Unitarity constraints, thermal masses, and their cylindrical diagrammatic representation in leptogenesis

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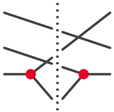
Viktor Zaujec, Tomáš Blažek, Peter Maták

Comenius University in Bratislava, Slovakia



Baryon and Lepton Number Violation
Université Libre de Bruxelles, September 2022

S-matrix unitarity and Holomorphic cutting rules

Perturbative S-matrix unitarity

$$SS^\dagger = S^\dagger S = 1 \quad \to \quad S^\dagger = 1 - \mathrm{i}T^\dagger = (1 + \mathrm{i}T)^{-1} \tag{1}$$

Iterative expansion $\mathrm{i}T^\dagger = \mathrm{i}T - (\mathrm{i}T)^2 + \ldots$ gives

$$\boxed{|T_{fi}|^2 = -\mathrm{i}T^\dagger_{if}\mathrm{i}T_{fi} = -\mathrm{i}T_{if}\mathrm{i}T_{fi} + \sum_n \mathrm{i}T_{in}\mathrm{i}T_{nf}\mathrm{i}T_{fi} - \ldots} \tag{2}$$

[Coster, Stapp, ’70]

Transition probability per unit volume per unit time

$$\frac{1}{V_4}|T_{fi}|^2 = (2\pi)^4\delta^{(4)}(p_f - p_i)|M_{fi}|^2. \tag{3}$$

CP asymmetries and unitarity constraints

$$\Delta|T_{fi}|^2 = \sum_n (\mathrm{i}T_{in}\mathrm{i}T_{nf}\mathrm{i}T_{fi} - \mathrm{i}T_{if}\mathrm{i}T_{fn}\mathrm{i}T_{ni}) - \sum_{n,m} (\mathrm{i}T_{in}\mathrm{i}T_{nm}\mathrm{i}T_{mf}\mathrm{i}T_{fi} - \mathrm{i}T_{if}\mathrm{i}T_{fm}\mathrm{i}T_{mn}\mathrm{i}T_{ni}) + \ldots \tag{4}$$

[Blažek, Maták '21a]

$$\boxed{\sum_f \Delta|T_{fi}|^2 = 0 \quad \xrightarrow{\text{detailed balance}} \quad \sum_f \Delta\mathring{\gamma}^{\mathrm{eq}}_{fi} = 0} \tag{5}$$

[Dolgov '79; Kolb, Wolfram '80]

Thermally averaged reaction rates

$$\mathring{\gamma}_{fi} = -\frac{1}{V_4}\int \prod_{\forall i} [d\mathbf{p}_i]\mathring{f}_i(p_i) \int \prod_{\forall f} [d\mathbf{p}_f] \left(\mathrm{i}T_{if}\mathrm{i}T_{fi} - \sum_n \mathrm{i}T_{in}\mathrm{i}T_{nf}\mathrm{i}T_{fi} + \ldots \right) \tag{6}$$

circled rates $=$ classical kinetic theory $+$ zero-temperature Feynman rules.

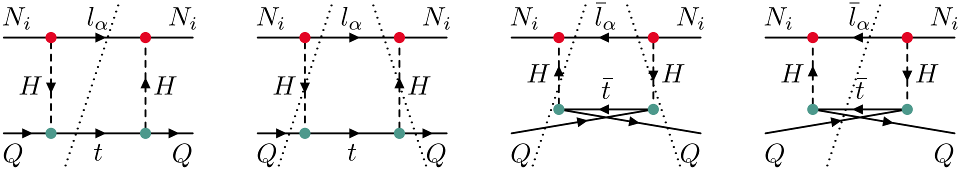
Seesaw type-I model and top-quark Yukawa interaction

The standard model Lagrangian extended with the right-handed neutrino Yukawa interaction

Considering top-quark Yukawa interaction for higher-order corrections

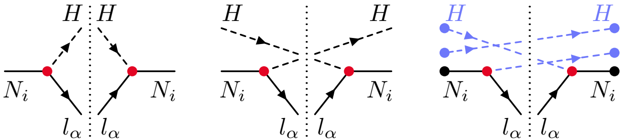
$$\mathcal{L} \supset -\frac{1}{2} M_i \bar{N}_i N_i - \left(Y_{\alpha i} \bar{N}_i P_L l_\alpha H + Y_t \bar{t} P_L Q H + \text{H.c.} \right) \tag{7}$$

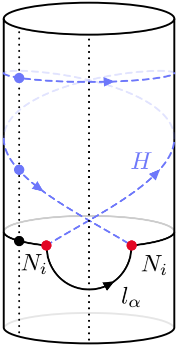
Right-handed neutrino and top quark scattering



[Racker '19; Blažek, Maták '22]

Winding on a cylinder





$$\sum_{w=0}^{\infty}\left(\mathring{f}_H^{\mathrm{eq}}\right)^w = \frac{1}{1-\exp\{-E_H/T\}} = \boxed{1+f_H^{\mathrm{eq}}} \qquad (8)$$

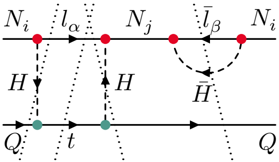
$$\gamma_{N_i\to lH} = \int \dots \boxed{f_{N_i}(1-f_l^{\mathrm{eq}})(1+f_H^{\mathrm{eq}})} \qquad (9)$$

Thermal propagators

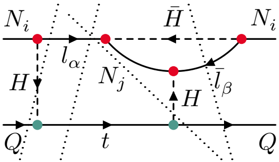
$$\frac{\mathrm{i}}{p_{\bar{H}}^2+\mathrm{i}\epsilon} + 2\pi\sum_{w=1}^{\infty}\mathring{f}_{\bar{H}}^{w}\,\theta(p_{\bar{H}}^0)\,\delta(p_{\bar{H}}^2) \quad\to\quad \frac{\mathrm{i}}{p_{\bar{H}}^2+\mathrm{i}\epsilon} + \boxed{2\pi f(p_{\bar{H}})\,\theta(p_{\bar{H}}^0)\,\delta(p_{\bar{H}}^2)} \qquad (10)$$

[Blažek, Maták ’21b]

Asymmetric right-handed neutrino and top quark scattering



(a)



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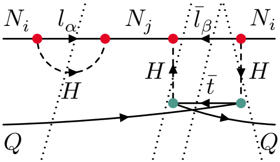
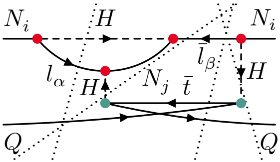


(c)



(d)

$$\boxed{\Delta\mathring{\gamma}^{\rm eq}_{N_iQ\to lt} + \Delta\mathring{\gamma}^{\rm eq}_{N_iQ\to lHQ} + \Delta\mathring{\gamma}^{\rm eq}_{N_iQ\to \bar{l}\bar{H}Q}} + \boxed{\Delta\mathring{\gamma}^{\rm eq}_{N_iQ\to \bar{l}QQ\bar{t}}} = 0 \qquad (11)$$

[Pilaftsis, Underwood '05; Abada, *et al.* '06;
Nardi, Racker, Roulet '07; Racker '19]

Unitarity constraints with quantum statistics

Higgs thermal mass terms and quantum statistics contributions

$$\Delta\mathring{\gamma}^{\text{eq}}_{N_iQ\to lt} + \Delta\mathring{\gamma}^{\text{eq}}_{N_iQ\to lHQ} + \Delta\mathring{\gamma}^{\text{eq}}_{N_iQ\to \bar{l}\bar{H}Q} + \boxed{\Delta\mathring{\gamma}^{\text{eq}}_{N_iQ\to \bar{l}QQ\bar{t}}} = 0 \tag{11}$$

Mass-derivative relation

$$\Delta\mathring{\gamma}^{\text{eq}}_{N_iQ\to lHQ} = \boxed{\Delta\mathring{\gamma}^{\text{eq}}_{N_i(Q)\to lH(Q)}} + \boxed{\frac{1}{4}\mathring{m}^2_{H,Y_t}(T)\frac{\partial}{\partial m_H^2}\bigg|_{m_H^2=0}\Delta\mathring{\gamma}^{\text{eq}}_{N_i\to lH}} \tag{12}$$

Including all windings

$$\boxed{\Delta\gamma^{\text{eq}}_{N_iQ\to lt} + \Delta\gamma^{\text{eq}}_{N_i(Q)\to lH(Q)} + \Delta\gamma^{\text{eq}}_{N_i(Q)\to \bar{l}\bar{H}(Q)} + \Delta\gamma^{\text{eq}}_{N_iQ\to \bar{l}QQ\bar{t}} = 0} \tag{13}$$

Conclusions

Anomalous thresholds $\rightarrow$ thermal masses

Cylindrical diagrammatic representation $\rightarrow$ quantum statistics

$\downarrow$

CPT and unitarity constraints for equilibrium CP asymmetries at finite-temperature

Thank you!

backup slides

Density-matrix evolution

$$\hat{\rho} = \frac{1}{Z} \exp\big\{ - \hat{F} \big\}, \quad \text{where} \quad Z = \mathrm{Tr} \exp\big\{ - \hat{F} \big\} \tag{14}$$

$$\hat{F} = \sum_p F_p a_p^\dagger a_p \tag{15}$$

Circled density

$$\mathring{f}(p) = \exp\big\{ - F_p \big\} = \frac{f(p)}{1 \pm f(p)} \tag{16}$$

[Blažek, Maták '21b]

Temporal evolution

$$\hat{\rho} \quad \to \quad \hat{S} \hat{\rho} \hat{S}^\dagger \tag{17}$$

[McKellar, Thomson '94]

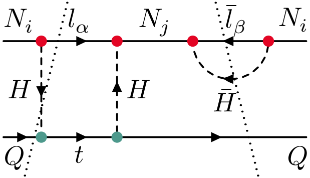
Summing over all possible winding numbers

$$f(p) = \sum_{w=1}^{\infty} (\pm 1)^{w-1} \mathring{f}(p)^w \tag{18}$$

Higgs thermal-mass contributions

Anomalous thresholds cuts at $\mathcal{O}(Y^4 Y_t^2)$ order

$\Delta\mathring{\gamma}^{(a)}_{N_i Q\to lHQ} =$



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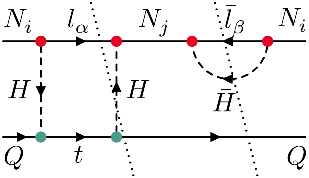

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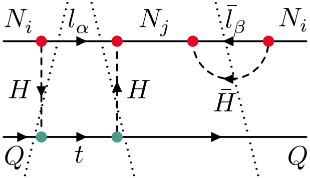
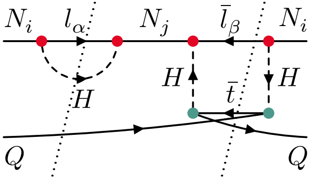


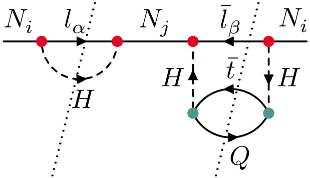
$-$ m.t. (19)

Thermal quark propagator

Other cuts



$$- \quad \text{m.t.} \tag{20}$$



$$- \quad \text{m.t.} \tag{21}$$

Thermal Q propagator

$$\frac{\mathrm{i}}{p_Q^2 + \mathrm{i}\epsilon} \quad \to \quad \frac{\mathrm{i}}{p_Q^2 + \mathrm{i}\epsilon} - 2\pi f_Q(|p_Q^0|)\delta(p_Q^2). \tag{22}$$