211}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓

$SO(10)$	$\xrightarrow[\text{Higgs}]{\text{defect}}$	G_4	$\xrightarrow[\text{Higgs}]{\text{defect}}$	G_3	$\xrightarrow[\text{Higgs}]{\text{defect}}$	G_2	$\xrightarrow[\text{Higgs}]{\text{defect}}$	G_1	$\xrightarrow[\text{Higgs}]{\text{defect}}$	G_{SM}	Observable strings?
IV1:	$\xrightarrow[\mathbf{54}]{\text{m,s}}$	G^C_{422}	$\xrightarrow[\mathbf{210}]{\text{m}}$	G^C_{3221}	$\xrightarrow[\mathbf{45}]{\text{w}}$	G_{3221}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{3211}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓
IV2:	$\xrightarrow[\mathbf{54}]{\text{m,s}}$	G^C_{422}	$\xrightarrow[\mathbf{210}]{\text{w}}$	G_{422}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{3221}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{3211}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓
IV3:	$\xrightarrow[\mathbf{54}]{\text{m,s}}$	G^C_{422}	$\xrightarrow[\mathbf{210}]{\text{w}}$	G_{422}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{421}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{3211}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓

Proton decay and GWs as complementary windows

$SO(10)$	$\xrightarrow[\text{Higgs}]{\text{defect}}$	G_1	$\xrightarrow[\text{Higgs}]{\text{defect}}$	G_{SM}	Observable strings?
I1:	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{3221}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓
I2:	$\xrightarrow[\mathbf{210}]{\text{m,s}}$	G^C_{3221}	$\xrightarrow[\mathbf{126}]{\text{s,w}}$		✗
I3:	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{421}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓
I4:	$\xrightarrow[\mathbf{210}]{\text{m}}$	G_{422}	$\xrightarrow[\mathbf{126,45}]{\text{m}}$		✗
I5:	$\xrightarrow[\mathbf{54}]{\text{m,s}}$	G^C_{422}	$\xrightarrow[\mathbf{126,45}]{\text{m,w}}$		✗
I6:	$\xrightarrow[\mathbf{210}]{\text{m}}$	G_{3211}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓

If unwanted defect created in final SSB $\Longrightarrow$ no GW else GW

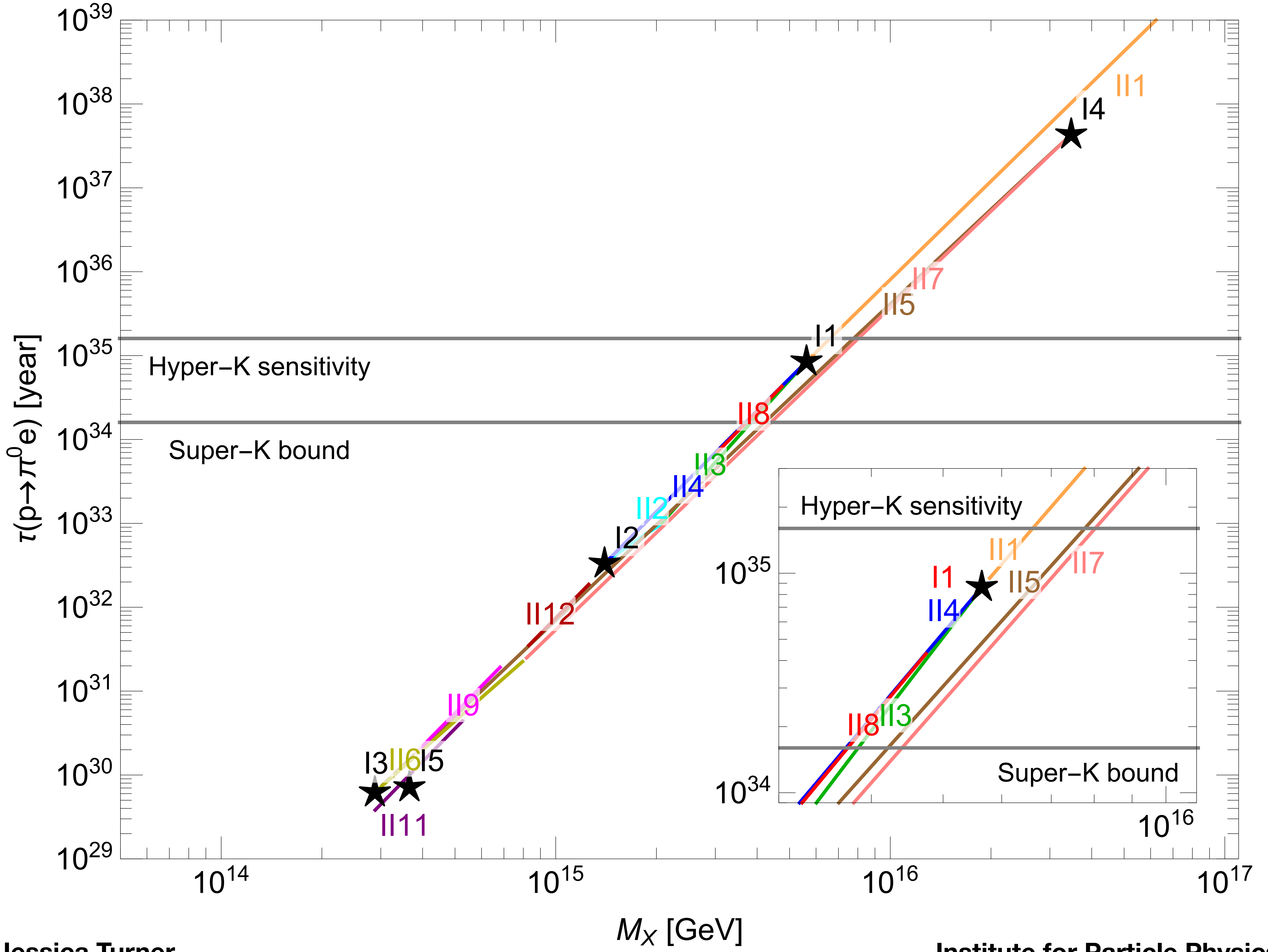
$SO(10)$	$\xrightarrow[\text{Higgs}]{\text{defect}}$	G_2	$\xrightarrow[\text{Higgs}]{\text{defect}}$	G_1	$\xrightarrow[\text{Higgs}]{\text{defect}}$	G_{SM}	Observable strings?
II1:	$\xrightarrow[\mathbf{210}]{\text{m}}$	G_{422}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{3221}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓
II2:	$\xrightarrow[\mathbf{54}]{\text{m,s}}$	G^C_{422}	$\xrightarrow[\mathbf{210}]{\text{m}}$	G^C_{3221}	$\xrightarrow[\mathbf{126}]{\text{s,w}}$		✗
II3:	$\xrightarrow[\mathbf{54}]{\text{m,s}}$	G^C_{422}	$\xrightarrow[\mathbf{45}]{\text{m,w}}$	G_{3221}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓
II4:	$\xrightarrow[\mathbf{210}]{\text{m,s}}$	G^C_{3221}	$\xrightarrow[\mathbf{45}]{\text{w}}$	G_{3221}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓
II5:	$\xrightarrow[\mathbf{210}]{\text{m}}$	G_{422}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{421}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓
II6:	$\xrightarrow[\mathbf{54}]{\text{m,s}}$	G^C_{422}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{421}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓
II7:	$\xrightarrow[\mathbf{54}]{\text{m,s}}$	G^C_{422}	$\xrightarrow[\mathbf{210}]{\text{w}}$	G_{422}	$\xrightarrow[\mathbf{126,45}]{\text{m}}$		✗
II8:	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{3221}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{3211}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓
II9:	$\xrightarrow[\mathbf{210}]{\text{m,s}}$	G^C_{3221}	$\xrightarrow[\mathbf{45}]{\text{m,w}}$	G_{3211}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓
II10:	$\xrightarrow[\mathbf{210}]{\text{m}}$	G_{422}	$\xrightarrow[\mathbf{210}]{\text{m}}$	G_{3211}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓
II11:	$\xrightarrow[\mathbf{54}]{\text{m,s}}$	G^C_{422}	$\xrightarrow[\mathbf{210}]{\text{m,w}}$	G_{3211}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓
II12:	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{421}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{3211}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓

$SO(10)$	$\xrightarrow[\text{Higgs}]{\text{defect}}$	G_3	$\xrightarrow[\text{Higgs}]{\text{defect}}$	G_2	$\xrightarrow[\text{Higgs}]{\text{defect}}$	G_1	$\xrightarrow[\text{Higgs}]{\text{defect}}$	G_{SM}	Observable strings?
III1:	$\xrightarrow[\mathbf{54}]{\text{m,s}}$	G^C_{422}	$\xrightarrow[\mathbf{210}]{\text{w}}$	G_{422}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{421}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓
III2:	$\xrightarrow[\mathbf{54}]{\text{m,s}}$	G^C_{422}	$\xrightarrow[\mathbf{210}]{\text{w}}$	G_{422}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{3221}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓
III3:	$\xrightarrow[\mathbf{54}]{\text{m,s}}$	G^C_{422}	$\xrightarrow[\mathbf{210}]{\text{w}}$	G_{422}	$\xrightarrow[\mathbf{210}]{\text{m}}$	G_{3211}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓
III4:	$\xrightarrow[\mathbf{54}]{\text{m,s}}$	G^C_{422}	$\xrightarrow[\mathbf{210}]{\text{m}}$	G^C_{3221}	$\xrightarrow[\mathbf{45}]{\text{w}}$	G_{3221}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓
III5:	$\xrightarrow[\mathbf{54}]{\text{m,s}}$	G^C_{422}	$\xrightarrow[\mathbf{210}]{\text{m}}$	G^C_{3221}	$\xrightarrow[\mathbf{45}]{\text{m,w}}$	G_{3211}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓
III6:	$\xrightarrow[\mathbf{54}]{\text{m,s}}$	G^C_{422}	$\xrightarrow[\mathbf{45}]{\text{m,w}}$	G_{3221}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{3211}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓
III7:	$\xrightarrow[\mathbf{210}]{\text{m,s}}$	G^C_{3221}	$\xrightarrow[\mathbf{45}]{\text{w}}$	G_{3221}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{3211}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓
III8:	$\xrightarrow[\mathbf{210}]{\text{m}}$	G_{422}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{3221}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{3211}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓
III9:	$\xrightarrow[\mathbf{54}]{\text{m,s}}$	G^C_{422}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{421}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{3211}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓
III10:	$\xrightarrow[\mathbf{210}]{\text{m}}$	G_{422}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{421}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{3211}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓

$SO(10)$	$\xrightarrow[\text{Higgs}]{\text{defect}}$	G_4	$\xrightarrow[\text{Higgs}]{\text{defect}}$	G_3	$\xrightarrow[\text{Higgs}]{\text{defect}}$	G_2	$\xrightarrow[\text{Higgs}]{\text{defect}}$	G_1	$\xrightarrow[\text{Higgs}]{\text{defect}}$	G_{SM}	Observable strings?
IV1:	$\xrightarrow[\mathbf{54}]{\text{m,s}}$	G^C_{422}	$\xrightarrow[\mathbf{210}]{\text{m}}$	G^C_{3221}	$\xrightarrow[\mathbf{45}]{\text{w}}$	G_{3221}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{3211}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓
IV2:	$\xrightarrow[\mathbf{54}]{\text{m,s}}$	G^C_{422}	$\xrightarrow[\mathbf{210}]{\text{w}}$	G_{422}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{3221}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{3211}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓
IV3:	$\xrightarrow[\mathbf{54}]{\text{m,s}}$	G^C_{422}	$\xrightarrow[\mathbf{210}]{\text{w}}$	G_{422}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{421}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{3211}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓

Proton decay and GWs as complementary windows

- Assume minimal survival hypothesis
- Perform two-loop RGE analysis to determine GUT scale (proton decay rate) in terms intermediate breaking scales (due to unification).



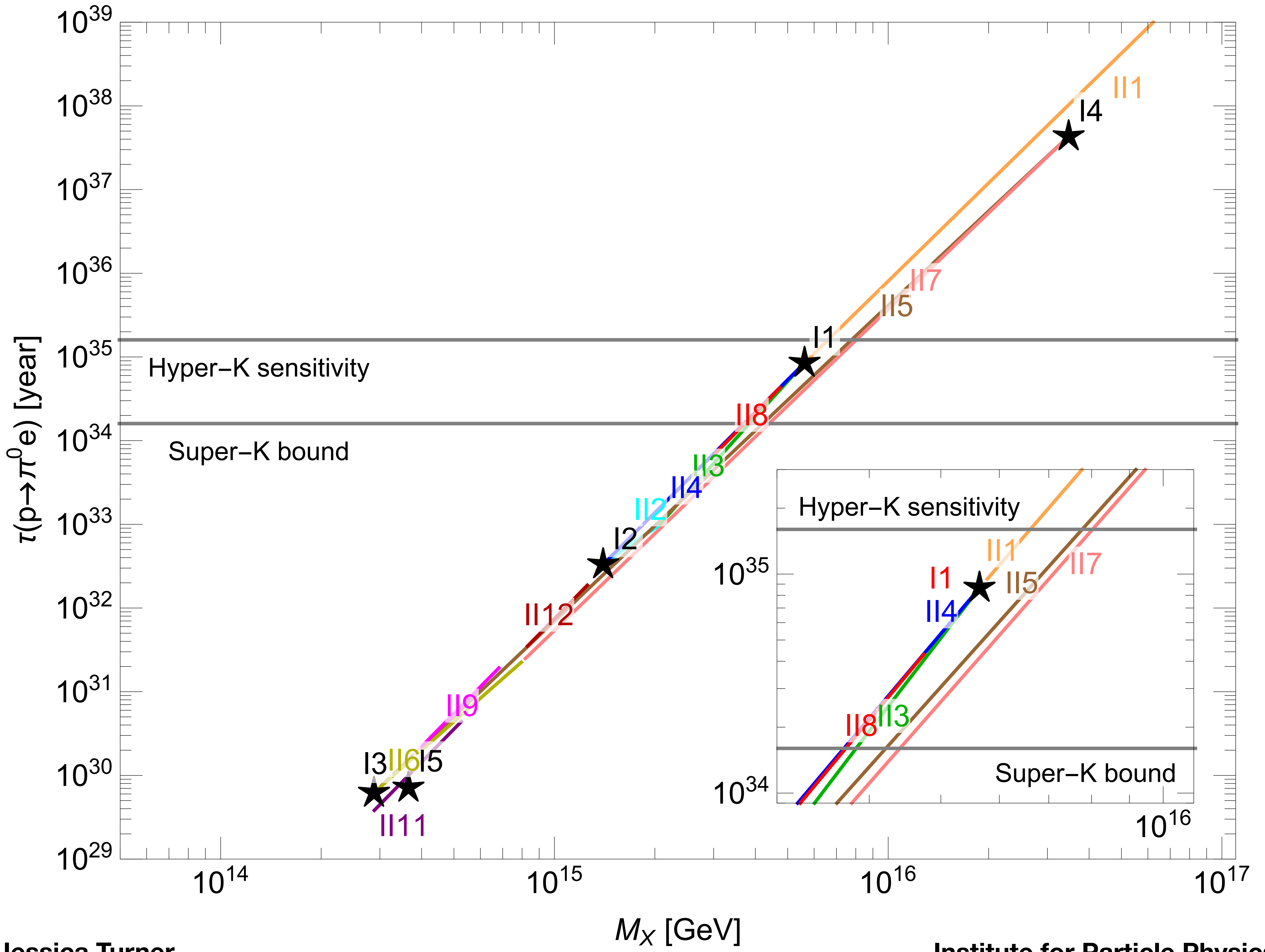
Breaking chains allowed by Super-K:
I1, I4, II1, 3, 4, 5, 7, 8

$$\mathrm{I1}: \mathrm{SO}(10) \xrightarrow{M_X} G_{3221} \xrightarrow{M_1} G_{SM}$$

$$\mathrm{II1}: \mathrm{SO}(10) \xrightarrow{M_X} G_{422} \xrightarrow{M_2} G_{3221} \xrightarrow{M_1} G_{SM}$$

Proton decay and GWs as complementary windows

- Assume minimal survival hypothesis
- Perform two-loop RGE analysis to determine GUT scale (proton decay rate) in terms intermediate breaking scales (due to unification).



Breaking chains allowed by Super-K:
I1, I4, II1, 3, 4, 5, 7, 8

$$\mathrm{I1}:\mathrm{SO}(10)\xrightarrow{M_X} G_{3221}\xrightarrow{M_1} G_{SM}$$

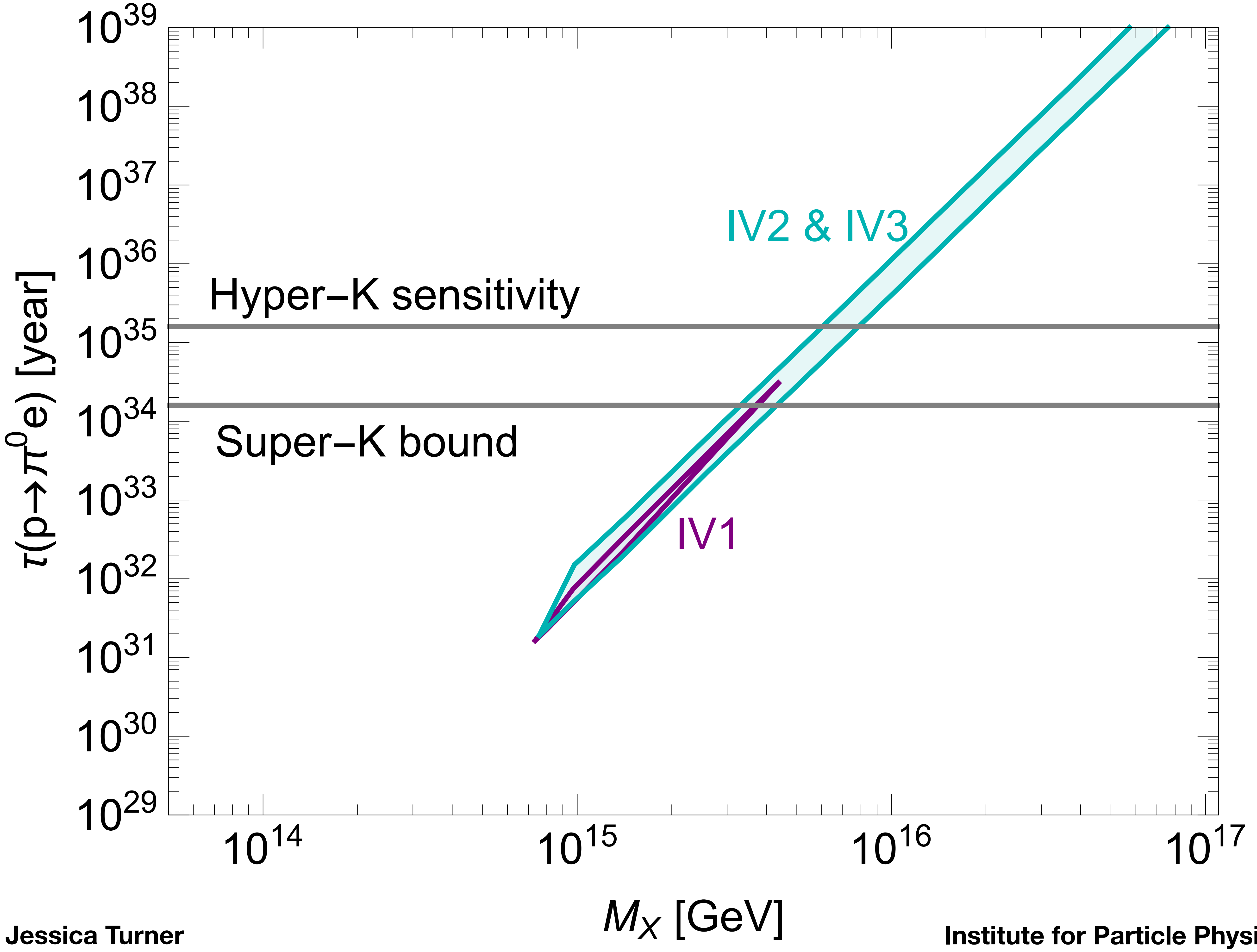
$$\mathrm{II1}:\mathrm{SO}(10)\xrightarrow{M_X} G_{422}\xrightarrow{M_2} G_{3221}\xrightarrow{M_1} G_{SM}$$

Parametrises PD/GUT scale

Parametrises GW scale

Proton decay and GWs as complementary windows

- Assume minimal survival hypothesis
- Perform two-loop RGE analysis to determine GUT scale (proton decay rate) in terms intermediate breaking scales (due to unification).



Breaking chains allowed by Super-K: IV2 & IV3

$$\text{IV2}: \ \text{SO}(10) \xrightarrow{M_X} G^C_{422} \xrightarrow{M_4} G_{422} \xrightarrow{M_3} G_{3221} \xrightarrow{M_2} G_{3211} \xrightarrow{M_1} G_{SM}$$

$$\text{IV3}: \ \text{SO}(10) \xrightarrow{M_X} G^C_{422} \xrightarrow{M_4} G_{422} \xrightarrow{M_3} G_{421} \xrightarrow{M_2} G_{3211} \xrightarrow{M_1} G_{SM}$$

Regions due to more free parameters

Proton decay and GWs as complementary windows

- RGE constrain GUT and intermediate scale symmetry breaking.
- For **type (c) chains** an observable **GW signal** is produced in the **final SSB**.
- We assume Nambu-Goto string $\Longrightarrow$ gravitational radiation primary emission.
- Determine $M_1 \Longrightarrow$ string tension

$$\mu \approx 2\pi v^2$$

$$v = |\langle \phi \rangle|$$

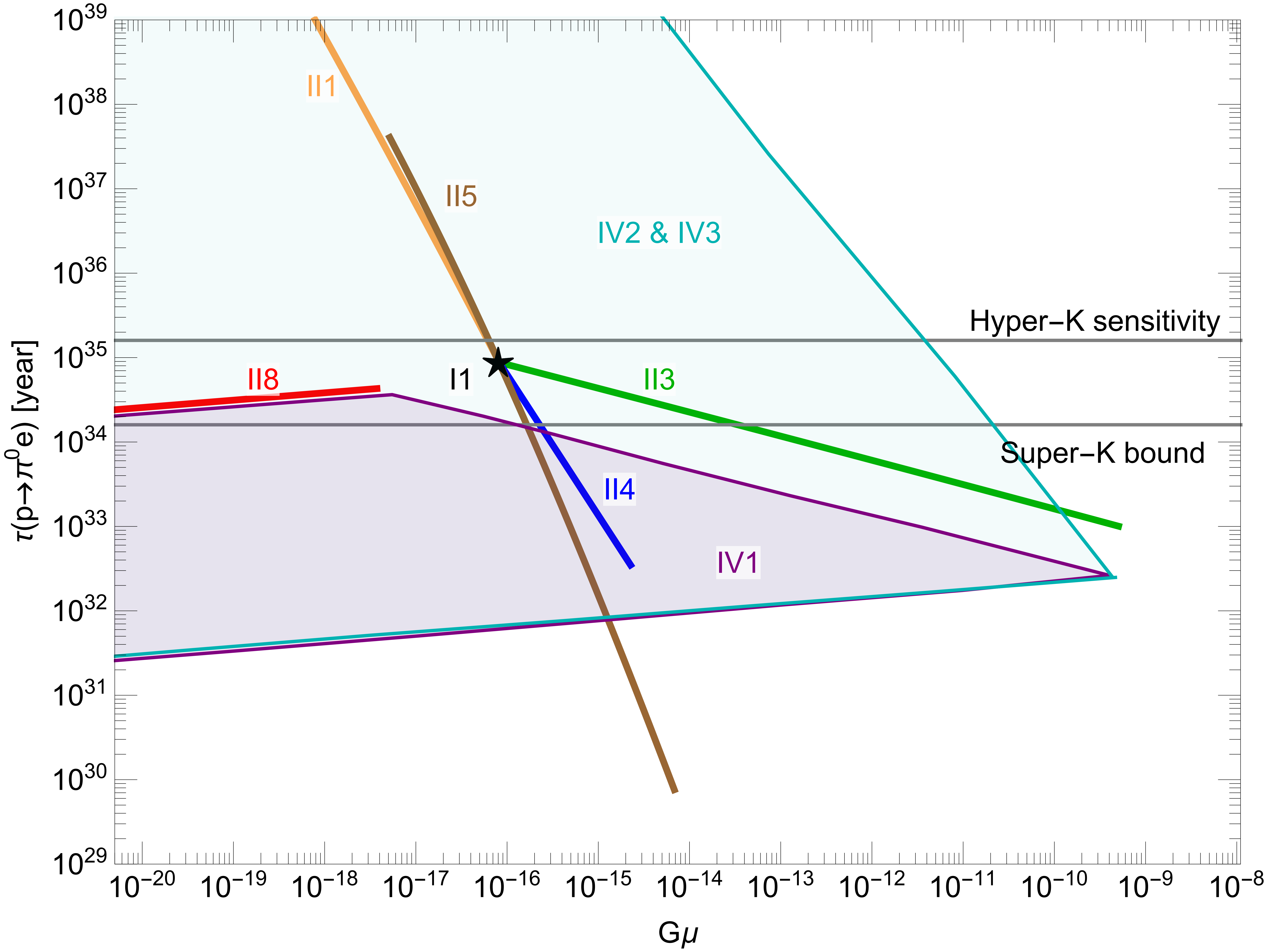
vev of Higgs that breaks U(1) gauge symmetry

$$M_1^2 = M_{Z'}^2 = 4\pi\alpha v^2 \Longrightarrow G\mu = \frac{M_1^2}{2\alpha M_{PL}^2}$$

α of U(1) that get broken

RGE + gauge unification correlates M_1 with M_X
$\Longrightarrow G\mu$ correlated $\tau(p \to e^+\pi^0)$

Proton decay and GWs as complementary windows



Summary

- non-SUSY $SO(10)$ Pati Salam type provide unification: **31 breaking chains**
- Two-loop RGE, **17 not excluded** by Super-K lower bound PD.

Chain	$G\mu$ after Hyper-K (no proton decay)
I1	excluded
II1:	$G\mu \lesssim 1.5 \times 10^{-17}$
II3:	excluded
II4:	excluded
II5:	$G\mu \simeq 5.1 \times 10^{-18}$–$6.3 \times 10^{-17}$
II8:	excluded
III1:	$G\mu \simeq 1.3 \times 10^{-18}$–$1.6 \times 10^{-15}$
III2:	$G\mu \lesssim 5.0 \times 10^{-12}$
III3:	$G\mu \lesssim 6.2 \times 10^{-14}$
III4:	excluded
III6:	excluded
III7:	excluded
III8:	excluded
III10:	$G\mu \lesssim 1.1 \times 10^{-21}$
IV1:	excluded
IV2:	$G\mu \lesssim 9.4 \times 10^{-13}$
IV3:	$G\mu \lesssim 9.4 \times 10^{-13}$

Testable by LIGO, DECIGO, AEDGE, C, ET, MAGIS..

- If HyperK **does not** observe PD $\Longrightarrow$ 9 chains excluded
- **8 survivors!** If we observe GW signal **larger than upper bounds** $\Longrightarrow$ exclude those breaking chains
- If we observe PD $\Longrightarrow$ M_1 determined and so is GW signal. Correlations matters!

Summary

- non-SUSY $SO(10)$ Pati Salam type provide unification: **31 breaking chains**
- Two-loop RGE, **17 not excluded** by Super-K lower bound PD.

Chain	$G\mu$ after Hyper-K (no proton decay)
I1	excluded
II1:	$G\mu \lesssim 1.5 \times 10^{-17}$
II3:	excluded
II4:	excluded
II5:	$G\mu \simeq 5.1 \times 10^{-18} - 6.3 \times 10^{-17}$
II8:	excluded
III1:	$G\mu \simeq 1.3 \times 10^{-18} - 1.6 \times 10^{-15}$
III2:	$G\mu \lesssim 5.0 \times 10^{-12}$
III3:	$G\mu \lesssim 6.2 \times 10^{-14}$
III4:	excluded
III6:	excluded
III7:	excluded
III8:	excluded
III10:	$G\mu \lesssim 1.1 \times 10^{-21}$
IV1:	excluded
IV2:	$G\mu \lesssim 9.4 \times 10^{-13}$
IV3:	$G\mu \lesssim 9.4 \times 10^{-13}$

Study specific breaking chain 2209.00021

Why? Can be excluded by Hyper-K & has an associated GW signal

SO(10) with leptogenesis

- Specific model of chain III4: **Fu, King, Marsili, Pascoli, JT, Zhou** 2209.00021
- See Dror also et al 1908.03227 which connected GUT with leptogenesis
- Fit SM fermion data to our model, perform RG running $\Longrightarrow$ scales of cosmic string formation, leptogenesis & proton decay determined

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$$
\begin{array}{c}
SO(10) \\
\mathbf{54} \downarrow \\
SU(4) \times SU(2)_L \times SU(2)_R \times Z_2^C \\
\mathbf{210} \downarrow \\
SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_X \times Z_2^C \\
\mathbf{45} \downarrow \\
SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \\
\overline{\mathbf{126}} \downarrow \\
SU(3)_c \times SU(2)_L \times U(1)_Y \, .
\end{array}
$$

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M_X (→ **54**)

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$$SO(10)$$

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M_X → Proton Decay

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$$SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_X \times Z_2^C$$

$$\mathbf{45} \downarrow$$

$$SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$$

$$\overline{\mathbf{126}} \downarrow$$

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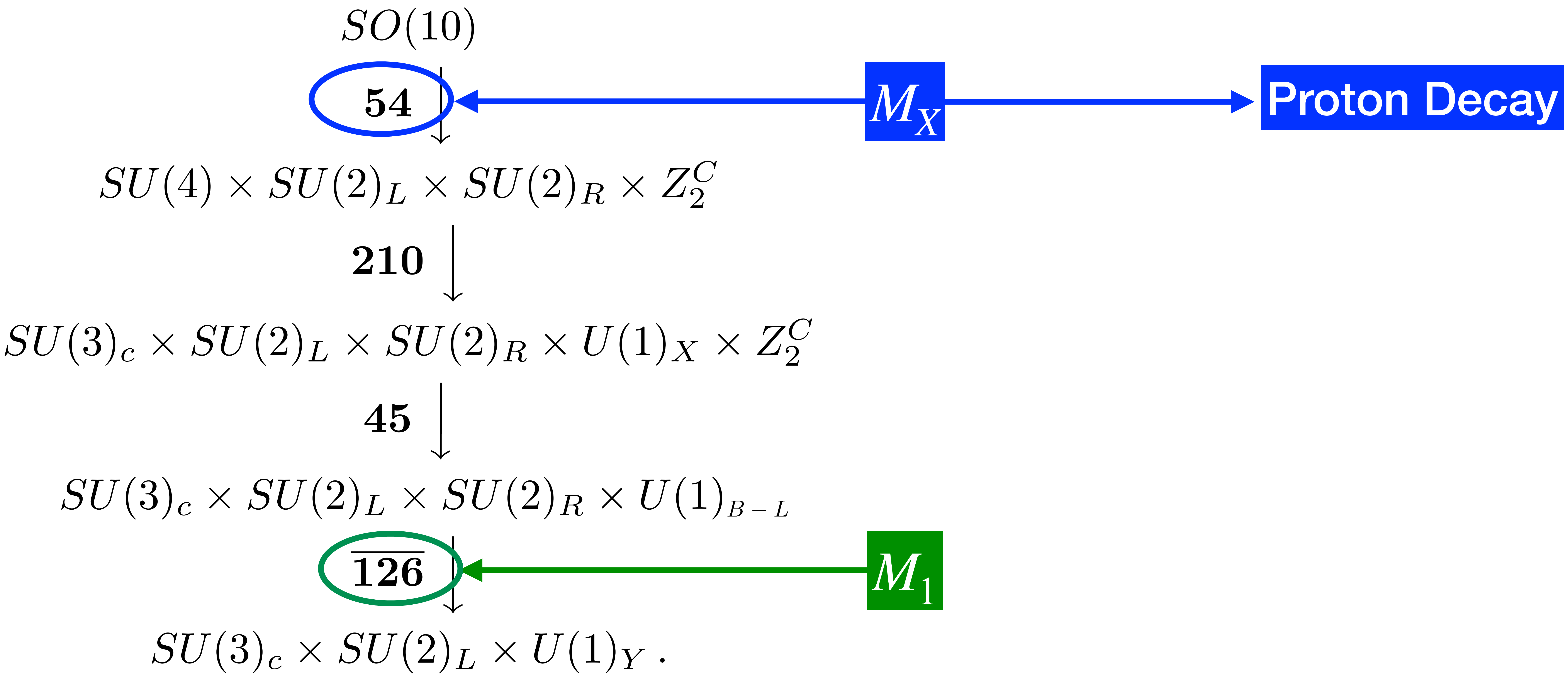

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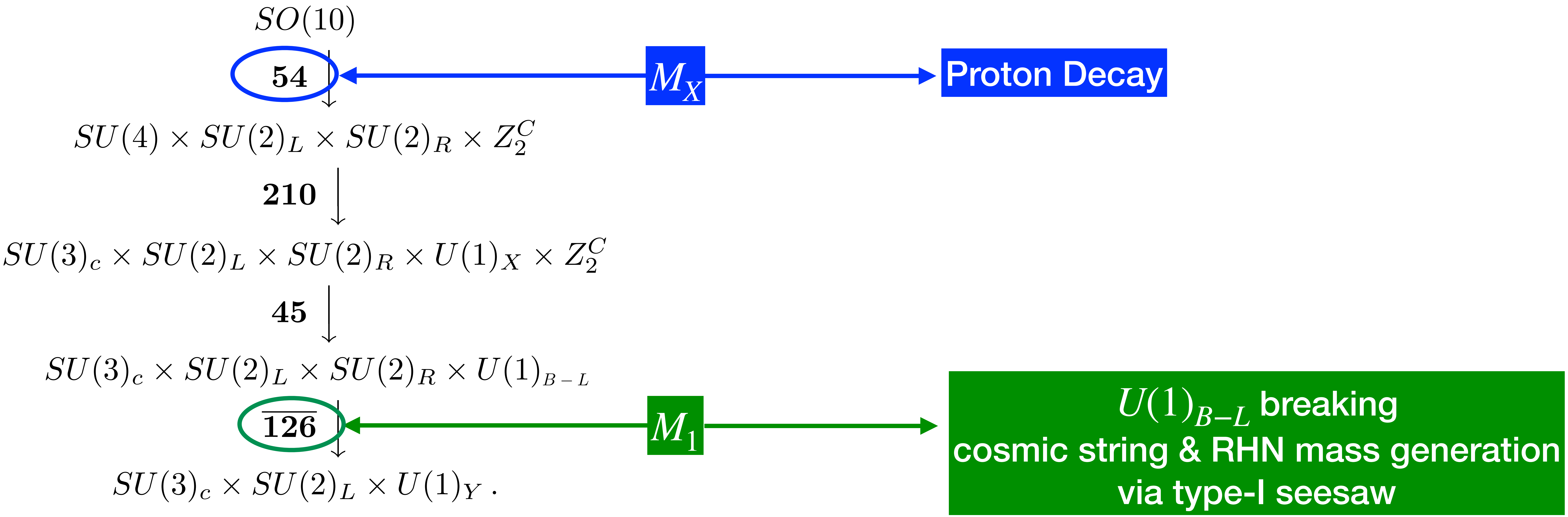

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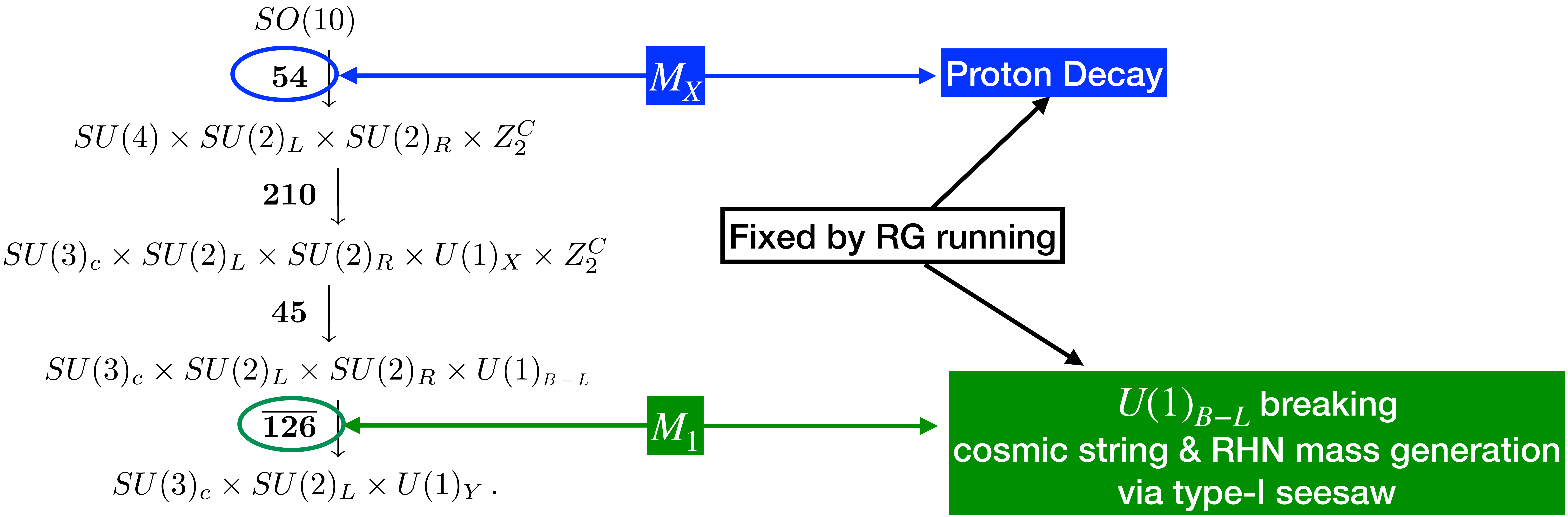


SO(10) with leptogenesis

- Up, down, neutrino, charged lepton Yukawa couplings and right-handed mass matrices parametrised in terms of $SO(10)$ model parameters **Altarelli et al 1012.2697**

$$\mathscr{P}_m \in \{a_1, a_2, r_1, c_e, c_\nu, m_0, \eta\}$$

- Quark & charged lepton sectors are inputs & neutrino sector is predicted:

$$\mathcal{O}_n \in \{\theta_{12}, \theta_{13}, \theta_{23}, \delta, \Delta m^2_{21}, \Delta m^2_{31}\}$$

- Perform (grid based) scan of model parameters to find Yukawa & mass matrices with low χ^2

SO(10) with leptogenesis

- For each point in scan we have RHN mass scale, Yukawa coupling of RHN to leptonic and Higgs doublet $\Longrightarrow$ thermal leptogenesis prediction

BP1

Inputs	a_1	a_2	c_ν	m_0	$(\eta_u, \eta_c, \eta_t; \eta_d, \eta_s, \eta_b)$
	63.57°	84.17°	-1.945	82.82 meV	$(+,+,-;+,-,+)$
Outputs	θ_{13}	θ_{12}	θ_{23}	δ	m_1
	8.53°	32.7°	41.9°	−125°	3.36 meV
$(\chi^2 = 0.33)$	$m_{\beta\beta}$		M_{N_1}	M_{N_2}	M_{N_3}
	5.83 meV		$4.23 \cdot 10^{11}$ GeV	$5.32 \cdot 10^{11}$ GeV	$1.66 \cdot 10^{13}$ GeV

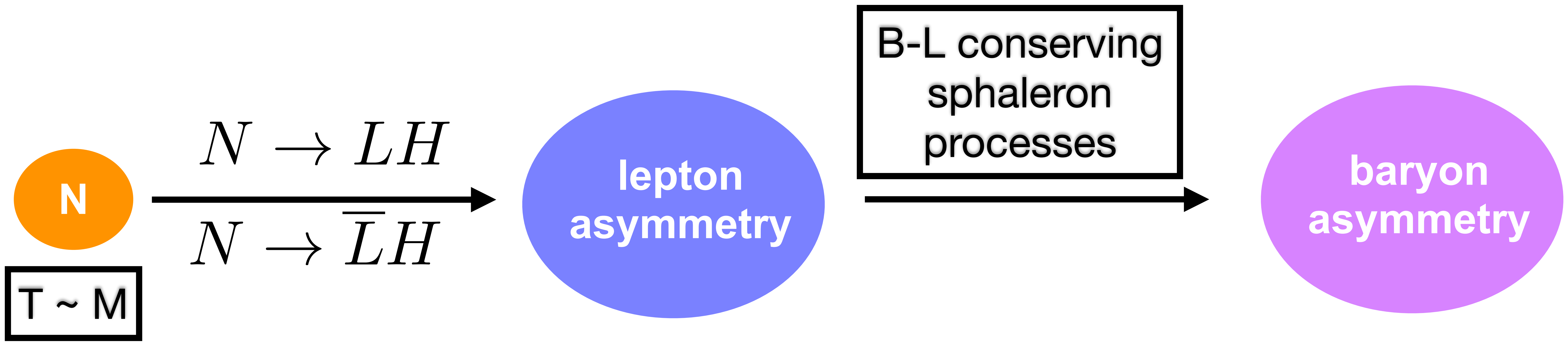
$$M_1 = 2 \times 10^{13}\text{GeV}, \quad M_2 = 5 \times 10^{13}\text{GeV} \quad M_3 = 7.55 \times 10^{13}\text{GeV}$$

$$M_X = 5.68 \times 10^{15}\text{GeV}, \quad \alpha_X = 0.0279$$

$$\mathcal{L} = i\overline{N_i}\partial\!\!\!/ N_i - \tilde{Y}_\nu \overline{L_\alpha}\tilde{\Phi}N_i - \frac{1}{2}M_i\overline{N_i^c}N_i$$

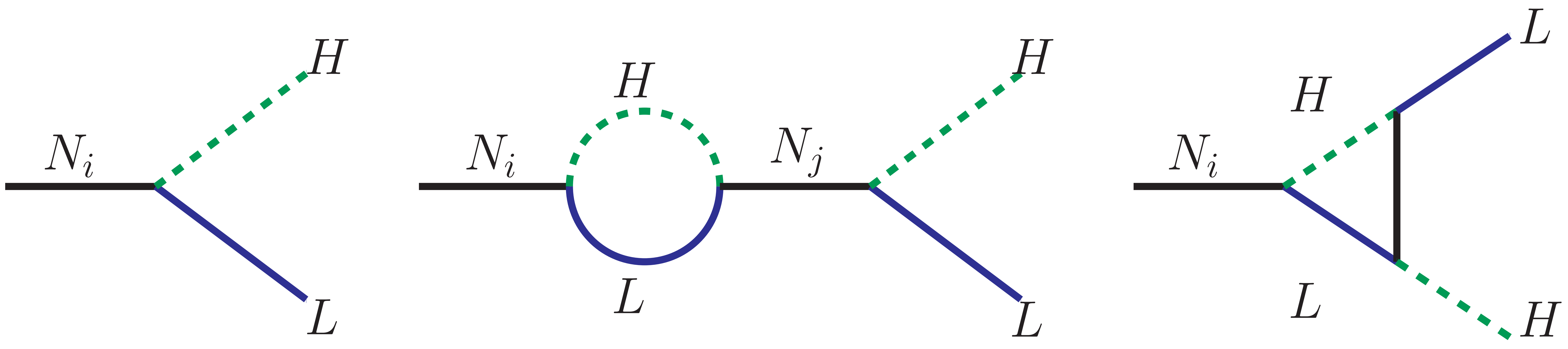
$$\tilde{Y}_\nu = 10^{-2} \cdot \begin{pmatrix} 0.0547 + 0.9061i & 0.2923 - 0.2626i & 0.1159 - 0.1146i \\ -0.0024 + 0.04351i & -1.8277 + 0.1813i & -0.4079 + 1.2977i \\ -0.7770 - 0.2221i & 0.5467 + 2.3425i & -6.8722 - 0.0676i \end{pmatrix}$$

Thermal leptogenesis



Decay asymmetry from interference between tree and loop level diagrams

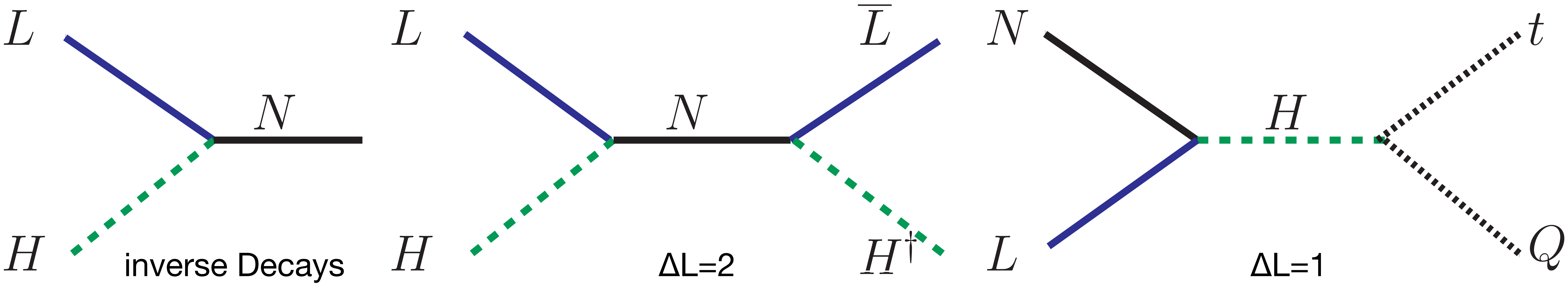
<u>Covi, Roulet, Vissani</u>

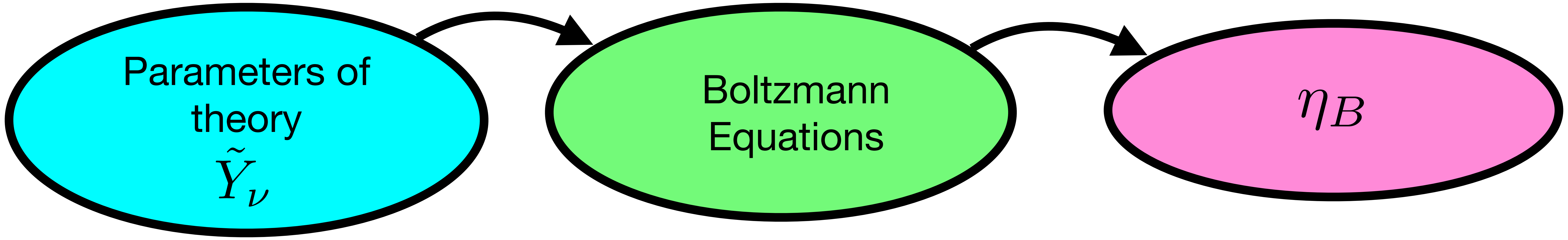


$$\epsilon_i = \frac{\Gamma_i - \overline{\Gamma_i}}{\Gamma_i + \overline{\Gamma_i}}$$

Thermal leptogenesis

Washout and scattering processes

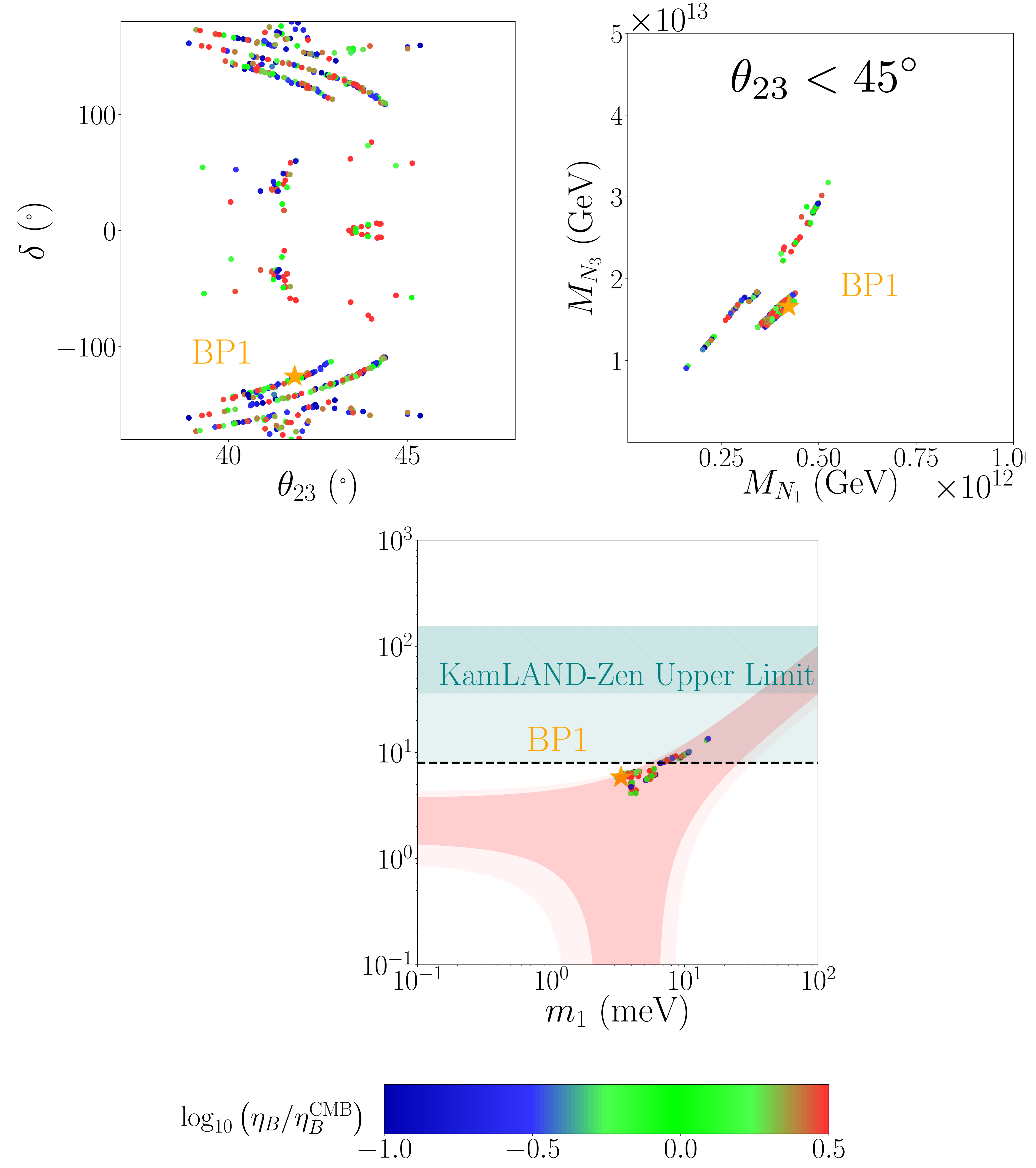




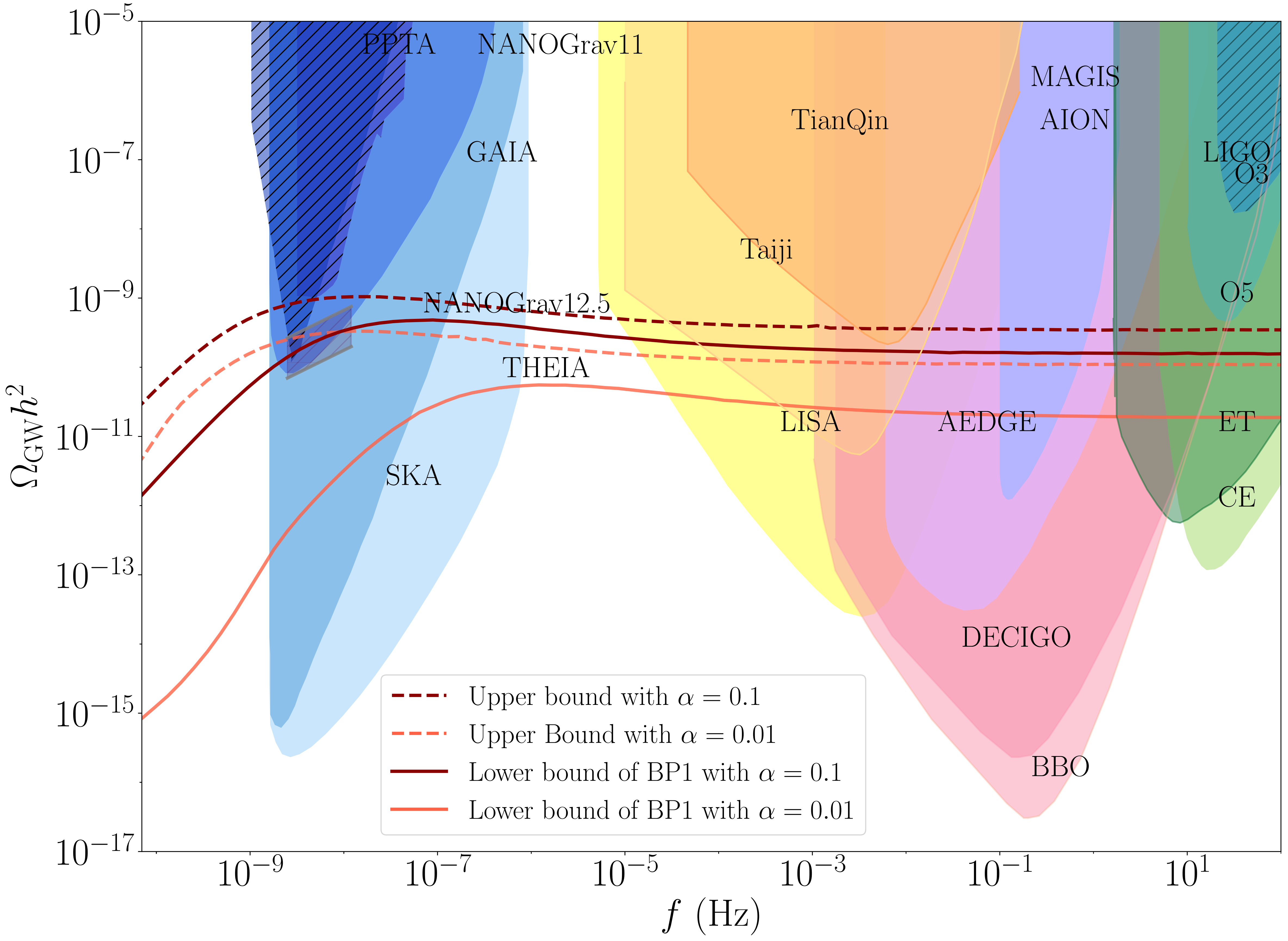
$$\frac{dn_{N_i}}{dz} = - D_i(n_{N_i} - n^{\rm eq}_{N_i}),$$

source **sink**

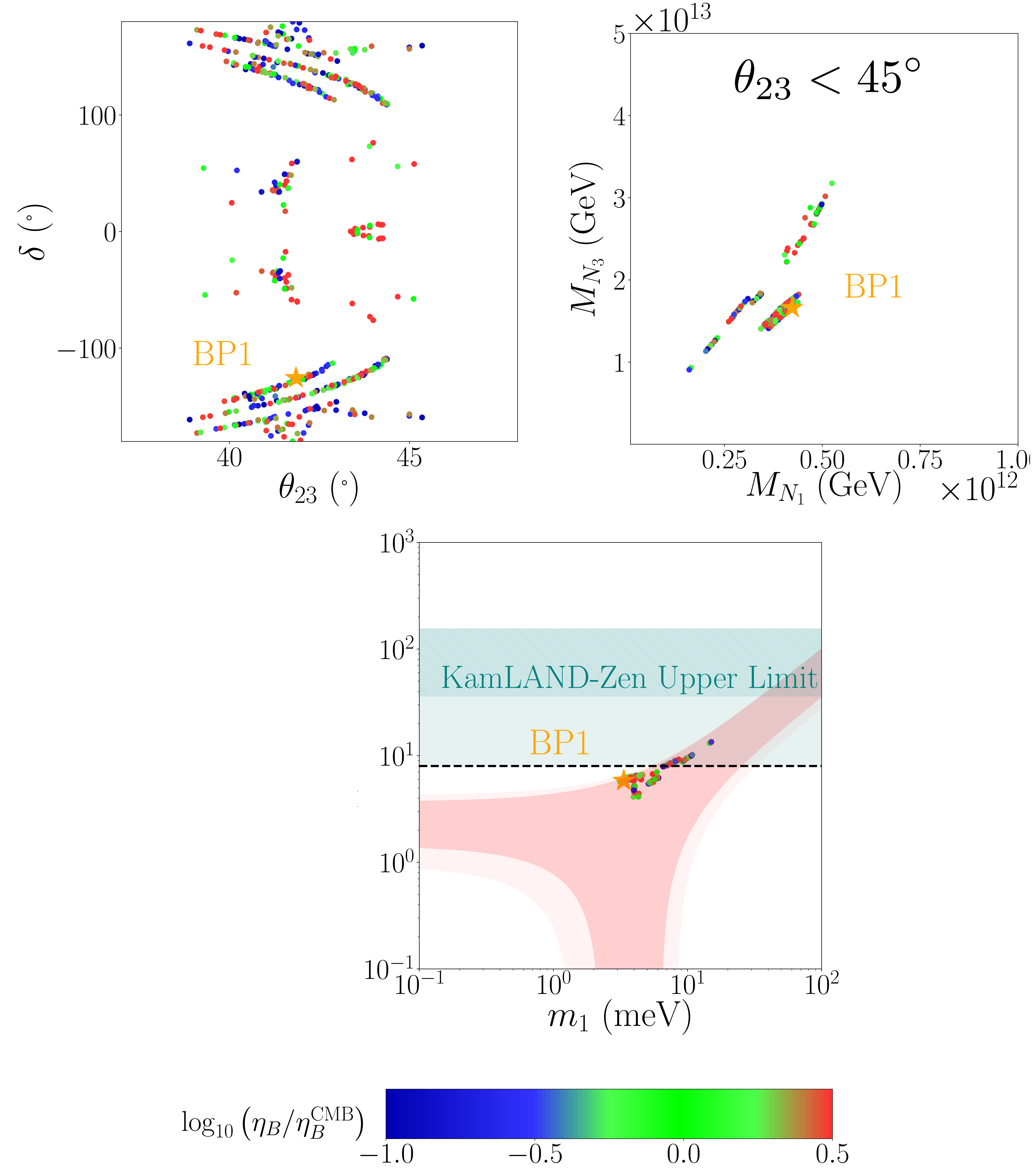
$$\frac{dn_{B-L}}{dz} = \sum_{i=1}^{3}\left(\epsilon^{(i)} D_i(n_{N_i} - n^{\rm eq}_{N_i}) - W_i n_{B-L}\right).$$

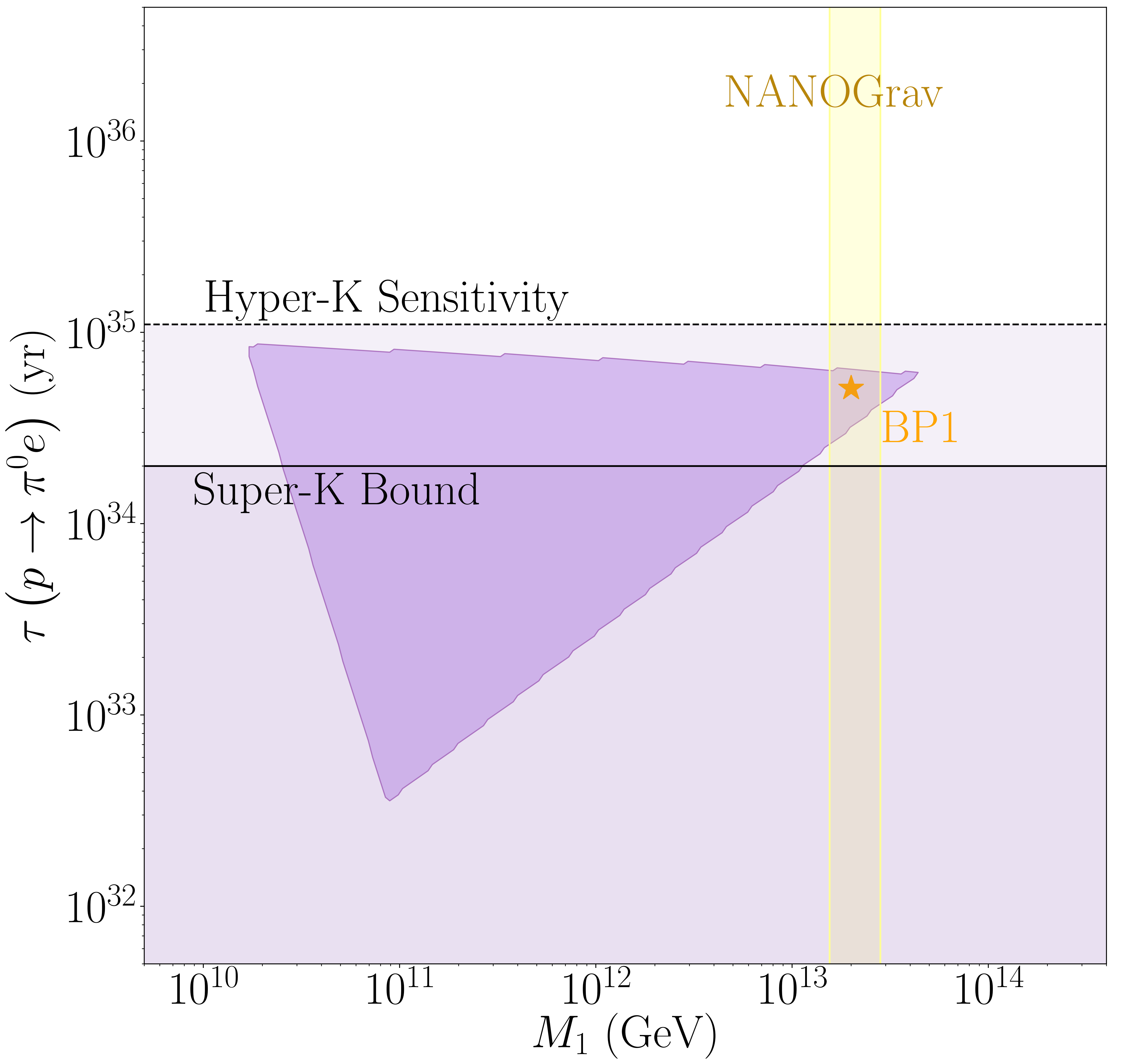


$\delta\ (^\circ)$
$\theta_{23}\ (^\circ)$
BP1
$\theta_{23} < 45^\circ$
M_{N_3} (GeV)
M_{N_1} (GeV)
$\times 10^{13}$
$\times 10^{12}$
KamLAND-Zen Upper Limit
m_1 (meV)
$\log_{10}(\eta_B/\eta_B^{\rm CMB})$



$\Omega_{\rm GW} h^2$
f (Hz)
PPTA
NANOGrav11
GAIA
NANOGrav12.5
THEIA
SKA
TianQin
Taiji
MAGIS
AION
LIGO
O3
O5
LISA
AEDGE
ET
CE
DECIGO
BBO
Upper bound with $\alpha = 0.1$
Upper Bound with $\alpha = 0.01$
Lower bound of BP1 with $\alpha = 0.1$
Lower bound of BP1 with $\alpha = 0.01$


δ (°)
θ23 (°)
BP1
θ23 < 45°
M_N3 (GeV)
M_N1 (GeV)
BP1
KamLAND-Zen Upper Limit
BP1
m1 (meV)
log10 (ηB/ηB^CMB)


NANOGrav
Hyper-K Sensitivity
Super-K Bound
BP1
τ (p → π0e) (yr)
M1 (GeV)

Summary

- GUTs generically predict nucleon decay and the formation of topological defects. Interplay of these observables is a more powerful way of constraining GUTs.

- Coming decade is an exciting time for GUTs as neutrino and GW experiments will constrain nucleon decay, the presence of GWs and neutrinoless double beta decay ($0\nu\beta\beta$).

- Studied a SO(10) breaking chain can be tested by Hyper-K, GW detectors and $0\nu\beta\beta$. Parameter space consistent with fermionic masses and mixing $\Longrightarrow$ successful leptogenesis

- For future: study interplay of inflationary scale, more breaking chains. Grid scans for d > 3 are hopeless and a more sophisticated machinery is required.

“we have entered an exciting era where new observations of GWs from the heavens and proton decay experiments from under the Earth can provide complementary windows to reveal the details of the unification of matter and forces at the highest energies.”


Merci!

Renormalisation Group Equations

Beta function coefficients 1 and 2-loop respectively

$$b_i = -\frac{11}{3} C_2(H_i) + \frac{2}{3} \sum_F T(F_i) + \frac{1}{3} \sum_S T(S_i)\,,$$

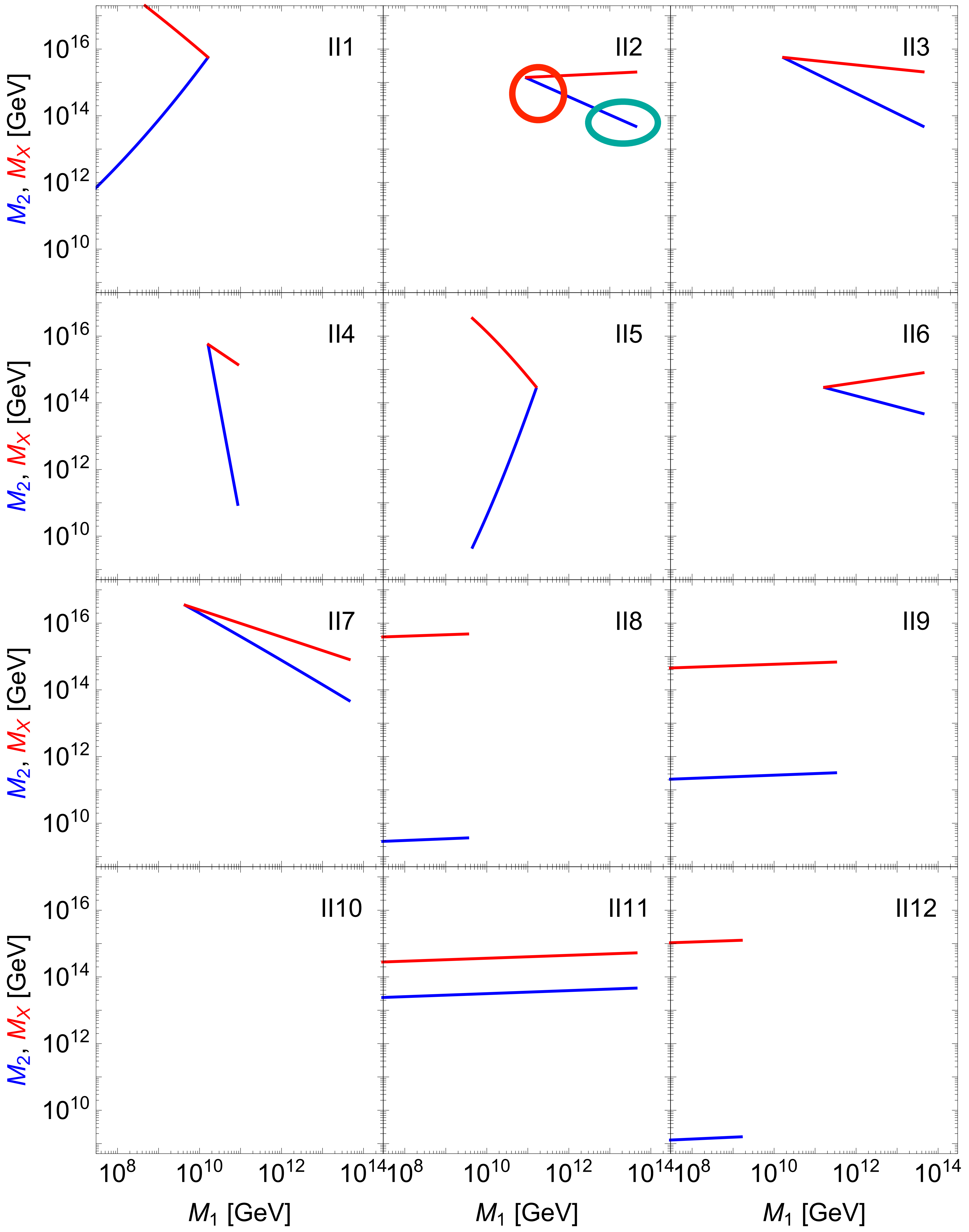
$$b_{ij} = -\frac{34}{3} [C_2(H_i)]^2 \delta_{ij} + \sum_F T(F_i)[2C_2(F_j) + \frac{10}{3} C_2(H_i)\delta_{ij}] + \sum_S T(S_i)[4C_2(S_j) + \frac{2}{3} C_2(H_i)\delta_{ij}]\,,$$

Two-loop RGE equation

$$\alpha_i(\mu)^{-1} = \alpha_i(\mu_0)^{-1} - \frac{b_i}{2\pi} \log \frac{\mu}{\mu_0} + \sum_j \frac{b_{ij}}{4\pi b_i} \log \left(1 - b_j \alpha_j(\mu_0) \log \frac{\mu}{\mu_0}\right)\,,$$

Matching condition

$$H_i \to H_j\,, \quad \frac{1}{\alpha_{H_i}(M_I)} - \frac{C_2(H_i)}{12\pi} = \frac{1}{\alpha_{H_j}(M_I)} - \frac{C_2(H_j)}{12\pi}\,.$$



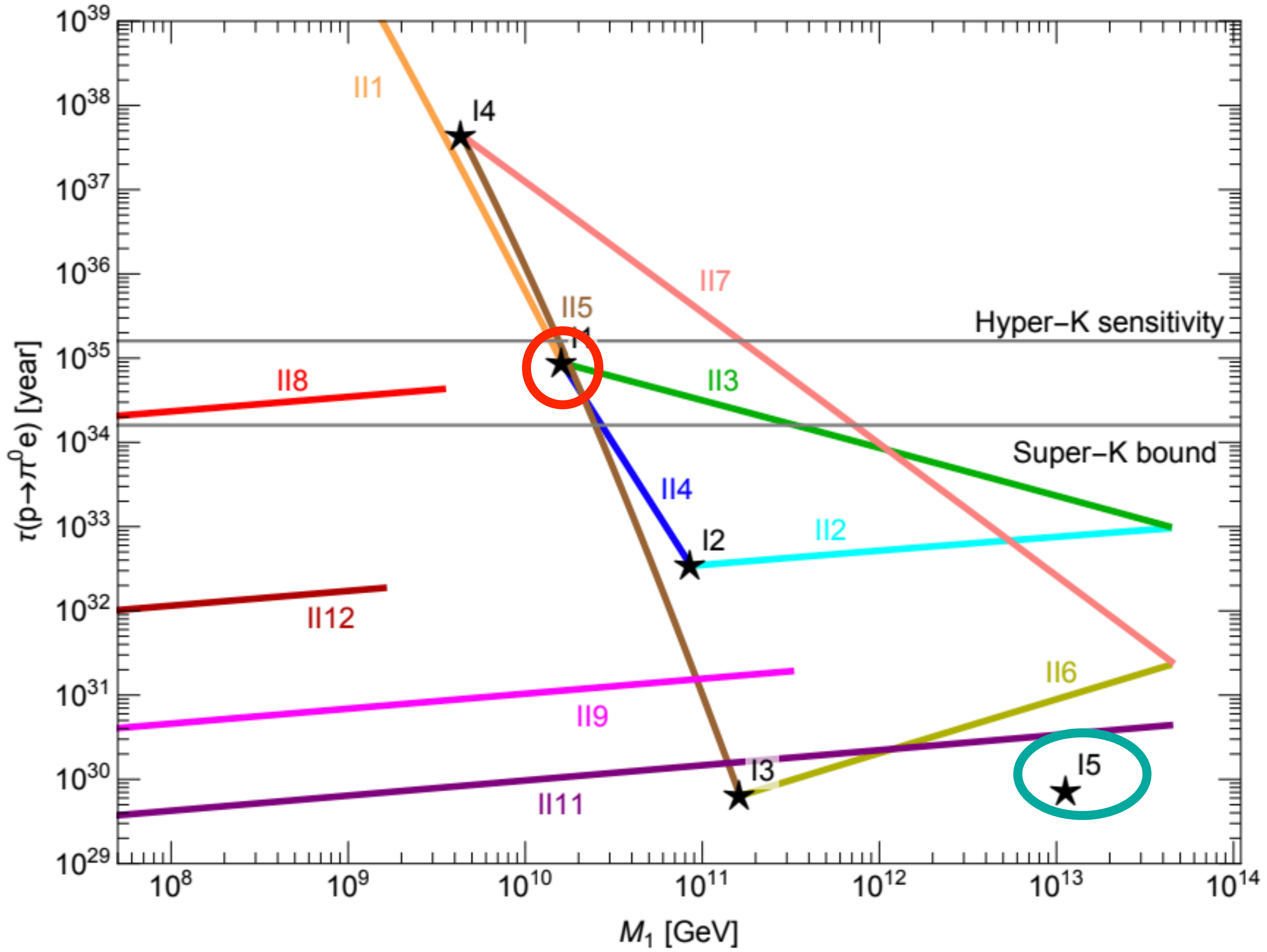
$$\mathrm{II2}: SO(10) \xrightarrow{M_X} G^C_{422} \xrightarrow{M_2} G^C_{3221} \xrightarrow{M_1} G_{\mathrm{SM}}$$

Intersection of M_2 and M_X reduces II2 to I2

$$\mathrm{I2}: SO(10) \xrightarrow{M_X} G^C_{3221} \xrightarrow{M_1} G_{\mathrm{SM}} \qquad M_X \equiv M_2$$

At right side blue curve II2 becomes I5

$$\mathrm{I5}: SO(10) \to G^C_{422} \to G_{\mathrm{SM}} \qquad M_2 \equiv M_1$$



Proton Lifetime

$$\epsilon^{ijk}\epsilon_{\alpha\beta}\Big(\frac{1}{\Lambda_1^2}(\overline{u_R^{jc}}\gamma^\mu Q_\alpha^k)(\overline{d_R^{ic}}\gamma_\mu L_\beta) + \frac{1}{\Lambda_1^2}(\overline{u_R^{jc}}\gamma^\mu Q_\alpha^k)(\overline{e_R^c}\gamma_\mu Q_\beta^i)$$
$$+\frac{1}{\Lambda_2^2}(\overline{d_R^{jc}}\gamma^\mu Q_\alpha^k)(\overline{u_R^{ic}}\gamma_\mu L_\beta) + \frac{1}{\Lambda_2^2}(\overline{d_R^{jc}}\gamma^\mu Q_\alpha^k)(\overline{\nu_R^c}\gamma_\mu Q_\beta^i) + \text{h.c.}\Big)$$

$$\Lambda_1 = \Lambda_2 \simeq (g_X M_X)/2$$

$$\Gamma(p \to \pi^0 + e^+) = \frac{m_p}{32\pi}\left(1 - \frac{m_{\pi^0}^2}{m_p^2}\right)^2 A_L^2 \times \Big[A_{SL}\Lambda_1^{-2}(1 + |V_{ud}|^2)\,|\langle\pi^0\,|(ud)_R u_L|p\rangle|^2$$
$$+A_{SR}(\Lambda_1^{-2} + |V_{ud}|^2\Lambda_2^{-2})\,\left|\langle\pi^0|(ud)_L u_L|p\rangle\right|^2\Big]$$

$$A_{SL(R)} = \prod_A^{M_Z \leqslant M_A \leqslant M_X} \prod_i \left[\frac{\alpha_i(M_{A+1})}{\alpha_i(M_A)}\right]^{\frac{\gamma_{iL(R)}}{b_i}}$$

Gravitational Wave Calculation

$$l(t) = l_i - \Gamma G\mu\,(t - t_i) \qquad l_i = \alpha t_i \text{ with } \alpha \simeq 0.1$$

Frequencies of GW released from the loops are given by $2k/l_i$ where $k = 1,2,\cdots$

Loops are found to emit energy in the form of gravitational radiation at a constant rate

$$\frac{dE}{dt} = -\Gamma G\mu^2 \qquad \Gamma \sim 50$$

Assuming the fraction of the energy transfer in the form of large loops is $F_\alpha \sim 0.1$

$$\Omega_{\rm GW}(f) = \sum_k \Omega_{\rm GW}^{(k)}(f) = \frac{1}{\rho_c}\frac{2k}{f}\frac{\mathcal{F}_\alpha \Gamma^{(k)} G\mu^2}{\alpha(\alpha + \Gamma G\mu)}$$

$$= \int_{t_F}^{t_0} dt \frac{C_{\rm eff}\left(t_i^{(k)}\right)}{t_i^{(k)4}} \frac{a^2(t) a^3\left(t_i^{(k)}\right)}{a^5\left(t_0\right)} \theta\left(t_i^{(k)} - t_F\right)$$

$$C_{\rm eff} = 5.7, 0.5$$

1101.5173 **1808.08968** **0003298**

GUT Model

In the Yukawa sector, couplings above the GUT scale are given by

$$Y^*_{\mathbf{10}}\, \mathbf{16}\cdot\mathbf{16}\cdot\mathbf{10} + Y^*_{\overline{\mathbf{126}}}\, \mathbf{16}\cdot\mathbf{16}\cdot\overline{\mathbf{126}} + Y^*_{\mathbf{120}}\, \mathbf{16}\cdot\mathbf{16}\cdot\mathbf{120} + \text{h.c.}\,,$$

After breaking to G_{SM}

$$Y_{\mathbf{10}}\Big[(\overline{Q}u_R + \overline{L}\nu_R)h^u_{\mathbf{10}} + (\overline{Q}d_R + \overline{L}e_R)h^d_{\mathbf{10}}\Big] + \frac{1}{\sqrt{3}}Y_{\overline{\mathbf{126}}}\left[(\overline{Q}u_R - 3\overline{L}\nu_R)h^u_{\overline{\mathbf{126}}} + (\overline{Q}d_R - 3\overline{L}e_R)h^d_{\overline{\mathbf{126}}}\right]$$

$$+Y_{\mathbf{120}}\Big[(\overline{Q}u_R + \overline{L}\nu_R)h^u_{\mathbf{120}} + (\overline{Q}d_R + \overline{L}e_R)h^d_{\mathbf{120}} + \frac{1}{\sqrt{3}}(\overline{Q}u_R - 3\overline{L}\nu_R)h^{u'}_{\mathbf{120}} + (\overline{Q}d_R - 3\overline{L}e_R)h^{d'}_{\mathbf{120}}\Big] + \text{h.c.}$$

Rotating the Higgs fields to their mass basis, we derive Yukawa couplings to the SM Higgs

$$Y_u\bar{Q}\tilde{h}_{\text{SM}}u_R + Y_d\bar{Q}h_{\text{SM}}d_R + Y_\nu\bar{L}\tilde{h}_{\text{SM}}\nu_R + Y_e\bar{L}h_{\text{SM}}e_R + \text{ h.c.}$$

$$Y_u = Y_{10}V^*_{11} + \frac{1}{\sqrt{3}}Y_{\overline{126}}V^*_{12} + Y_{120}\left(V^*_{13} + \frac{1}{\sqrt{3}}V^*_{14}\right)$$

$$Y_d = Y_{10}V_{15} + \frac{1}{\sqrt{3}}Y_{\overline{126}}V_{16} + Y_{120}\left(V_{17} + \frac{1}{\sqrt{3}}V_{18}\right)$$

$$Y_\nu = Y_{10}V^*_{11} - \sqrt{3}Y_{\overline{126}}V^*_{12} + Y_{120}\left(V^*_{13} - \sqrt{3}V^*_{14}\right)$$

$$Y_e = Y_{10}V_{15} - \sqrt{3}Y_{\overline{126}}V_{16} + Y_{120}\left(V_{17} - \sqrt{3}V_{18}\right).$$

GUT Model

$$Y_u = Y_{10}V_{11}^* + \frac{1}{\sqrt{3}}Y_{\overline{126}}V_{12}^* + Y_{120}\left(V_{13}^* + \frac{1}{\sqrt{3}}V_{14}^*\right)$$
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$$Y_\nu = Y_{10}V_{11}^* - \sqrt{3}Y_{\overline{126}}V_{12}^* + Y_{120}\left(V_{13}^* - \sqrt{3}V_{14}^*\right)$$
$$Y_e = Y_{10}V_{15} - \sqrt{3}Y_{\overline{126}}V_{16} + Y_{120}\left(V_{17} - \sqrt{3}V_{18}\right).$$

$$Y_u = h + r_2 f + i r_3 h', \quad Y_d = r_1\left(h + f + ih'\right), \quad Y_\nu = h - 3r_2 f + i c_\nu h'$$
$$Y_e = r_1\left(h - 3f + i c_e h'\right), \quad M_{\nu_R} = f\,\frac{\sqrt{3}r_1}{V_{16}}v_S$$

$$h = Y_{\mathbf{10}}V_{11}\,,\ f = Y_{\overline{\mathbf{126}}}\frac{V_{16}}{\sqrt{3}}\frac{V_{11}^*}{V_{15}}\,,\ c_e = \frac{V_{17} - \sqrt{3}V_{18}}{V_{17} + V_{18}/\sqrt{3}}\,, \qquad c_\nu = \frac{V_{13}^* - \sqrt{3}V_{14}^*}{V_{17} + V_{18}/\sqrt{3}}\frac{V_{15}}{V_{11}^*}\,,$$
$$r_1 = \frac{V_{15}}{V_{11}^*}\,, \quad r_2 = \frac{V_{12}^*}{V_{16}}\frac{V_{15}}{V_{11}^*}\,, \qquad r_3 = \frac{V_{13}^* + V_{14}^*/\sqrt{3}}{V_{17} + V_{18}/\sqrt{3}}\frac{V_{15}}{V_{11}^*}\,,\ h' = -i\,Y_{\mathbf{120}}\left(V_{17} + V_{18}/\sqrt{3}\right)\frac{V_{11}^*}{V_{15}}\,,$$

GUT Model

$$Y_\nu = -\frac{3r_2+1}{r_2-1} Y_u + \frac{4r_2}{r_1\left(r_2-1\right)} \operatorname{Re} Y_d + i\frac{c_\nu}{r_1} \operatorname{Im} Y_d$$

$$Y_e = -\frac{4r_1}{r_2-1} Y_u + \frac{r_2+3}{r_2-1} \operatorname{Re} Y_d + ic_e \operatorname{Im} Y_d$$

$$M_\nu = m_0 \left(\frac{8r_2\left(r_2+1\right)}{r_2-1} Y_u - \frac{16r_2^2}{r_1\left(r_2-1\right)} \operatorname{Re} Y_d + \frac{r_2-1}{r_1}\left(r_1 Y_u + ic_\nu \operatorname{Im} Y_d\right)\left(r_1 Y_u - \operatorname{Re} Y_d\right)^{-1}\left(r_1 Y_u - ic_\nu \operatorname{Im} Y_d\right) \right)$$

GUT Model Particle Content

	Multiplet	Role in the model
Fermions	**16**	Contains all SM fermions and RH neutrinos
Higgses	**10**	Generates fermion masses
	45	Triggers intermediate symmetry breaking
	54	Triggers GUT symmetry breaking
	120	Generates fermion masses
	$\overline{\mathbf{126}}$	Generates fermion masses & intermediate symmetry breaking
	210	Triggers intermediate symmetry breaking

$SO(10)$	**54**	**210**	**45**	$\overline{\mathbf{126}}$
G_3	$(\mathbf{1},\mathbf{1},\mathbf{1})$	$(\mathbf{15},\mathbf{1},\mathbf{1})_1$	$(\mathbf{15},\mathbf{1},\mathbf{1})_2$	$(\mathbf{10},\mathbf{1},\mathbf{3})+(\overline{\mathbf{10}},\mathbf{3},\mathbf{1})$
G_2	–	$(\mathbf{1},\mathbf{1},\mathbf{1},0)_1$	$(\mathbf{1},\mathbf{1},\mathbf{1},0)_2$	$(\mathbf{1},\mathbf{1},\mathbf{3},-1)+(\mathbf{1},\mathbf{3},\mathbf{1},1)$
G_1	–	–	$(\mathbf{1},\mathbf{1},\mathbf{1},0)_2$	$(\mathbf{1},\mathbf{1},\mathbf{3},-1)$
$G_{\rm SM}$	–	–	–	$(\mathbf{1},\mathbf{1},0)_S$

SO(10) Higgs reps for SSB

$SO(10)$	**16**
G_3	$(\mathbf{4},\mathbf{2},\mathbf{1})_L+(\overline{\mathbf{4}},\mathbf{1},\mathbf{2})_{R^c}$
G_2	$(\mathbf{3},\mathbf{2},\mathbf{1},1/6)_{Q_L}+(\overline{\mathbf{3}},\mathbf{1},\mathbf{2},-1/6)_{Q^c_R}$ $+(\mathbf{1},\mathbf{2},\mathbf{1},-1/2)_{l_L}+(\mathbf{1},\mathbf{1},\mathbf{2},1/2)_{l^c_R}$
G_1	$(\mathbf{3},\mathbf{2},\mathbf{1},1/6)_{Q_L}+(\overline{\mathbf{3}},\mathbf{1},\mathbf{2},-1/6)_{Q^c_R}$ $+(\mathbf{1},\mathbf{2},\mathbf{1},-1/2)_{l_L}+(\mathbf{1},\mathbf{1},\mathbf{2},1/2)_{l^c_R}$
$G_{\rm SM}$	$(\mathbf{3},\mathbf{2},1/6)_{Q_L}+(\overline{\mathbf{3}},\mathbf{1},-2/3)_{u^c_R}+(\overline{\mathbf{3}},\mathbf{1},1/3)_{d^c_R}$ $+(\mathbf{1},\mathbf{2},-1/2)_{l_L}+(\mathbf{1},\mathbf{1},0)_{\nu^c_R}+(\mathbf{1},\mathbf{1},1)_{e^c_R}$

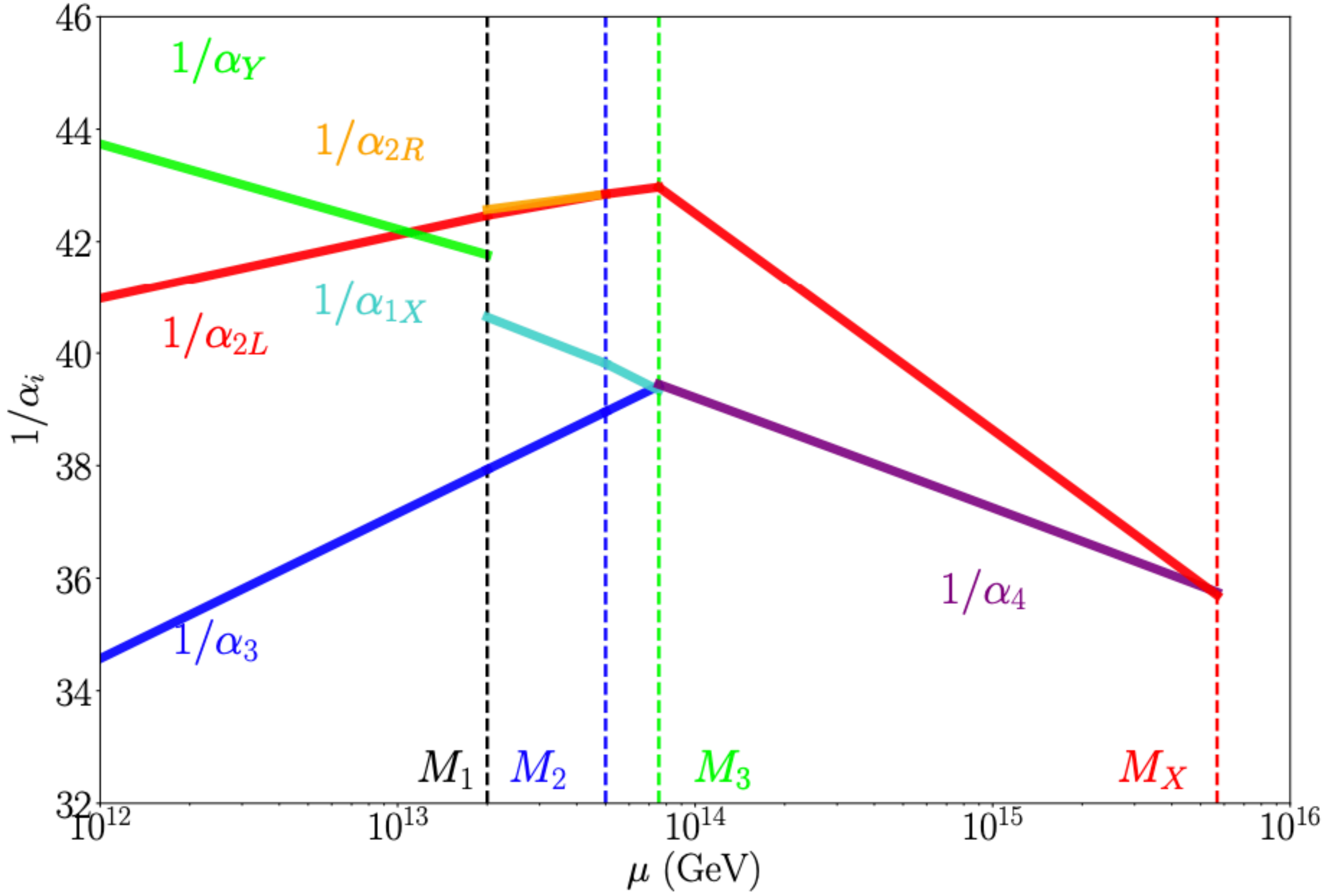
Matter field decomposition

$SO(10)$	**10**	$\overline{\mathbf{126}}$	**120**
G_3	$(\mathbf{1},\mathbf{2},\mathbf{2})_1$	$(\mathbf{15},\mathbf{2},\mathbf{2})_1$ $+(\mathbf{10},\mathbf{1},\mathbf{3})+(\overline{\mathbf{10}},\mathbf{3},\mathbf{1})$	$(\mathbf{1},\mathbf{2},\mathbf{2})_2+(\mathbf{15},\mathbf{2},\mathbf{2})_2$
G_2	$(\mathbf{1},\mathbf{2},\mathbf{2},0)_1$	$(\mathbf{1},\mathbf{2},\mathbf{2},0)_2$ $+(\mathbf{1},\mathbf{1},\mathbf{3},-1)+(\mathbf{1},\mathbf{3},\mathbf{1},1)$	$(\mathbf{1},\mathbf{2},\mathbf{2},0)_{3,4}$
G_1	$(\mathbf{1},\mathbf{2},\mathbf{2},0)_1$	$(\mathbf{1},\mathbf{2},\mathbf{2},0)_2$ $+(\mathbf{1},\mathbf{1},\mathbf{3},-1)$	$(\mathbf{1},\mathbf{2},\mathbf{2},0)_{3,4}$
$G_{\rm SM}$	$(\mathbf{1},\mathbf{2},-1/2)_{h^u_{\mathbf{10}}}$ $+(\mathbf{1},\mathbf{2},+1/2)_{h^d_{\mathbf{10}}}$	$(\mathbf{1},\mathbf{2},-1/2)_{h^u_{\overline{\mathbf{126}}}}$ $+(\mathbf{1},\mathbf{2},+1/2)_{h^d_{\overline{\mathbf{126}}}}$ $+(\mathbf{1},\mathbf{1},0)_S$	$(\mathbf{1},\mathbf{2},-1/2)_{h^u_{\mathbf{120}},h^{u'}_{\mathbf{120}}}$ $+(\mathbf{1},\mathbf{2},+1/2)_{h^d_{\mathbf{120}},h^{d'}_{\mathbf{120}}}$

SO(10) Higgs reps for fermion mass generation

GUT Model Particle Content

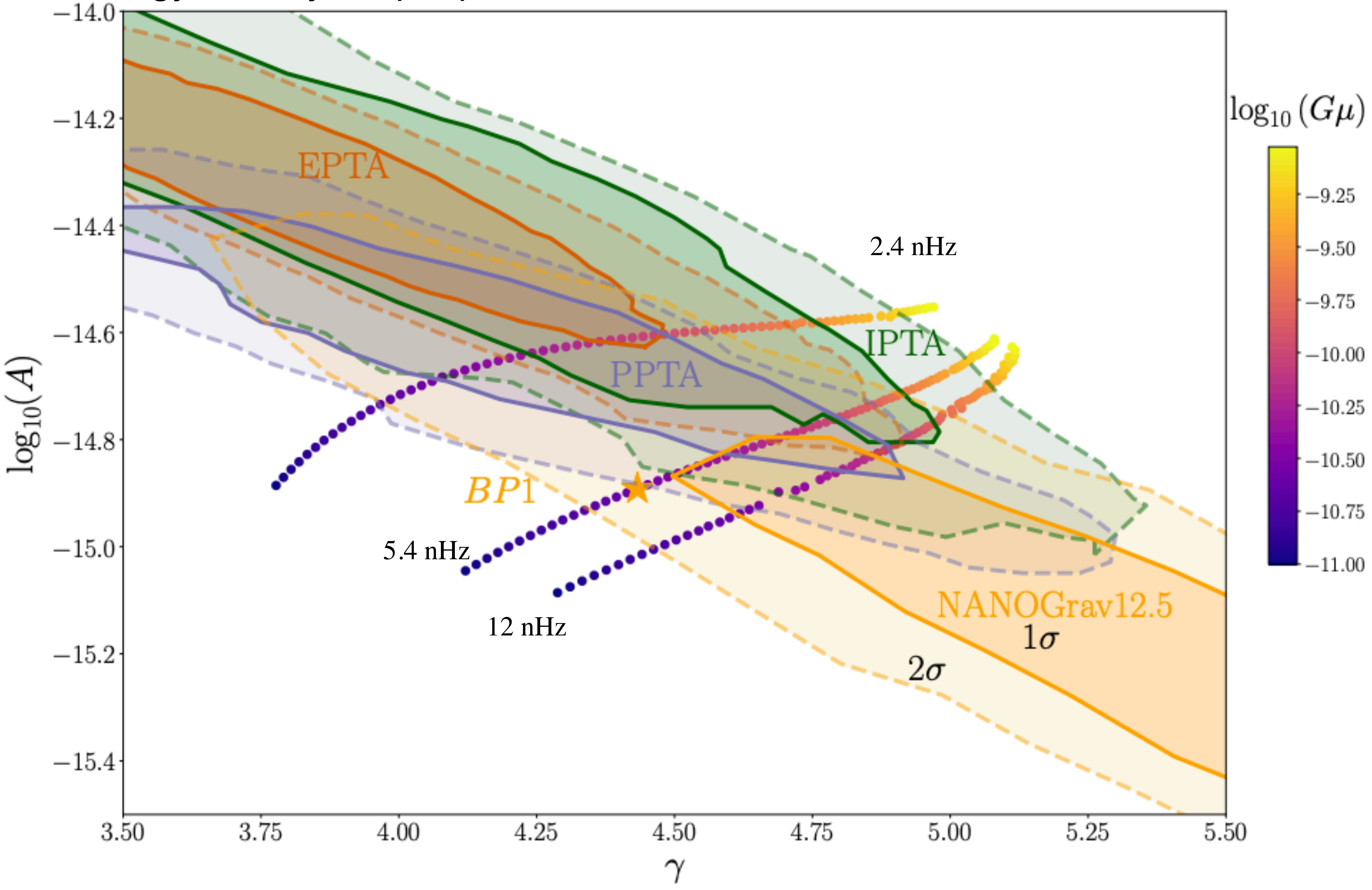
Benchmark 1 RGE



Overlap with PTA experiments

A ≡ amplitude parameter of correlation between pulsars.

γ ≡ related to GW energy density freq dependence



Leptogenesis Equations

$$\frac{dN^{B-L}_{\alpha\beta}}{dz} = \sum_{i=1}^{3} \varepsilon^{(i)}_{\alpha\beta} D_i \left(N_{N_i} - N^{\text{eq}}_{N_i} \right) - \frac{1}{2} W_i \left\{ \mathcal{P}^{(i)0}, N^{B-L} \right\}_{\alpha\beta}$$
$$- \frac{\text{Im}\left(\Lambda_\tau\right)}{Hz} \left[\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \left[\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, N^{B-L} \right] \right]_{\alpha\beta}$$
$$- \frac{\text{Im}\left(\Lambda_\mu\right)}{Hz} \left[\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \left[\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, N^{B-L} \right] \right]_{\alpha\beta} ,$$

$$N^{B-L} = \begin{pmatrix} N_{\tau\tau} & N_{\tau\mu} & N_{\tau e} \\ N_{\mu\tau} & N_{\mu\mu} & N_{\mu e} \\ N_{e\tau} & N_{e\mu} & N_{ee} \end{pmatrix} , \quad \mathcal{P}^{(i)0} = \frac{1}{\left(\tilde{Y_\nu}^\dagger \tilde{Y_\nu}\right)_{ii}} \begin{pmatrix} \left|\tilde{Y}_{\nu\tau i}\right|^2 & \tilde{Y}_{\nu\tau i}\tilde{Y}^\star_{\nu\mu i} & \tilde{Y}_{\nu\tau i}\tilde{Y}^\star_{\nu e i} \\ \tilde{Y}^\star_{\nu\tau i}\tilde{Y}_{\nu\mu i} & \left|\tilde{Y}_{\nu\mu i}\right|^2 & \tilde{Y}^\star_{\nu\tau i}\tilde{Y}_{\nu e i} \\ \tilde{Y}_{\nu e i}\tilde{Y}^\star_{\nu\tau i} & \tilde{Y}_{\nu\mu i}\tilde{Y}^\star_{\nu\tau i} & \left|\tilde{Y}_{\nu e i}\right|^2 \end{pmatrix} .$$