# **Confronting GUTs with Proton Decay and Gravitational Waves** Jessica Turner

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# 2022 International Workshop on **B**aryon and Lepton Number Violation





## **Motivation for Grand Unified Theories**

- 19 free and seemingly arbitrary Standard Model parameters
- GUT unifies SM gauge interactions into a single gauge group
- SM fermionic multiplet  $\implies$  single GUT irrep  $\implies$ reduces # parameters
- Many GUTs predict non-zero neutrino masses, dark matter candidate etc



### **GUT Predictions - Proton Decay**

- GUTs unify leptons and quarks into common multiplets.
- GUTs spontaneously broken to SM gauge group  $\implies$  heavy gauge boson integrated out  $\implies$  baryon number violating process e.g. proton decay

$$\underbrace{\frac{\epsilon_{\alpha\beta}}{\Lambda_{1}^{2}} \left[ \left(\overline{u_{R}^{c}} \gamma^{\mu} Q_{\alpha}\right) \left(\overline{d_{R}^{c}} \gamma_{\mu} L_{\beta}\right) + \left(\overline{u_{R}^{c}} \gamma^{\mu} Q_{\alpha}\right) \left(\overline{e_{R}^{c}} \gamma_{\mu} Q_{\beta}\right) \right] }_{+ \frac{\epsilon_{\alpha\beta}}{\Lambda_{2}^{2}} \left[ \left(\overline{d_{R}^{c}} \gamma^{\mu} Q_{\alpha}\right) \left(\overline{u_{R}^{c}} \gamma_{\mu} L_{\beta}\right) + \left(\overline{d_{R}^{c}} \gamma^{\mu} Q_{\alpha}\right) \left(\overline{\nu_{R}^{c}} \gamma_{\mu} Q_{\beta}\right) \right] }$$

$$\underbrace{SU(3)_{C} \times SU(2)_{L} \times U(1)_{Y} \text{ invariant but BN}}_{C}$$



<u>Weinberg, 1979</u>





### **GUT Predictions - Proton Decay**

- GUTs unify leptons and quarks into common multiplets.
- GUTs spontaneously broken to SM gauge group  $\implies$  heavy gauge boson integrated out  $\implies$  baryon number violating process e.g. proton decay

$$\frac{\frac{\epsilon_{\alpha\beta}}{\Lambda_{1}^{2}} \left[ \left( \overline{u_{R}^{c}} \gamma^{\mu} Q_{\alpha} \right) \left( \overline{d_{R}^{c}} \gamma_{\mu} L_{\beta} \right) + \left( \overline{u_{R}^{c}} \gamma^{\mu} Q_{\alpha} \right) \left( \overline{e_{R}^{c}} \gamma_{\mu} Q_{\beta} \right) \right]}{+ \frac{\epsilon_{\alpha\beta}}{\Lambda_{2}^{2}} \left[ \left( \overline{d_{R}^{c}} \gamma^{\mu} Q_{\alpha} \right) \left( \overline{u_{R}^{c}} \gamma_{\mu} L_{\beta} \right) + \left( \overline{d_{R}^{c}} \gamma^{\mu} Q_{\alpha} \right) \left( \overline{\nu_{R}^{c}} \gamma_{\mu} Q_{\beta} \right) \right]} \\ p \left( u \right) \\ p \left( u \right) \\ \frac{1}{d} \left( u$$

<u>Weinberg, 1979</u>



olden Channel non-SUSY GUTs



### **World Leading Limit on Proton Decay**

### Neutrino experiments are large vats of proton sitting around for a long time. $\tau_{\pi^0 e^+} > 1.6 \times 10^{34} \text{ years}$ <u>SK (1610.03597)</u>





### **Future Limits from Neutrino Experiments**



Water Cherenkov sensitive to  $p \rightarrow e^+ \pi^0$  LArTPC more sensitive to  $p \rightarrow K^+ \nu$ 

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# **GUT Predictions - Topological Defects** During SSB from $G_{GUT} \rightarrow \cdots \rightarrow G_{SM}$ topological defects may form.





 $\pi_0(G/H) \neq 0$ 

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Monopoles

 $\pi_1(G/H) \neq 0$ 



 $\pi_2(G/H) \neq 0$ 

cambridge cosmic





# **GUT Predictions - Topological Defects**





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During SSB from  $G_{GUT} \rightarrow \cdots \rightarrow G_{SM}$  topological defects may form.

**Cosmic strings** 

 $\pi_1(G/H) \neq 0$ 





During SSB from  $G_{GUT} \rightarrow \cdots \rightarrow G_{SM}$  topological defects may form.



### Cosmic strings induced via U(1) breaking are ubiquitously as GUT breaks to SM

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### **GUT Predictions - Topological Defects**





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### **GUT Predictions - Topological Defects**

 $V(\phi) = \frac{\lambda}{4} \left( |\phi|^2 - \eta^2 \right)^2$ 







### **GUT Predictions - Topological Defects**

 $V(\phi) = \frac{\lambda}{\Lambda} \left( |\phi|^2 - \eta^2 \right)^2$ 



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### **GUT Predictions - Topological Defects**

 $S_{U(1)} = \int d^4x \left[ \partial_\mu \phi \partial^\mu \phi^* - V \left( |\phi|^2 \right) \right]$  $V(\phi) = \frac{\lambda}{4} \left( |\phi|^2 - \eta^2 \right)^2$ **Institute for Particle Physics Phenomenology** 





### **GUT Predictions - Topological Defects**

 $S_{U(1)} = \int d^4x \left[ \partial_\mu \phi \partial^\mu \phi^* - V \left( |\phi|^2 \right) \right]$ 

 $V(\phi) = \frac{\lambda}{\Lambda} \left( |\phi|^2 - \eta^2 \right)^2$ 

Kibble, & Nielsen, Ole

 $\frac{\rho_{\rm string}}{\rho_{\rm tot}} \propto G\mu \sim \left(\frac{\langle \phi \rangle}{M_{\rm pl}}\right)^2$  $\mu \equiv \text{string parameter}$ 





### **GUT Predictions - Cosmic Strings**

- Inflation occurs before string formation → string network gives "scaling" solution
- Inflation occurs after string formation → string network diluted and no GW signal
- broken power law behaviour (Cui, Lewicki, Morrissey) <u>1912.08832</u>



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• Inflation occurs during string formation  $\rightarrow$  partly diluted string network  $\rightarrow$  GW spectrum

$$\Lambda_{\rm cs} = 10^{14} \,{\rm GeV}$$
$$G\mu = 0.7 \times 10^{-10}$$

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# Topological defects in non-supersymmetric SO(10)

unwanted

ects SY

Б

non-

Bu



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 $G_x = G_{3221} \text{ or } G_{421}$ 

2005.13549 King, Pascoli, JT, Zhou use PD and GWs to examine viable non-SUSY SO(10) GUT breaking chains.

# $G_r = G_{3221} \text{ or } G_{421}$

- Assume inflation at highest scale to remove unwanted defects and preserve cosmic strings
- GW & GUTs also explored by Buchmuller et al <u>1912.03695</u>









![](_page_15_Picture_7.jpeg)

![](_page_16_Figure_1.jpeg)

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	CO(10)	defect	$\mathcal{O}$	defect	$\mathcal{O}$	Observ	vable
	50(10	) Higgs	$G_1$	Higgs	$G_{\rm SM}$	string	gs?
	I1:	$\xrightarrow{\mathrm{m}}$	$G_{3221}$	$\xrightarrow{\mathrm{S}}$		$\checkmark$	
	I2:	$\begin{array}{c} 45 \\ \xrightarrow{\mathrm{m,s}} \\ 210 \end{array}$	$G_{3221}^{C}$	$\begin{array}{c} 126 \\ \xrightarrow{\mathrm{S,W}} \\ \xrightarrow{126} \end{array}$		X	
	I3:	$\xrightarrow{\mathrm{m}}$	$G_{421}$	$\xrightarrow{\mathrm{S}}$		$\checkmark$	
	I4:	$\begin{array}{c} {}^{45} \\ {}^{\mathrm{m}} \\ {}^{210} \end{array}$	$G_{422}$	$egin{array}{c} 126 \\ \stackrel{\mathrm{m}}{\longrightarrow} \\ \overline{126}.45 \end{array}$		×	
	I5:	$\xrightarrow[]{\text{m,s}}{\textbf{54}}$	$G^{C}_{422}$	$\xrightarrow[]{\text{m,w}}{126,45}$		×	
	I6:	$\xrightarrow[]{\text{m}}{\textbf{210}}$	$G_{3211}$	$\xrightarrow[]{\text{S}}{126}$		✓	
SO(	(10) $\xrightarrow{\text{defect}}_{\text{Higgs}}$	$G_3 \stackrel{\mathrm{defe}}{=}_{\mathrm{Higg}}$	$\stackrel{\mathrm{ct}}{\to} G_2$	$\stackrel{\text{defect}}{\longrightarrow} G$ $\underset{\text{Higgs}}{\text{defect}}$	defec 1 Higg	$\stackrel{\text{t}}{{\rightarrow}} G_{\text{SM}}$	Dbservable strings?
III1:	$: \xrightarrow{\mathrm{m,s}} 54$	$G_{422}^C = \frac{W}{210}$	$\rightarrow G_{422}$	$\xrightarrow{\mathrm{m}}$ G	$7421  \frac{s}{126}$	>	1
III2:	$: \qquad \xrightarrow{\mathrm{m,s}} 54$	$G_{422}^C = \frac{w}{210}$	$\rightarrow G_{422}$	$\xrightarrow{\mathrm{m}}$ G	$73221  \frac{s}{126}$	>	$\checkmark$
III3:	$: \qquad \xrightarrow{\mathrm{m,s}} 54$	$G_{422}^C = \frac{w}{210}$	$\rightarrow G_{422}$	m (	y S		1
III4:	• =	411		$\overrightarrow{210}$ 0	73211 - 73211	<b>≻</b>	$\checkmark$
	$: \qquad \xrightarrow{\mathrm{m,s}} 54$	$G_{422}^C = \frac{m}{210}$	$\rightarrow G^C_{3221}$	$\begin{array}{c} \overrightarrow{210} \\ \overrightarrow{210} \\ \overrightarrow{W} \\ \overrightarrow{45} \end{array} \qquad G$	$73211$ $\overline{126}$ $73221$ $\overline{\frac{s}{126}}$	→	✓ ✓
III5:	$\begin{array}{c} \xrightarrow{\mathrm{m,s}} \\ 54 \\ \vdots \\ \xrightarrow{\mathrm{m,s}} \\ 54 \end{array}$	$ \begin{array}{ccc}  & 210 \\ G_{422}^{C} & \frac{m}{210} \\ G_{422}^{C} & \frac{m}{210} \end{array} $	$ \begin{array}{l} \rightarrow & G^C_{3221} \\ \rightarrow & G^C_{3221} \\ \rightarrow & G^C_{3221} \end{array} $	$\begin{array}{c} \overrightarrow{210} \\ \overrightarrow{210} \\ \overrightarrow{W} \\ \overrightarrow{45} \\ \overrightarrow{M} \\ \overrightarrow{45} \end{array} \qquad G$	$\begin{array}{c} 3211 \\ \hline 126 \\ \hline 3221 \\ \hline 3221 \\ \hline 3211 \\ \hline \\ 3211 \\ \hline \\ 126 \\ \hline \end{array}$		✓ ✓ ✓
III5: III6:	$\begin{array}{c} \xrightarrow{\mathrm{m,s}} \\ 54 \\ \xrightarrow{\mathrm{m,s}} \\ 54 \\ \xrightarrow{\mathrm{m,s}} \\ 54 \\ \xrightarrow{\mathrm{54}} \end{array}$	$ \begin{array}{cccc}  & & & & & \\ G_{422}^C & & & & \\ \end{array} $	$\begin{array}{c} & G^{C}_{3221} \\ \rightarrow & G^{C}_{3221} \\ \rightarrow & G^{C}_{3221} \\ \leftarrow & G_{3221} \end{array}$	$\begin{array}{c} \overrightarrow{210} \\ \overrightarrow{210} \\ \overrightarrow{W} \\ \overrightarrow{45} \\ \overrightarrow{M} \\ \overrightarrow{45} \\ \overrightarrow{M} \\ \overrightarrow{45} \end{array} \qquad G$	$   \begin{array}{c}       3211 \\       \overline{126} \\       3221 \\       \overline{126} \\   \end{array} $	→	✓ ✓ ✓
III5: III6: III7:	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{c} & \mathbf{Z}_{422}^{\mathrm{C}} & \frac{\mathrm{m}}{210} \\ G_{422}^{C} & \frac{\mathrm{m}}{210} \\ G_{422}^{C} & \frac{\mathrm{m}}{210} \\ G_{422}^{C} & \frac{\mathrm{m},\mathrm{w}}{45} \\ G_{3221}^{C} & \frac{\mathrm{w}}{45} \end{array}$	$\begin{array}{c} \bullet \\ \bullet $	$\begin{array}{c} \overrightarrow{210} \\ \overrightarrow{W} \\ \overrightarrow{45} \\ \overrightarrow{H} \\ \overrightarrow{45} \\ \overrightarrow{H} \\ \overrightarrow{45} \\ \overrightarrow{H} \\ \overrightarrow{45} \\ \overrightarrow{H} \\ \overrightarrow{45} $ \overrightarrow{45}	$ \begin{array}{c} 3211 \\ \hline 126 \\ 3221 \\ \hline 3221 \\ \hline 3211 \\ $		✓ ✓ ✓ ✓
<ul><li>III5:</li><li>III6:</li><li>III7:</li><li>III8:</li></ul>	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{c} & \mathbf{G}_{422}^{C} & \frac{\mathbf{m}}{210} \\ G_{422}^{C} & \frac{\mathbf{m}}{210} \\ G_{422}^{C} & \frac{\mathbf{m}}{210} \\ G_{422}^{C} & \frac{\mathbf{m}, \mathbf{w}}{45} \\ G_{3221}^{C} & \frac{\mathbf{w}}{45} \\ G_{422}^{C} & \frac{\mathbf{m}}{45} \end{array}$	$\begin{array}{c} & G^{C}_{3221} \\ \rightarrow & G^{C}_{3221} \\ \rightarrow & G^{C}_{3221} \\ \rightarrow & G_{3221} \\ \rightarrow & G_{3221} \\ \rightarrow & G_{3221} \\ \rightarrow & G_{3221} \end{array}$	$\begin{array}{c} \overrightarrow{210} \\ \overrightarrow{W} \\ \overrightarrow{45} \\ \overrightarrow{45} \\ \overrightarrow{M} \\ \overrightarrow{45} \\ \overrightarrow{45} \\ \overrightarrow{M} \\ \overrightarrow{45} \\ \overrightarrow{45} \\ \overrightarrow{M} \\ \overrightarrow{45} $ \overrightarrow{45} \overrightarrow{45} \overrightarrow{45}  \overrightarrow{45} 45	$ \begin{array}{c} 3211 \\ \hline 126 \\ \hline 3221 \\ \hline 3221 \\ \hline 126 \\ \hline 3211 \\ \hline 3211 \\ \hline \hline $	$\Rightarrow$ $\Rightarrow$ $\Rightarrow$	
<ul> <li>III5</li> <li>III6</li> <li>III7</li> <li>III8</li> <li>III8</li> <li>III9</li> </ul>	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{c} & \begin{array}{c} & \begin{array}{c} & \begin{array}{c} & \begin{array}{c} & \\ & \\ & \\ & \\ & \end{array} \end{array} \\ & & \\ & & \\ & \\$	$\begin{array}{c} & G^{C}_{3221} \\ \hline & G^{C}_{3221} \\ \hline & G^{C}_{3221} \\ \hline & G_{3221} \\ \hline & G_{421} \end{array}$	$\begin{array}{c} \overrightarrow{210} & G \\ \overrightarrow{210} & G \\ \hline \end{array} \\ \overrightarrow{45} & G \\ \hline \end{array} $	$ \begin{array}{c} 3211 \\ \hline 126 \\ 3221 \\ \hline 3221 \\ \hline 3211 \\ $	$\Rightarrow$ $\Rightarrow$ $\Rightarrow$ $\Rightarrow$	

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SO(10)IV1: IV2: IV3:

SO(10)	defect	C	defect	C	defect	C	Observable
50(10)	Higgs	$G_2$	Higgs	$G_1$	Higgs	$G_{\rm SM}$	strings?
II1:	$\stackrel{ m m}{\longrightarrow}$ 210	$G_{422}$	$rac{\mathrm{m}}{45}$	$G_{3221}$	$\xrightarrow{\mathrm{S}}{\overline{126}}$		$\checkmark$
II2:	$\xrightarrow[]{\text{m,s}}{\textbf{54}}$	$G^C_{422}$	$\stackrel{ m m}{\longrightarrow}$ 210	$G_{3221}^{C}$	$\xrightarrow{\overline{s,w}}$		×
II3:	$\xrightarrow[54]{\mathrm{m,s}}$	$G_{422}^{C}$	$\stackrel{\mathrm{m,w}}{\longrightarrow}$ 45	$G_{3221}$	$\xrightarrow{\text{S}}{\overline{126}}$		$\checkmark$
II4:	$\xrightarrow[]{\text{m,s}}{\textbf{210}}$	$G_{3221}^{C}$	$rac{\mathrm{w}}{45}$	$G_{3221}$	$\xrightarrow{s}{\overline{126}}$		✓
II5:	$\stackrel{ m m}{\longrightarrow}$ 210	$G_{422}$	$\stackrel{ m m}{\longrightarrow}$ 45	$G_{421}$	$\xrightarrow{s}{\overline{126}}$		✓
II6:	$\xrightarrow[]{\text{m,s}}{\textbf{54}}$	$G_{422}^{C}$	$\stackrel{ m m}{\longrightarrow}$ 45	$G_{421}$	$\xrightarrow{s}{\overline{126}}$		✓
II7:	$\xrightarrow[]{\text{m,s}}{\textbf{54}}$	$G_{422}^{C}$	$\stackrel{ ext{W}}{ ext{210}}$	$G_{422}$	$\xrightarrow{\frac{m}{126}}{45}$		×
II8:	$\stackrel{\mathrm{m}}{\longrightarrow}$ 45	$G_{3221}$	$\stackrel{\mathrm{m}}{\longrightarrow}$ 45	$G_{3211}$	$\xrightarrow{s}{\overline{126}}$		$\checkmark$
II9:	$\xrightarrow[]{\text{m,s}}{\textbf{210}}$	$G_{3221}^{C}$	$\stackrel{\mathrm{m,w}}{\longrightarrow}$	$G_{3211}$	$\xrightarrow{s}{\overline{126}}$		✓
II10:	$\stackrel{\mathrm{m}}{\longrightarrow}$ 210	$G_{422}$	$\stackrel{\mathrm{m}}{\longrightarrow}$ 210	$G_{3211}$	$\xrightarrow{s}{\overline{126}}$		✓
II11:	$\xrightarrow[54]{\mathrm{m,s}}$	$G_{422}^{C}$	$\xrightarrow{\mathrm{m,w}}210$	$G_{3211}$	$\frac{\xrightarrow{s}}{126}$		✓
II12:	$rac{\mathrm{m}}{45}$	$G_{421}$	$rac{\mathrm{m}}{45}$	$G_{3211}$	$\xrightarrow[]{s}{126}$		✓

$\stackrel{\text{defect}}{\longrightarrow}_{\text{Higgs}}$	$G_4$	$\stackrel{\text{defect}}{\longrightarrow}_{\text{Higgs}}$	$G_3$	$\stackrel{\text{defect}}{\longrightarrow}_{\text{Higgs}}$	$G_2$	$\stackrel{\text{defect}}{\longrightarrow}_{\text{Higgs}}$	$G_1$	$\stackrel{\text{defect}}{\longrightarrow}_{\text{Higgs}}$	$G_{\rm SM}$	Observable strings?
$\xrightarrow[]{\text{m,s}}{54}$	$G^{C}_{422}$	$\xrightarrow[]{\text{m}}{\textbf{210}}$	$G_{3221}^{C}$	$rac{\mathrm{w}}{45}$	$G_{3221}$	$rac{\mathrm{m}}{45}$	$G_{3211}$	$\xrightarrow{\text{S}} \overline{126}$		<ul> <li>Image: A start of the start of</li></ul>
$\xrightarrow[]{\text{m,s}}{\textbf{54}}$	$G_{422}^{C}$	$\stackrel{ ext{W}}{ ext{210}}$	$G_{422}$	$rac{\mathrm{m}}{45}$	$G_{3221}$	$rac{\mathrm{m}}{45}$	$G_{3211}$	$\xrightarrow{s}{\overline{126}}$		✓
$\xrightarrow[]{\text{m,s}}{\textbf{54}}$	$G_{422}^{C}$	$\stackrel{ ext{W}}{ ext{210}}$	$G_{422}$	$rac{\mathrm{m}}{45}$	$G_{421}$	$rac{\mathrm{m}}{45}$	$G_{3211}$	$\xrightarrow{s}{\overline{126}}$		✓

![](_page_17_Picture_8.jpeg)

]		defec		d	foot		Obcorve	hlo	Mone	nole				_								
	SO(10)	$) \frac{\text{derec}}{}$	$\stackrel{_{\mathcal{T}}}{\rightarrow} G_1$	ae	$\stackrel{\text{eiect}}{\longrightarrow} G$	SM	Observa		WOIR	pole					SO(10)	$\stackrel{\rm defect}{\longrightarrow}$	$C_{-2}$	$\stackrel{\rm defect}{\longrightarrow}$	$C_{1}$	$\stackrel{\rm defect}{\longrightarrow}$	$C_{\alpha}$	Observable
		' Higg	S I	Н	iggs		strings	5?							50(10)	Higgs	02	Higgs	ΟŢ	Higgs	USM	strings?
	I1:	m	$\rightarrow G_{32}$	21 -	S										II1:	$\xrightarrow{\mathrm{m}}$	$G_{422}$	$\xrightarrow{\mathrm{m}}$	$G_{3221}$	$\xrightarrow{\mathrm{S}}$		$\checkmark$
		<b>45</b>		$\overline{1}$	26										110.	210 m,s	$C^{C}$	45 m	$C^{C}$	126 s,w		¥
	I2:	<u> </u>	$\rightarrow G^C_{32}$	21 _			×								112.	54	G <sub>422</sub>	<b>210</b>	G <sub>3221</sub>	$\overline{126}$		r
	To	210 m		- 1	. <b>26</b>				cosm	ic					II3:	$\xrightarrow{11,8}{54}$	$G_{422}^{C}$	$\xrightarrow{111,w}$	$G_{3221}$	$\xrightarrow{S}{\overline{100}}$		$\checkmark$
	13:	<u></u>	$\rightarrow G_{42}$	1 -	$\rightarrow$				string	<b>js</b>					II4∙	$\xrightarrow{\mathrm{m,s}}$	$G^{C}_{aaaa}$	$\xrightarrow{W}$	$G_{2221}$	$\xrightarrow{\text{S}}$		
	Τ1.	<b>40</b> m		T	. <b>26</b> m		V			-						<b>210</b> <sup>′</sup>	3221	<b>45</b> ′ m	G 3221	$\overline{126}'$		•
	14.	$\overline{210}$	$\rightarrow$ G <sub>42</sub>	$2 \overline{1}$	$\overrightarrow{26} 45$		^		Dor	nain					II5:	$\stackrel{ ext{m}}{ ext{210}}$	$G_{422}$	$rac{11}{45}$	$G_{421}$	$\frac{3}{126}$		
	15.	m,s	$\sim C^{C}$	]	m,w		Y			alle					II6:	$\xrightarrow{\mathrm{m,s}}$	$G_{422}^{C}$	$\xrightarrow{\mathrm{m}}$	$G_{421}$	$\xrightarrow{s}$		$\checkmark$
	10.	$\overline{54}$	7 G <sub>42</sub>	$\frac{2}{12}$	$\overline{\overline{26.45}}$				vvc						TT-7	<b>54</b> m,s	CC	45 W	$\overline{C}$	<b>126</b> m		V
	I6·	m	$\rightarrow G_{22}$	11 -	S ́		1								117:	$\overrightarrow{54}$	$G_{422}^{\circ}$	$\overrightarrow{210}$	$G_{422}$	$\overrightarrow{126,45}$		~
	10.	<b>210</b>	/ 0.32	$1$ $\overline{1}$	.26		•								II8:	$\xrightarrow{\mathrm{m}}$	$G_{3221}$	$\xrightarrow{\mathrm{m}}$	$G_{3211}$	$\xrightarrow{\mathrm{S}}$		$\checkmark$
0.0	defect	d	efect	def	fect	defect	Ob	oservable							110.	45 m,s	$C^{C}$	<b>45</b> m,w	$C_{-2214}$	126 		
SO(	10) $$ Higgs	G3 E	$$ $G_2$	Hig	$\xrightarrow{ggs} G_1$	$\rightarrow$ Higgs	$G_{ m SM}$ s	trings?							113.	<b>210</b>	G <sub>3221</sub>	<b>45</b>	G3211	$\overline{126}'$		V
III1:	$\xrightarrow{\mathrm{m,s}}$	$\overline{G^{C}_{422}}$	$\xrightarrow{\mathrm{w}} G_{42}$	$\frac{n}{22}$	$\xrightarrow{\mathrm{n}} G_{421}$	$\xrightarrow{s}$		<ul> <li>Image: A start of the start of</li></ul>							II10:	$rac{11}{210}$	$G_{422}$	$\stackrel{ ext{in}}{ ext{210}}$	$G_{3211}$	$\xrightarrow{5}{126}$		$\checkmark$
III9-	54 m,s	$C^{C}$	$\xrightarrow{W} C $	$\frac{4}{n}$	$\stackrel{5}{\longrightarrow} C_{2222}$	126									II11:	$\xrightarrow{\mathrm{m,s}}$	$G^{C}_{422}$	$\overset{\mathrm{m,w}}{\longrightarrow}$	$G_{3211}$	$\xrightarrow{s}$		$\checkmark$
1114	<b>54</b> m s		210 U4	<sup>22</sup> 4	5	$1 \overline{126}$		•							TT10	54 m	$\alpha$	210 m	0-11	$\overline{126}_{\mathrm{S}}$		
III3:	$\begin{array}{c} \xrightarrow{11,3} \\ 54 \end{array}$	$G_{422}^C$	$\xrightarrow{w} G_{42}$	$22  \frac{1}{21}$	$\stackrel{\mathrm{n}}{\longrightarrow} G_{321}$	$1 \xrightarrow{3} \overline{126}$									1112:	$\overrightarrow{45}$	$G_{421}$	$\overrightarrow{45}$	$G_{3211}$	$\stackrel{\longrightarrow}{\overline{126}}$		$\checkmark$
III4:	$\xrightarrow{\mathrm{m,s}}$	$G^{C}_{422}$ :	$\xrightarrow{\mathrm{m}} G_{32}^C$	$\frac{v}{221}$ - $\frac{v}{4}$	$\xrightarrow{v} G_{322}$	$1 \xrightarrow{s}{s}$		1			d of o of		d of o of		defect		d of o of		defe	- <b>-</b>	$\cap$	beorvabla
III5	$\begin{array}{c} 54\\ \mathbf{m,s}\\ \xrightarrow{\mathbf{m}}\end{array}$	$C^{C}$	$\xrightarrow{\mathrm{m}} C^{C}$	4 m	$\stackrel{\mathbf{b}}{\xrightarrow{,\mathrm{W}}} G_{221}$	126				SO(10)	$) \xrightarrow{\text{delect}}$	$G_4$	$\xrightarrow{\text{delect}}$	$G_3$	$\xrightarrow{\text{delect}}$	$G_2$	$\xrightarrow{\text{defect}}$	$G_1$		$\stackrel{\text{\tiny CL}}{\rightarrow} G_{\text{SI}}$	M U	
1110	<b>54</b> m.s		<b>210</b>	<sup>221</sup> 4	<b>5</b>	$1 \overline{126}$		•		, , ,	Higgs	~	Higgs		Higgs		Higgs		Higg	S		strings?
III6:	$\begin{array}{c} \xrightarrow{11,3} \\ 54 \end{array}$	$G_{422}^{C}$ -	$\xrightarrow{\mathrm{d}a,\mathrm{d}} G_{32}$	$221 \frac{1}{4}$	$\stackrel{\mathrm{n}}{\rightarrow} G_{321}$	$1 \xrightarrow[]{126}$				IV1:	$\xrightarrow{111,5}{54}$	$G_{422}^{C}$	$\xrightarrow{111}$ 210	$G_{3221}^{C}$	$\xrightarrow{W}$	$G_{3221}$	$\xrightarrow{111}$	$G_{322}$	$11 \frac{s}{10}$	$\frac{1}{2}$		$\checkmark$
III7:	$\xrightarrow{\mathrm{m,s}}$	$G^{C}_{3221}$ .	$\xrightarrow{\mathrm{w}} G_{32}$	$\frac{n}{4}$	$\xrightarrow{\mathrm{n}} G_{321}$	$1 \xrightarrow{s}$		✓			m,s	$C^{C}$	ZIU W	$C_{122}$	40 m	$C_{2224}$	40 m	$C_{\alpha\alpha}$	126 S			./
III8	$\xrightarrow{m}$	$G_{ADD}$ .	$\xrightarrow{\mathrm{m}} G_{2^{\prime}}$	4 n 201 —	$\stackrel{0}{\longrightarrow} G_{201}$	$126$ $_{1} \xrightarrow{\mathrm{S}}$					$\overline{54}$	$\sigma_{422}$	$\overrightarrow{210}$	U422	$\overline{45}$	G3221	$\overline{45}$	G32	$11 \frac{11}{126}$	7		V
	<b>210</b> m,s	$\sim 422$	45 ~ ~ ~	²∠ı 4	$5$ $\sim$ $321$	$\overline{126}'_{\mathrm{S}}$				IV3:	$\xrightarrow{\mathrm{m,s}}$	$G_{422}^{C}$	$\xrightarrow{W}$	$G_{422}$	$\xrightarrow{\mathrm{m}}$	$G_{421}$	$\xrightarrow{\mathrm{m}}$	$G_{322}$	$11  \underline{\hspace{1cm}}^{s}$	$\rightarrow$		$\checkmark$
1119:	$rac{}{54}$	$G_{422}^{\cup}$	$\rightarrow G_{45}$	$21 \frac{1}{4}$	$\rightarrow$ $G_{321}$	$1 \xrightarrow{1} \overline{126}$		✓			<b>54</b>	± <b>=  =</b>	210		45		45		126			
III1	): $\xrightarrow{\mathrm{m}}_{210}$	$G_{422}$ -	$\xrightarrow{\mathrm{m}} G_{45}$	$21  \frac{n}{4}$	$\xrightarrow{n} G_{321}$	$1 \xrightarrow{\mathrm{S}} 1 \xrightarrow{\mathrm{S}}$		✓ <b>→</b>														
				-	-	1 <b>4</b> 0																

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SO(10)	defect	C	defect	C	defect	C	Observable
50(10)	Higgs	$G_2$	Higgs	$G_1$	Higgs	$G_{\rm SM}$	strings?
II1:	$\stackrel{ m m}{\longrightarrow}$ 210	$G_{422}$	$rac{\mathrm{m}}{45}$	$G_{3221}$	$\xrightarrow{\mathrm{S}}$ $\overline{126}$		$\checkmark$
II2:	$\xrightarrow[]{\text{m,s}}{\textbf{54}}$	$G^C_{422}$	$\stackrel{ m m}{\longrightarrow}$ 210	$G_{3221}^{C}$	$\xrightarrow{\overline{s,w}}$		×
II3:	$\xrightarrow[54]{\mathrm{m,s}}$	$G_{422}^{C}$	$\stackrel{\mathrm{m,w}}{\longrightarrow}$ 45	$G_{3221}$	$\xrightarrow{\text{S}}{\overline{126}}$		✓
II4:	$\xrightarrow[]{\text{m,s}}{\textbf{210}}$	$G_{3221}^{C}$	$rac{\mathrm{w}}{45}$	$G_{3221}$	$\xrightarrow{s}{\overline{126}}$		✓
II5:	$\stackrel{ m m}{\longrightarrow}$ 210	$G_{422}$	$\stackrel{ m m}{\longrightarrow}$ 45	$G_{421}$	$\xrightarrow{s}{\overline{126}}$		✓
II6:	$\xrightarrow[]{\text{m,s}}{\textbf{54}}$	$G_{422}^{C}$	$\stackrel{ m m}{\longrightarrow}$ 45	$G_{421}$	$\xrightarrow{s}{\overline{126}}$		✓
II7:	$\xrightarrow[]{\text{m,s}}{\textbf{54}}$	$G_{422}^{C}$	$\stackrel{ ext{W}}{ ext{210}}$	$G_{422}$	$\xrightarrow{\frac{m}{126}}{45}$		×
II8:	$\stackrel{\mathrm{m}}{\longrightarrow}$ 45	$G_{3221}$	$\stackrel{\mathrm{m}}{\longrightarrow}$ 45	$G_{3211}$	$\xrightarrow{s}{\overline{126}}$		$\checkmark$
II9:	$\xrightarrow[]{\text{m,s}}{\textbf{210}}$	$G_{3221}^{C}$	$\stackrel{\mathrm{m,w}}{\longrightarrow}$	$G_{3211}$	$\xrightarrow{s}{\overline{126}}$		✓
II10:	$\stackrel{\mathrm{m}}{\longrightarrow}$ 210	$G_{422}$	$\stackrel{\mathrm{m}}{\longrightarrow}$ 210	$G_{3211}$	$\xrightarrow{s}{\overline{126}}$		✓
II11:	$\xrightarrow[54]{\mathrm{m,s}}$	$G_{422}^{C}$	$\xrightarrow{\mathrm{m,w}}210$	$G_{3211}$	$\xrightarrow{s}{\overline{126}}$		✓
II12:	$rac{\mathrm{m}}{45}$	$G_{421}$	$rac{\mathrm{m}}{45}$	$G_{3211}$	$\xrightarrow[]{s}{126}$		✓

![](_page_18_Picture_6.jpeg)

ſ		defect		defect		Obsorvabla						_									
	SO(10)	$\xrightarrow{\text{defect}}$	$G_1$	$\xrightarrow{\text{delect}}$	$G_{ m SM}$	Observable							SO(10)	$\stackrel{\text{defect}}{\longrightarrow}$	$G_{2}$	$\stackrel{\rm defect}{\longrightarrow}$	$G_1$	$\stackrel{\rm defect}{\longrightarrow}$	Gan	Observab	le
		Higgs	Ŧ	Higgs		strings?							50(10)	Higgs	02	Higgs	ΟŢ	Higgs	USM	strings?	
	I1:	$\xrightarrow{\text{m}}$	$G_{3221}$	$\xrightarrow{S}$		$\checkmark$							II1:	$\stackrel{ m m}{\longrightarrow}$ 210	$G_{422}$	$rac{\mathrm{m}}{45}$	$G_{3221}$	$\xrightarrow[]{\text{S}}{126}$		$\checkmark$	
	I2:	$\xrightarrow{\mathrm{m,s}}$	$G^{C}_{3221}$	$\xrightarrow{120}{\mathrm{s,w}}$		×							II2:	$\xrightarrow[54]{\mathrm{m,s}}$	$G_{422}^{C}$	$\xrightarrow[]{\text{m}}{\textbf{210}}$	$G_{3221}^{C}$	$\xrightarrow[]{\text{s,w}}{\overline{\textbf{126}}}$		×	
	TO	<b>210</b> m	$\overline{0}$	$\overline{126}_{S}$									II3:	$\xrightarrow{\mathrm{m,s}}$ 54	$G_{422}^{C}$	$\xrightarrow{\mathrm{m,w}}$ <b>45</b>	$G_{3221}$	$\xrightarrow{\mathrm{S}}$ $\overrightarrow{\mathrm{126}}$		$\checkmark$	
	13:	$\overrightarrow{45}$	$G_{421}$	$\stackrel{\longrightarrow}{\overline{126}}$			<b>SO</b> (10	) Higgs					II4:	$\xrightarrow{\text{m,s}}{210}$	$G_{3221}^{C}$	$\xrightarrow{W}{45}$	$G_{3221}$	$\xrightarrow{S}{\overline{126}}$		$\checkmark$	
	I4:	$\stackrel{ m m}{\longrightarrow}$ 210	$G_{422}$	1264!	5	×	mult	iplets					II5:	$\stackrel{ ext{m}}{ ext{210}}$	$G_{422}$	$\stackrel{ m m}{\longrightarrow}$ 45	$G_{421}$	$\xrightarrow{\text{S}}$ $\overrightarrow{126}$		$\checkmark$	
	I5:	$\xrightarrow{\mathrm{m,s}}$	$G_{422}^{C}$			X							II6:	$\xrightarrow[54]{\mathrm{m,s}}$	$G_{422}^{C}$	$rac{\mathrm{m}}{45}$	$G_{421}$	$\xrightarrow{\frac{S}{126}}$		$\checkmark$	
	IC.	54 m	0	$\overline{126},\!$	5								II7:	$\stackrel{\mathrm{m,s}}{\longrightarrow}$	$G_{422}^{C}$	$\stackrel{ ext{W}}{ ext{210}}$	$G_{422}$	$\xrightarrow{\text{m}}$ $\overrightarrow{126}$ $4^{\text{p}}$	ς.	×	
	10:	$\overrightarrow{210}$	G <sub>3211</sub>	$\overrightarrow{126}$		✓							II8:	$\xrightarrow{\mathrm{m}}$ 45	$G_{3221}$	$\xrightarrow{\mathrm{m}}$ 45	$G_{3211}$	$\xrightarrow{S}{\overline{126}}$	,	$\checkmark$	
SO(	10) $\stackrel{\text{defect}}{\longrightarrow}_{\text{Higgs}} C$	$G_3 \stackrel{\text{defe}}{=}_{\text{Hig}}$	$\stackrel{\mathrm{ct}}{\to} G_2$	$\stackrel{\text{defect}}{\longrightarrow} C$ Higgs	$\vec{r}_1$ $\frac{\text{defec}}{\text{Higg}}$	$\stackrel{\text{t}}{\stackrel{\text{s}}{\Rightarrow}} G_{\text{SM}} \stackrel{\text{Observal}}{\text{strings}}$	ole						II9:	$\xrightarrow[]{\text{m,s}}{\textbf{210}}$	$G_{3221}^{C}$	$\stackrel{ m m,w}{ m 45}$	$G_{3211}$	$ \xrightarrow{\text{S}} \overline{126} $		$\checkmark$	
III1:	$\xrightarrow{\mathrm{m,s}}$ (	$\overline{\mathcal{G}_{422}^C}  \frac{\mathrm{w}}{\mathrm{210}}$	$\rightarrow G_{422}$	$\xrightarrow{\mathrm{m}}$ (	$\frac{s}{421}$ $\frac{s}{122}$	>							II10:	$\stackrel{ m m}{\longrightarrow}$ 210	$G_{422}$	$\stackrel{ m m}{\longrightarrow}$ 210	$G_{3211}$	$\stackrel{ m S}{\longrightarrow}$ 126		$\checkmark$	
III2:	$\begin{array}{c} \begin{array}{c} 34 \\ \mathbf{m,s} \\ 54 \end{array} \end{array} $	$\widehat{\mathcal{F}}_{422}^C = \frac{\mathbb{W}}{210}$	$\rightarrow G_{422}$	$\xrightarrow{\text{m}}$ (	$73221  \frac{s}{126}$	$\rightarrow$							II11:	$\xrightarrow[]{\text{m,s}}{\textbf{54}}$	$G_{422}^{C}$	$\stackrel{\mathrm{m,w}}{\longrightarrow}$ <b>210</b>	$G_{3211}$	$\stackrel{\mathrm{S}}{\longrightarrow}$ 126		$\checkmark$	
III3:	$\begin{array}{c} \overset{\mathrm{orl}}{\longrightarrow} \\ \overline{54} \end{array} $	$\mathcal{G}_{422}^C  \frac{\mathbf{w}}{210}$	$\rightarrow G_{422}$	$\frac{m}{210}$ (	$73211 \frac{120}{126}$	→							II12:	$\stackrel{\mathrm{m}}{\longrightarrow}$ 45	$G_{421}$	$\xrightarrow{\mathrm{m}}$ <b>45</b>	$G_{3211}$	$\stackrel{\mathrm{S}}{\longrightarrow}$ 126		$\checkmark$	
III4:	$\xrightarrow{\mathrm{m,s}}$ (	$G_{422}^C = \frac{m}{210}$	$\rightarrow G^C_{3221}$	$\frac{\mathrm{w}}{45}$ (	$73221  \frac{-5}{126}$	$\rightarrow$		CO(10)	defect	~	defect		defecț	~	defect		defe	ct a	0	bservable	
III5:	$\stackrel{\mathrm{m,s}}{\longrightarrow}$ (	$\mathcal{G}_{422}^C  \frac{\mathrm{m}}{210}$	$\rightarrow G^C_{3221}$	$\xrightarrow{\mathrm{m,w}}$ $45$	$\frac{1}{7}3211  \frac{1}{126}$			SO(10)	$\rightarrow$ Higgs	$G_4$	$\rightarrow$ Higgs	$G_3$	Higgs	$G_2$	$\rightarrow$ Higgs	$G_1$	Hig	$\rightarrow G_{S}$	SM	strings?	
III6:	$\xrightarrow{\mathrm{m,s}}$ $C$	$\mathcal{G}_{422}^C = \frac{m,v}{45}$	$\stackrel{\scriptscriptstyle{\mathrm{W}}}{\rightarrow} G_{3221}$	$\frac{\mathrm{m}}{45}$ (	$\frac{1}{2}3211$ $\frac{s}{126}$	$\rightarrow$		IV1:	$\xrightarrow[]{\text{m,s}}{54}$	$G^C_{422}$	$\xrightarrow[]{\text{m}}{\textbf{210}}$	$G_{3221}^{C}$	$45 \rightarrow$	$G_{3221}$	$\begin{array}{c} \xrightarrow{\mathrm{m}} \\ 45 \end{array}$	$G_{32}$	$11 \frac{s}{12}$	$\rightarrow \overline{3}$		1	
III7:	$\begin{array}{c} \xrightarrow{\mathrm{m,s}} \\ 210 \\ \end{array} \qquad \qquad$	$\mathcal{G}_{3221}^C \xrightarrow{\mathrm{W}} 45$	$\rightarrow G_{3221}$	$\begin{array}{c} \xrightarrow{\mathrm{m}} & 0 \\ 45 \\ \mathbf{m} \end{array} $	$\frac{1}{2}3211$ $\frac{1}{126}$			IV2:	$\xrightarrow{\mathrm{m,s}}_{F4}$	$G_{422}^{C}$	$\xrightarrow{W}$ 210	$G_{422}$	$\xrightarrow{\mathrm{m}}$	$G_{3221}$	$\xrightarrow{\text{m}}$	$G_{32}$	11	$\rightarrow$		$\checkmark$	
III8:	$\begin{array}{c} \stackrel{\mathrm{m}}{\longrightarrow} \\ 210 \\ \mathrm{m \ s} \end{array} $	$\begin{array}{c} \gamma \\ \mu 422 \\ 45 \\ \mu 6 \end{array}$	$\rightarrow G_{3221}$	$\begin{array}{c} \xrightarrow{\mathrm{m}} \\ 45 \\ \mathrm{m} \end{array}$	$\overrightarrow{3211}  {126}$			IV3:	$\xrightarrow{\text{m,s}}$	$G^{C}_{_{A22}}$	$\xrightarrow{W}$	$G_{422}$	$\xrightarrow{\text{m}}$	$G_{421}$	$\xrightarrow{\text{m}}$	$G_{32}$	$\begin{array}{c} 120\\ \text{s}\\ 11 \end{array}$	$\dot{o}$		1	
III9:	$\stackrel{\mathrm{m},\mathrm{s}}{54}$ (	$\mathcal{G}_{422}^{U} = \frac{11}{45}$	$\rightarrow G_{421}$	$\frac{11}{45}$ (	$73211 \frac{3}{126}$	→			54	<u>'t</u> 22	210	± <b>=                                   </b>	45	± <b>=</b> ±	45		120	3			
III1(	): $\frac{m}{210}$ (	$   \widehat{J}_{422}  \frac{\mathrm{m}}{45} $	$\rightarrow G_{421}$	$\frac{\mathrm{m}}{45}$ (	$\frac{1}{7}3211  \frac{1}{126}$																

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SO(10)	defect	C	defect	C	defect	C	Observable
50(10)	Higgs	$G_2$	Higgs	$G_1$	Higgs	$G_{\rm SM}$	strings?
II1:	$\stackrel{ m m}{\longrightarrow}$ 210	$G_{422}$	$rac{\mathrm{m}}{45}$	$G_{3221}$	$\xrightarrow{\mathrm{S}}{\overline{126}}$		$\checkmark$
II2:	$\xrightarrow[]{\text{m,s}}{\textbf{54}}$	$G^C_{422}$	$\stackrel{ m m}{\longrightarrow}$ 210	$G_{3221}^{C}$	$\xrightarrow{\overline{s,w}}$		×
II3:	$\xrightarrow[54]{\mathrm{m,s}}$	$G_{422}^{C}$	$\stackrel{\mathrm{m,w}}{\longrightarrow}$ 45	$G_{3221}$	$\xrightarrow{\text{S}}{\overline{126}}$		✓
II4:	$\xrightarrow[]{\text{m,s}}{\textbf{210}}$	$G_{3221}^{C}$	$rac{\mathrm{w}}{45}$	$G_{3221}$	$\xrightarrow{s}{\overline{126}}$		✓
II5:	$\stackrel{ m m}{\longrightarrow}$ 210	$G_{422}$	$\stackrel{ m m}{\longrightarrow}$ 45	$G_{421}$	$\xrightarrow{s}{\overline{126}}$		✓
II6:	$\xrightarrow[]{\text{m,s}}{\textbf{54}}$	$G_{422}^{C}$	$\stackrel{ m m}{\longrightarrow}$ 45	$G_{421}$	$\xrightarrow{s}{\overline{126}}$		✓
II7:	$\xrightarrow[]{\text{m,s}}{\textbf{54}}$	$G_{422}^{C}$	$\stackrel{ ext{W}}{ ext{210}}$	$G_{422}$	$\xrightarrow{\frac{m}{126}}{45}$		×
II8:	$\stackrel{\mathrm{m}}{\longrightarrow}$ 45	$G_{3221}$	$\stackrel{\mathrm{m}}{\longrightarrow}$ 45	$G_{3211}$	$\xrightarrow{s}{\overline{126}}$		$\checkmark$
II9:	$\xrightarrow[]{\text{m,s}}{\textbf{210}}$	$G_{3221}^{C}$	$\stackrel{\mathrm{m,w}}{\longrightarrow}$ 45	$G_{3211}$	$\xrightarrow{s}{\overline{126}}$		✓
II10:	$\stackrel{\mathrm{m}}{\longrightarrow}$ 210	$G_{422}$	$\stackrel{\mathrm{m}}{\longrightarrow}$ 210	$G_{3211}$	$\xrightarrow{s}{\overline{126}}$		✓
II11:	$\xrightarrow[54]{\mathrm{m,s}}$	$G_{422}^{C}$	$\xrightarrow[]{\text{m,w}}{\textbf{210}}$	$G_{3211}$	$\frac{\xrightarrow{s}}{126}$		✓
II12:	$rac{\mathrm{m}}{45}$	$G_{421}$	$rac{\mathrm{m}}{45}$	$G_{3211}$	$\xrightarrow[]{s}{126}$		✓

![](_page_19_Picture_6.jpeg)

	CO(10)	defect	C	defect	Q	Observ	vable						Г		defect		defect		defect		Observable	٦
	SO(10)	) Higgs	$G_1$	Higgs	$G_{\rm SM}$	strin	gs?							SO(10)	$) \xrightarrow{\text{defect}}_{\text{Higgs}}$	$G_2$	$\xrightarrow{\text{Higgs}}$	$G_1$	$\xrightarrow{\text{Higgs}}$	$G_{\rm SM}$	strings?	
	I1:	$\xrightarrow{\mathrm{m}}$	$G_{3221}$	$\xrightarrow{s}$		1								II1:	$\xrightarrow[]{\text{m}}{\textbf{210}}$	$G_{422}$	$\xrightarrow{\mathrm{m}}$ <b>45</b>	$G_{3221}$	$\xrightarrow{\text{S}}$ 126		$\checkmark$	
	I2:	$\xrightarrow{\begin{array}{c} 45 \\ \mathbf{m,s} \\ \longrightarrow \end{array}}$	$G^{C}_{2221}$	$\xrightarrow{126}_{\mathrm{S,W}}$		x		If upw	anted	dofo	ct cr	roate	bd	II2:	$\xrightarrow[]{\text{m,s}}{\textbf{54}}$	$G_{422}^{C}$	$\xrightarrow{\mathrm{m}}$ <b>210</b>	$G_{3221}^{C}$	$\xrightarrow{\text{S,W}} \overline{126}$		×	
	10	<b>210</b> m	⊂ <u>3221</u>	$\overline{126}$						uere				II3:	$\xrightarrow{\mathrm{m,s}}$ 54	$G_{422}^{C}$	$\xrightarrow{\mathrm{m,w}}$	$G_{3221}$	$\xrightarrow{s}{100}$		$\checkmark$	
	13:	$rac{45}{45}$	$G_{421}$	$\stackrel{\sim}{\overline{126}}$				in tina	I 22R =	$ \rightarrow $	no G	i VV		II4:	$\xrightarrow{\text{m,s}}$	$G_{3221}^{C}$	$\xrightarrow{W}$	$G_{3221}$	$\xrightarrow{s}$		$\checkmark$	
	I4:	$\xrightarrow{\mathrm{m}}$ <b>210</b>	$G_{422}$	$\xrightarrow{\mathrm{m}}$ $\overrightarrow{126}$ $4^{\mathrm{p}}$	5	×		else G	VV					II5:	210 $\xrightarrow{\text{m}}$ 210	$G_{422}$	$\xrightarrow{\begin{array}{c} 45\\ m\\ \hline 45\end{array}}$	$G_{421}$	$\begin{array}{c} 126 \\ \xrightarrow{\mathrm{S}} \\ \hline 126 \end{array}$		$\checkmark$	
	I5:	$\xrightarrow{\mathrm{m,s}}$	$G_{422}^{C}$	$\xrightarrow{\text{m,w}}$		×								II6:	$\xrightarrow[\mathbf{m,s}]{54}$	$G_{422}^{C}$	$\xrightarrow{\mathrm{m}}{45}$	$G_{421}$	$ \xrightarrow{\text{S}} \overline{126} $		$\checkmark$	
	IG.	54 m	C	126,48	5									II7:	$\xrightarrow[54]{\mathrm{m,s}}$	$G_{422}^{C}$	$\stackrel{ ext{w}}{\longrightarrow}$ 210	$G_{422}$	$\xrightarrow{\text{m}}$ $\overrightarrow{126}$ 45		×	
	10.	$\overrightarrow{210}$	G3211	$\overrightarrow{126}$		V								II8:	$\xrightarrow{\mathrm{m}}$ 45	$G_{3221}$	$\xrightarrow{\mathrm{m}}$ 45	$G_{3211}$	$\xrightarrow{s}{\overline{126}}$		$\checkmark$	
SO(	$10) \stackrel{\text{defect}}{\longrightarrow} 0$	$G_3  \stackrel{\mathrm{def}}{\underset{\mathrm{Hig}}{-}}$	$\stackrel{\text{fect}}{\to} G_2$	$\stackrel{\text{defect}}{\longrightarrow} C$ Higgs	$     \vec{J}_1 \xrightarrow{\text{defec}}_{\text{Higgs}} $	$\stackrel{\mathrm{t}}{\simeq} G_{\mathrm{SM}}$	Observable strings?							II9:	$\xrightarrow[]{\text{m,s}}{\textbf{210}}$	$G_{3221}^{C}$	$\xrightarrow[]{\text{m,w}}{45}$	$G_{3211}$	$ \xrightarrow{\text{S}} \overline{126} $		$\checkmark$	
III1:	$\xrightarrow{\text{m,s}}$	$\overline{G^C_{422}}$ $\frac{\mathbf{v}}{\mathbf{v}}$	$\xrightarrow{v} G_{422}$	$\xrightarrow{\mathrm{m}}$ (	$G_{421} \xrightarrow{s}$	•	<u>√</u>	_						II10:	$\stackrel{ m m}{\longrightarrow}$ 210	$G_{422}$	$\xrightarrow{\mathrm{m}}$ <b>210</b>	$G_{3211}$	$\xrightarrow{s}{\overline{126}}$		$\checkmark$	
III2:	$\xrightarrow{\text{m,s}}^{54}$	$G_{422}^C = \frac{v}{21}$	$\stackrel{\text{v}}{\to} G_{422}$	$\xrightarrow{\text{m}}$ (	$ \begin{array}{c}     126 \\     _{3221} \xrightarrow{\mathrm{s}} \end{array} $	•	<b>√</b>							II11:	$\xrightarrow{\mathrm{m,s}}{54}$	$G^C_{422}$	$\stackrel{\mathrm{m,w}}{\longrightarrow}$	$G_{3211}$	$\xrightarrow{s}{\overline{126}}$		$\checkmark$	
III3:	$egin{array}{c} {f 54} \ {f m,s} \ {f 54} \ {f 54} \end{array}$	$G_{422}^C = rac{v}{21}$	$\stackrel{\text{lo}}{\longrightarrow} G_{422}$	$\begin{array}{c} 45 \\ \mathbf{\frac{m}{210}} \end{array} $	$ \begin{array}{c} \begin{array}{c} \begin{array}{c} 126\\ s\\ \end{array}\\ 3211 \end{array} \xrightarrow{s}\\ 126 \end{array} $		$\checkmark$							II12:	$\xrightarrow{\mathrm{m}}{45}$	$G_{421}$	$\xrightarrow{\mathrm{m}}$ 45	$G_{3211}$	$\xrightarrow[]{S}{126}$		$\checkmark$	
III4:	$rac{\mathrm{m,s}}{54}$ (	$G_{422}^C = \frac{n}{21}$	$\stackrel{\text{a}}{\longrightarrow} G^C_{3221}$	$\frac{\mathrm{w}}{45}$ (	$\widetilde{f}_{3221} \xrightarrow[]{s}{126}$	•	$\checkmark$			defect		defect		defect		defect		defe	ct	O	bservable	
III5:	$\xrightarrow{\mathrm{m,s}}$ (	$G_{422}^C = \frac{n}{21}$	$\stackrel{\text{a}}{\longrightarrow} G^C_{3221}$	$\begin{array}{c} \xrightarrow{\mathrm{m,w}} & \mathbf{C} \\ 45 \end{array}$	$G_{3211} \xrightarrow[]{s}{126}$	•	$\checkmark$		SO(10)	$\longrightarrow$ Higgs	$G_4$	$\longrightarrow$ Higgs	$G_3$	Higgs	$G_2$	$\longrightarrow$ Higgs	$G_1$	Higg	$\stackrel{\rightarrow}{}_{\mathrm{gs}} G_{\mathrm{SI}}$	M s	strings?	
III6:	$rac{\mathrm{m,s}}{54}$ (	$G_{422}^C  \frac{\mathrm{m}}{4}$	$\xrightarrow{\mathbf{w}} G_{3221}$	$\frac{\mathrm{m}}{45}$ (	$\overrightarrow{J}_{3211} \xrightarrow{\mathrm{s}} \overrightarrow{\mathrm{126}}$	•	$\checkmark$		IV1:	$\xrightarrow{\text{m,s}} 54$	$G_{422}^{C}$	$\xrightarrow{\mathrm{m}}$ 210	$G_{3221}^{C}$	$\xrightarrow{W}$	$G_{3221}$	$\xrightarrow{\text{m}}$	$G_{321}$	$11 \frac{s}{100}$	$\rightarrow$		✓	
III7:	$\stackrel{\mathrm{m,s}}{\longrightarrow}$ (	$G_{3221}^C - \frac{v}{4}$	$\xrightarrow{v} G_{3221}$	$\frac{\mathrm{m}}{45}$ (	$   \widehat{J}_{3211} \xrightarrow{\mathrm{s}} \overline{126} $	•	$\checkmark$		IV2:	$\xrightarrow{\mathrm{m,s}}$	$G_{422}^{C}$	$\xrightarrow{W}$	$G_{422}$	$\xrightarrow{\mathrm{m}}$	$G_{3221}$	$\xrightarrow{\mathrm{m}}$	$G_{321}$		$\rightarrow$		✓	
III8:	$\frac{\mathrm{m}}{210}$ (	$G_{422} = \frac{n}{4}$	$\xrightarrow{n}{5} G_{3221}$	$\frac{m}{45}$ (	$   \widehat{J}_{3211} \xrightarrow{\mathrm{s}} \overline{126} $	•	$\checkmark$		IV3·	$\stackrel{54}{\longrightarrow}$	$G^{C}_{422}$	$\begin{array}{c} 210 \\ \xrightarrow{\mathrm{W}} \end{array}$	$G_{499}$	$\stackrel{\textbf{45}}{\longrightarrow}$	$G_{491}$	$\begin{array}{c} 45 \\ \mathrm{m} \\ \longrightarrow \end{array}$	$G_{201}$	126	<b>6</b> →		<u>,</u>	
III9:	$rac{\mathrm{m,s}}{54}$ (	$G_{422}^C - \frac{1}{4}$	$\xrightarrow{n}{5} G_{421}$	$\frac{\mathrm{m}}{45}$ (	$   \widehat{f}_{3211} \xrightarrow{\mathrm{s}} \overline{126} $	•	$\checkmark$			54	₩422	$210^{'}$	₩422	45	₩421	$45^{'}$	₩ 32]	$11 \overline{126}$	<u>;</u>		▼	J
III10	$\therefore  \frac{\mathrm{m}}{210}  0$	$G_{422} = \frac{n}{4}$	$\xrightarrow{n} G_{421}$	$\begin{array}{c} { m m} { m \overline{45}} \end{array} \qquad \qquad$	$   \widehat{J}_{3211} \xrightarrow{s}{126} $	•	✓															

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![](_page_20_Picture_5.jpeg)

- Assume minimal survival hypothesis
- intermediate breaking scales (due to unification).

![](_page_21_Figure_3.jpeg)

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Perform two-loop RGE analysis to determine GUT scale (proton decay rate) in terms

![](_page_21_Picture_7.jpeg)

![](_page_21_Picture_8.jpeg)

- Assume minimal survival hypothesis
- intermediate breaking scales (due to unification).

![](_page_22_Figure_3.jpeg)

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Perform two-loop RGE analysis to determine GUT scale (proton decay rate) in terms

![](_page_22_Picture_7.jpeg)

- Assume minimal survival hypothesis
- intermediate breaking scales (due to unification).

![](_page_23_Figure_3.jpeg)

**Jessica Turner** 

Perform two-loop RGE analysis to determine GUT scale (proton decay rate) in terms

### Breaking chains allowed by Super-K: IV2 & IV3

IV2 : SO (10)  $\xrightarrow{M_X} G_{422}^C \xrightarrow{M_4} G_{422} \xrightarrow{M_3} G_{3221} \xrightarrow{M_2} G_{3211} \xrightarrow{M_1} G_{SM}$ IV3 : SO(10)  $\xrightarrow{M_X} G_{422}^C \xrightarrow{M_4} G_{422} \xrightarrow{M_3} G_{421} \xrightarrow{M_2} G_{3211} \xrightarrow{M_1} G_{SM}$ 

# Regions due to more free parameters

10<sup>17</sup>

![](_page_23_Picture_10.jpeg)

![](_page_23_Picture_11.jpeg)

- RGE constrain GUT and intermediate scale symmetry breaking.
- For type (c) chains an observable GW signal is produced in the final SSB.
- We assume Nambu-Goto string  $\implies$  gravitational radiation primary emission.
- Determine  $M_1 \implies$  string tension

![](_page_24_Figure_5.jpeg)

![](_page_24_Picture_11.jpeg)

![](_page_24_Picture_12.jpeg)

![](_page_25_Figure_1.jpeg)

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![](_page_25_Picture_5.jpeg)

- non-SUSY SO(10) Pati Salam type provide unification: 31 breaking chains
- Two-loop RGE, 17 not excluded by Super-K lower bound PD.

Chain	$G\mu$ after Hyper-K (no proton decay)
I1	excluded
II1:	$G\mu \lesssim 1.5 \times 10^{-17}$
II3:	excluded
II4:	excluded
II5:	$G\mu \simeq 5.1 \times 10^{-18} - 6.3 \times 10^{-17}$
II8:	excluded
III1:	$G\mu \simeq 1.3 \times 10^{-18} - 1.6 \times 10^{-15}$
III2:	$G\mu \lesssim 5.0 \times 10^{-12}$
III3:	$G\mu \lesssim 6.2 \times 10^{-14}$
III4:	excluded <b>Testable by LIGO</b> ,
III6:	excluded <b>DECIGO, AEDGE,</b>
III7:	excluded <b>C, ET, MAGIS.</b>
III8:	excluded
III10:	$G\mu \lesssim 1.1 \times 10^{-21}$
IV1:	excluded
IV2:	$G\mu \lesssim 9.4 \times 10^{-13}$
IV3:	$G\mu \lesssim 9.4 \times 10^{-13}$

### Summary

- If HyperK does not observe PD  $\implies$  9 chains excluded
- 8 survivors! If we observe GW signal larger than upper bounds  $\implies$  exclude those breaking chains
- If we observe PD  $\Longrightarrow M_1$  determined and so is GW signal. Correlations matters!

![](_page_26_Picture_11.jpeg)

![](_page_26_Picture_12.jpeg)

![](_page_26_Picture_13.jpeg)

- non-SUSY SO(10) Pati Salam type provide unification: 31 breaking chains
- Two-loop RGE, 17 not excluded by Super-K lower bound PD.

Chain	Gu after Hyper-K (no proton decay)	
	$\frac{O\mu}{1} \frac{\operatorname{arter}(\operatorname{IIO} \operatorname{proton} \operatorname{uccay})}{1}$	
	excluded	
II1:	$G\mu \lesssim 1.5 \times 10^{-17}$	
II3:	excluded	
II4:	excluded	
II5:	$G\mu \simeq 5.1 \times 10^{-18} - 6.3 \times 10^{-17}$	
II8:	excluded	St
III1:	$G\mu \simeq 1.3 \times 10^{-18} - 1.6 \times 10^{-15}$	•••
III2:	$G\mu \lesssim 5.0 \times 10^{-12}$	
III3:	$G\mu \lesssim 6.2 \times 10^{-14}$	۱۸/
III4:	excluded	vv
III6:	excluded	an
III7:	excluded	
III8:	excluded	
III10:	$G\mu \lesssim 1.1 \times 10^{-21}$	
IV1:	excluded	
IV2:	$G\mu \lesssim 9.4 \times 10^{-13}$	
IV3:	$G\mu \lesssim 9.4 \times 10^{-13}$	

### Summary

udy specific breaking chain <u>2209.00021</u>

hy? Can be excluded by Hyper-K & has associated GW signal

![](_page_27_Picture_11.jpeg)

- Specific model of chain III4: Fu, King, Marsili, Pascoli, JT, Zhou 2209.00021
- See Dror also et al <u>1908.03227</u> which connected GUT with leptogenesis
- formation, leptogenesis & proton decay determined

• Fit SM fermion data to our model, perform RG running  $\implies$  scales of cosmic string

![](_page_28_Picture_9.jpeg)

- Specific model of chain III4: Fu, King, Marsili, Pascoli, JT, Zhou 2209.00021
- See Dror also et al <u>1908.03227</u> which connected GUT with leptogenesis
- formation, leptogenesis & proton decay determined

![](_page_29_Figure_4.jpeg)

• Fit SM fermion data to our model, perform RG running  $\implies$  scales of cosmic string

![](_page_29_Picture_10.jpeg)

- Specific model of chain III4: Fu, King, Marsili, Pascoli, JT, Zhou 2209.00021
- See Dror also et al <u>1908.03227</u> which connected GUT with leptogenesis
- formation, leptogenesis & proton decay determined

![](_page_30_Figure_4.jpeg)

• Fit SM fermion data to our model, perform RG running  $\implies$  scales of cosmic string

![](_page_30_Picture_10.jpeg)

- Specific model of chain III4: Fu, King, Marsili, Pascoli, JT, Zhou 2209.00021
- See Dror also et al <u>1908.03227</u> which connected GUT with leptogenesis
- Fit SM fermion data to our model, perform RG running  $\implies$  scales of cosmic string formation, leptogenesis & proton decay determined

![](_page_31_Figure_4.jpeg)

![](_page_31_Picture_9.jpeg)

![](_page_31_Picture_10.jpeg)

- Specific model of chain III4: Fu, King, Marsili, Pascoli, JT, Zhou <u>2209.00021</u>
- See Dror also et al <u>1908.03227</u> which connected GUT with leptogenesis
- Fit SM fermion data to our model, perform RG running  $\implies$  scales of cosmic string formation, leptogenesis & proton decay determined

![](_page_32_Figure_4.jpeg)

![](_page_32_Picture_9.jpeg)

![](_page_32_Picture_10.jpeg)

- Specific model of chain III4: Fu, King, Marsili, Pascoli, JT, Zhou 2209.00021
- See Dror also et al <u>1908.03227</u> which connected GUT with leptogenesis
- Fit SM fermion data to our model, perform RG running  $\implies$  scales of cosmic string formation, leptogenesis & proton decay determined

![](_page_33_Figure_4.jpeg)

![](_page_33_Picture_9.jpeg)

 $U(1)_{R-I}$  breaking cosmic string & RHN mass generation via type-I seesaw

![](_page_33_Picture_12.jpeg)

![](_page_33_Picture_13.jpeg)

- Specific model of chain III4: Fu, King, Marsili, Pascoli, JT, Zhou 2209.00021
- See Dror also et al <u>1908.03227</u> which connected GUT with leptogenesis
- Fit SM fermion data to our model, perform RG running  $\implies$  scales of cosmic string formation, leptogenesis & proton decay determined

![](_page_34_Figure_4.jpeg)

![](_page_34_Picture_10.jpeg)

 Up, down, neutrino, charged lepton Yukawa couplings and right-handed mass matrices parametrised in terms of SO(10) model parameters <u>Altarelli et al 1012.2697</u>

$$\mathcal{P}_m \in \left\{a_1, a_2, r_1, c_e, c_\nu, m_0, \eta\right\}$$

Quark & charged lepton sectors are inputs & neutrino sector is predicted:

$$\mathcal{O}_n \in \left\{\theta_{12}, \theta_{13}, \theta_{23}, \delta, \Delta m_{21}^2, \Delta m_{31}^2\right\}$$

Perform (grid based) scan of model parameters to find Yukawa & mass matrices with low

![](_page_35_Picture_9.jpeg)

![](_page_35_Picture_10.jpeg)

and Higgs doublet  $\implies$  thermal leptogenesis prediction

RD1	Inputs	$a_1$	$a_2$	$c_{ u}$	$m_0$	$(\eta_u,\eta_c,\eta_t;\eta_d,\eta_s,\eta_b)$
		$63.57^{\circ}$	$84.17^{\circ}$	-1.945	$82.82\mathrm{meV}$	(+,+,-;+,-,+)
	Outputs	$ heta_{13}$	$ heta_{12}$	$ heta_{23}$	δ	$m_1$
		$8.53^{\circ}$	$32.7^{\circ}$	$41.9^{\circ}$	$-125^{\circ}$	$3.36 \mathrm{meV}$
	$(\chi^2=0.33)$	$m_{etaeta}$		$M_{N_1}$	$M_{N_2}$	$M_{N_3}$
		$5.83\mathrm{meV}$		$4.23\cdot 10^{11}\mathrm{GeV}$	$5.32\cdot10^{11}\mathrm{GeV}$	$1.66\cdot 10^{13}{ m GeV}$

 $M_1 = 2 \times 10^{13} \text{GeV}, \quad M_2 = 5 \times 10^{13} \text{GeV} \quad M_3 = 7.55 \times 10^{13} \text{GeV}$  $M_{\rm X} = 5.68 \times 10^{15} {\rm GeV}, \quad \alpha_X = 0.0279$ 

$$\mathcal{L} = i\overline{N_i}\partial N_i - \tilde{Y}_{\nu}\overline{L_{\alpha}}\tilde{\Phi}N_i - \frac{1}{2}M_i\overline{N_i^c}N_i$$

 $\tilde{Y}_{\nu} = 10^{-2} \cdot \begin{pmatrix} 0.0547 + 0.9061i & 0.2923 - 0.2626i & (\\ -0.0024 + 0.04351i & -1.8277 + 0.1813i & -\\ -0.7770 - 0.2221i & 0.5467 + 2.3425i & - \end{pmatrix}$ 

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• For each point in scan we have RHN mass scale, Yukawa coupling of RHN to leptonic

$$0.1159 - 0.1146i$$
  
 $-0.4079 + 1.2977i$   
 $-6.8722 - 0.0676i$ 

![](_page_36_Picture_12.jpeg)

![](_page_37_Figure_1.jpeg)

### **Decay asymmetry from interference between tree** and loop level diagrams

### Covi, Roulet, Vissani

![](_page_37_Figure_4.jpeg)

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### **Thermal leptogenesis**

![](_page_37_Picture_8.jpeg)

![](_page_37_Picture_9.jpeg)

### Washout and scattering processes

![](_page_38_Figure_2.jpeg)

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### **Thermal leptogenesis**

![](_page_38_Picture_6.jpeg)

![](_page_39_Figure_0.jpeg)

![](_page_39_Figure_3.jpeg)

![](_page_39_Picture_4.jpeg)

![](_page_39_Picture_5.jpeg)

![](_page_40_Figure_0.jpeg)

![](_page_40_Figure_3.jpeg)

![](_page_40_Picture_4.jpeg)

- observables is a more powerful way of constraining GUTs.
- Coming decade is an exciting time for GUTs as neutrino and GW experiments will constrain nucleon decay, the presence of GWs and neutrinoless double beta decay  $(0\nu\beta\beta)$ .
- space consistent with fermionic masses and mixing  $\implies$  successful leptogenesis
- For future: study interplay of inflationary scale, more breaking chains. Grid scans for d > 3are hopeless and a more sophisticated machinery is required.

"we have entered an exciting era where new observations of GWs from the heavens and proton decay experiments from under the Earth can provide complementary windows to reveal the details of the unification of matter and forces at the highest energies."

### Summary

• GUTs generically predict nucleon decay and the formation of topological defects. Interplay of these

• Studied a SO(10) breaking chain can be tested by Hyper-K, GW detectors and  $0\nu\beta\beta$ . Parameter

![](_page_41_Picture_13.jpeg)

![](_page_41_Figure_14.jpeg)

![](_page_41_Picture_15.jpeg)

![](_page_42_Picture_0.jpeg)

# Merci!

![](_page_42_Picture_2.jpeg)

HI

Ш

A REAL PROPERTY

I

### **Renormalisation Group Equations**

### **Beta function coefficients 1 and 2-loop respectively**

$$b_{i} = -\frac{11}{3}C_{2}(H_{i}) + \frac{2}{3}\sum_{F}T(F_{i}) + \frac{1}{3}\sum_{S}T(S_{i}),$$
  
$$b_{ij} = -\frac{34}{3}[C_{2}(H_{i})]^{2}\delta_{ij} + \sum_{F}T(F_{i})[2C_{2}(F_{j}) + \frac{10}{3}C_{2}(H_{i})\delta_{ij}] + \sum_{S}T(S_{i})[4C_{2}(S_{j}) + \frac{2}{3}C_{2}(H_{i})\delta_{ij}],$$

### **Two-loop RGE equation**

$$\alpha_i(\mu)^{-1} = \alpha_i(\mu_0)^{-1} - \frac{b_i}{2\pi} \log \frac{\mu}{\mu_0} + \sum_j \frac{b_{ij}}{4\pi b_i} \log \left(1 - b_j \alpha_j(\mu_0) \log \frac{\mu}{\mu_0}\right) ,$$

### **Matching condition**

$$H_i \to H_j, \quad \frac{1}{\alpha_{H_i}(M_I)} - \frac{C}{C}$$

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 $\frac{C_2(H_i)}{12\pi} = \frac{1}{\alpha_{H_j}(M_I)} - \frac{C_2(H_j)}{12\pi}$ 

![](_page_43_Picture_11.jpeg)

![](_page_44_Figure_0.jpeg)

II2 :  $SO(10) \xrightarrow{M_X} G_{422}^C \xrightarrow{M_2} G_{3221}^{M_1} \xrightarrow{M_1} G_{SM}$ Intersection of  $M_2$  and  $M_X$  reduces II2 to I2 I2 :  $SO(10) \xrightarrow{M_X} G_{3221}^C \xrightarrow{M_1} G_{SM}$   $M_X \equiv M_2$ At right side blue curve II2 becomes I5 I5 :  $SO(10) \rightarrow G_{422}^C \rightarrow G_{SM}$  $M_2 \equiv M_1$ 10<sup>38</sup> 10<sup>37</sup> 10<sup>36</sup> Hyper-K sensitivity (b→10<sup>35</sup> (Jear) (a<sup>0</sup> 10<sup>34</sup> 10<sup>34</sup> 10<sup>33</sup> Super-K bound 10<sup>32</sup> II12 10<sup>31</sup> 119 ★15 10<sup>30</sup> 1111 10<sup>29</sup> 

*M*<sub>1</sub> [GeV]

10<sup>11</sup>

10<sup>12</sup>

10<sup>13</sup>

10<sup>14</sup>

10<sup>10</sup>

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 $10^{8}$ 

10<sup>9</sup>

![](_page_44_Picture_6.jpeg)

### **Proton Lifetime**

$$\begin{split} \epsilon^{ijk} \epsilon_{\alpha\beta} \Big( \frac{1}{\Lambda_1^2} (\overline{u_R^{jc}} \gamma^{\mu} Q_{\alpha}^k) (\overline{d_R^{ic}} \gamma_{\mu} L_{\beta}) + \frac{1}{\Lambda_1^2} (\overline{u_R^{jc}} \gamma^{\mu} Q_{\alpha}^k) (\overline{e_R^c} \gamma_{\mu} Q_{\beta}^i) \\ &+ \frac{1}{\Lambda_2^2} (\overline{d_R^{jc}} \gamma^{\mu} Q_{\alpha}^k) (\overline{u_R^{ic}} \gamma_{\mu} L_{\beta}) + \frac{1}{\Lambda_2^2} (\overline{d_R^{jc}} \gamma^{\mu} Q_{\alpha}^k) (\overline{\nu_R^c} \gamma_{\mu} Q_{\beta}^i) + \text{h.c.} \Big) \\ &\Lambda_1 = \Lambda_2 \simeq (g_X M_X) \\ \rightarrow \pi^0 + e^+) = \frac{m_p}{32\pi} \left( 1 - \frac{m_{\pi^0}^2}{m_p^2} \right)^2 A_L^2 \times \left[ A_{SL} \Lambda_1^{-2} (1 + |V_{ud}|^2) |\langle \pi^0 | (ud)_R u_L | p \rangle |^2 \right. \\ &+ A_{SR} (\Lambda_1^{-2} + |V_{ud}|^2 \Lambda_2^{-2}) \left| \langle \pi^0 | (ud)_L u_L | p \rangle |^2 \right] \end{split}$$

$$\begin{aligned} \epsilon^{ijk} \epsilon_{\alpha\beta} \Big( \frac{1}{\Lambda_1^2} (\overline{u_R^{jc}} \gamma^{\mu} Q_{\alpha}^k) (\overline{d_R^{ic}} \gamma_{\mu} L_{\beta}) + \frac{1}{\Lambda_1^2} (\overline{u_R^{jc}} \gamma^{\mu} Q_{\alpha}^k) (\overline{e_R^c} \gamma_{\mu} Q_{\beta}^i) \\ + \frac{1}{\Lambda_2^2} (\overline{d_R^{jc}} \gamma^{\mu} Q_{\alpha}^k) (\overline{u_R^{ic}} \gamma_{\mu} L_{\beta}) + \frac{1}{\Lambda_2^2} (\overline{d_R^{jc}} \gamma^{\mu} Q_{\alpha}^k) (\overline{\nu_R^c} \gamma_{\mu} Q_{\beta}^i) + \text{h.c.} \Big) \\ \Lambda_1 = \Lambda_2 \simeq (g_X M_X) \\ \Gamma(p \to \pi^0 + e^+) = \frac{m_p}{32\pi} \left( 1 - \frac{m_{\pi^0}^2}{m_p^2} \right)^2 A_L^2 \times \left[ A_{SL} \Lambda_1^{-2} (1 + |V_{ud}|^2) |\langle \pi^0 | (ud)_R u_L | p \rangle |^2 \\ + A_{SR} (\Lambda_1^{-2} + |V_{ud}|^2 \Lambda_2^{-2}) \left| \langle \pi^0 | (ud)_L u_L | p \rangle |^2 \right] \end{aligned}$$

$$A_{SL(R)} = \prod_{A \in A} M_X = \prod_{i} M_{i}$$

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$$\left[\frac{\alpha_i(M_{A+1})}{\alpha_i(M_A)}\right]^{\frac{\gamma_{iL(R)}}{b_i}}$$

![](_page_45_Picture_8.jpeg)

![](_page_45_Picture_9.jpeg)

### **Gravitational Wave Calculation**

$$l(t) = l_i - \Gamma G \mu \left( t - t_i \right)$$

Frequencies of GW released from the loops are given by  $2k/l_i$  where  $k = 1, 2, \cdots$ Loops are found to emit energy in the form of gravitational radiation at a constant rate

$$\frac{dE}{dt} = -\Gamma G \mu^2 \qquad \Gamma \sim 50$$

Assuming the fraction of the energy transfer in the form of large loops is  $F_{lpha} \sim 0.1$ 

$$\Omega_{\rm GW}(f) = \sum_{k} \Omega_{\rm GW}^{(k)}(f) = \frac{1}{\rho_c} \frac{2k}{f} \frac{\mathcal{F}_{\alpha} \Gamma^{(k)} G \mu^2}{\alpha (\alpha + \Gamma G \mu)}$$
$$= \int_{t_F}^{t_0} dt \frac{C_{\rm eff}\left(t_i^{(k)}\right)}{t_i^{(k)4}} \frac{a^2(t) a^3\left(t_i^{(k)}\right)}{a^5(t_0)} \theta\left(t_i^{(k)} - t_F\right)$$

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 $l_i = \alpha t_i$  with  $\alpha \simeq 0.1$ 

$$rac{\mu^2}{\mu)}$$

$$C_{\rm eff} = 5.7, 0.5$$

<u>1101.5173</u> <u>1808.08968</u> <u>0003298</u>

![](_page_46_Picture_15.jpeg)

In the Yukawa sector, couplings above the GUT scale are given by  $Y_{10}^* \mathbf{16} \cdot \mathbf{16} \cdot \mathbf{10} + Y_{\overline{126}}^* \mathbf{16} \cdot \mathbf{16} \cdot \overline{\mathbf{126}} + Y_{120}^* \mathbf{16} \cdot \mathbf{16} \cdot \mathbf{120} + \text{h.c.},$ 

After breaking to  $G_{SM}$ 

$$Y_{10} \Big[ (\overline{Q}u_R + \overline{L}\nu_R) h_{10}^u + (\overline{Q}d_R + \overline{L}e_R) h_{10}^d \Big] + \frac{1}{\sqrt{3}} Y_{\overline{126}} \Big[ (\overline{Q}u_R - 3\overline{L}\nu_R) h_{\overline{126}}^u + (\overline{Q}d_R - 3\overline{L}e_R) h_{\overline{126}}^d \Big]$$
$$+ Y_{120} \Big[ (\overline{Q}u_R + \overline{L}\nu_R) h_{120}^u + (\overline{Q}d_R + \overline{L}e_R) h_{120}^d + \frac{1}{\sqrt{3}} (\overline{Q}u_R - 3\overline{L}\nu_R) h_{120}^{u'} + (\overline{Q}d_R - 3\overline{L}e_R) h_{120}^{d'} \Big]$$
$$+ h.c.$$

Rotating the Higgs fields to their mass basis, we derive Yukawa couplings to the SM Higgs  $J_{L}\bar{L}h_{SM}\nu_{R} + Y_{e}\bar{L}h_{SM}e_{R} + h.c.$ 

$$Y_u \bar{Q} \tilde{h}_{\rm SM} u_R + Y_d \bar{Q} h_{\rm SM} d_R + Y_\nu$$

$$\begin{split} Y_u &= Y_{10}V_{11}^* + \frac{1}{\sqrt{3}}Y_{\overline{126}}V_{12}^* + Y_{120}\left(V_{13}^* + \frac{1}{\sqrt{3}}V_{14}^*\right) \\ Y_d &= Y_{10}V_{15} + \frac{1}{\sqrt{3}}Y_{\overline{126}}V_{16} + Y_{120}\left(V_{17} + \frac{1}{\sqrt{3}}V_{18}\right) \\ Y_\nu &= Y_{10}V_{11}^* - \sqrt{3}Y_{\overline{126}}V_{12}^* + Y_{120}\left(V_{13}^* - \sqrt{3}V_{14}^*\right) \\ Y_e &= Y_{10}V_{15} - \sqrt{3}Y_{\overline{126}}V_{16} + Y_{120}\left(V_{17} - \sqrt{3}V_{18}\right). \end{split}$$

$$Y_{u} = Y_{10}V_{11}^{*} + \frac{1}{\sqrt{3}}Y_{\overline{126}}V_{12}^{*} + Y_{120}\left(V_{13}^{*} + \frac{1}{\sqrt{3}}V_{14}^{*}\right)$$
$$Y_{d} = Y_{10}V_{15} + \frac{1}{\sqrt{3}}Y_{\overline{126}}V_{16} + Y_{120}\left(V_{17} + \frac{1}{\sqrt{3}}V_{18}\right)$$
$$Y_{\nu} = Y_{10}V_{11}^{*} - \sqrt{3}Y_{\overline{126}}V_{12}^{*} + Y_{120}\left(V_{13}^{*} - \sqrt{3}V_{14}^{*}\right)$$
$$Y_{e} = Y_{10}V_{15} - \sqrt{3}Y_{\overline{126}}V_{16} + Y_{120}\left(V_{17} - \sqrt{3}V_{18}\right).$$

$$Y_{u} = Y_{10}V_{11}^{*} + \frac{1}{\sqrt{3}}Y_{\overline{126}}V_{12}^{*} + Y_{120}\left(V_{13}^{*} + \frac{1}{\sqrt{3}}V_{14}^{*}\right)$$
$$Y_{d} = Y_{10}V_{15} + \frac{1}{\sqrt{3}}Y_{\overline{126}}V_{16} + Y_{120}\left(V_{17} + \frac{1}{\sqrt{3}}V_{18}\right)$$
$$Y_{\nu} = Y_{10}V_{11}^{*} - \sqrt{3}Y_{\overline{126}}V_{12}^{*} + Y_{120}\left(V_{13}^{*} - \sqrt{3}V_{14}^{*}\right)$$
$$Y_{e} = Y_{10}V_{15} - \sqrt{3}Y_{\overline{126}}V_{16} + Y_{120}\left(V_{17} - \sqrt{3}V_{18}\right).$$

$$Y_{u} = Y_{10}V_{11}^{*} + \frac{1}{\sqrt{3}}Y_{\overline{126}}V_{12}^{*} + Y_{120}\left(V_{13}^{*} + \frac{1}{\sqrt{3}}V_{14}^{*}\right)$$

$$Y_{d} = Y_{10}V_{15} + \frac{1}{\sqrt{3}}Y_{\overline{126}}V_{16} + Y_{120}\left(V_{17} + \frac{1}{\sqrt{3}}V_{18}\right)$$

$$Y_{\nu} = Y_{10}V_{11}^{*} - \sqrt{3}Y_{\overline{126}}V_{12}^{*} + Y_{120}\left(V_{13}^{*} - \sqrt{3}V_{14}^{*}\right)$$

$$Y_{e} = Y_{10}V_{15} - \sqrt{3}Y_{\overline{126}}V_{16} + Y_{120}\left(V_{17} - \sqrt{3}V_{18}\right).$$

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### **GUT Model**

$$Y_{u} = Y_{10}V_{11}^{*} + \frac{1}{\sqrt{3}}Y_{\overline{126}}V_{12}^{*} + Y_{120}\left(V_{13}^{*} + \frac{1}{\sqrt{3}}V_{14}^{*}\right)$$
$$Y_{d} = Y_{10}V_{15} + \frac{1}{\sqrt{3}}Y_{\overline{126}}V_{16} + Y_{120}\left(V_{17} + \frac{1}{\sqrt{3}}V_{18}\right)$$
$$Y_{\nu} = Y_{10}V_{11}^{*} - \sqrt{3}Y_{\overline{126}}V_{12}^{*} + Y_{120}\left(V_{13}^{*} - \sqrt{3}V_{14}^{*}\right)$$
$$Y_{e} = Y_{10}V_{15} - \sqrt{3}Y_{\overline{126}}V_{16} + Y_{120}\left(V_{17} - \sqrt{3}V_{18}\right).$$

$$\begin{aligned} Y_u &= h + r_2 f + i r_3 h', \quad Y_d = r_1 \left( h + f + i h' \right), \quad Y_\nu = h - 3 r_2 f + i c_\nu h' \\ Y_e &= r_1 \left( h - 3 f + i c_e h' \right), \quad M_{\nu_R} = f \frac{\sqrt{3} r_1}{V_{16}} v_S \end{aligned}$$

$$h = Y_{10}V_{11}, f = Y_{\overline{126}}\frac{V_{16}}{\sqrt{3}}\frac{V_{11}^*}{V_{15}}, c_e = \frac{V_{17} - \sqrt{3}V_{18}}{V_{17} + V_{18}/\sqrt{3}}, \qquad c_\nu = \frac{V_{13}^* - \sqrt{3}V_{14}^*}{V_{17} + V_{18}/\sqrt{3}}\frac{V_{15}}{V_{11}^*}, r_1 = \frac{V_{15}}{V_{11}^*}, \qquad r_2 = \frac{V_{12}^*}{V_{16}}\frac{V_{15}}{V_{11}^*}, \qquad r_3 = \frac{V_{13}^* + V_{14}^*/\sqrt{3}}{V_{17} + V_{18}/\sqrt{3}}\frac{V_{15}}{V_{11}^*}, \quad h' = -iY_{120}\left(V_{17} + V_{18}/\sqrt{3}\right)\frac{V_{11}^*}{V_{15}},$$

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### **GUT Model**

$$\begin{split} Y_{\nu} &= -\frac{3r_2 + 1}{r_2 - 1}Y_u + \frac{4r_2}{r_1 \left(r_2 - 1\right)}\operatorname{Re} Y_d + i\frac{c_{\nu}}{r_1}\operatorname{Im} Y_d \\ Y_e &= -\frac{4r_1}{r_2 - 1}Y_u + \frac{r_2 + 3}{r_2 - 1}\operatorname{Re} Y_d + ic_e\operatorname{Im} Y_d \end{split}$$

$$M_{\nu} = m_0 \left( \frac{8r_2 (r_2 + 1)}{r_2 - 1} Y_u - \frac{16r_2^2}{r_1 (r_2 - 1)} \operatorname{Re} Y_d + \frac{r_2 - 1}{r_1} (r_1 Y_u + ic_{\nu} \operatorname{Im} Y_d) (r_1 Y_u - \operatorname{Re} Y_d)^{-1} (r_1 Y_u - ic_{\nu} \operatorname{Im} Y_d) \right)$$

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### **GUT Model**

	Multiplet	Role in the model
Fermions	16	Contains all SM fermions and RH neutrinos
	10	Generates fermion masses
	<b>45</b>	Triggers intermediate symmetry breaking
Higgses	<b>54</b>	Triggers GUT symmetry breaking
	120	Generates fermion masses
	$\overline{126}$	Generates fermion masses & intermediate symmetry breaking
	210	Triggers intermediate symmetry breaking

SO(10)	16
$G_3$	$({f 4},{f 2},{f 1})_L+(\overline{f 4},{f 1},{f 2})_{R^c}$
$G_2$	$egin{aligned} & (3,2,1,1/6)_{Q_L}+(\overline{3},1,2,-1/6)_{Q_R^c} \ & +(1,2,1,-1/2)_{l_L}+(1,1,2,1/2)_{l_R^c} \end{aligned}$
$G_1$	$egin{aligned} & (3,2,1,1/6)_{Q_L}+(\overline{3},1,2,-1/6)_{Q_R^c} \ & +(1,2,1,-1/2)_{l_L}+(1,1,2,1/2)_{l_R^c} \end{aligned}$
$G_{ m SM}$	$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$

### Matter field decomposition

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### **GUT Model Particle Content**

SO(10)	54	210	45	126
$G_3$	$({f 1},{f 1},{f 1})$	$({f 15},{f 1},{f 1})_1$	$({f 15},{f 1},{f 1})_2$	$({f 10},{f 1},{f 3})+(\overline{{f 10}},{f 3},{f 1})$
$G_2$	_	$({f 1},{f 1},{f 1},0)_1$	$({f 1},{f 1},{f 1},0)_2$	(1, 1, 3, -1) + (1, 3, 1, 1)
$G_1$	_	_	$({f 1},{f 1},{f 1},0)_2$	$({f 1},{f 1},{f 3},-1)$
$G_{ m SM}$	_	_	_	$({f 1},{f 1},0)_S$

### SO(10) Higgs reps for SSB

SO(10)	10	$\overline{126}$	120
C	$({f 1},{f 2},{f 2})_1$	$({f 15},{f 2},{f 2})_1$	$({f 1},{f 2},{f 2})_2+({f 15},{f 2},{f 2})_2$
G3		$+ ({f 10},{f 1},{f 3}) + (\overline{f 10},{f 3},{f 1})$	
$G_2$	$({f 1},{f 2},{f 2},0)_1$	$({f 1},{f 2},{f 2},0)_2$	$({f 1},{f 2},{f 2},0)_{3,4}$
		+(1, 1, 3, -1) + (1, 3, 1, 1)	
$G_1$	$({f 1},{f 2},{f 2},0)_1$	$({f 1},{f 2},{f 2},0)_2$	$({f 1},{f 2},{f 2},0)_{3,4}$
		$+({f 1},{f 1},{f 3},-1)$	
	$(1, 2, -1/2)_{h_{10}^u}$	$(1,2,-1/2)_{h^u_{126}}$	$(1, 2, -1/2)_{h_{120}^u, h_{120}^{u'}}$
$G_{ m SM}$	$+(1,2,+1/2)_{h_{10}^d}$	$+({f 1},{f 2},+1/2)_{hrac{d}{{f 126}}}$	$+(1,2,+1/2)_{h_{120}^{d},h_{120}^{d'}}^{h_{120}^{d}}$
		$+(1,1,0)_S$	

SO(10) Higgs reps for fermion mass generation

### **Benchmark 1 RGE**

![](_page_51_Figure_2.jpeg)

### **GUT Model Particle Content**

## **Overlap with PTA experiments**

 $A \equiv$  amplitude parameter of correlation between pulsars.

![](_page_52_Figure_2.jpeg)

Institute for Particle Physics Phenomenology

### Leptogenesis Equations

$$\begin{split} \frac{dN_{\alpha\beta}^{B-L}}{dz} &= \sum_{i=1}^{3} \varepsilon_{\alpha\beta}^{(i)} D_{i} \left( N_{N_{i}} - N_{N_{i}}^{\mathrm{eq}} \right) - \frac{1}{2} W_{i} \left\{ \mathcal{P}^{(i)0}, N^{B-L} \right\}_{\alpha\beta} \\ &- \frac{\mathrm{Im} \left( \Lambda_{\tau} \right)}{Hz} \left[ \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \left[ \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, N^{B-L} \right] \right]_{\alpha\beta} \\ &- \frac{\mathrm{Im} \left( \Lambda_{\mu} \right)}{Hz} \left[ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \left[ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, N^{B-L} \right] \right]_{\alpha\beta}, \end{split}$$

$$N^{B-L} = \begin{pmatrix} N_{\tau\tau} & N_{\tau\mu} & N_{\tau e} \\ N_{\mu\tau} & N_{\mu\mu} & N_{\mu e} \\ N_{e\tau} & N_{e\mu} & N_{ee} \end{pmatrix}, \quad \mathcal{P}^{(i)0} = \frac{1}{\left(\tilde{Y}_{\nu}^{\dagger} \tilde{Y}_{\nu}\right)_{ii}} \begin{pmatrix} \left|\tilde{Y}_{\nu\tau i}\right|^{2} & \tilde{Y}_{\nu\tau i} \tilde{Y}_{\nu\mu i} & \tilde{Y}_{\nu\tau i} \tilde{Y}_{\nu ei} \\ \tilde{Y}_{\nu\tau i} \tilde{Y}_{\nu\mu i} & \left|\tilde{Y}_{\nu\mu i}\right|^{2} & \tilde{Y}_{\nu\tau i} \tilde{Y}_{\nu ei} \\ \tilde{Y}_{\nu ei} \tilde{Y}_{\nu\tau i} & \tilde{Y}_{\nu\mu i} \tilde{Y}_{\nu\tau i} & \left|\tilde{Y}_{\nu ei}\right|^{2} \end{pmatrix}$$

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