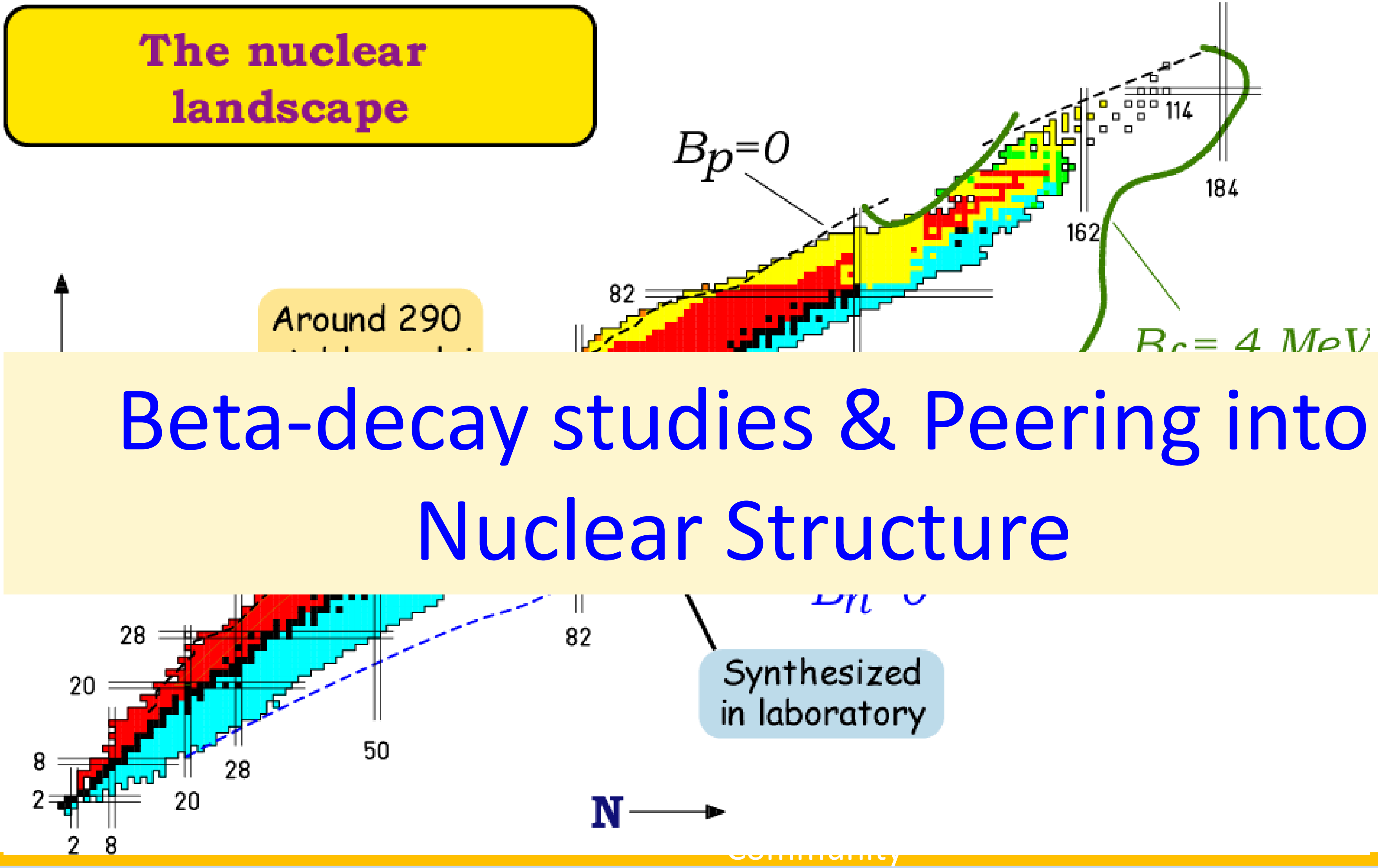
Beta-decay studies & Peering into Nuclear Structure



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Useful References

Books

- ✓ “Handbook of nuclear spectroscopy”, J. Kantele, 1995
- ✓ “Radiation detection and measurements”, G.F. Knoll, 1989
- ✓ “Alpha-, Beta- and Gamma-ray Spectroscopy”, Ed. K. Siegbahn, 1965
- ✓ “Introductory Nuclear Physics”, K. S. Krane, 1988
- ✓ “Basic Ideas and Concepts in Nuclear Physics” K. Heyde IOP Publ. Ltd. 1994
- ✓ “Particle Emission from Nuclei” Ed. D.N. Poenaru & M.S. Ivaçcu CRD 1989 Vol I, II, III

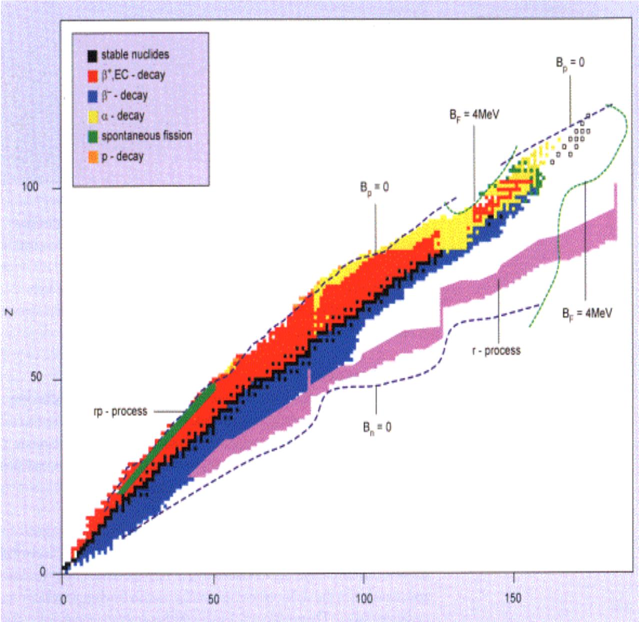
Journal Articles

- ✓ Euroschool on Exotic Beams, Lectures Notes: “Decay Studies of N~Z Nuclei”, E. Roeckl, Vol I, “Beta “Decay of exotic Nuclei”, B. Rubio & W. Gelletly, Vol III
- ✓ B. Blank and M.J.G. Borge, Prog Part and Nuc. Phys 60 (2008) **403**
- ✓ M. Pfützner, L.V. Grigorenco, M. Karny & K. Riisager, Rev. Mod. Phys,ArXiV:1111.0482
- ✓ V.I. Goldanskii , Ann. Rev. Nucl. Sci. 16 (1966)1
- ✓ P.I. Woods, C.N. Davids, Ann.Rev.Nucl.Part.Sci 47 (1997)541
- ✓ P. Buford Price, Ann.Rev.Nucl.Part.Sci. **39** (1989) 19

Plenty of information available on the Web

Atomic Mass Model

Relationship with Nuclear Decay Models



- **265 Stable nuclei**
  - 157 e-e
  - 4 o-o
  - 104 e-o

60 Primordial ($T_{1/2} > 10^9$y)

~ 2500 produced in nuclear reactions

- **Decay characteristics of most radioactive nuclei determined by β-decay i.e. weak interaction**
- **For heavier nuclei → Electromagnetic interaction important →**
  - **α-decay**
  - **fission**
- **Adding protons or neutrons** new nuclei are created **from the stable nuclei →**

until the particle drip-lines (S_p = 0 or S_n = 0).

Nuclei beyond drip-line are unbound to nucleon emission, i.e. Strong interaction cannot bind one more nucleon to the nucleus

Binding Energy

- Strong interaction acts at very short distance.
- Naively one would expect A(A-1)/2 bonds and each E_{bond} ~constant thus giving:

  $\mathbf{BE({}^{A}_{Z}X_{N})/A} \propto E_2\,(A-1)/2$

- Experimentally $\mathbf{BE({}^{A}_{Z}X_{N})/A} \propto$ 8 MeV over the full region indicating
  - Nuclear and charge independent
  - Saturation of Nuclear Forces: $\rho_o \approx 0.17\ N/fm^3$
  - The less bound nucleon has an energy of ~ 8 MeV independent of the number of nucleons

  → **The independent particle picture holds : nucleons move in an average potential**

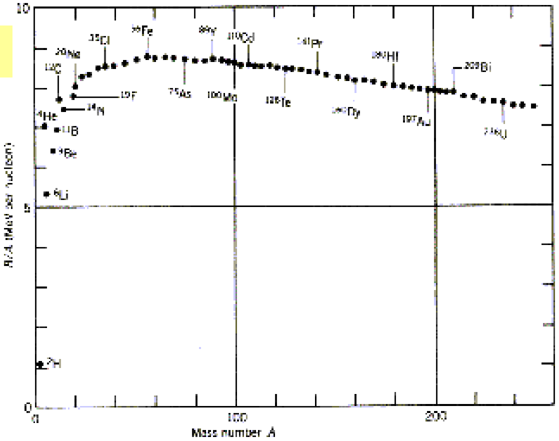
Nuclear density is independent of A and 10^{14} times normal density

- **BE/A as function of A has its maximum around**

  A = 56-60 (^{62}Ni)

  → Source of energy production

  - **Fission of heavy nuclei**
  - **Fusion of light nuclei**



Nuclear stability

$$BE(A,Z) = ZMpc^2 + NMnc^2 - M'({}^{A}_{Z}X_{N})c^2$$

Using the Bethe-Weizsäcker mass equation for BE(A,Z)

$$M'({}^{A}_{Z}X_{N})c^2 = ZMpc^2 + NMnc^2 - a_vA + a_sA^{2/3} + a_cZ(Z-1)A^{-1/3} + a_A(A-2Z)^2/A - a_pA^{-1/2}$$

For each A value this represents a quadratic equation in Z

$$M'({}^{A}_{Z}X_{N})c^2 = xA + yZ + zZ^2 + 0(\pm\delta) \qquad \left.\begin{aligned} x &= Mnc^2 - a_v + a_A + a_sA^{1/3} \\ y &= (Mp-Mn)c^2 - 4a_A - a_cA^{1/3} \\ z &= a_cA^{1/3} + 4a_A/A \end{aligned}\right\}$$

$$\frac{\partial M'}{\partial Z} = 0$$

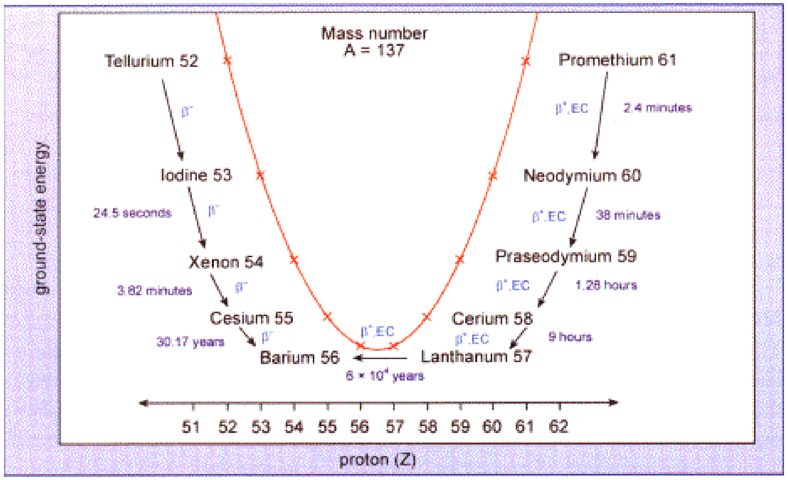
$$Z_0 \approx \frac{A/2}{1+0.007A^{2/3}}$$

Thus for each A-value one can calculate the nucleus with lowest mass (largest binding energy):

For a given A a parabolic behaviour of the nuclear masses show up.

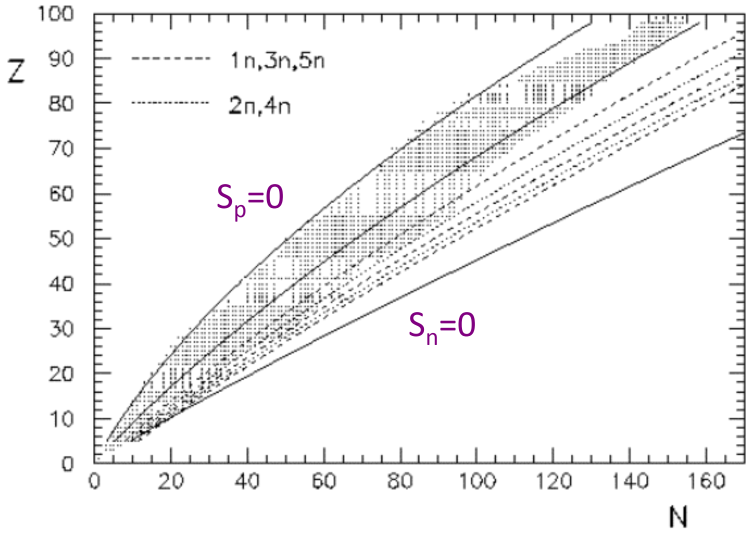
odd-A only one stable nucleus. The rest β± decay towards the only stable nucleus.

even A both even-even and odd-odd ⇒2 parabolas implied by the mass equation.



Stability Against Radioactive Decay

Last stable nuclei A≈210



The conditions $S_n = 0$ and $S_p = 0$ establishes the drip-lines

Spontaneus α-decay ($S_\alpha = 0$) correspond to

$$BE({}^{A}_{Z}X_{N}) - [\, BE({}^{A-4}_{Z-2}X_{N-2}) + BE({}^{4}He)\,] = 0$$

The half-lives becomes short in the actinide region $A \approx 210$

The energy release in nuclear fission:

$$E_{fission} = M^{1}\left({}^{A}_{Z}X_{N}\right)c^{2} - 2M'\left({}^{A/2}_{Z/2}X_{N/2}\right)c^{2}$$

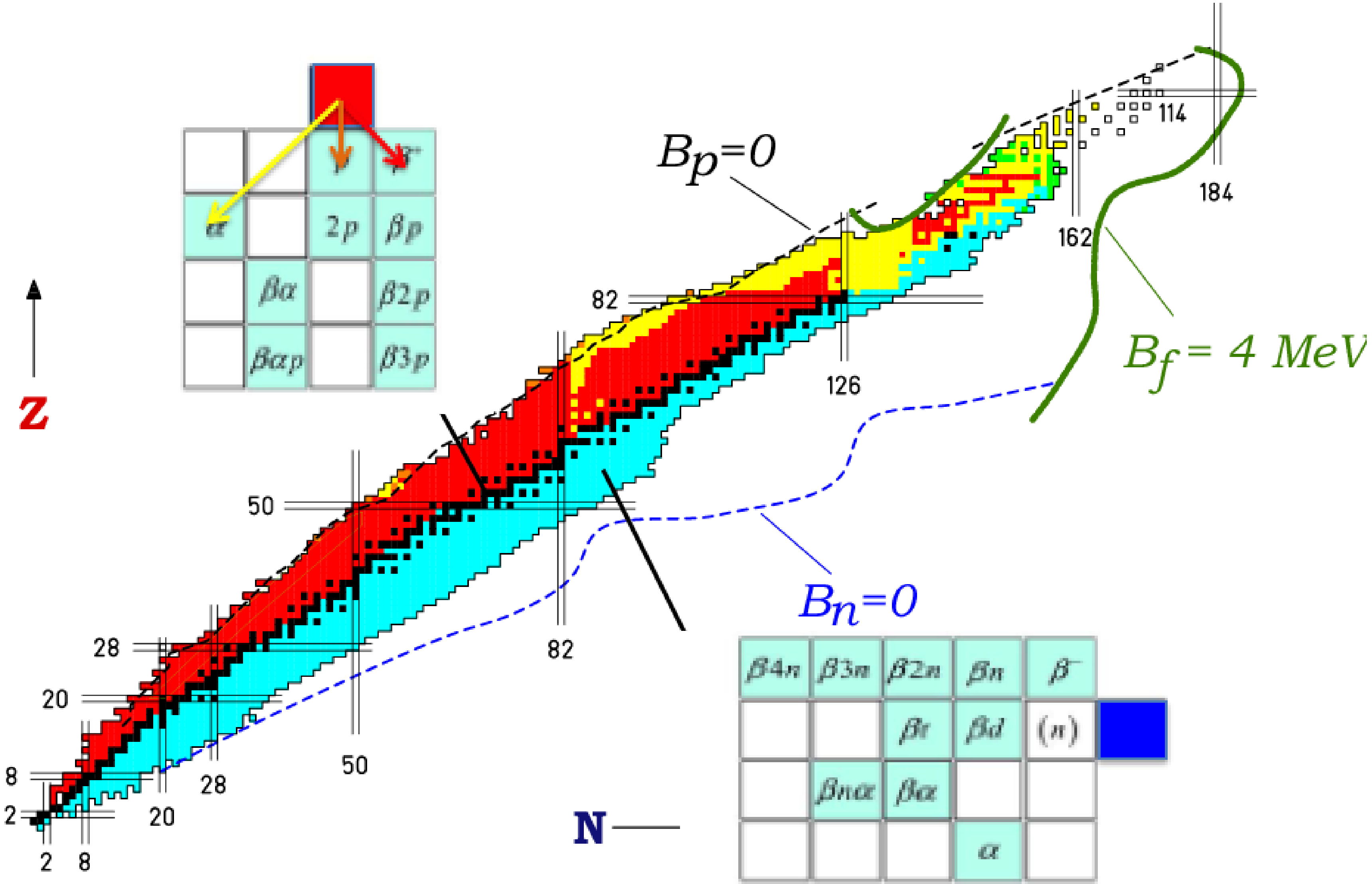
Using a simplified mass eq. where $Z(Z-1) \approx Z^2$ and neglecting the pairing corrections δ:

$$E_{fission} = [\, -5.12\, A^{2/3} + 0.28\, Z^2 A^{-1/3}\,]\, c^2$$

$E_{fission} > 0$ for $A \approx 90$ and $E_{fission}$ = 185 MeV for ${}^{238}U$.

The fission products, neutron rich nuclei, mainly $\beta^- \Rightarrow$ good source of electron anti-neutrinos.

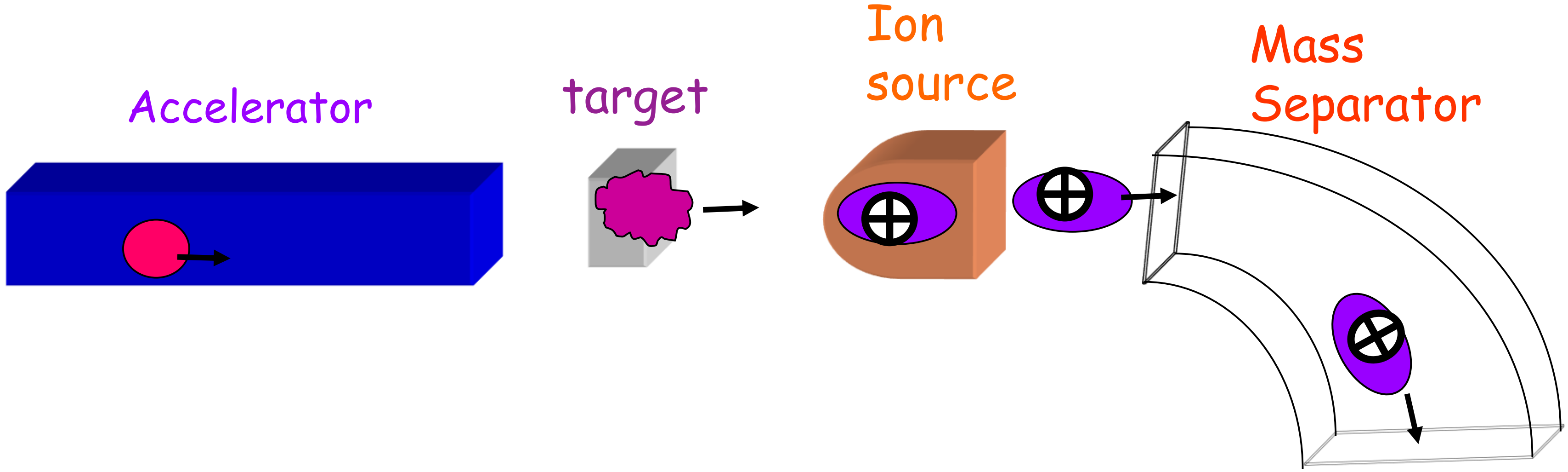
Different decay modes



Beta-decay

- Introduction
- Formalism
- Beta-decay and fundamental interactions
- Beta decay and the structure of the nucleus

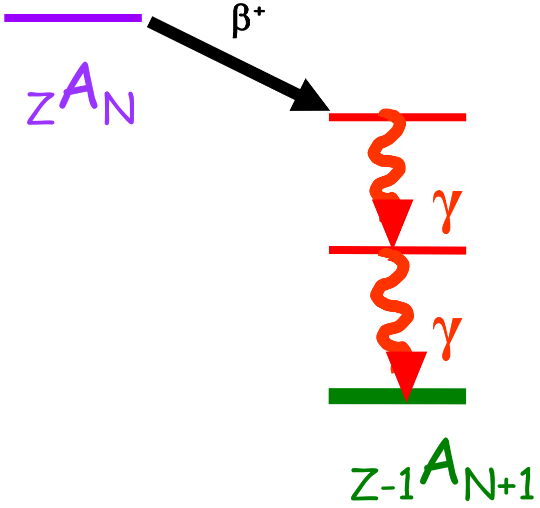
Beta decay Studies

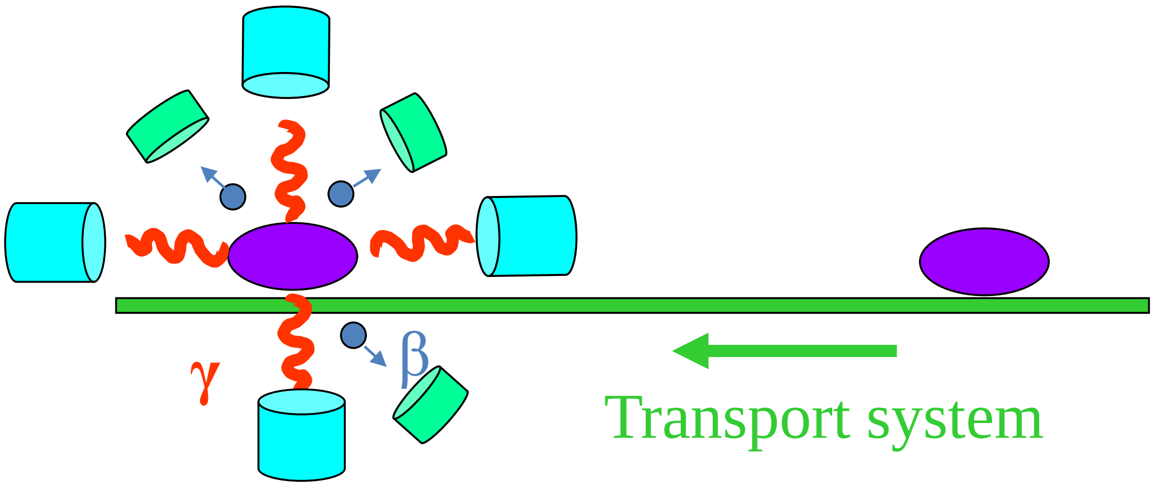

Accelerator

target

Ion source

Mass Separator





Transport system

Introduction

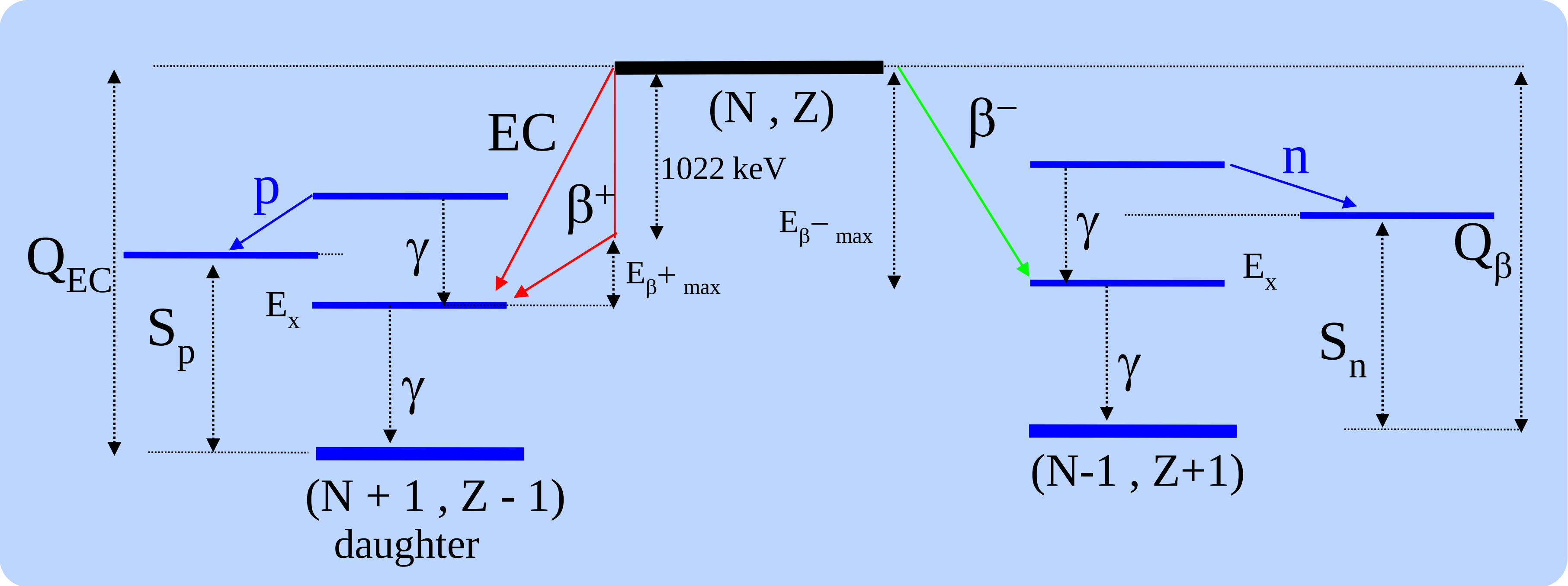
Process mediated by the weak interaction between two isobars

$(N , Z) \overset{\beta^-}{\text{--->}} (N-1, Z+1) + e^- + \nu$

$M(Z) - M(Z+1)= E_\beta + E_\nu + Ex$

$(N , Z) \overset{\beta^+ + EC}{\text{--->}} (N+1, Z-1) + e^+ + \nu$

$M(Z) - M(Z-1)= E_{\beta+} + E_\nu + 1022+ Ex$

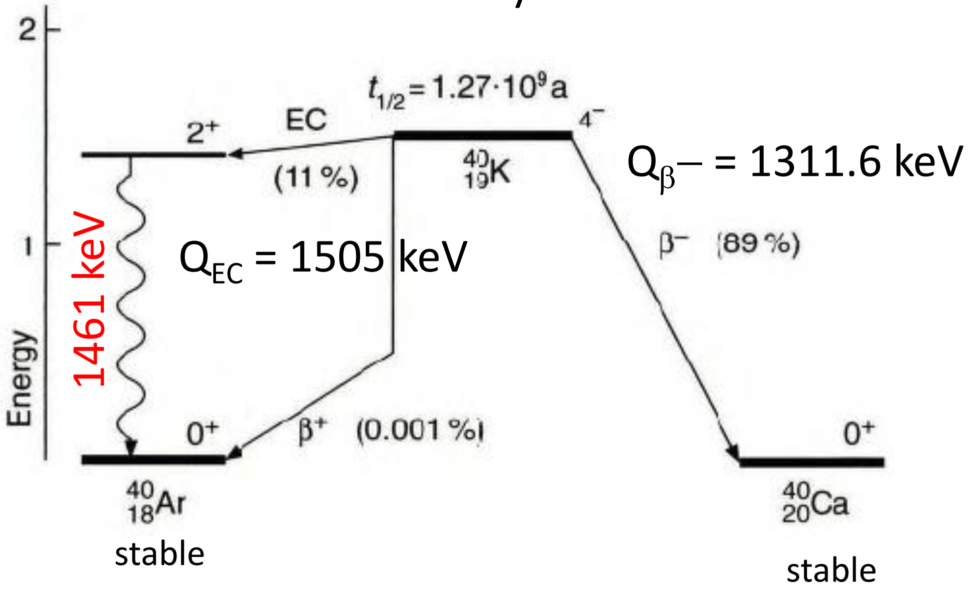


The decay of ^{40}K

- Radioactive decay :
  - probability per unit time λ
  - lifetime τ, half-life $t_{1/2}$
  - activity A (decays per unit time)

$$\begin{aligned} \tau &= 1/\lambda \\ t_{1/2} &= \ln 2/\lambda \\ A(t) &= \lambda\, N(t) \;=\; \lambda\, N_0\, \mathrm{e}^{-\lambda t} \\ 1\ \mathrm{Bq} &= 1\ \mathrm{decay/s} \end{aligned}$$

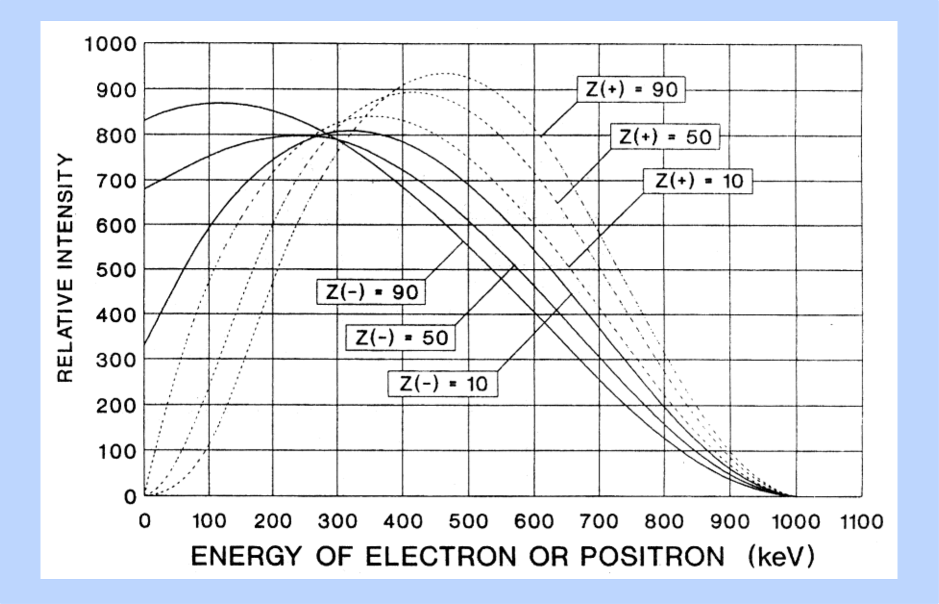
^{40}K all 3 decays



- ^{40}K is 0.01% of natural $^{39-41}$K :
  - ⤳ K^+ signal transmitter in nervous system
  - ⤳ 16% of human radiation exposure !
  - ⤳ 70 kg human = 4,400 decays/s !
  - ⤳ K-Ar dating method for rocks

Introduction (II)

Spectra $\beta^{\pm}$



Expand in a large E-scale

E_{β}–= 18,6 keV (^{3}H, β^{-})

$E_{\beta-}$= 22800 keV (^{22}N, β^{-})

Half-life

$T_{1/2}$: ms --> 10^{15} years

^{35}Na, $T_{1/2}$ = 1,5 ms

^{148}Sm, $T_{1/2}$ = 7 10^{15} years

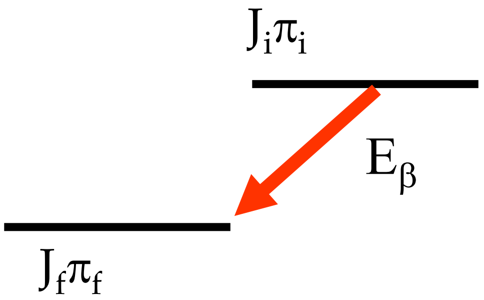
Emission of delayed particles

P_p = 6 10^{-6} (^{151}Lu) to 100 % (^{31}Ar)

P_n = 5,5 10^{-4} (^{79}Ge) to 99 % (^{11}Li)

β p, β2p, β3p, ...βn, β2n ...

Classification of β-decay transitions



$$J_i = J_f + L_b + S_b$$

$$\pi_i \pi_f = (-1)^{L_\beta}$$

$$L_b = l_{b-(b+)} + l_{\tilde{n}(n)} = 0,1,2$$

$$S_b = s_{b-(b+)} + s_{\tilde{n}(n)} = \begin{cases} 0 & - \\ 1 & \text{— } or \text{ - -} \end{cases}$$

L_β = defines the degree of forbiddeness

allowed

when $L_\beta=0$ and $\pi_i\pi_f=+1$

$$\Delta I = \left| I_i - I_f \right| \equiv 0,1$$

forbidden

when the angular momentum conservation requires that

$L_\beta>0$ and/or $\pi_i\pi_f=-1$

Allowed transitions

$J_i = J_f + L_{e\nu} + S_{e\nu}$

$L_{e\nu} = 0$

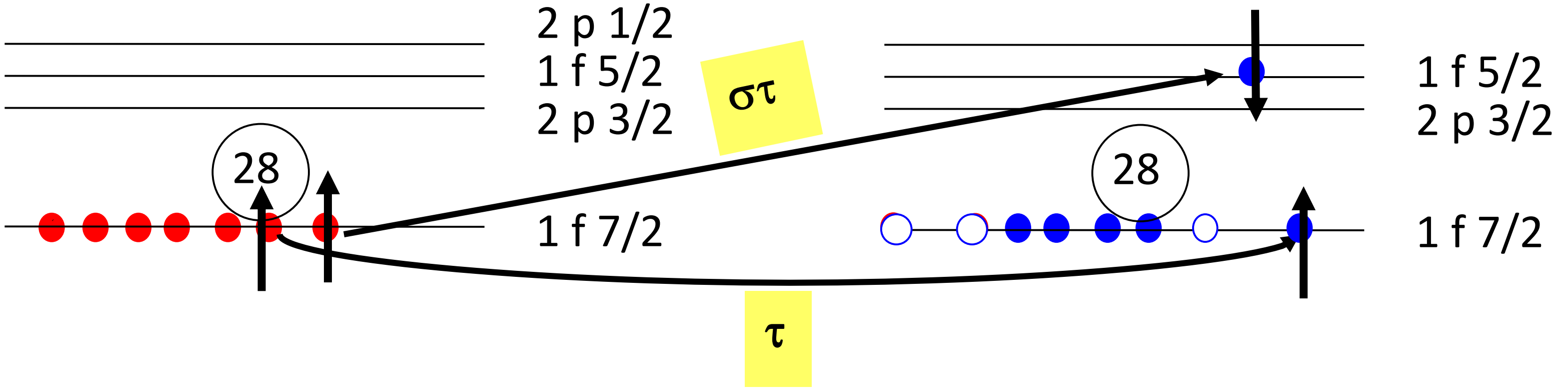
$\pi^i = \pi^f (-1)^{L e\nu}$

spins ν & electron ↑↑ $S_{e\nu} \neq 0$ → **transition type Gamow Teller (GT)**

access to the structure of the nucleus

spins ν & electron ↑↓ $S_{e\nu} = 0$ → **transition du type Fermi (F)**

access to the weak interaction



Beta-decay Formalism

Fermi gold rule

| i > ---> | f >

Density of final states

$p = 2\pi/h\, |M_{if}|^2\, dn/dE$

Transition probability

$M_{if} = \int \phi_f\, H\, \phi_i\, dv$; where H?

$\phi_f = \varphi_e\, \varphi_n\, \varphi_{\text{daughter}}$

$$\varphi_e(r) = \frac{1}{\sqrt{V}} e^{ip.r/\hbar} = \frac{1}{\sqrt{V}}\left[1 + \frac{i\vec{p}.\vec{r}}{\hbar} + ...\right] \cong \frac{1}{\sqrt{V}}$$

Energy conservation

$dn = dn_e\,.\,dn_\nu = \frac{(4p)^2 V^2 p^2 dp q^2 dq}{h^6}$

Radioactive decay constant: $\lambda = {}_0\int^{Po} p\, dp$

$$d\lambda = \frac{2\pi}{\hbar} g^2 |M_{if}|^2 (4\pi)^2 \frac{p^2 dp q^2}{h^6} \frac{dq}{dE_f}$$

For a certain β transition

Fermi function

$\lambda\, t = \text{Log}2 = \text{Cte}\, |M_{if}|^2\; f(Z, E_\beta)\, t$

~ 1 for Z <10

↘ for Z ↗, > 1 β^+

↗ for Z ↗, < 1 β^-

Radioactive constant

t partial half-life

$t = \frac{T_{1/2}}{\%b}$

% β feeding

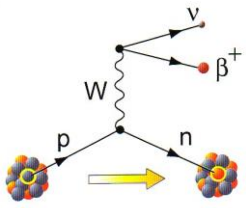
→ $f(Z, E_\beta)\, t = \text{Cte} / |M_{if}|^2$

Classification of the transitions & Spin-Parity

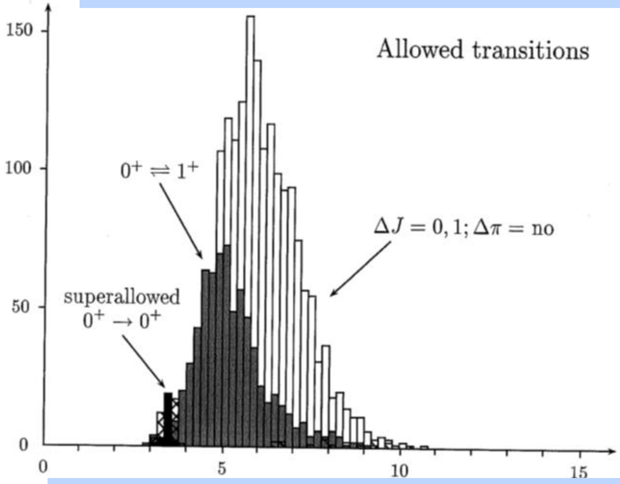
~3900 cases -> gives centroids and widths

log f t

**Independent of
Energy range
and Z**



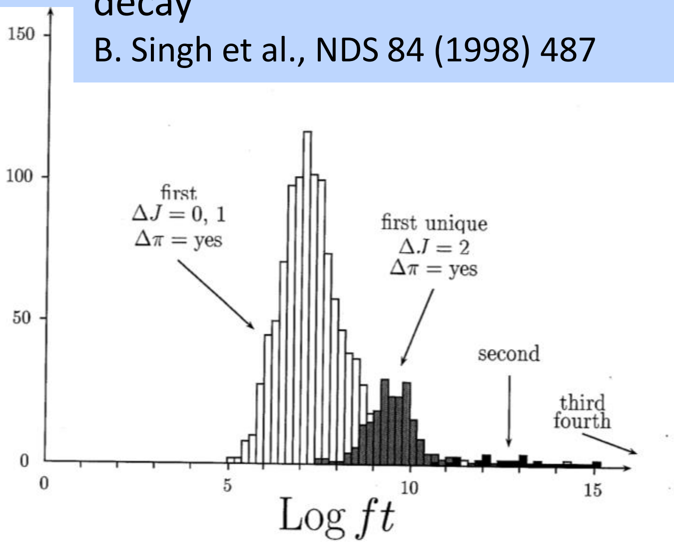
log f t	*Transition type*
< 3,8	super allowed, Fer
< 5,9	Allowed, Gamow
> 6	“special allowed”
7 (1)	first forbidden
8,5 (5)	first forbidden
~ 13	second forbidden
~ 18	Third forbidden



“Review of Log ft Values in β decay”
B. Singh et al., NDS 84 (1998) 487

http://www

$$f\,(Z, E_\beta)\ t = K \,/\, |\,M_{if}\,|^2 = C \,/\, (\,B(F) + B(\,GT\,)\,)$$



- ❑ Only a few cases where from logft unambiguous spir
- ❑ “pandemonium effect” – neutron rich nuclei – log *ft* is
- ❑ needs to know the decay scheme and its properties ac

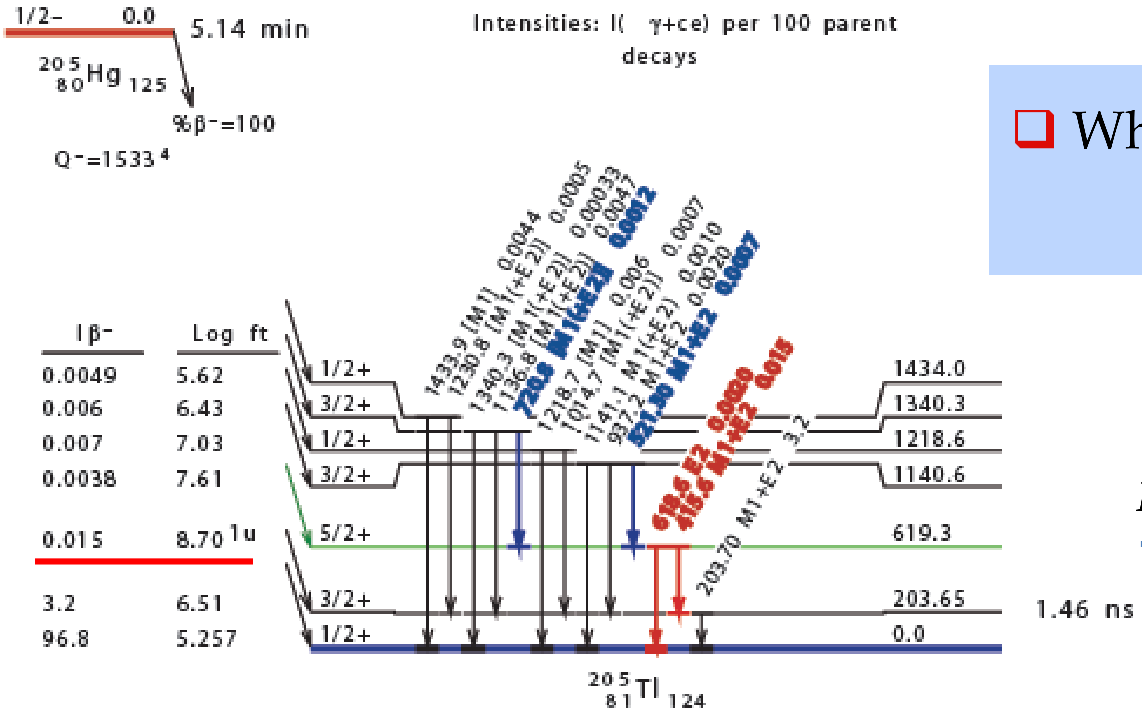
Practical example

$$t \equiv T_{1/2}^{\beta_i} = \frac{T_{1/2}^{\exp}}{P_{\beta_i}}$$

$$P_{\beta_i} = \eta[I^{tot}(out) - I^{tot}(in)]$$

$$I^{tot}(out/in) = \sum_i I_{\gamma_i}(1+\alpha_{Ti})$$

$$\alpha_T(M1+E2) = \frac{\alpha_T(M1) + \delta^2 \alpha_T(E2)}{1+\delta^2}$$



- ☐ What we want to know accurately
  - ✓$T_{1/2}$, I_γ, α_T & δ

In

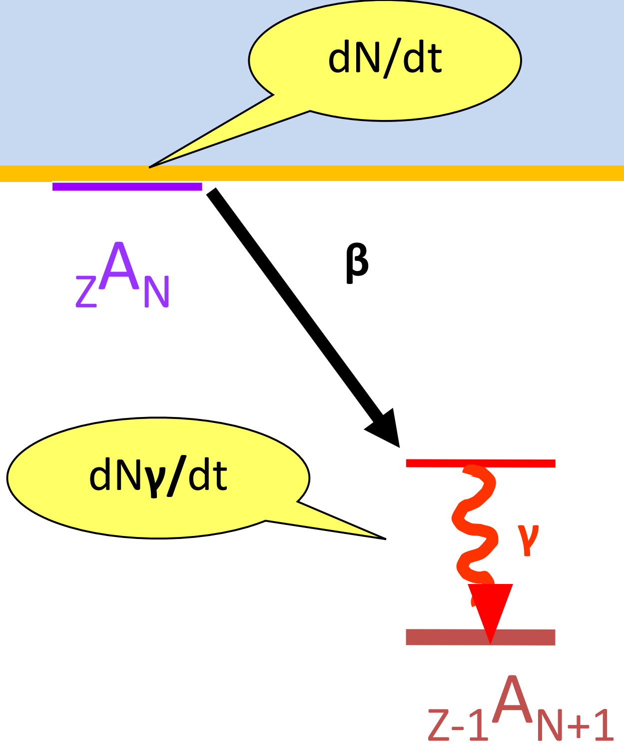
$$\frac{I^{tot}(521+721) = 0.086(16)}{I^{tot}(416+619) = 0.78(10)} = 0.69(10)$$

(net)

Out

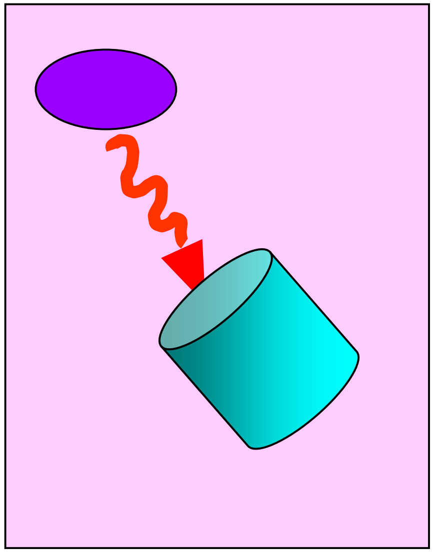
$$\eta = 0.0022 \rightarrow t = 2.056\times10^6[s] \rightarrow \log t = 6.31 \rightarrow \log f = 2.386 \rightarrow \log ft = 8.7$$

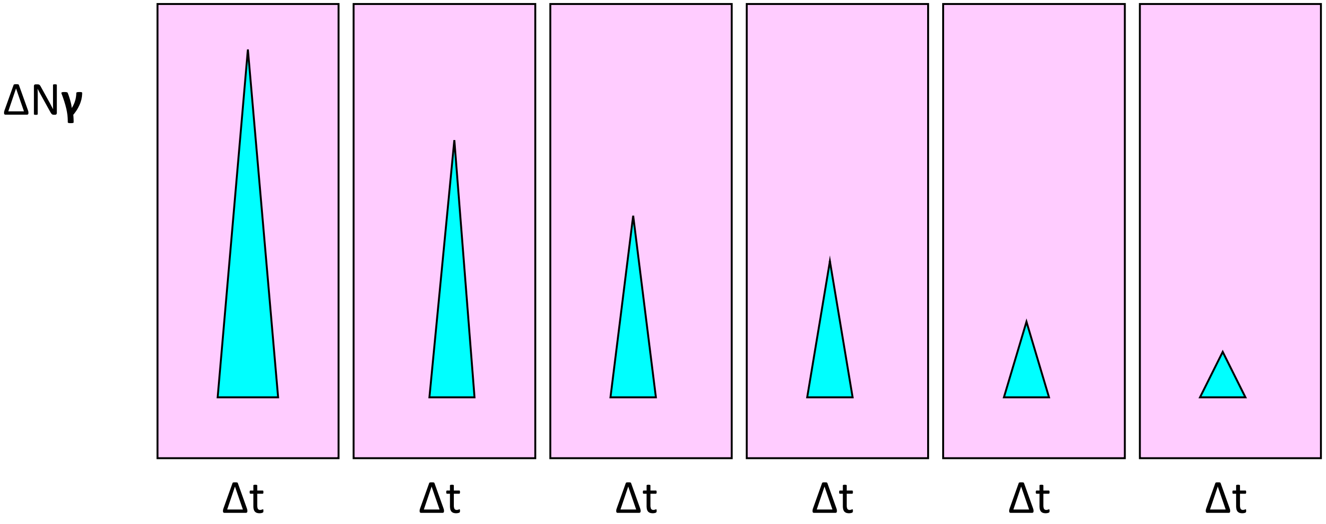
Half-life measurement

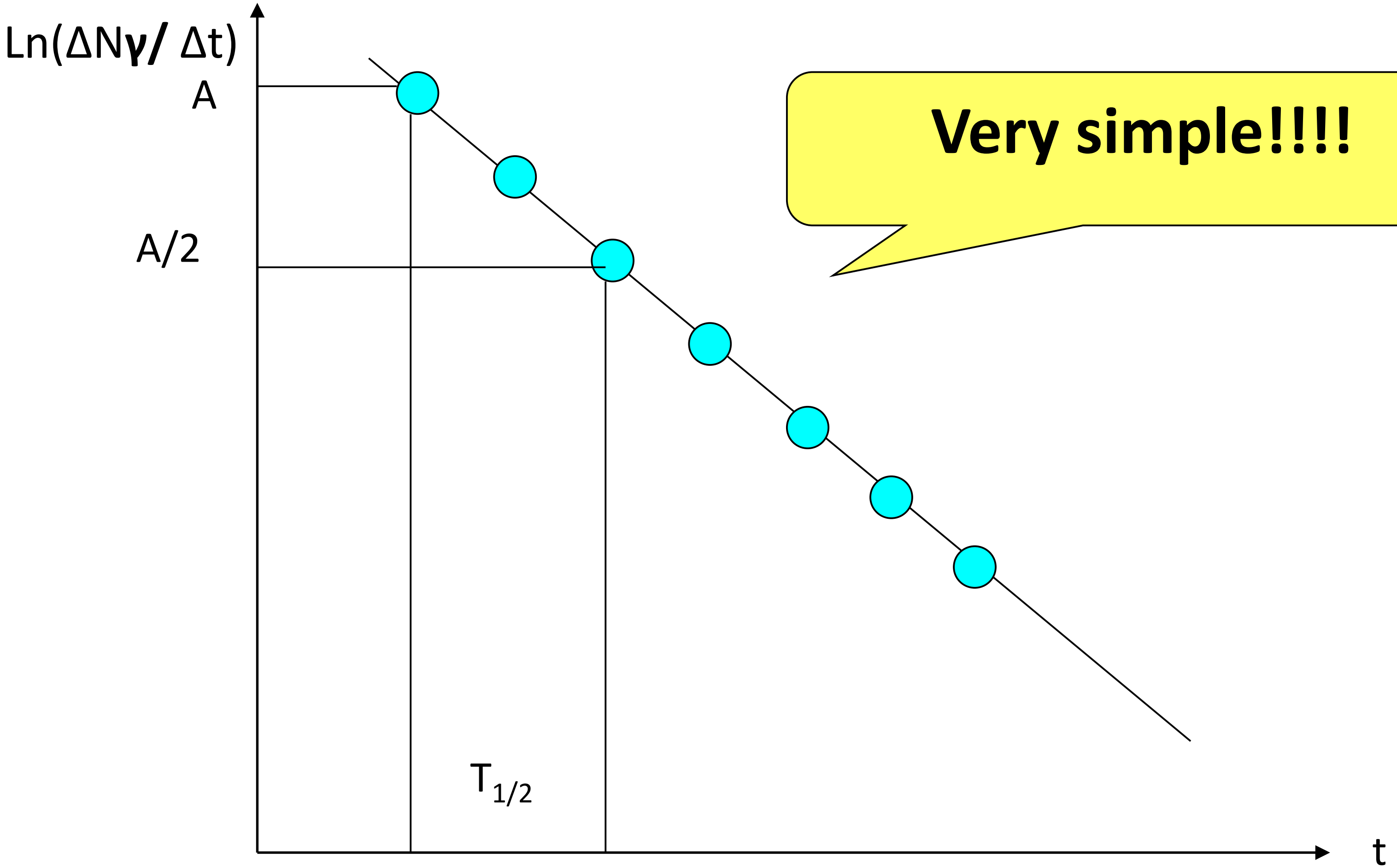


$$\frac{dN}{dt} = \frac{dN_0}{dt} e^{-\lambda t}$$

$$\lambda = \frac{1}{\tau} = \frac{Ln2}{T_{1/2}}$$

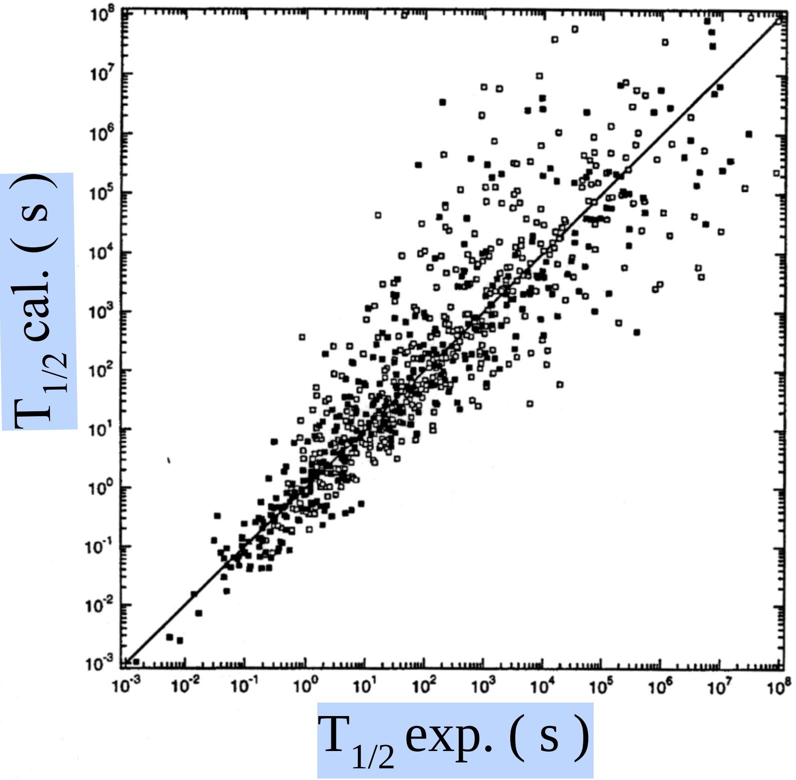


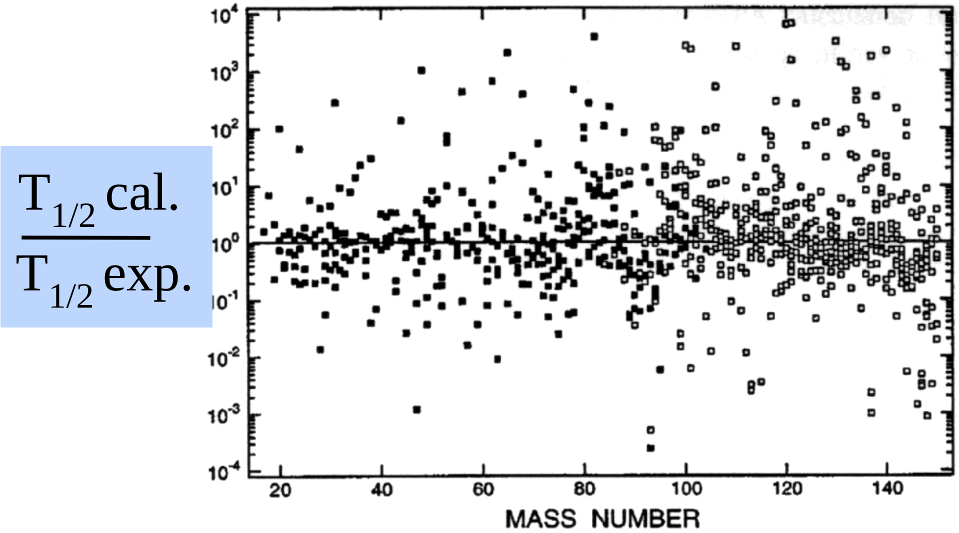




Very simple!!!!

Half-live: First Glance into Nuclear Structure





The isospin formalism:

p and n are the same kind of particles with a different isospin state (T)

The third component T_z is very clear:

τ Fermi Transition

It can only change the third component of isospin:
Only one state called Isobaric Analog State (IAS)

$$B_F = \left| \langle \psi_f | \sum \tau^{\pm} | \psi_i \rangle \right|^2$$

Fermi Strength independent of Nuclear Structure

$$B_F^+ - B_F^- = Z - N$$

B(F) = T (T + 1) - Tz_i Tz_f

$\sigma\tau$ Gamow-Teller

Can change the spin and the isospin:
Many possible final states

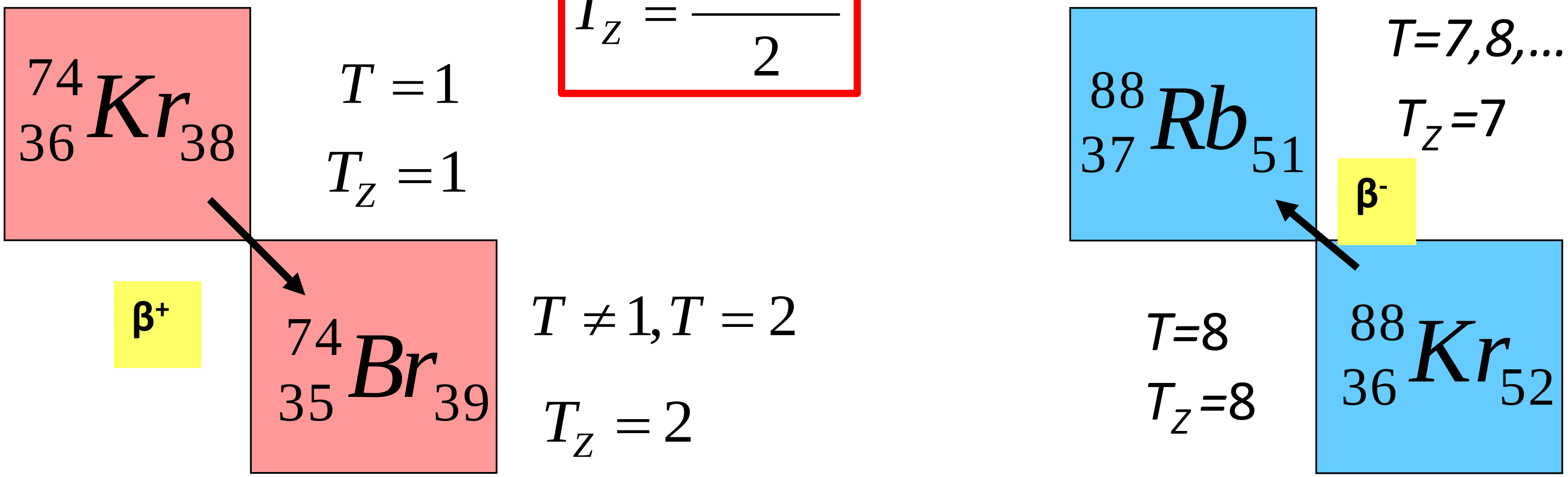
$$B_{GT} = \left| \langle \psi_f | \sum \sigma\tau^{\pm} | \psi_i \rangle \right|^2$$

Gamow-Teller strength obeys the Ikeda sum Rule

$$SB_{GT}^- - SB_{GT}^+ = 3(N - Z)$$

Fermi & Gamow Teller transitions

$$T_Z = \frac{N-Z}{2}$$



$^{74}_{36}Kr_{38}$

$T = 1$

$T_Z = 1$

β⁺

$^{74}_{35}Br_{39}$

$T \neq 1, T = 2$

$T_Z = 2$

$^{88}_{37}Rb_{51}$

T=7,8,…

$T_Z=7$

β⁻

T=8

$T_Z=8$

$^{88}_{36}Kr_{52}$

In β^+ Fermi, forbidden for N>Z

In β^- allowed but energetically difficult

In β^+ Gamow Teller "allowed"

In β^- Gamow Teller "allowed"

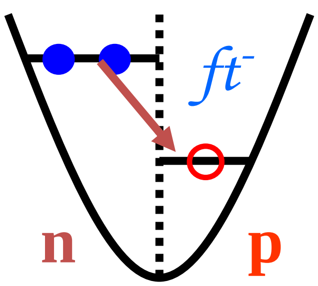
Mirror Asymmetry & Systematics

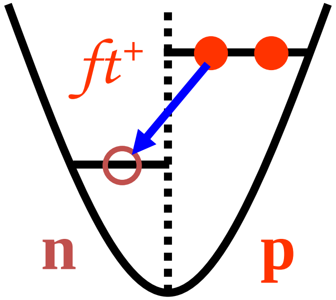
β^- : n $\rightarrow$ p + e^- + $\bar{\nu}$

β^+ : p $\rightarrow$ n + e^+ + ν

E.C. : p + e^- $\rightarrow$ n + ν

$$\boxed{\delta = \frac{ft^+}{ft^-} - 1}$$





$\delta = \delta_{nuc} + \delta_{SCC}$

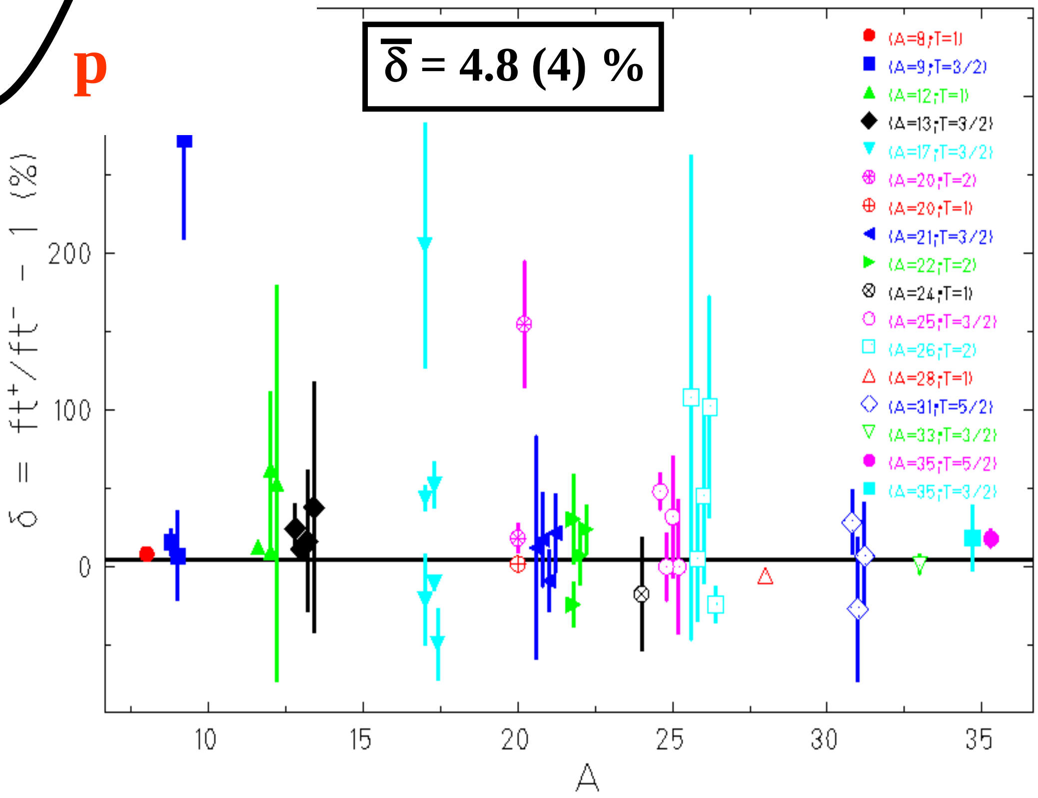
Thomas et al., AIP Conf. Proc 681, p. 235

- **Allowed Gamow-Teller transitions (log(ft)<6)**
  - →**17 couples of nuclei**
  - →**46 mirror transitions**

Average asymmetry δ :

11 (1) % in the 1p shell (A<17)

0 (1) % in the (2s,1d) shell (17<A<40)



Beta-decay and Nuclear Structure: Observables

Mass Originally determine by the Q_β-endpoint

→ Measurement of Q_β $\left\{ \begin{array}{l} \text{Direct measure of } E_{Q\beta} \\ \text{coincidences } \beta.\gamma,\ \beta.n,\ \beta p \end{array} \right\}$ Precision ~ 400 keV

Use of Local Ma

00 keV

Wigner in 1957

that the

members of an

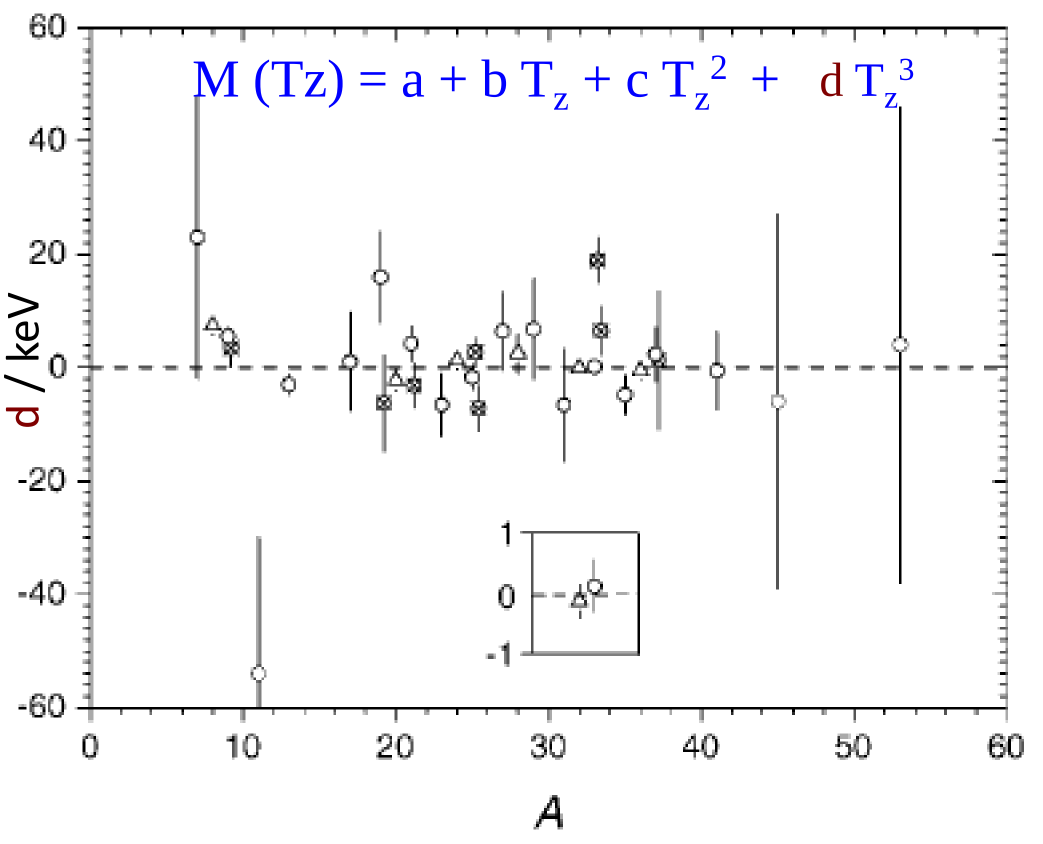
→ IMME

25 (1998)

33, 34, 4

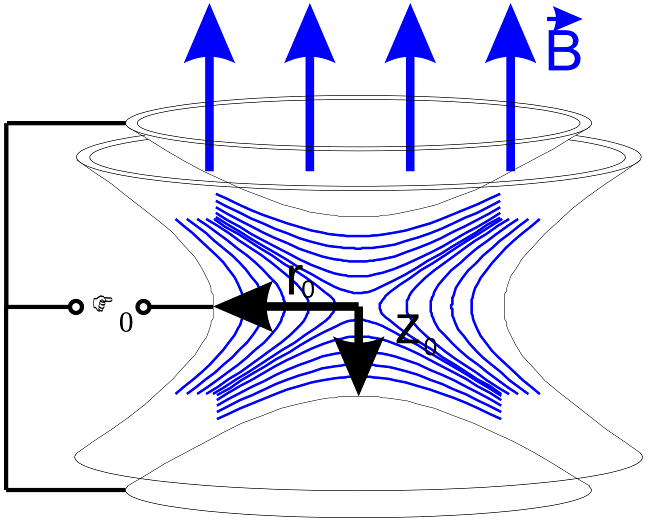
1 (2001)

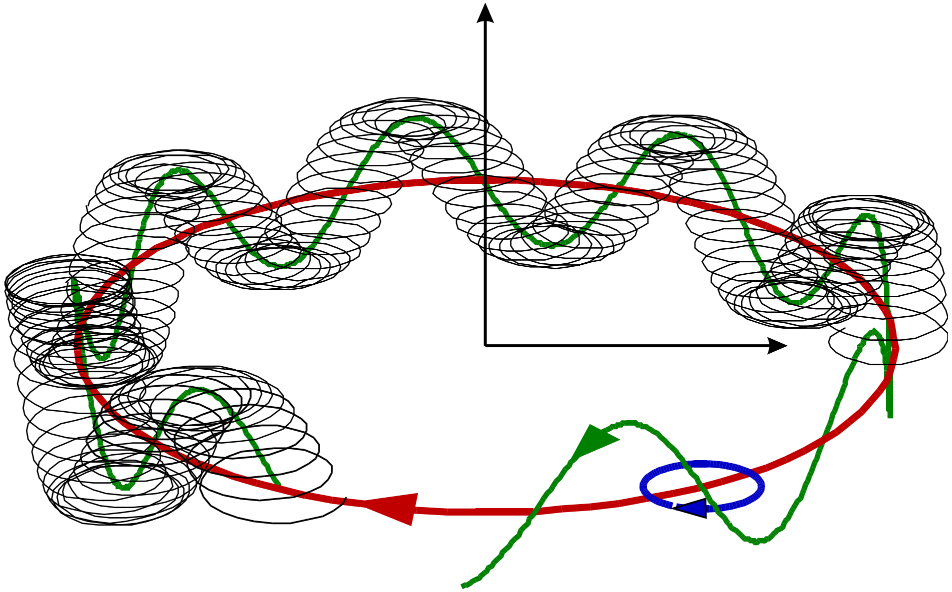
Penning trap
level of 2 ke



K. Blaum et al. PRL 91, 260801 (2003)

Principles of the Penning trap





A Penning trap can be defined as the superposition of a homogeneous magnetic field and an electrostatic quadrupole field.

$$\omega_c = \frac{Q}{m} B$$

Precision of 1 keV even for nuclei of 100 ms $T_{1/2}$

Mass measurements at storage rings

"Recent trends in the determination of nuclear masses" *Review: D. Lunney et al, Rev. Mod. Phys. 75, 1021 (2003)*

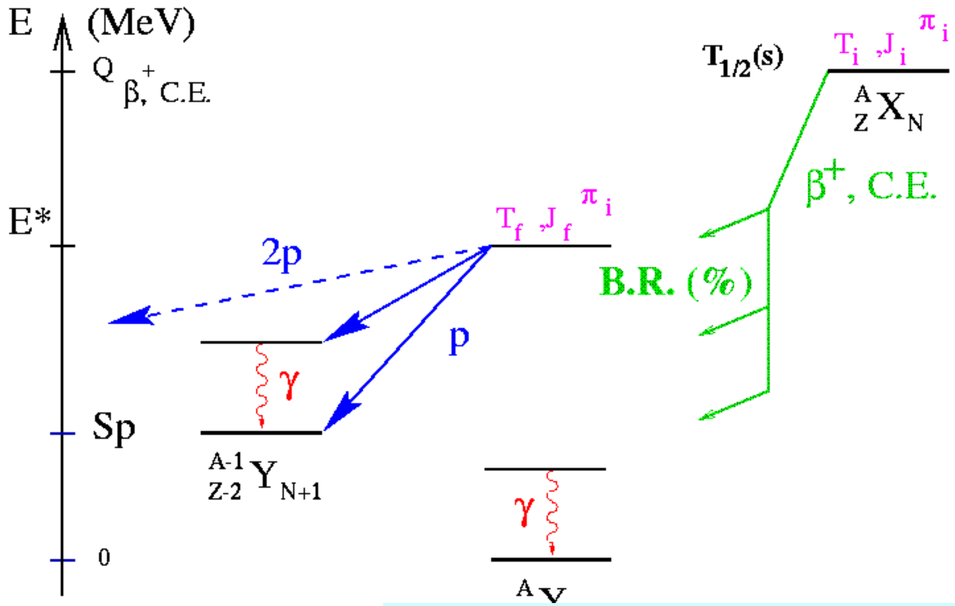
Decay properties of exotic nuclei

- **1916** **Rutherford & Wood** **βα** [*Philos. Mag.* ***31*** *(1916) 379*]
- **1963** **Barton & Bell** identified ^{25}Si as **βp**

Global properties

- *Short half-lives (~ms)*
- *High Q_β values*
- *Low $S_{p/n}$ values*

→ β-delayed particle emission



Very Selective probe

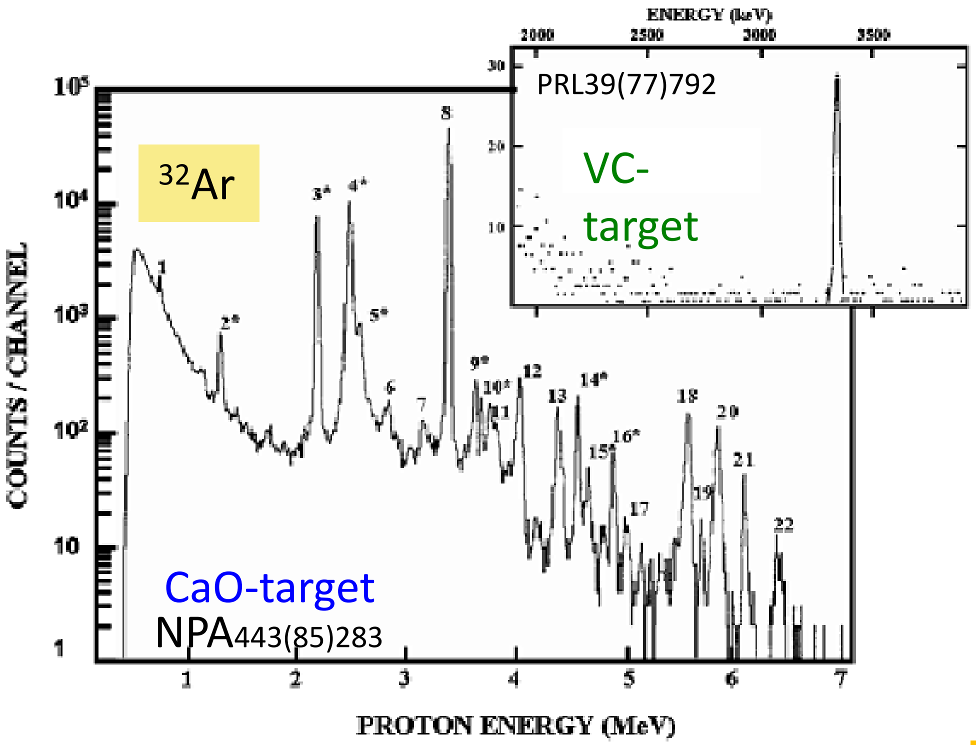
- *Reduced transition probability:*

$$ft = f * \frac{T_{1/2}}{B.R.} = \frac{K}{G_V^2|\tau|^2 + G_A^2|\sigma\tau|^2} = \frac{C}{B(F) + B(GT)}$$

- Particle energy spectrum determined by 2 factors
  - 1-intensity of β-decay branches from precursor to the emitter
  - 2-probability of emission by proton rather gamma

Beta-proton emitters

- ✓ More than 160 precursors identified
- ✓ For every element up to Z = 73 at least one proton precursor
- ✓ The βp spectrum depends on the Z and A of the precursor and differs in the different mass region due to diferences in level density in the Q-Sp window
- ✓ Properties of βp well understood → large variety of spectroscopic information



^{32}Ar

PRL39(77)792

VC-target

CaO-target

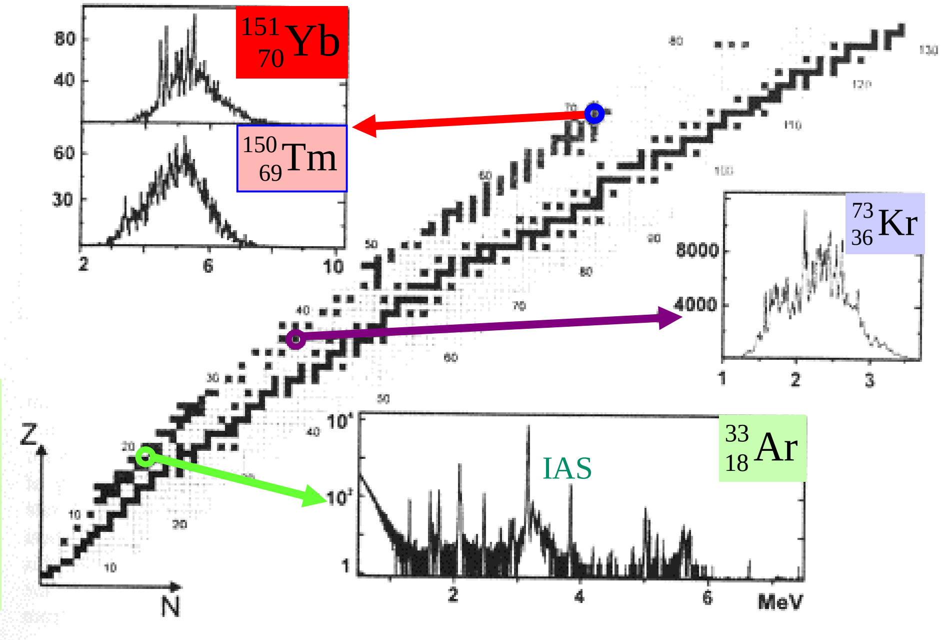
NPA443(85)283

- ✓For light nuclei with Z ≥ 8, the IAS within the Qec window.
- ✓ From βp energy of IAS ➔Q_{EC}-Sp deduced.
- ✓Test Isobaric Multiplet Mass eq. $M(A,T,T_z) = a + bT_z + cT_z^2 + \delta(dT_z^3 + eT_z^4)$
- ✓ If strength to IAS ≠ B_F ⇔ Isospin Mixing
- ✓If IAS in the middle of the Q_{EC} large part of the GTGR available => quenching factor deduced
- ✓Test of Mirror Symmtry

Beta Delayed Proton Emission (TODAY)

Today more than 134 precursor known

- Properties well understood
- This spectroscopic tool is often the only way to identify exotic nuclei
- Data provide large spectroscopic information
  - Level density
  - Spin, isospin
  - Width & density
  - β-decay properties



- In ${}^{33}\text{Ar}$ ⇒ low level density, spectrum marked for proton peaks
  Cut off at low energy at the Coulomb barrier
  IAS (only in precursors with $T_Z \leq -3/2$)

- In the rest bellshape spectrum with superimpose peak structure
  ⇒ no individual transition rather cluster of them atributed to Porter-Thomas fluctuations

- Notice differences
  - ${}^{150}_{69}\text{Tm}_{81}$ Emitter even-even Q_{EC} and B_p large ⇒ populate high excited states
    ⇒ rather smooth spectrum
  - ${}^{151}_{70}\text{Yb}$ Emitter even-odd B_p low ⇒ proton emitted from low states
    ⇒ fluctuations more pronunced

^{31}Ar @ the dripline: 18p + 13n

Unique Spectroscopic Information

$Q_{2p} \Rightarrow E_{IAS}$ = 12322(2)(50) keV

$Q_{EC} = E_{IAS} + \Delta Ec - \Delta np$

ΔEc =7045 keV

Q_{EC} = 18490(110) keV

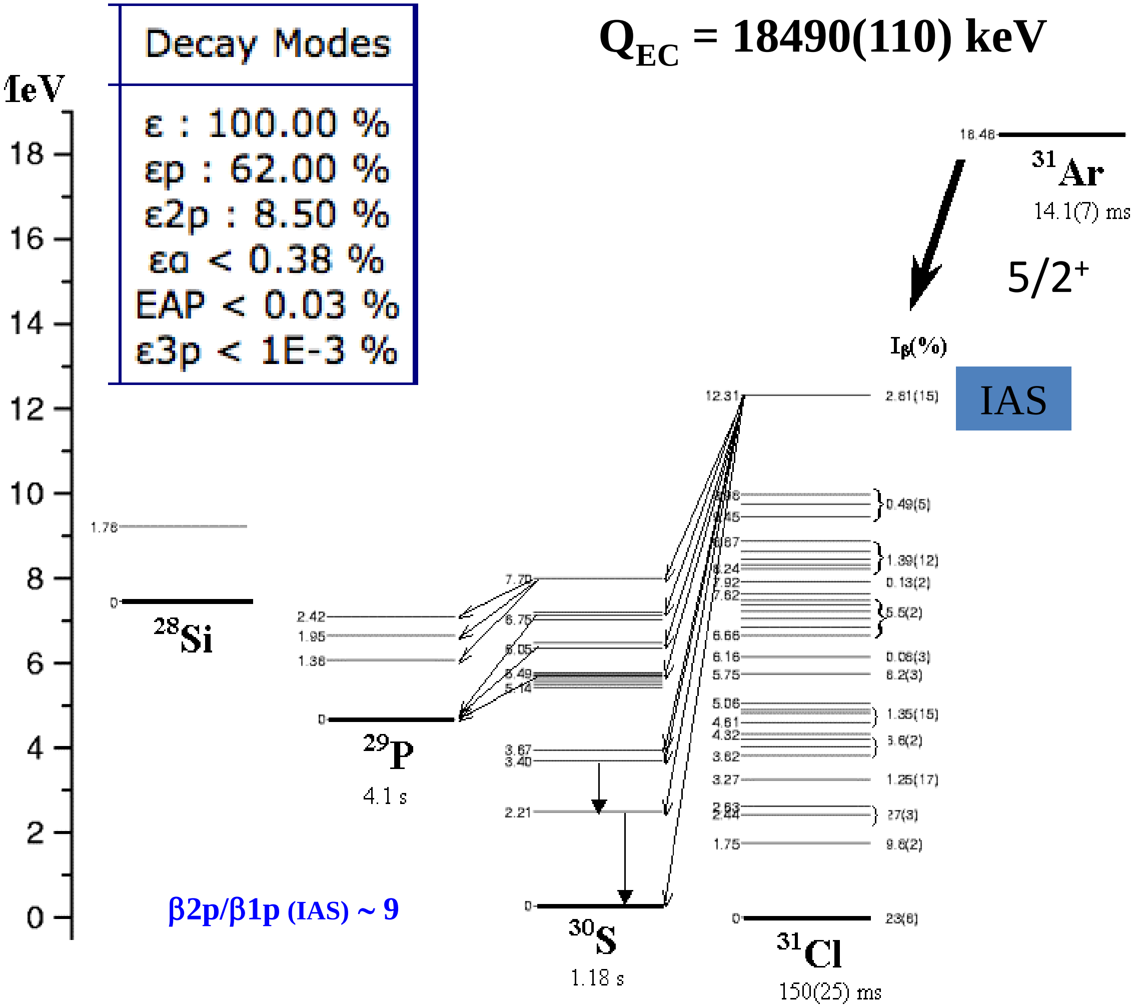
$f(E_{\beta IAS})t_{IAS}$ = 6145(4) s / [B(F) + B(GT)]

b.r.(IAS) = $T_{1/2} / t_{IAS}$

$B(F) = [T(T+1)-T_{zi}T_{zf}]\delta_{if} = 5$

Expected b.r. (IAS) = 4.35(31)%

Experimentally: b.r. (IAS) = 4.25(30) %



Decay Modes

ε : 100.00 %
εp : 62.00 %
ε2p : 8.50 %
εα < 0.38 %
EAP < 0.03 %
ε3p < 1E-3 %

Q_{EC} = 18490(110) keV

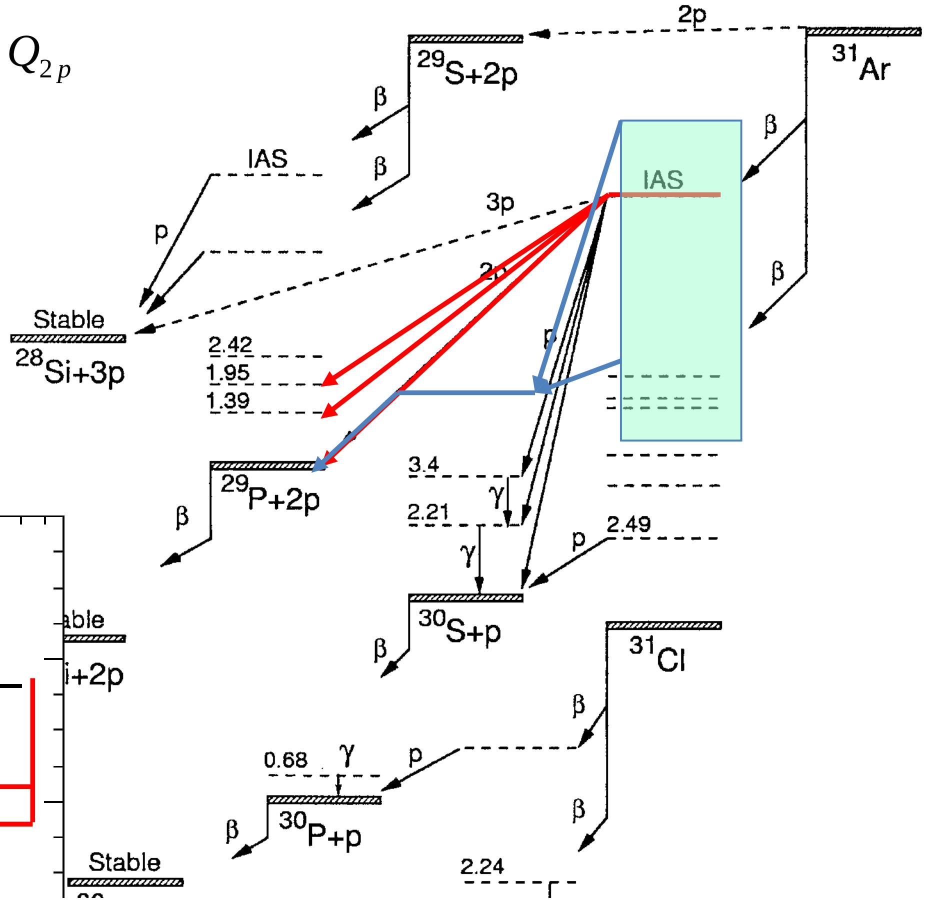
2p emission from ^{31}Ar IAS

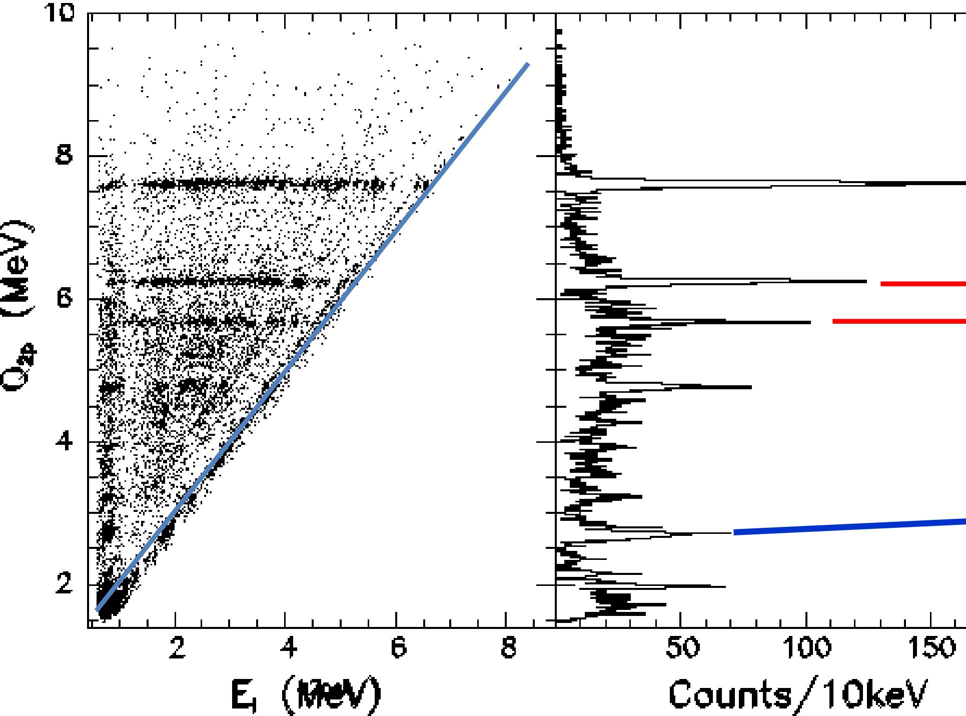
a) Energy Conservation $$\frac{\vec{P}_1^{\,2}}{2m_P}+\frac{\vec{P}_2^{\,2}}{2m_P}+\frac{\vec{P}_r^{\,2}}{2m_r}=Q_{2p}$$

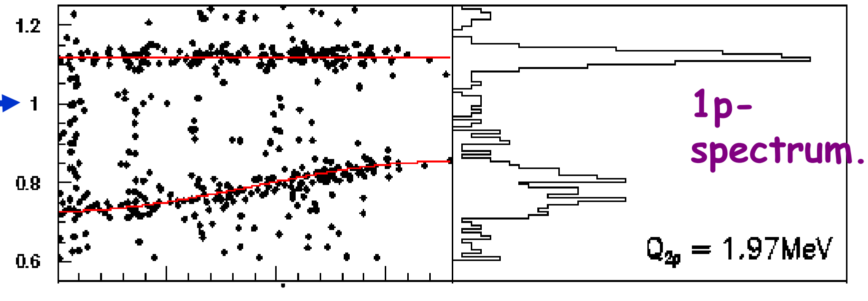
b) Momentum Conservation $$\vec{P}_1^{\,2}+\vec{P}_2^{\,2}+\vec{P}_r^{\,2}=0$$

$$E_1=\frac{M_{D1}}{M_{D1}+m_p}Q_1$$

$$Q_{2p}=E_1+E_2+\frac{m_p}{m_r}\left(E_1+E_2+2\sqrt{E_1E_2}\cos\theta_{2p}\right)$$

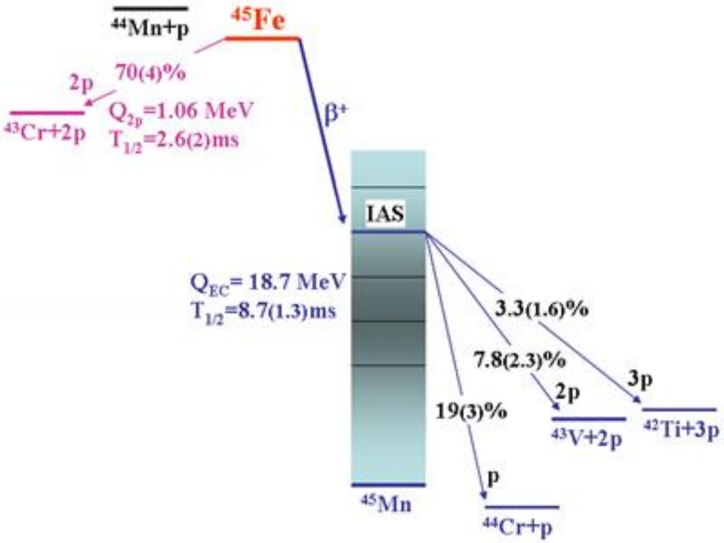




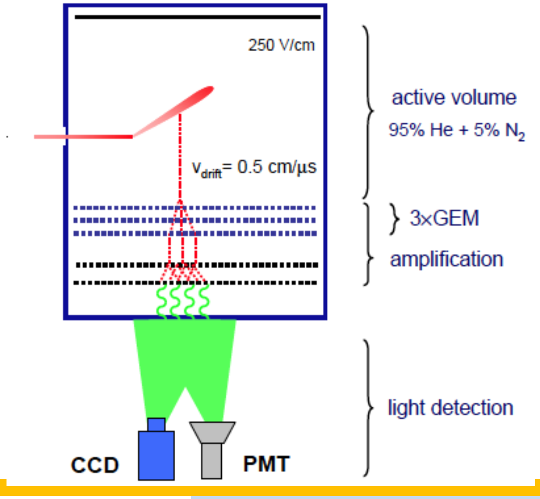


1p-spectrum.

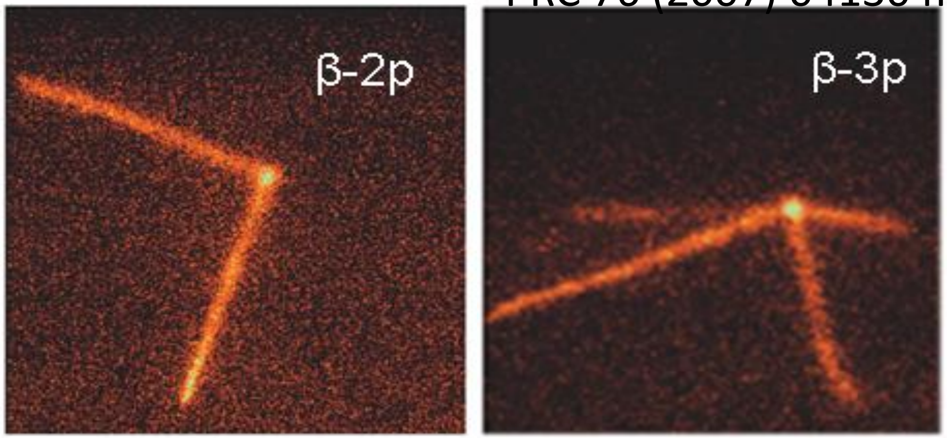
β-delayed 3p-emitters



Decay mode search for in ^{31}Ar where the Q3p is around 4.8 MeV



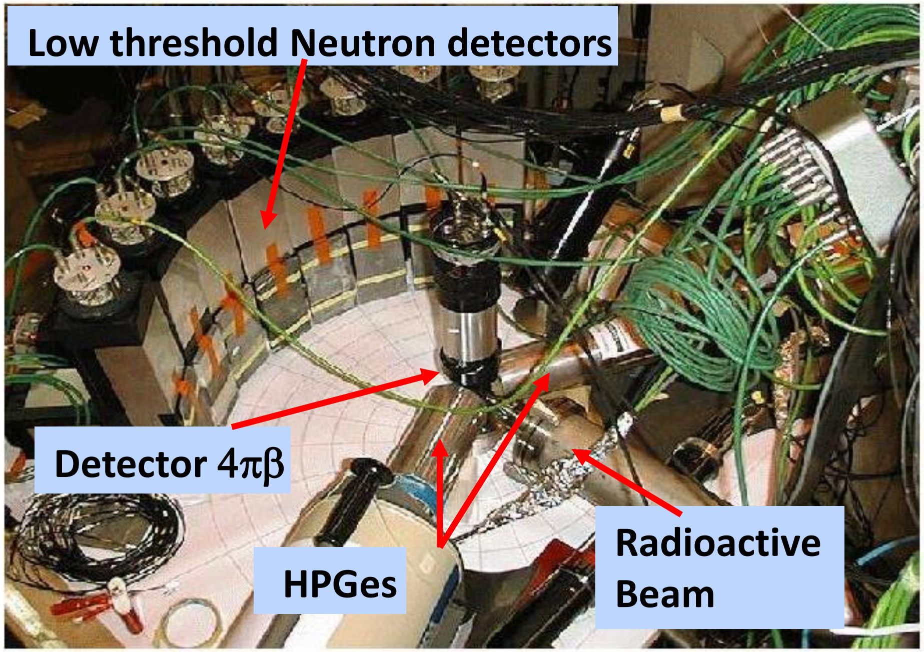
PRC 76 (2007) 041304R

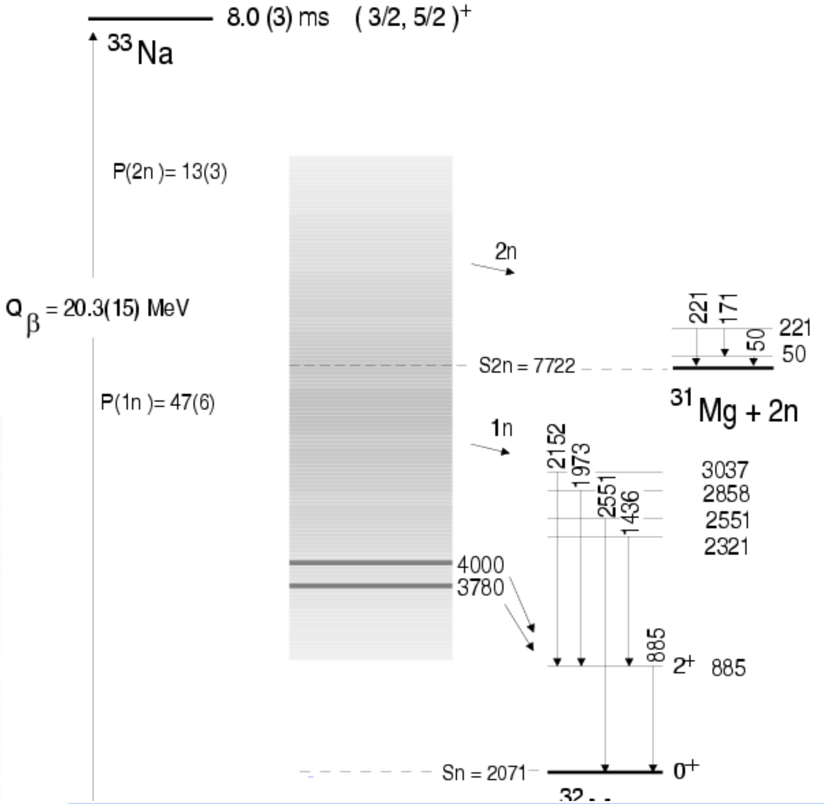


Decay Scheme → Structure Information (N= 20)

^{33}Na

ISOLDE
fragmentation U (46g/cm^2) 2000°
1,4 GeV protons 3 10^{13} / pulse (1,2s) ^{33}Na 2 at / s





^{33}Na $T_{1/2}$ = 8.0 (3) ms

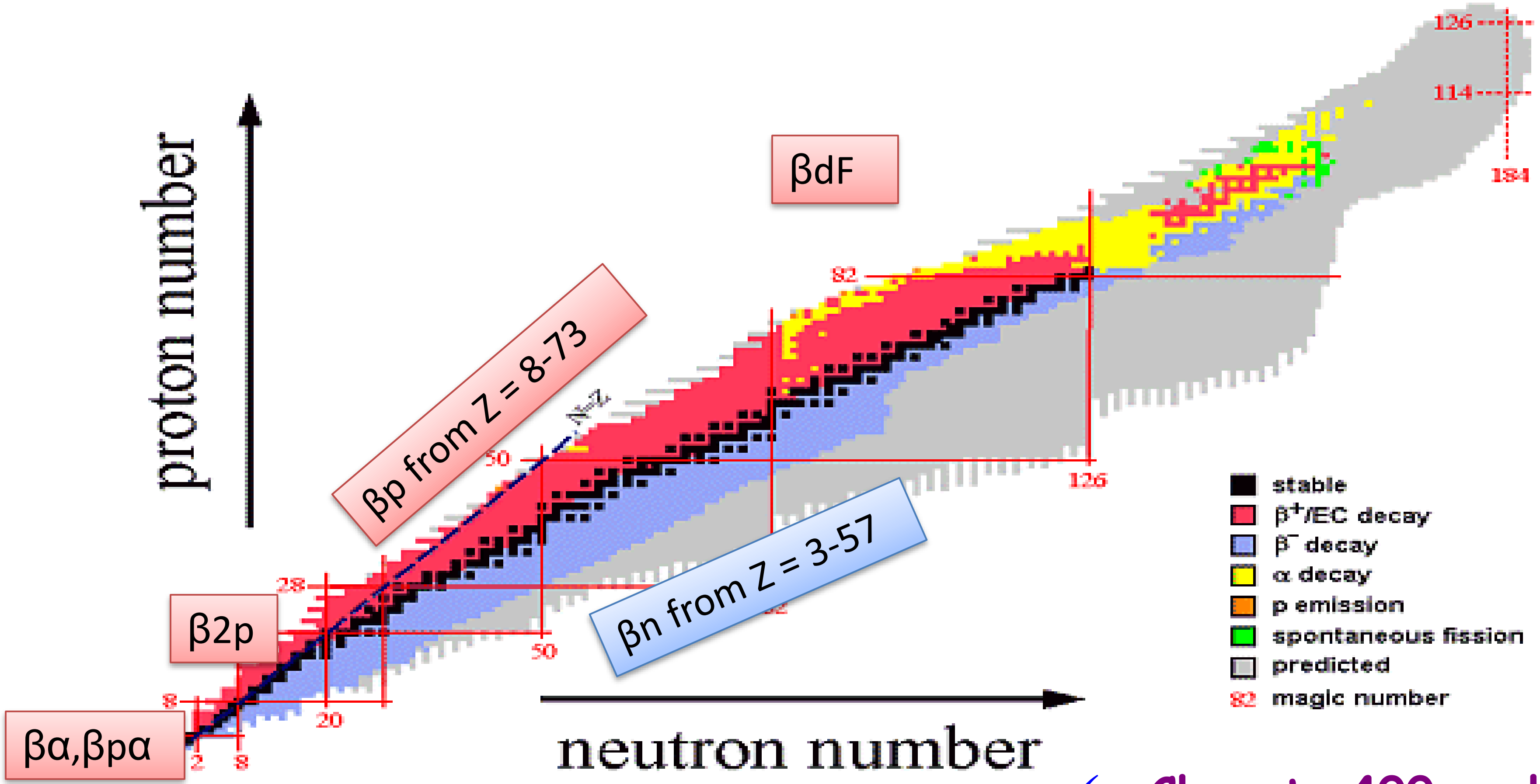
Detailed Level Scheme

inversion of $3/2^+$ $7/2^-$ orbits in ^{33}Mg

exp. : coinc. β neutrons β.γ.n

M. Langevin et al NP A414 151 (1984)
S. Nummela et al PRC64 054313 (2001)

Nuclear Landscape



βdF

βp from Z = 8-73

βn from Z = 3-57

β2p

βα,βpα

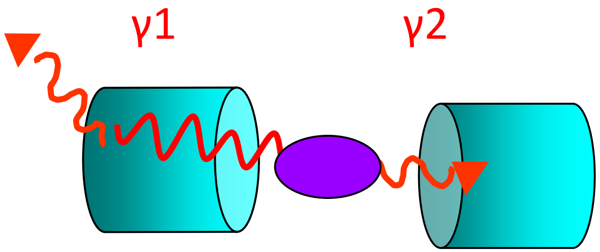
βd, βt

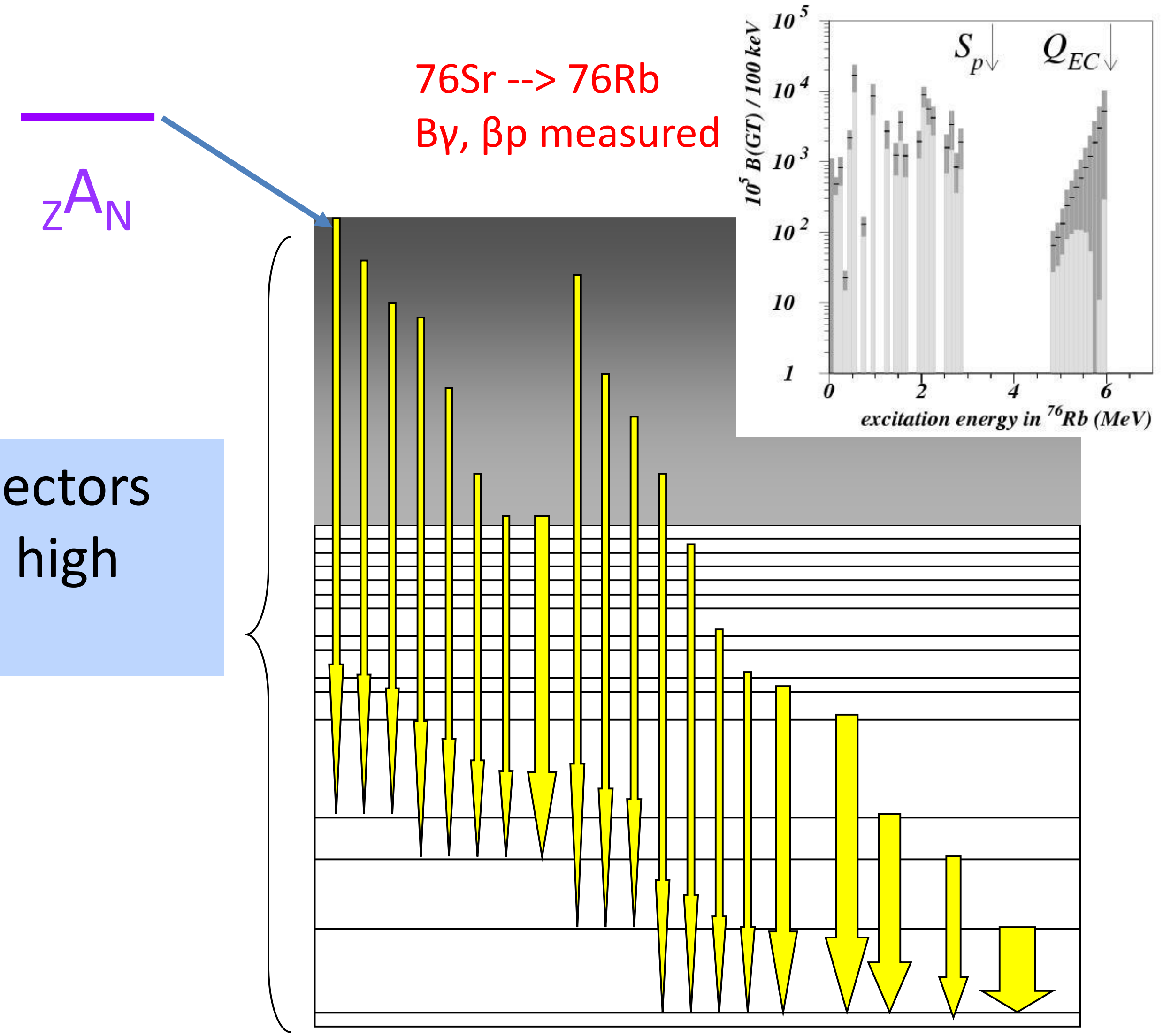
Near the drip lines ➡

- ✓ Close to 400 nuclei emit delayed particles
- ✓ Exotic decays

Beta-decay Limitations: beta feeding

Traditionally

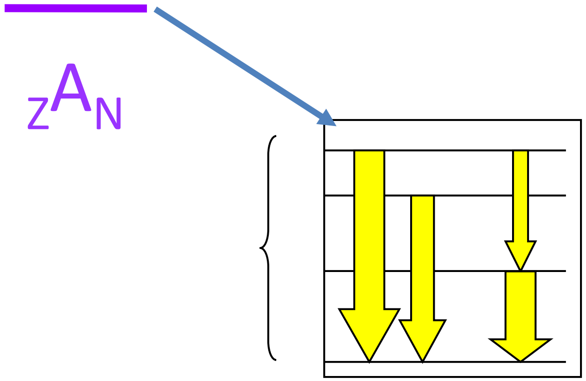




76Sr --> 76Rb

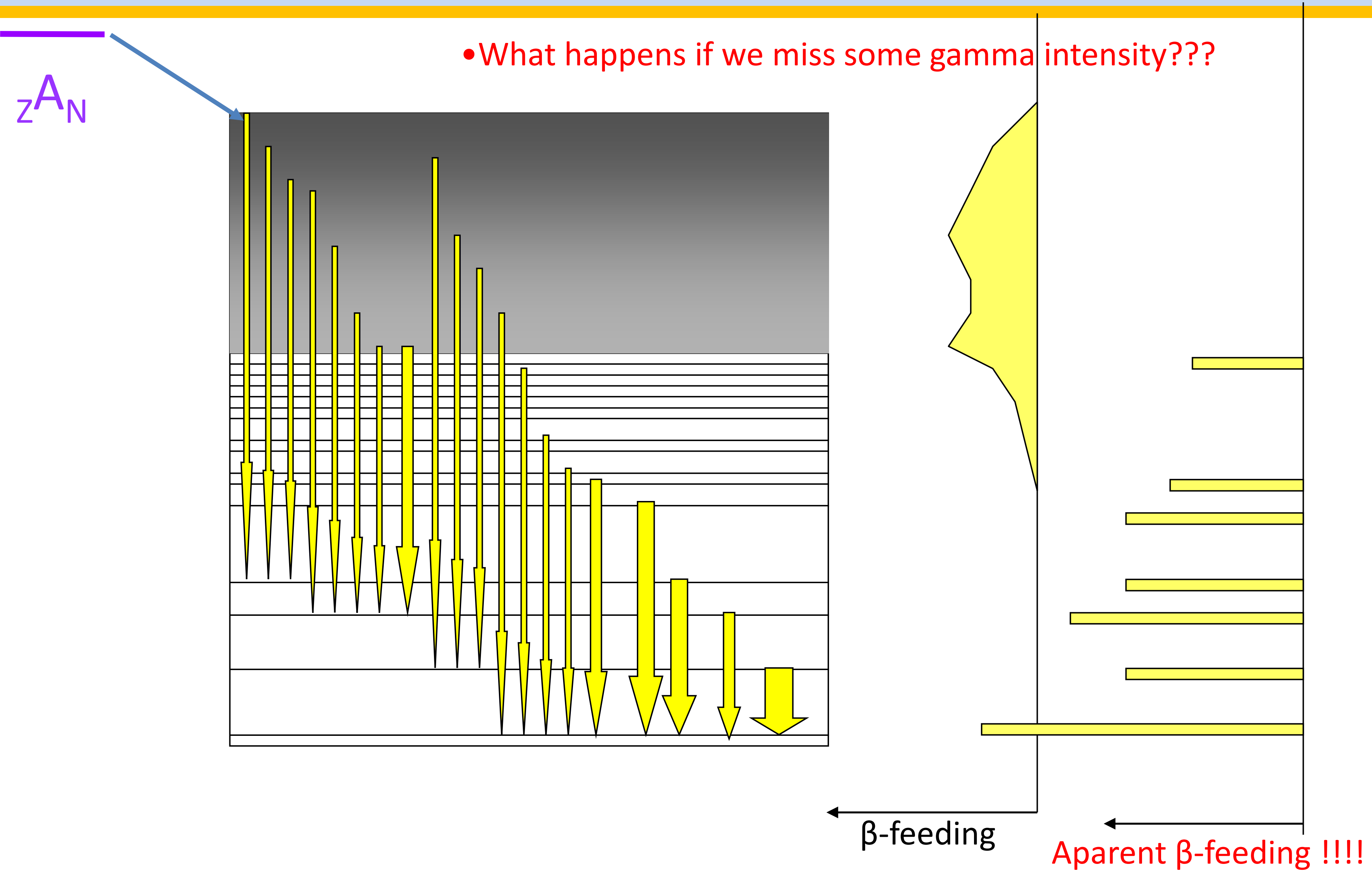
Bγ, βp measured

For high Q-values, Ge detectors fail to detect β-feeding at high excitation energy!!!

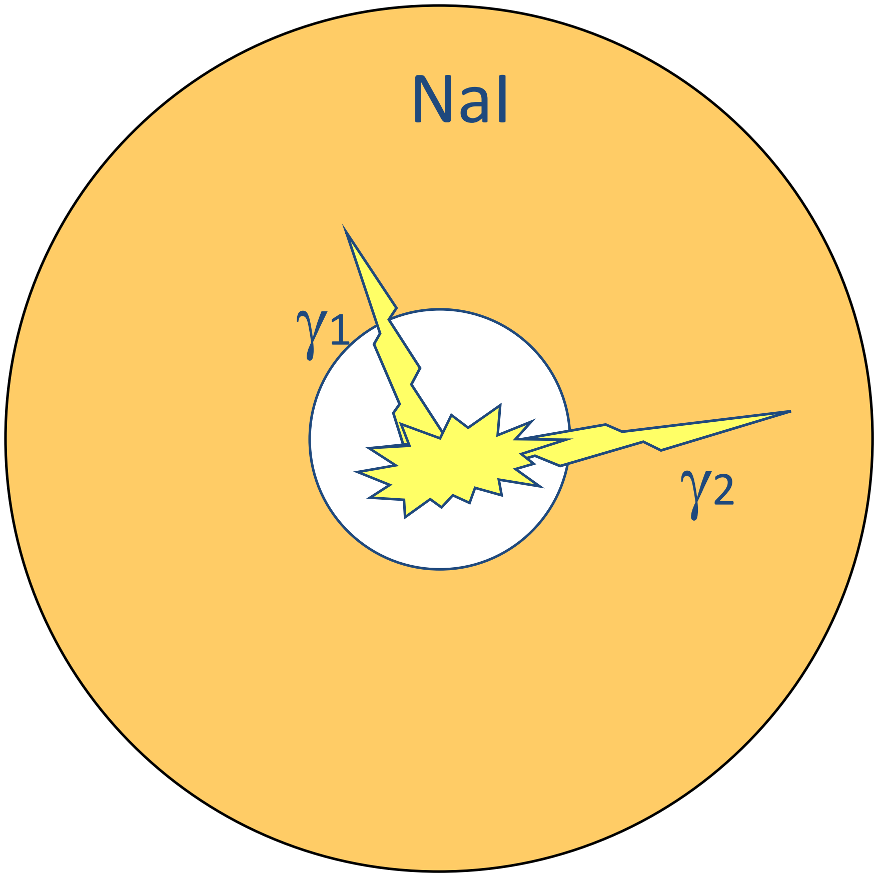


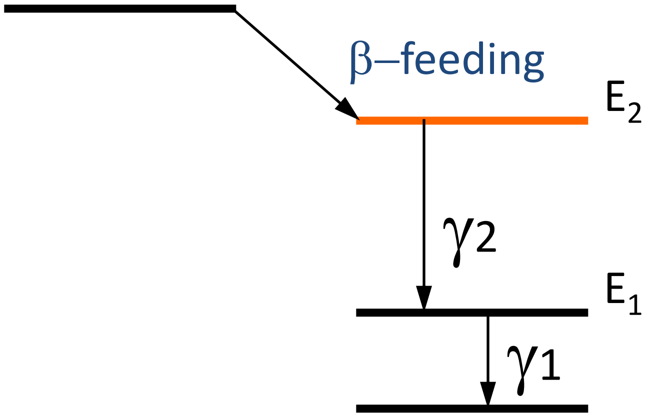
- We use Ge detectors to construct the decay scheme
  - From the γ-balance we extract the β –feeding

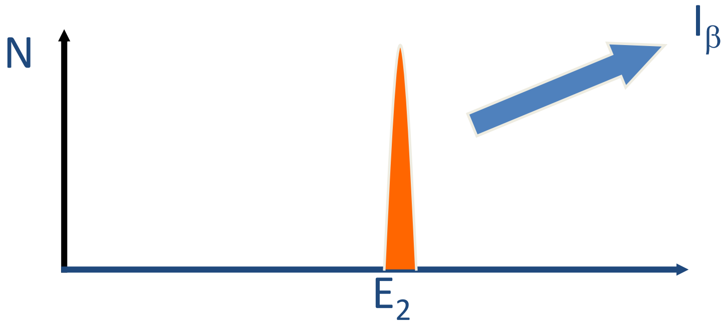
- What happens if we miss some gamma intensity???



Total Absorption spectroscopy





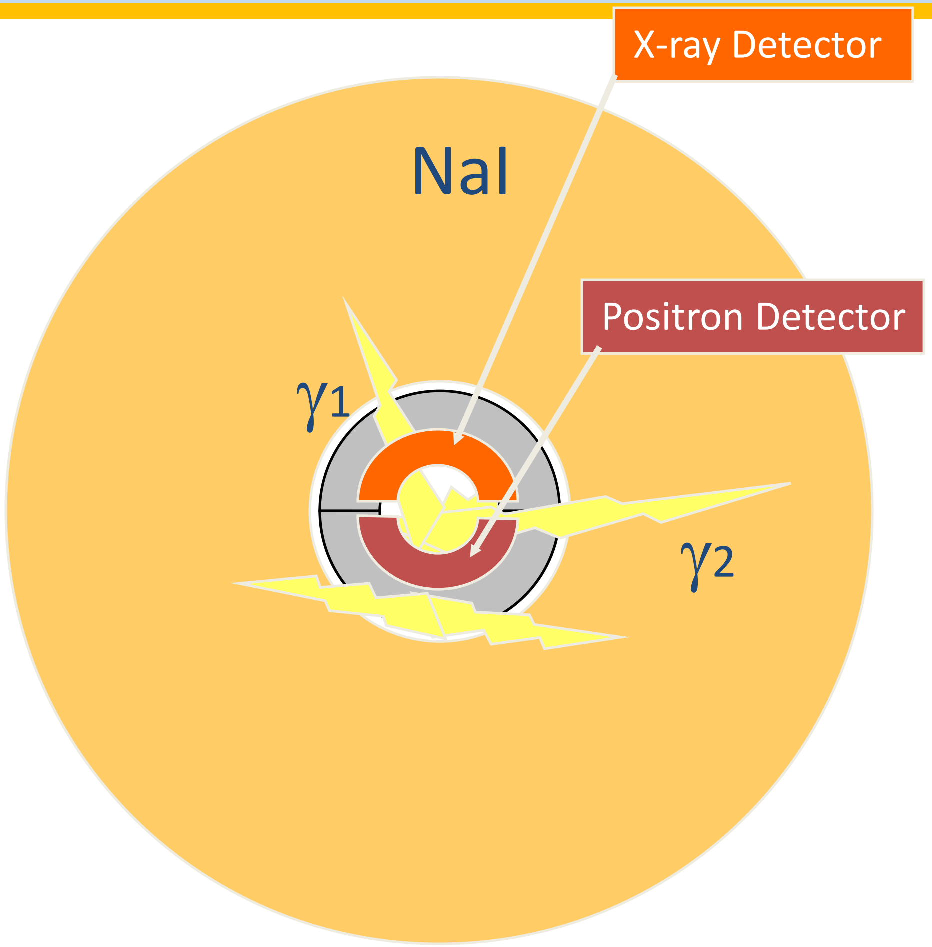


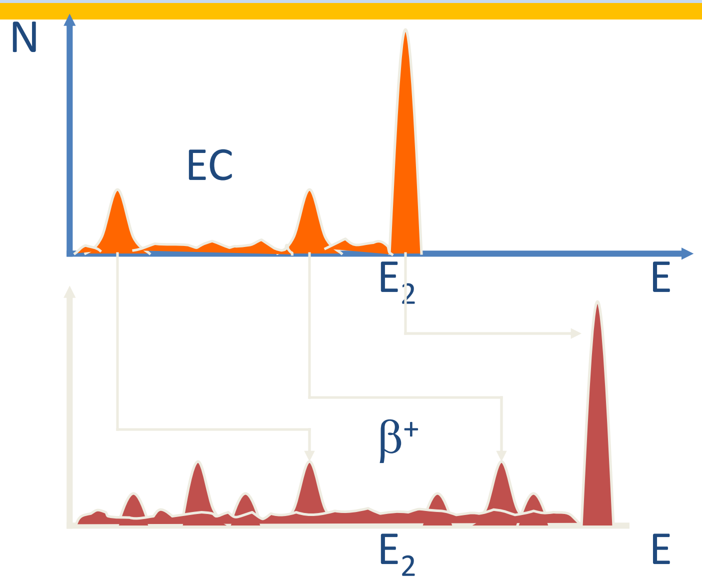
Ex in the daughter

Ideal case

By B. Rubio

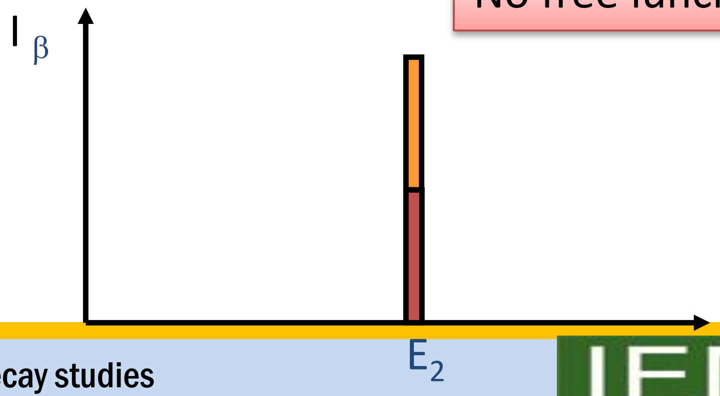
Total absorption spectroscopy





After Deconvolution and sum

No free lunch!!



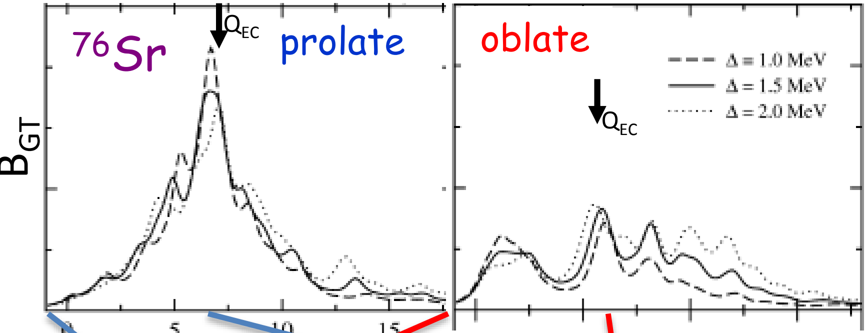
Real case

By B. Rubio

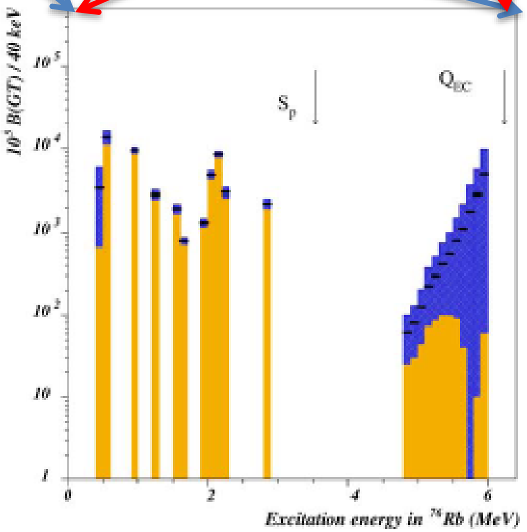
Deformation in the region N ~Z with 70 < A < 80

High resolution measurements: β,βγ,βp

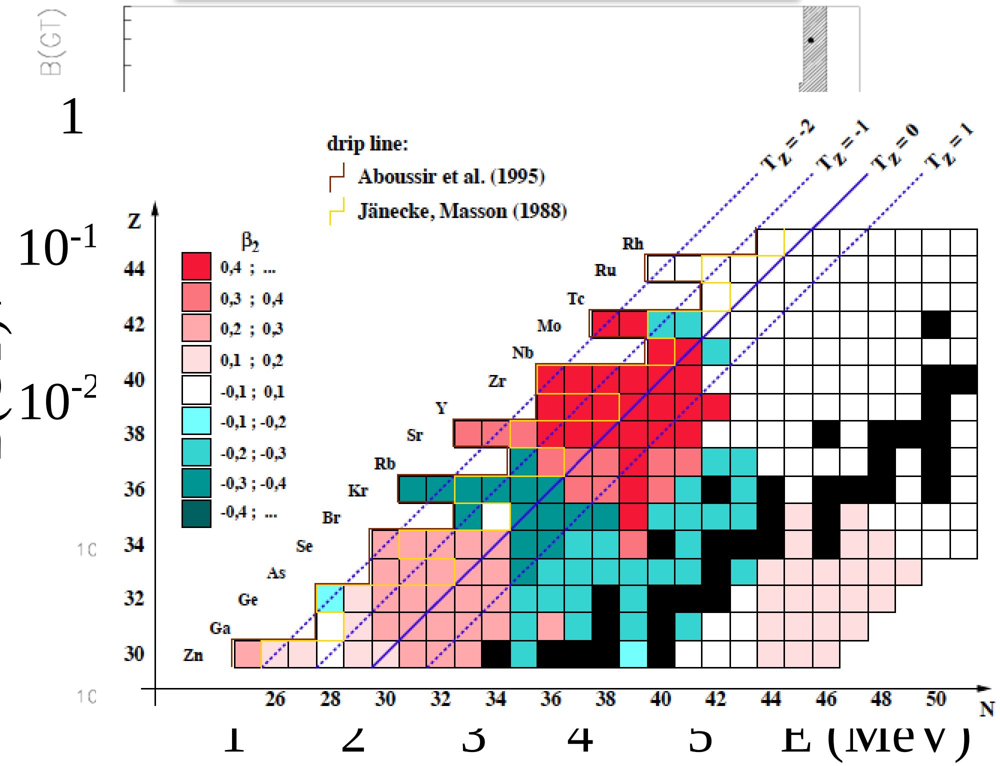
$^{76}\text{Sr} \xrightarrow{\beta^{+}\text{EC}} {}^{76}\text{Rb}$



Sarriguren NPA 691(2001)631



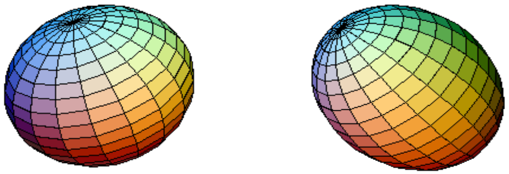
After the TAgS measurements

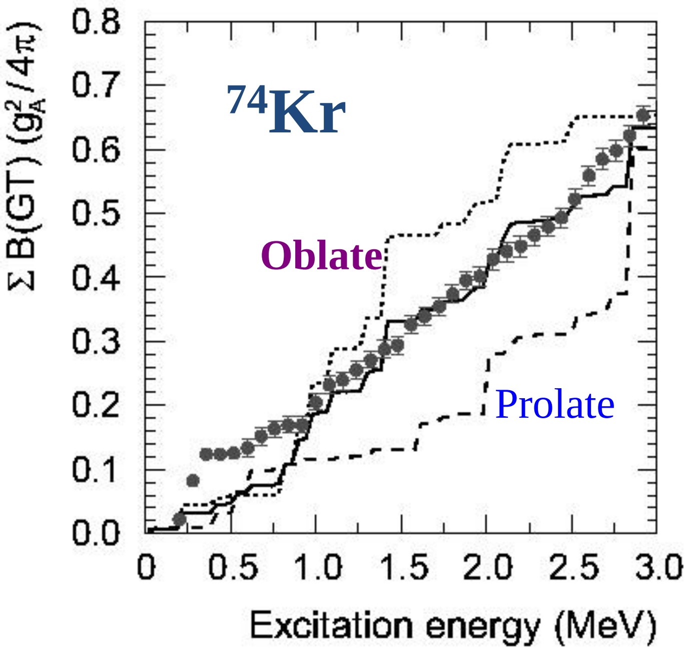


Mass ~70 : Strong Deformation & Shape Coexistence

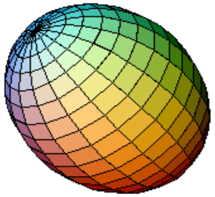
^{74}Kr, shape admixture

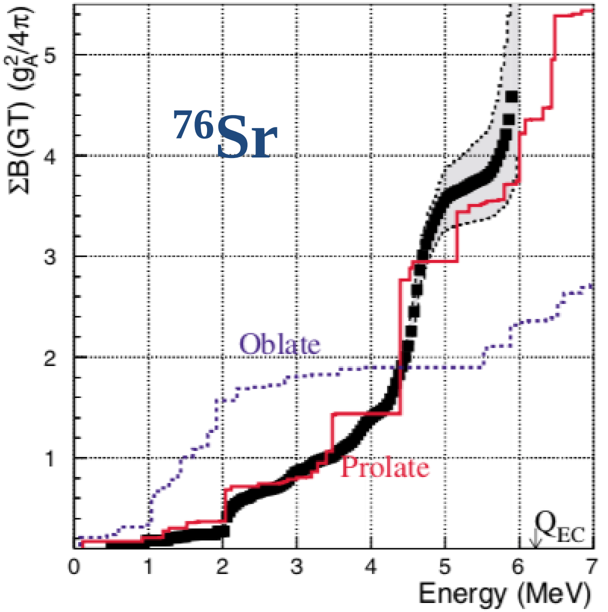
^{76}Sr clearly prolate





Poirier et al., PRC 69 (2004) 034307

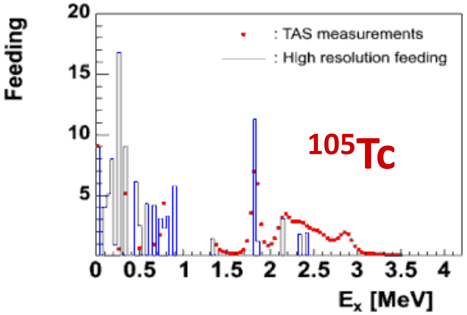




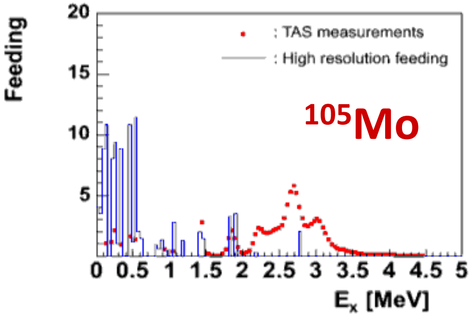
Nácher et al., PRL 92 (2004) 232501

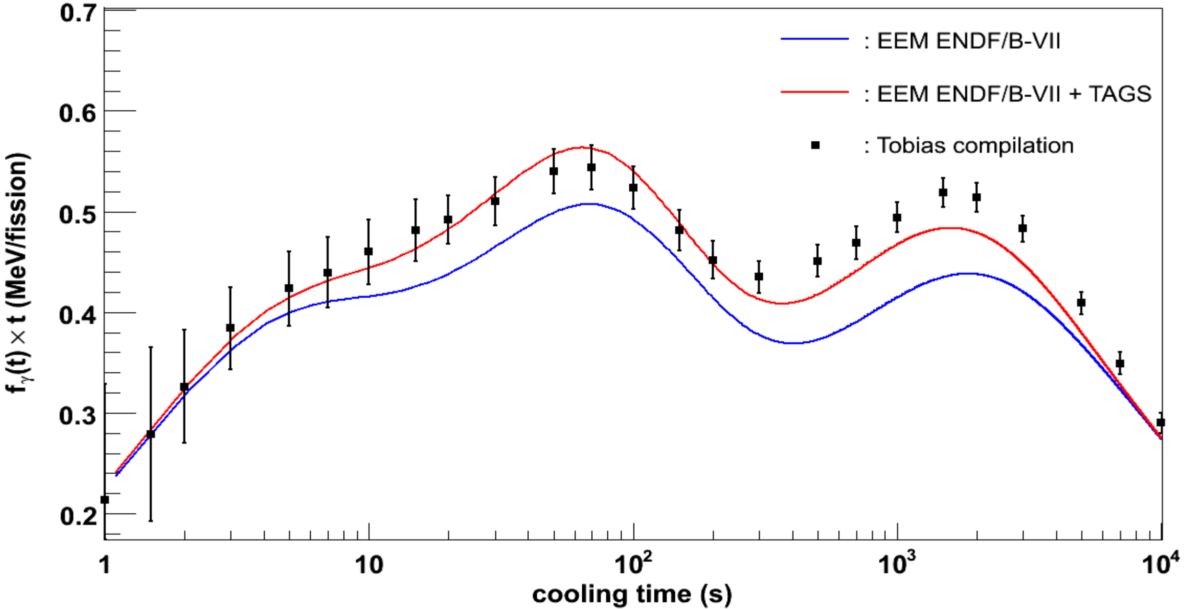
New results on Reactor Decay Heat discrepancies

- Experiment at IGISOL-JYFL (Jyvaskyla), A. Algora et al. Phys. Rev. Lett 105(2010) 202501
- Total Absorption Gamma-ray Spectroscopy (TAGS) technique: IFIC & CIEMAT
- First use of a Penning Trap with TAGS to purify samples



- The new data on the decay of Mo, Tc and Nb isotopes helps to solve a large fraction of the discrepancy between calculated and measured decay heat



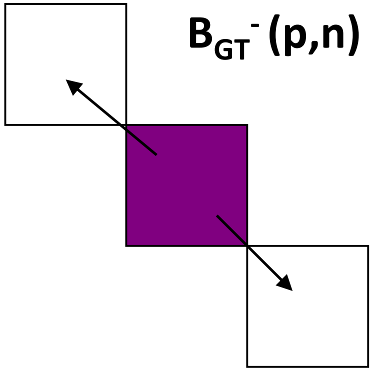


Charge exchange reactions ↔ Beta decay process

Beta decay and Charge Exchange are two processes governed by the same **στ (τ)** operator

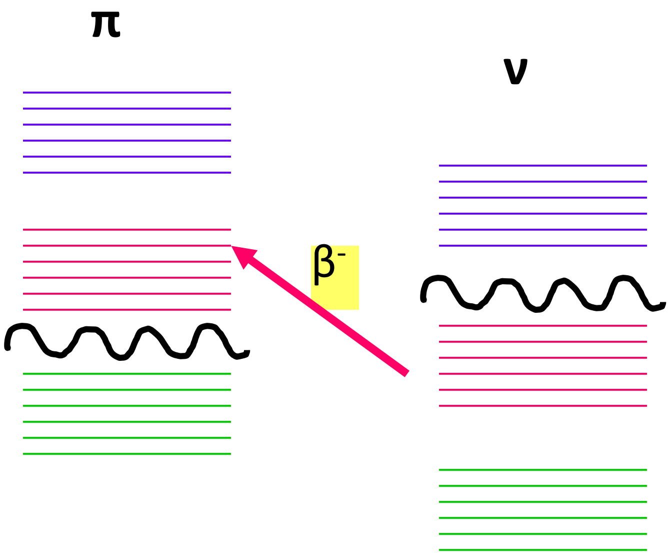
The Ikeda sum rule: Independent

$$\mathbf{S^- - S^+ = B_{GT}^- - B_{GT}^+ = 3(N-Z)}$$



B_{GT}^- (p,n)

B_{GT}^+(n,p)



In principle β⁻ decay is more interesting because most of the nuclei have more neutrons than protons, and then most of the Ikeda sum rule is in the β⁻ side.

The "experimental B_{GT}" is obtained from the reaction cross section, with all the problems and ambiguities associated (back ground, L transfer, target, current normalisation, detector efficiency….)

Beta decay : Advantages & disadvantages

- Mechanism under control
- No background ambiguities
- No normalisation ambiguities
- β^+ or β^- given by nature, β^- almost always bigger than β^+
- Q_β given by nature limiting the states that can be populated
- The further from stability the bigger the Q_β window
- At some moment β delayed protons and β delayed neutrons set in

Charge exchange reactions: (p,n), (^{3}He,t)

Decay: Excitation energy range limited→ Q-window limitation

(p,n) reaction at intermediate energies (E = 100 – 500) MeV/u
“proportionality “ of B(GT) and cross section at 0º

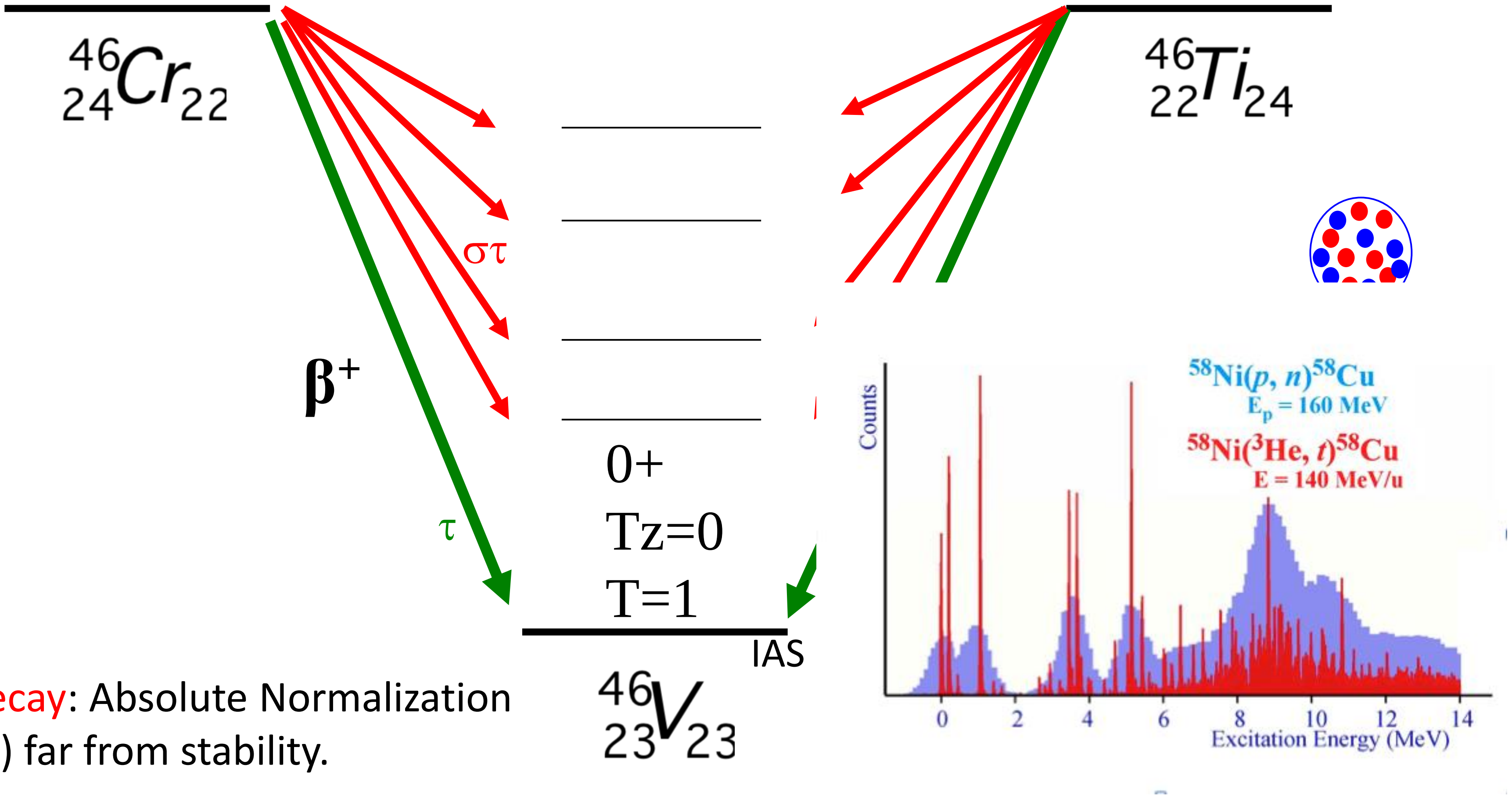
$$\boxed{\sigma\ (0º) = K N\sigma\tau\ |\ J\sigma\tau\ (0º)\ |^2\ B(GT)}$$

Breakthrough againts “Q-window-limitation”
But poor resolution (E = 200-400 keV)

(3He,t) reactions at intermediate energies (E = 130-150 MeV/u)
“high resolution” (E < 50 keV)
Magnetic spectrometer, matching technique
Good proporcionality (B(GT) > 0.03, observed)

→Breakthrough against “Energy resolution Limitations”
→Reliable B(GT) values for individual transitions

If isospin symmetry holds, mirror nuclei should populate the same states with the same probability, in the daughter nucleus, in the two mirror processes


Tz=-1
T=1

$^{46}_{24}Cr_{22}$

Tz=+1
T=1

$^{46}_{22}Ti_{24}$

$\sigma\tau$

β^+

τ

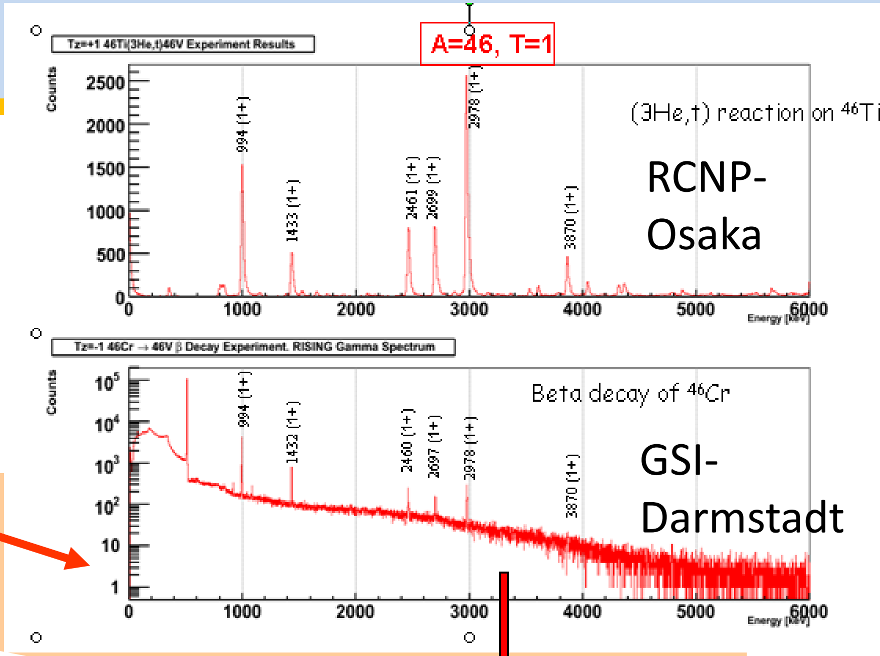
0+
Tz=0
T=1

IAS

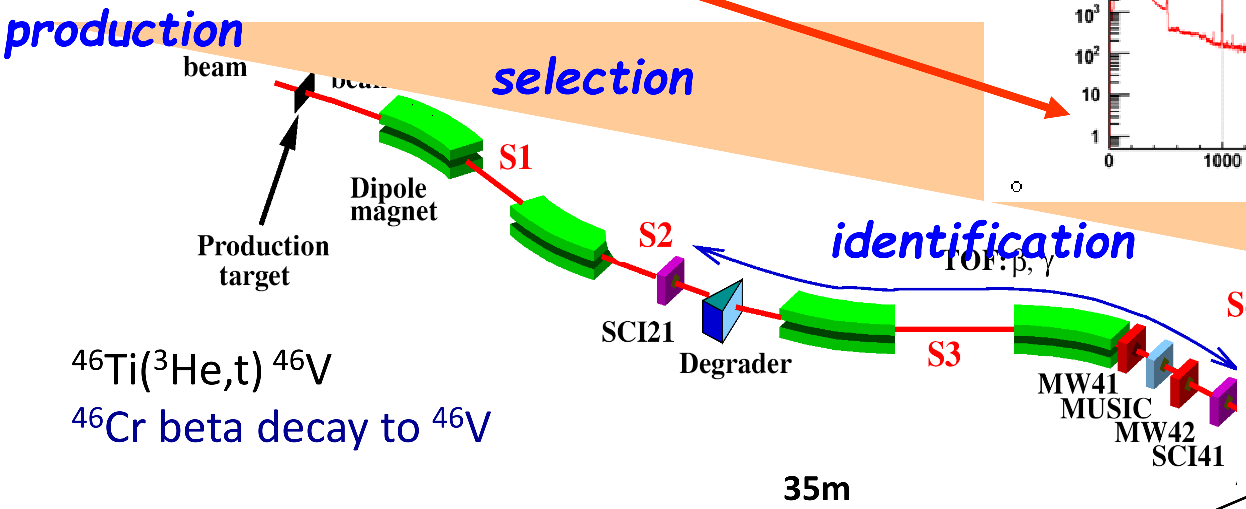
$^{46}_{23}V_{23}$

Beta Decay: Absolute Normalization of B(GT) far from stability.

Nuclear Isospin Symmetry Studies using the Weak and Strong Interactions

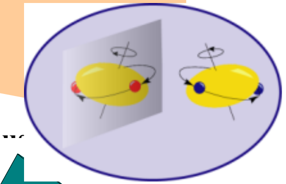


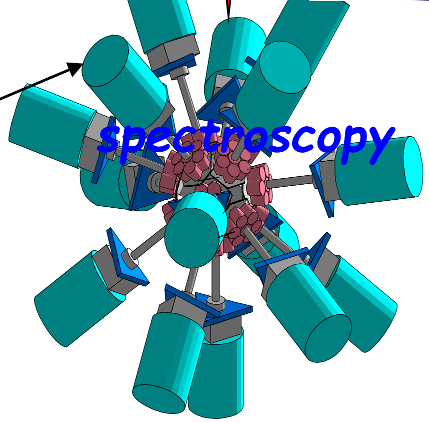
- GSI experiments (towards DESPEC-FAIR), Co-Spokesperson: B. Rubio
- Fragment Separator (FRS) + Ge- Array RISING



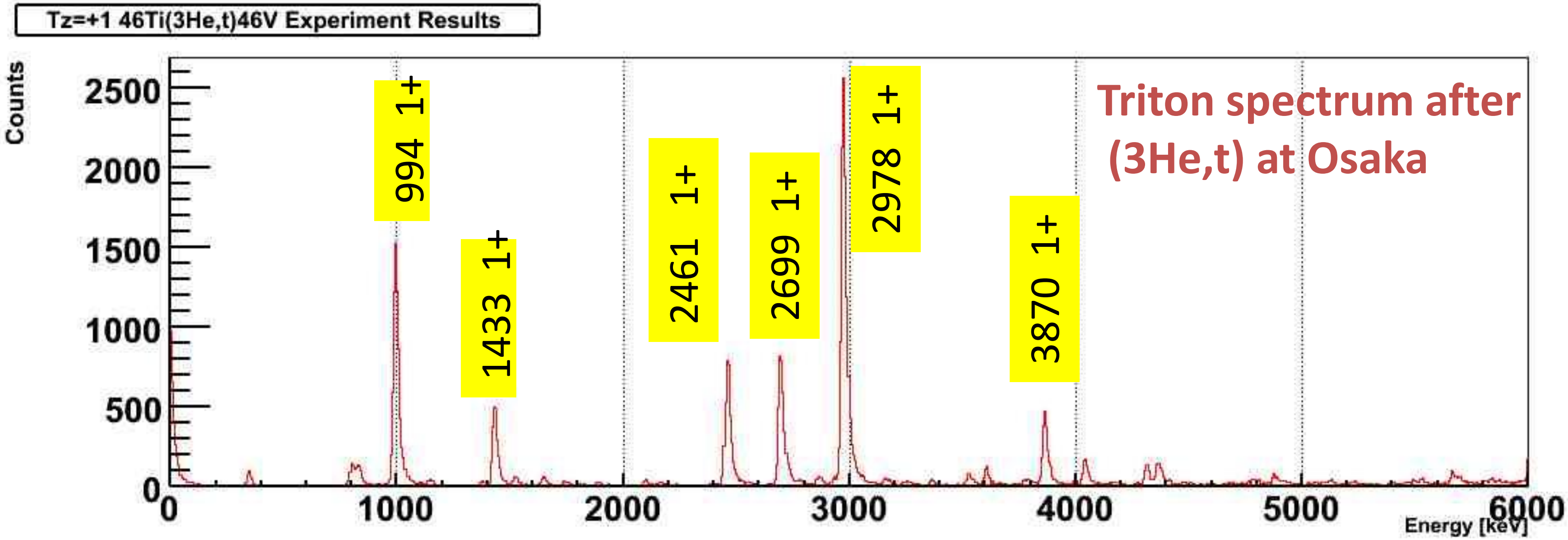
^{46}Ti(^{3}He,t) ^{46}V

^{46}Cr beta decay to ^{46}V



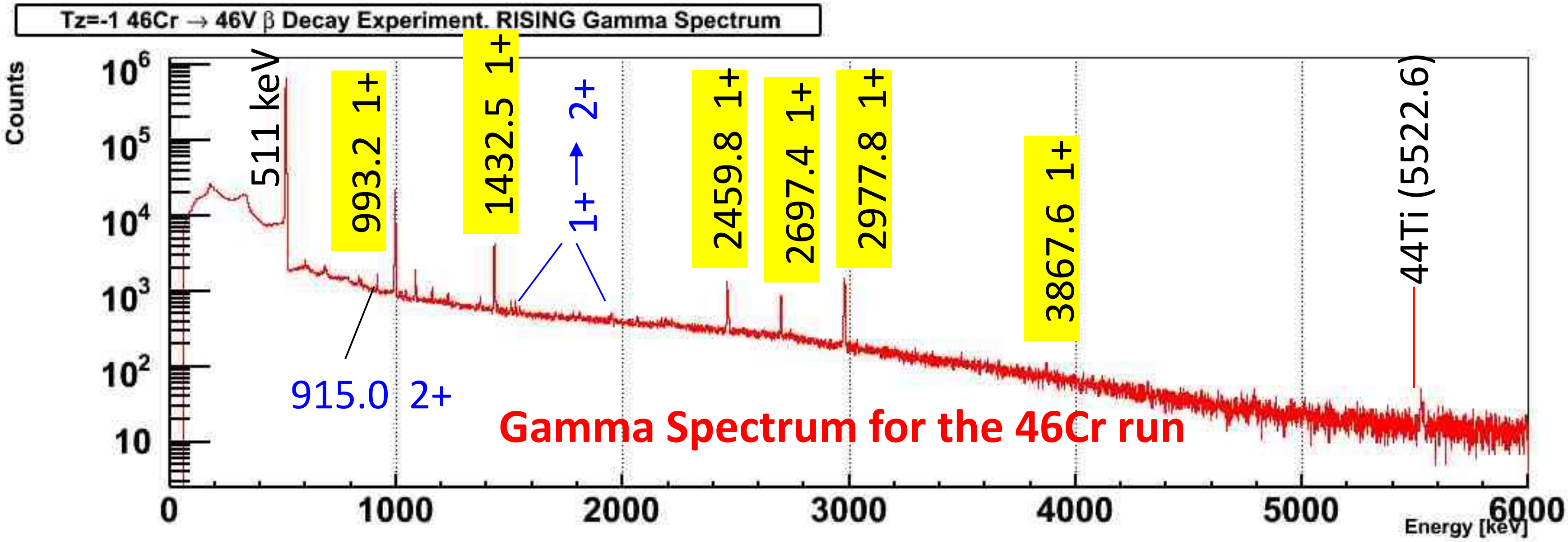


- The states observed in the charge exchange reaction have been also seen for the first time in the beta decay process (mirror transitions)



Tz=+1 46Ti(3He,t)46V Experiment Results

Triton spectrum after (3He,t) at Osaka



Tz=-1 46Cr → 46V β Decay Experiment. RISING Gamma Spectrum

Gamma Spectrum for the 46Cr run

Double- β Decay

Of interest: *Particle Physics*
Nuclear Physics

ββ2ν : Predicted by the Standard Model

$$(Z, A) \dashrightarrow (Z+2, A) + 2\ e^{-} + 2\bar{\nu}$$

S.M. (*E. Caurier et al. PRL 77, 1954 1996*)

QRPA (*J.Engel et al. PRC 37, 731 1988*)

$T_{1/2\ \text{calc.}}$ ~ 0.3 - 1

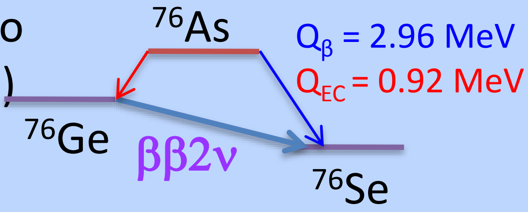
N nda osc

calc

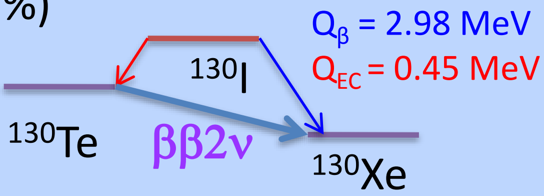
exp

Future in Gran Sasso

GERDA (^{76}Ge, 7.6 %)

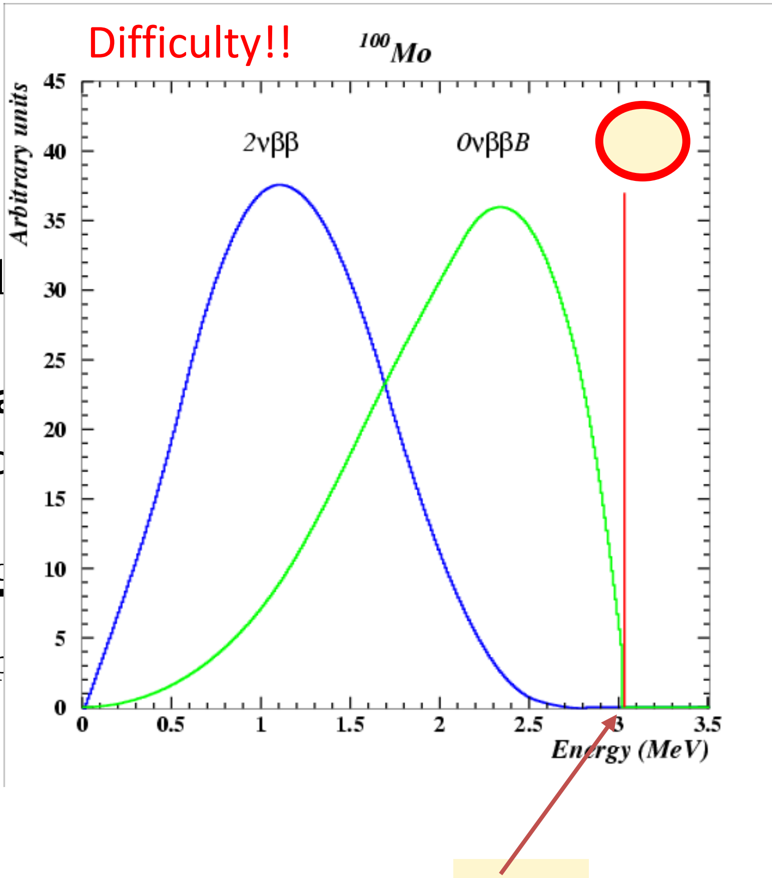


CUORE (^{130}Te, 34.1%)



Super Nemo

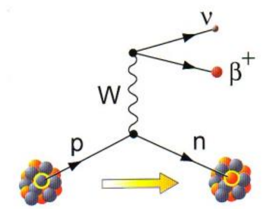
NEXT(^{134,136}Xe(20%) TPC, ββ2ν not yet measured)



$Q_{\beta\beta}$

Superallowed Fermi transitions

For pure Fermi Transition 0+ ➔ 0+



$$f\,(Z, E_b)\,t = K / |M_{if}|^2 = \frac{K}{G_v^{\,2}\,|M_F|^2}$$

B(F) = I M_F I^2 = T (T + 1) - Tz_i Tz_f

Hypothesis of the « Conserved Vector Current »

$$f\,(Z, E_b)\,(1 + \delta_R)\,t\ \ (1 - \delta_C) = \frac{K}{G_v^{\,2}\,(1 + \Delta_R)\,|M_F|^2}$$

Identical for all transitions estimation of G_V

corrections

radiatives	Δ_R (2,5 %)	**Independent of nucleus function of model**
	δ_R (1,5 %)	**Exchange of photons between e$^+$ and nucleus** **Depend of the nucleus**
Isospin impurities	δ_C (0, 2 – 4 %)	**For states with isospin mixing**

A. Sirlin et al., NP B71, 29 (1974)

D.H. Wilkinson et al., NIM A 335, 172 (1993)

W.E. Ormand et al., PRC 52 2455 (1995)

Beta-decay and fundamental interactions

$$F\,t = f\,(Z, E_b)\,t\,(1 + \delta_R)\;(1 - \delta_C) \rightarrow F\,t = \frac{K}{2\,G_v'^{\,2}}$$

Tz = -1 ➔ B(F) = 1.(1+1)-(-1)0) = 2

β-decay ➔ access to the dominant term V_{ud} of the Cabibbo Kobayashi Maskawa (CKM) Matrix

$$\left\{\begin{matrix} d' \\ s' \\ b' \end{matrix}\right\} = \left\{\begin{matrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{matrix}\right\} \left\{\begin{matrix} d \\ s \\ b \end{matrix}\right\}$$

$$G_v'^{\,2} \,/\, G_\mu'^{\,2} = V^2_{ud}\,(1 + \Delta_R)$$

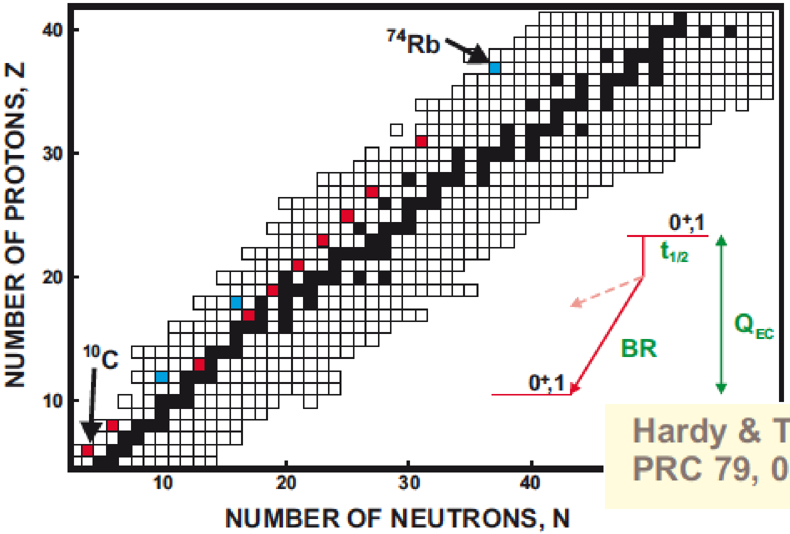
μ–decay

$$V^2_{ud}\,| + |\,V^2_{us}\,| + |\,V^2_{ub}\,| = 1?$$

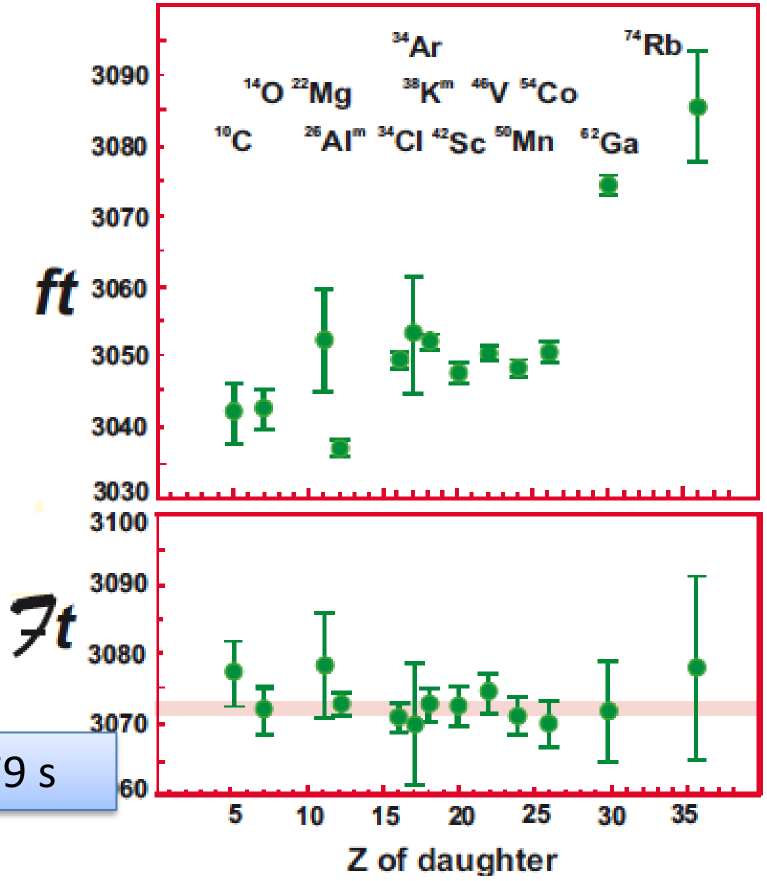
Unitarity of the C K M Matrix ?

D.H. Wilkinson NIM A 488, 654 (2002

World data for $0^+ \rightarrow 0^+$ transitions, 2009



Hardy & Towner
PRC 79, 055502 (2009)



- 10 cases with *ft*-values measured to ~0.1% precision; 3 more cases with <0.3% precision.

- ~150 individual measurements with compatible precision

Ft = 3072.08 ± 0.79 s

$$\mathcal{F}t = ft\,(1+\delta_R')[1-(\delta_C-\delta_{NS})] = \frac{K}{2G_V^2\,(1+\Delta_R)}$$

1) G_V constant ✓ verified to ± 0.013%

2) $|V_{ud}| = G_V/G_\mu$ = 0.97425 ± 0.00022

3) CKM unitarity established ✓

$$V_{ud}^2 + V_{us}^2 + V_{ub}^2 = 0.99990 \pm 0.00060$$

Summary

- The study of beta-decay is a powerful tool for nuclear structure.

- Very far from stability new exotic decay modes appear

- Beta-delayed particles decay is a consecuence of the high Q_β-values and low binding energies for the last nucleon and has paved the way to the discovery of proton and two-proton radiactivity.

- I hope I have convience you of the richness of nuclear structure information one can extract from these studies.

Alpha decay

Spontaneus α-decay ($S_\alpha = 0$) correspond to

$BE({}^{A}_{Z}X_{N}) - [\, BE({}^{A-4}_{Z-2}X_{N-2}) + BE({}^{4}He)\,] = 0$

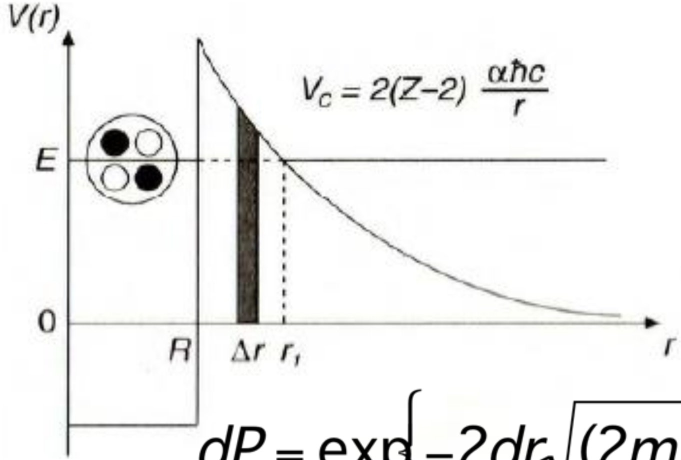
$${}^{A}_{Z}X_{N} \rightarrow {}^{A-4}_{Z-2}X_{N-2} + \alpha$$

▶ α tunnelling :

$\lambda = FP$

P = Prob Transmission

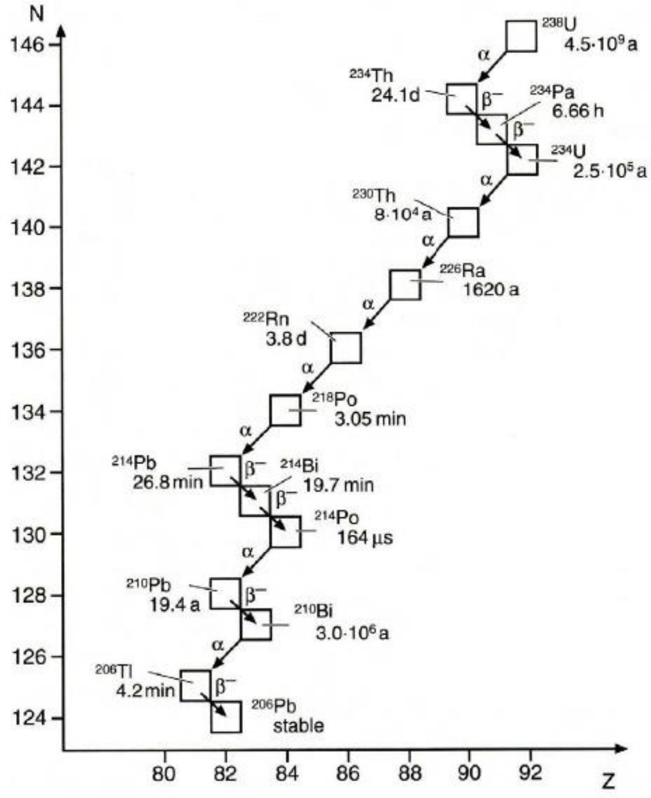
F = frecuency to reach the barrier



$$dP = \exp\left\{-2dr\sqrt{(2m/\hbar[V(r)-E]}\right\}$$

P = exp(-2G) and; G = Gamow factor

$$G = \sqrt{\frac{2m}{h^2}}\int_R^{r_1}[V(r)-E]^{1/2}dr = \sqrt{\frac{2m}{h^2E}}\,\frac{zZe^2}{4\pi\varepsilon_0}\left[\arccos\sqrt{x}-\sqrt{x(1-x)}\right]$$

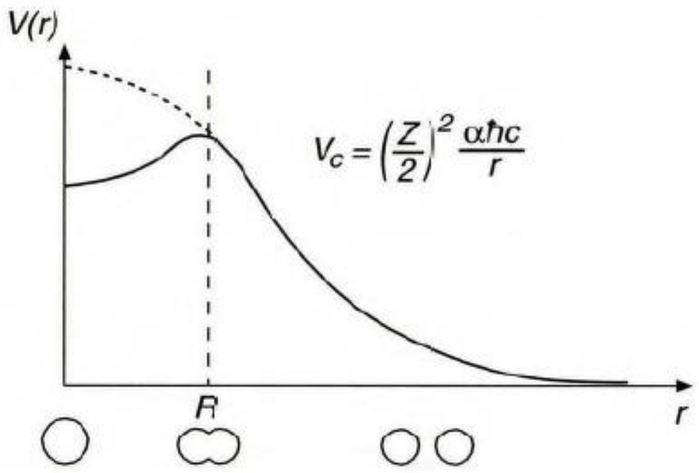


$x = R/r = E / V(R)$ ➔ $G \propto Z/E^{1/2}$ ➔ $\lambda \propto v_o/2R \exp(-2G)$

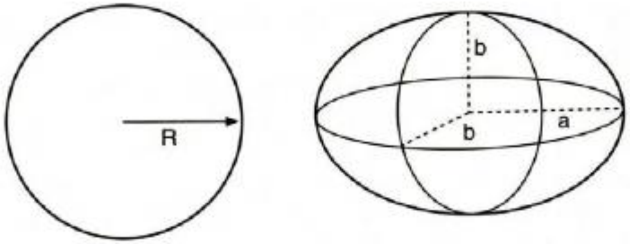
$\tau \approx$ from ns to 10^{17} years!

Nuclear fission

Potential during Spontaneous Fission



Deformed Sphere into ellipsoid



$$\left.\begin{array}{l} a = R\,(1+\varepsilon) \\ b = R\,(1-\varepsilon/2) \end{array}\right\} \; ab^2 \approx R^3$$

$$E_s = a_s\, A^{2/3}\left[1 + \frac{2}{5}\varepsilon^2 + \cdots\right]$$

$$E_c = a_c\,\frac{Z^2}{A^{1/3}}\left[1 - \frac{1}{5}\varepsilon^2 + \cdots\right]$$

▷ small deformation ε changes E by :

$$\Delta E \approx \frac{\varepsilon^2}{5}\left[2a_s\, A^{2/3} - a_c\, Z^2 A^{-1/3}\right]$$

▷ fission barrier disappears for :

$$\frac{Z^2}{A} \gtrsim \frac{2a_s}{a_c} \approx 48$$

$\rightsquigarrow$ about $Z > 114$ and $A > 270$ …

Induced Fission:

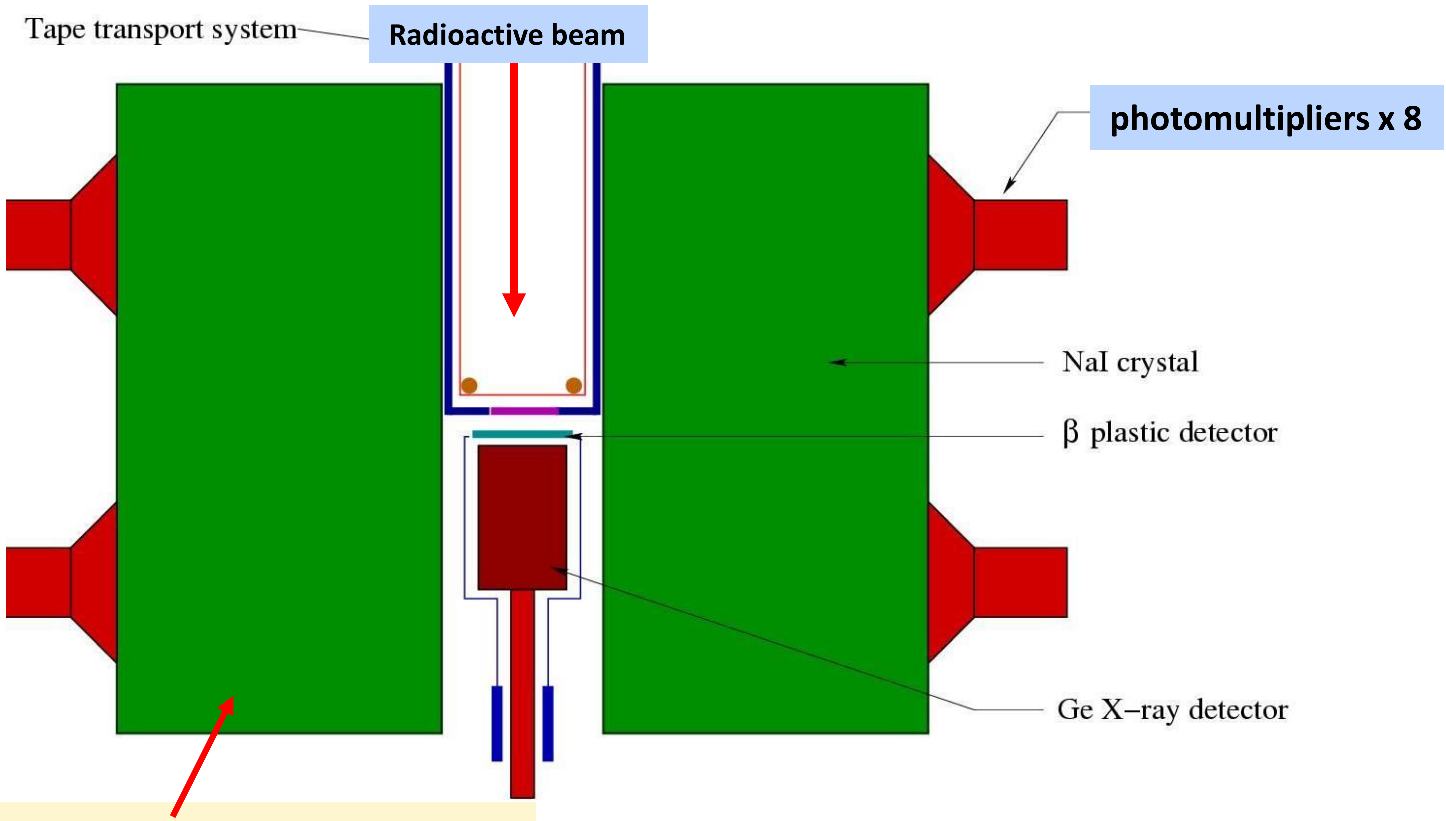
Z ≈ 92: barrier ~ 6 MeV

N capture by odd N Nuclei ➔ δ-term +δ

^{235}U (not ^{238}U) , ^{233}Th, ^{239}Pu….

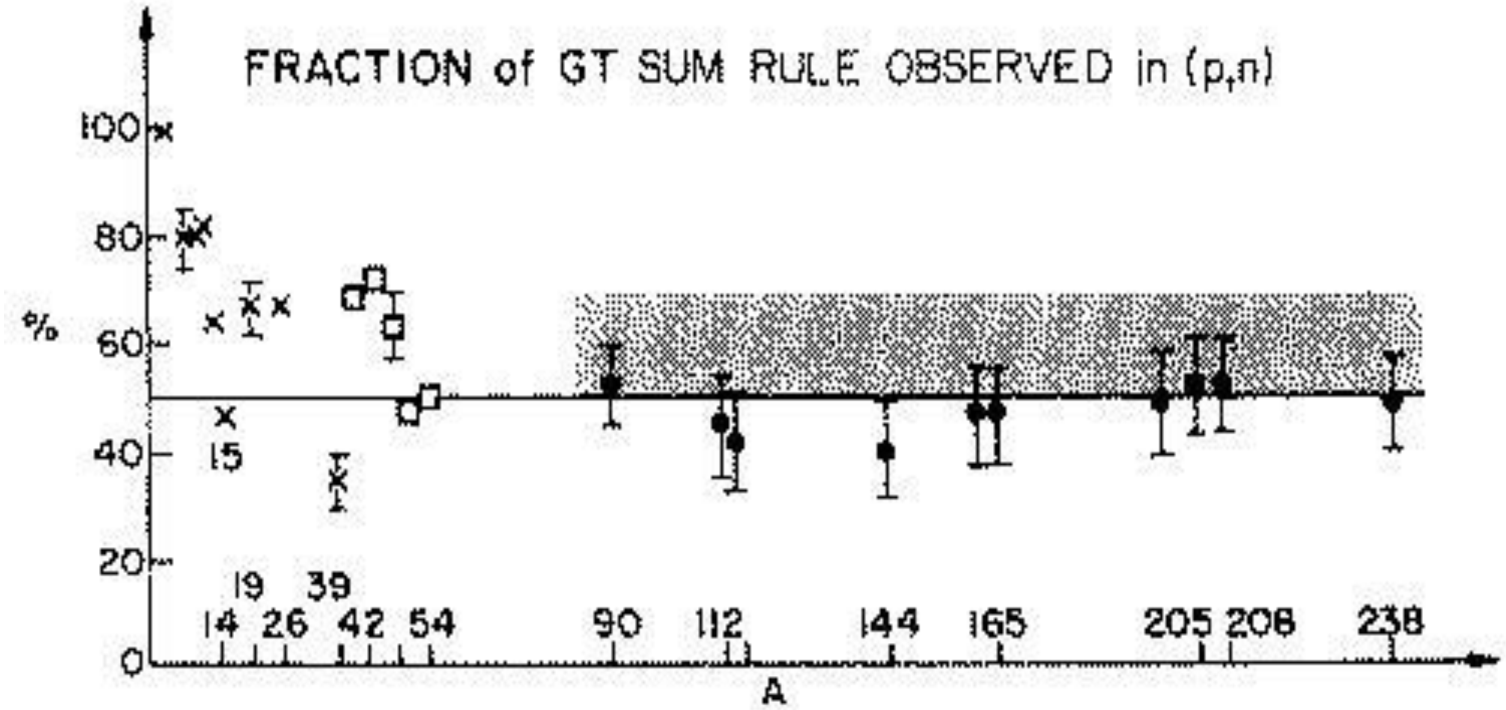
Total absorption Spectrometer (TAS) @ ISOLDE

Aim to measure the total β Strength



NaI crystal of 38cm diameter & L=38cm

Quenching of the GT Strength



Two possible explanations:

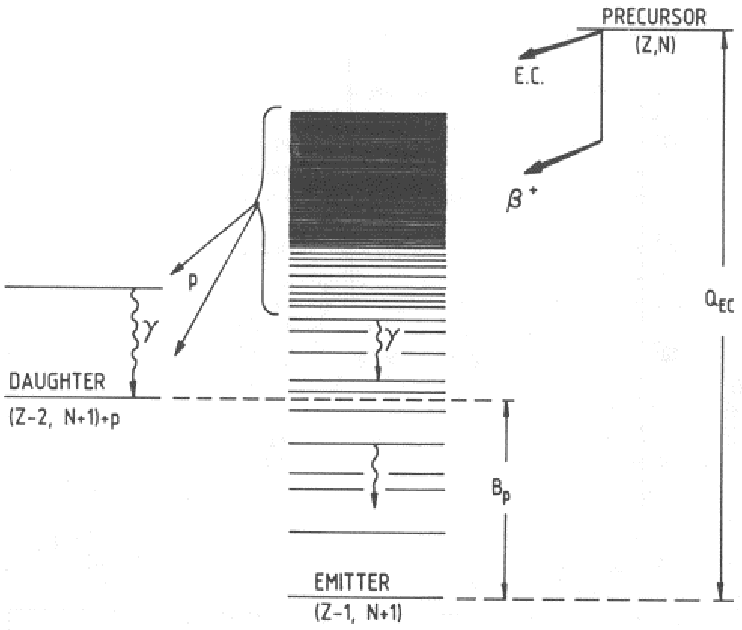
- The Δ-Resonance at 1232 MeV (internal degrees of freedom of nucleon)
- Higher order configuration mixing:

- Experiments in ^{90}Zr(p,n) and ^{90}Zr(n,p) proved that by exploring energies well beyond the GT-resonance they recover 95 % of Sum Rule

Beta Delayed Proton Emission

- **1963** **Barton &Bell** in McGill identify ^{25}Si as first proton precursor thanks to the used of Si-surface barrier detectors

- **Decay Scheme of β-delayed proton precursor**

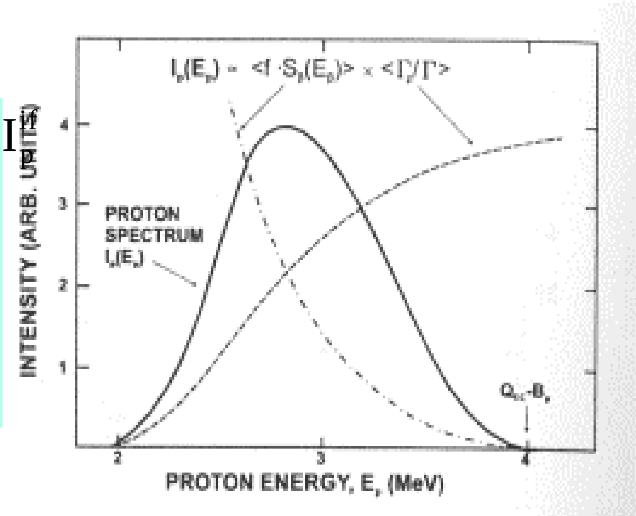


- Particle energy spectrum determined by 2 factors
  - 1-intensity of β-decay branches from precursor to the emitter
  - 2-probability of emission by proton rather gamma

$$\boxed{I_p^{if} = I_\beta^i \frac{\Gamma_p^{if}}{\Gamma^{if}}}$$

Formula valid for light precursor when individual transitition are resolved

- For heavier precursors, I_p^{if} is statistically averaged over an energy range with Bell shape (neglecting nuclear structure)



The Building Blocks

Electron

- In 1897, Thomson produces beams of particles in discharge tubes :
  - by deflecting them : $(v, M/Q)$
    - $\rightsquigarrow$ a universal constituent of matter !
  - then measures Q : $M = 511$ keV/c^2

Proton

E. Rutherf

- In 1911, Rutherford finds a central Coulomb field in the atom caused by a massive, positively charged nucleus …

- Bombarding nuclei with α's :

$$^{14}\mathrm{N} + {}^{4}\mathrm{He} \rightarrow {}^{17}\mathrm{O} + p$$

he observes positively charged particles with a very long range !

- $\rightsquigarrow$ Hydrogen nuclei ?
- $\rightsquigarrow$ elementary constituent of nuclei !

Neutron

- A "neutral radiation" had been observed but not understood …
- In 1932, Chadwick irradiates Beryllium with α's from Polonium source :
  - radiation collides with several nuclei that recoil in ionisation chamber :
    - $\rightsquigarrow$ mass similar to that of the proton
    - $\rightsquigarrow$ new constituent, the "neutron" !

Binding Energy

- Once the constituents known, the forces holding them could be investigated …
  - stronger than atomic forces :
    - $\rightsquigarrow$ need energetic α's to break up
  - mass defect of the order of 1% :

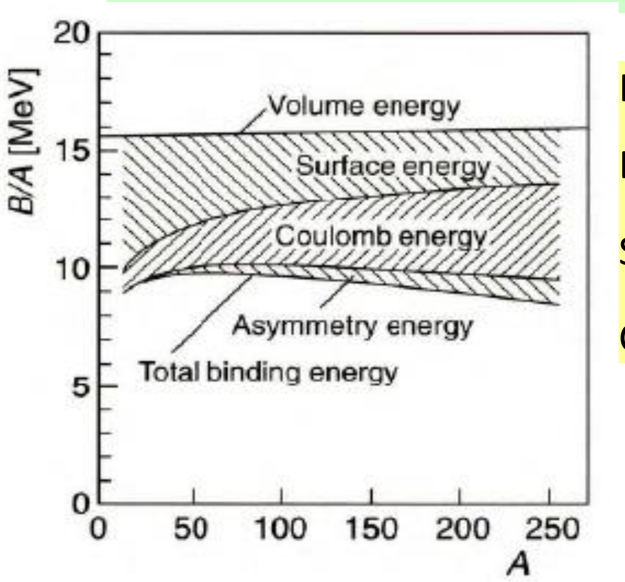
Semi Empirical Mass Formula

The variation of BE with A and Z is described by the Liquid Drop Model with some Shell Model correction.

- **Volume** saturation of forces **BE $\propto a_V A$ (not to $A(A-1) \approx A^2$)**
- **Surface** less binding at surface (few neighbors) $\propto$ **$a_S A^{1/3}$** as the Nuclear surface $\propto 4\pi R^2$
- **Coulomb** effect $\propto$ **$a_C Z(Z-1)A^{-1/3}$**

→ **$BE(A,Z) = a_V A - a_S A^{1/3} - a_V Z(Z-1)A^{-1/3}$**

$\left\{ \begin{array}{ll} \text{A small} & \rightarrow \text{surface correction dominate} \\ \text{A big} & \rightarrow \text{Coulomb correction dominate} \end{array} \right.$



Deformed nuclei both surface and Coulomb corrections change:

Ellipsoidal deformation at constant volume: $a = R(1+ \varepsilon)$

$b = R / (\mathrm{sqrt}(1+ \varepsilon))$

Surface part: $= a_S A^{2/3} (1 + 2/5\, \varepsilon^2)$

Coulomb part: $= a_C Z(Z-1)A^{-1/3} (1 - 1/5\, \varepsilon^2)$

$\Delta E = \Delta E_S + \Delta E_C > 0$ → stable spherical shape $Z^2/A < 49$

Shell Model Corrections

Symmetry energy

Pauli principle prevents occupation of certain orbitals

Favours $Z = N = A / Z$ → parities $\begin{cases} N = A/Z + \nu \\ N = A/Z - \nu \end{cases}$

$a_v = 15.85$ MeV
$a_s = 18.34$ MeV
$a_c = 0.71$ MeV
$a_A = 23.21$ MeV
$a_p = 12$ MeV

The average energy between adjacent orbitals is Δ;

→ $$\Delta E_{bind} = \nu(\Delta\nu/2); \text{ where } \nu = (N-Z)/2$$

As the potential depth Uo describing the nuclear well is approximately the same from ^{16}O to ^{208}Pb ($\Delta U_0 < 10$ %). Average energy spacing between orbitals is $\Delta \propto 1/A$

→ $$\Delta E_{bind} = 1/8(N-Z)^2 \Delta = 1/8(A-2Z)^2 \Delta$$

Pairing energy

Nucleus preferentially form pairs under influence of the short range nucleon-nucleon atractive force

$$\Delta E_{pair} \begin{cases} +\delta & (e-e) \\ 0 & (e-0) \\ -\delta & (0-0) \end{cases}$$

$$\delta \approx a_p A^{-1/2}$$

Bethe-Weizsäcker mass equation (1935-1936)

$$BE(A,Z) = a_v A - a_s A^{2/3} - a_c Z(Z-1)A^{-1/3} - a_A (A-2Z)^2/A + a_p A^{-1/2}$$

Binding Energy (I)

- Strong interaction acts at very short distance.
- Naively one would expect A(A-1)/2 bonds and each E_{bond} ~constant thus giving:

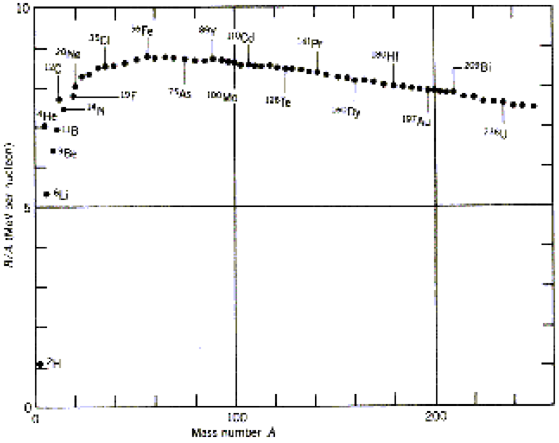
  $BE({}^{A}_{Z}X_{N})/A$ $\propto E_2\,(A-1)/2$

- Experimentally **$BE({}^{A}_{Z}X_{N})/A$** $\propto$ 8 MeV over the full region indicating
  - Nuclear and charge independent
  - Saturation of Nuclear Forces: $\rho_o \approx 0.17\ N/fm^3$
  - The less bound nucleon has an energy of ~ 8 MeV independent of the number of nucleons

  → **The independent particle picture holds : nucleons move in an average potential**

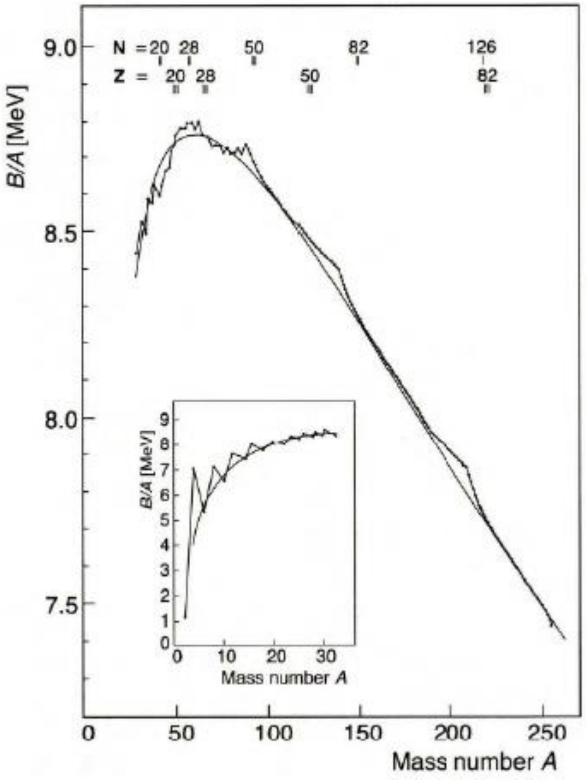
- **BE/A as function of A has its maximum around A = 56-60 (^{62}Ni)**

  → Source of energy production
  - **Fission of heavy nuclei**
  - **Fusion of light nuclei**



Binding Energy (II)

Under assumption of saturation and charge independence. Each nucleon occupies an almost equal size within the nucleus ***the elementary radius*** r_0



$$V = \frac{4}{3}\pi r_0^3 A \quad \begin{cases} r_0 = 1.2 \text{ fm for charge radius} \\ r_0 = 1.4 \text{ fm for matter radius} \end{cases}$$

$$\rho(r) = \frac{\rho_0}{1+e^{(r-R_0)/a}} \quad \begin{cases} \rho_0 = \text{central density} \\ R_0 = \text{Radius at half density} \\ a = \text{diffuseness of nuclear surface} \end{cases}$$

Nuclear density is independent of A and 10^{14} times normal density

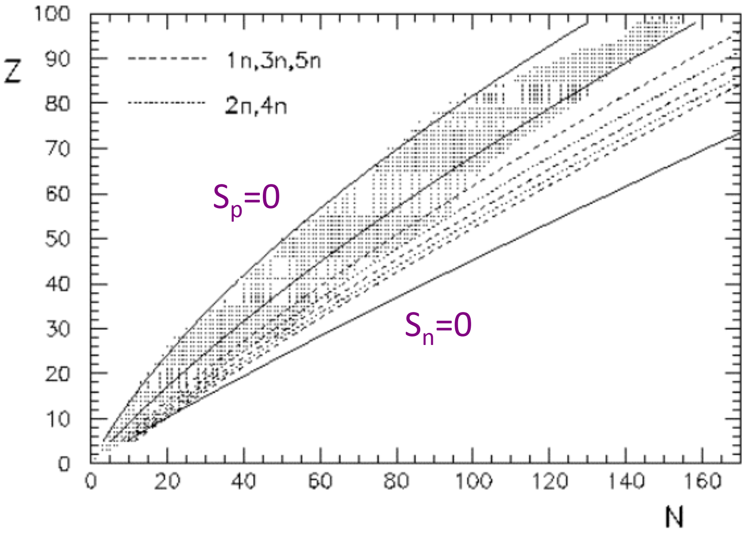
The liquid drop model was first to describe the nuclear properties.

- saturation of nuclear forces gives BE/A = constant

-Nucleus presents low compressibility and well defined surface.

Stability Against Radioactive Decay

Last stable nuclei A≈210



Spontaneus α-decay (S_α = 0) correspond to

$BE({}^A_Z X_N) - [\, BE({}^{A-4}_{Z-2}X_{N-2}) + BE({}^4He)\,] = 0$

The half-lives becomes short in the actinide region A ≈ 210

The conditions S_n = 0 and S_p = 0 establishes the drip-lines

The energy release in nuclear fission:

$$E_{fission} = M^1\left({}^A_Z X_N\right)c^2 - 2M'\left({}^{A/2}_{Z/2}X_{N/2}\right)c^2$$

Using a simplified mass eq. where Z(Z-1) ≈ Z^2 and neglecting the pairing corrections δ:

$$E_{fission} = [\, -5.12\, A^{2/3} + 0.28\, Z^2 A^{-1/3}\,]\, c^2$$

$E_{fission}$ > 0 for A≈ 90 and $E_{fission}$ = 185 MeV for ${}^{238}U$.

The fission products, neutron rich nuclei, mainly β^- ⇒ good source of electron anti-neutrinos.

Definition

Beta Decay: **universal term for all weak-interaction transitions between two neighboring isobars**

Takes place is 3 different forms

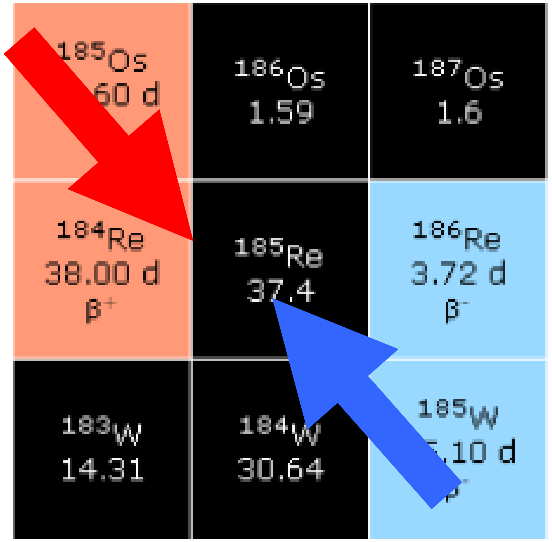
β^-, β^+ & EC (capture of an atomic electron)

β^+: p → n + e^+ + ν

$$^{A}_{Z}X_{N} \rightarrow {}^{A}_{Z-1}X^{*}_{N+1} + e^{+} + \nu_{e}$$

EC: p + e^- → n + ν

$$^{A}_{Z}X_{N} + e^{-} \rightarrow {}^{A}_{Z-1}X^{*}_{N+1} + \nu_{e} + X_{ray}$$



^{185}Os 60 d	^{186}Os 1.59	^{187}Os 1.6
^{184}Re 38.00 d β^+	^{185}Re 37.4	^{186}Re 3.72 d β^-
^{183}W 14.31	^{184}W 30.64	^{185}W .10 d

β^-: n → p + e^- + $\tilde{\nu}$

$$^{A}_{Z}X_{N} \rightarrow {}^{A}_{Z+1}X^{*}_{N-1} + e^{-} + \overline{\nu}_{e}$$

a nucleon inside the nucleus is transformed into another

Beta-decay lifetime

$t \equiv T_{1/2}^{\beta_i} = \frac{T_{1/2}^{\exp}}{P_{\beta_i}}$ partial half-life of a given β^- (β^+,EC) decay branch (*i*)

$$\frac{\ln 2}{T_{1/2}^{n}} = \frac{g^2}{2\pi^3}\int_1^{W} p_e W_e (W_0 - W_e)^2 F(Z,W_e) C_n \, dW_e$$

Assuming
$F(Z,W) = 1$ & $Q \gg m_e c^2$
$f = W_o^5/30$ (β^+)
$f = (W_o+1)^5/30$ (β^-)

g – weak interaction coupling constant

p_e – momentum of the β particle

W_e – total energy of the β particle

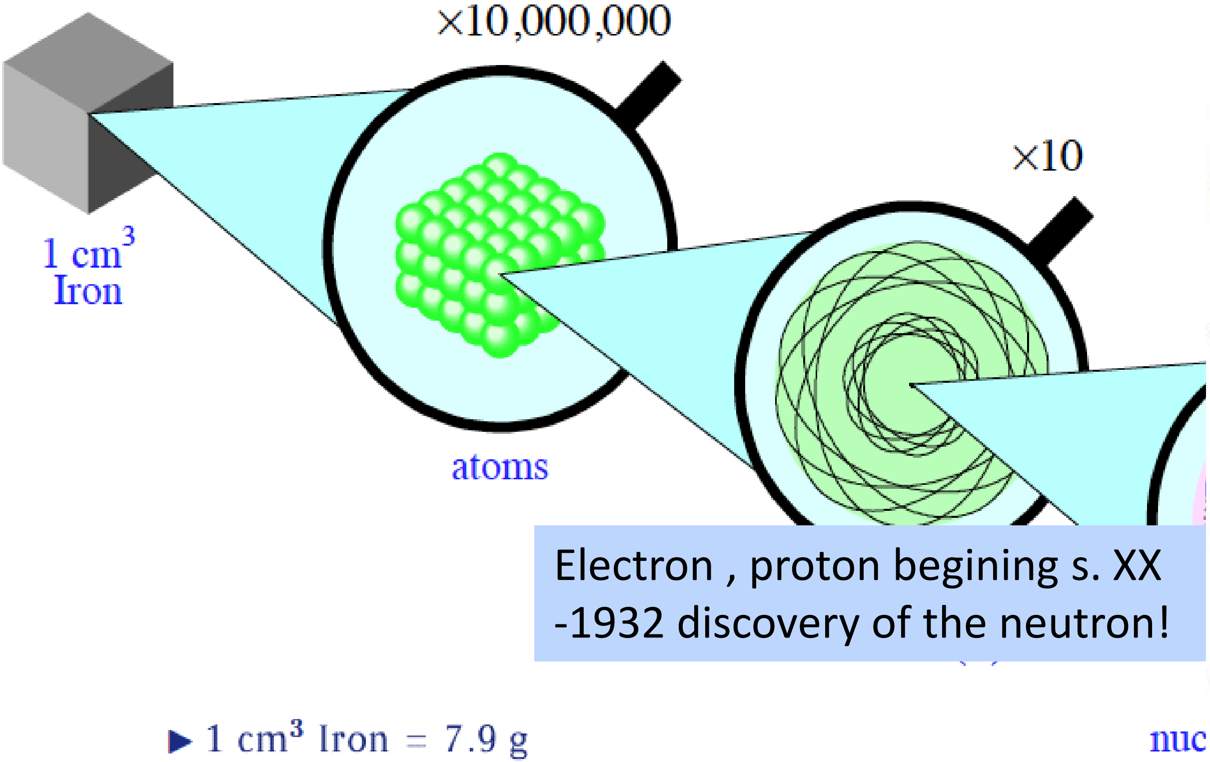
W_0 – maximum energy of the β particle

$F(Z,W_e)$ – Fermi function – distortion of the β particle wave function by the nuclear charge

C_n – shape factor ≠ 1 for forbidden transitions = C(p,q)

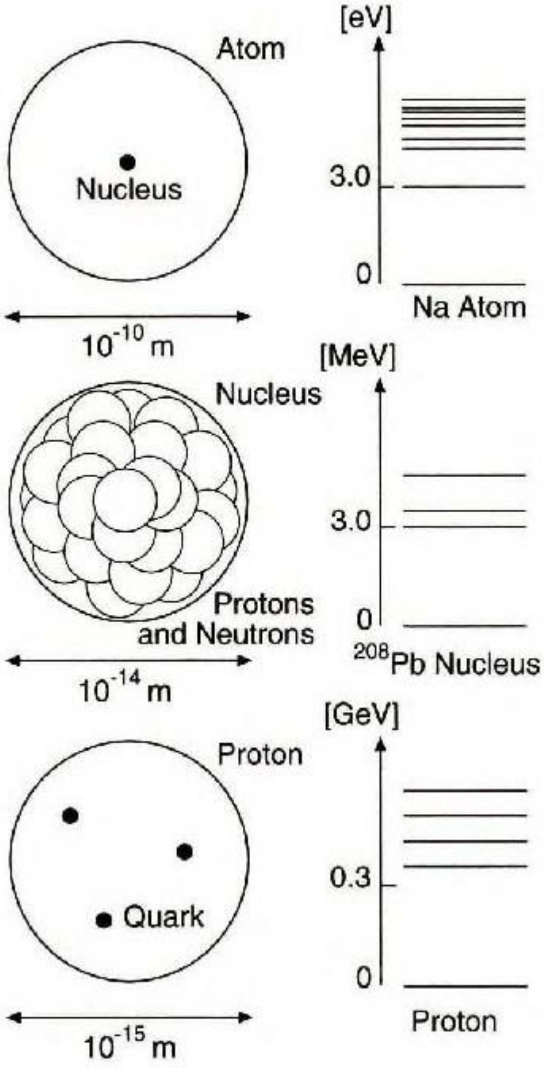
Z – atomic number

The structure of the Matter



Electron , proton begining s. XX
-1932 discovery of the neutron!

Energy Scale!



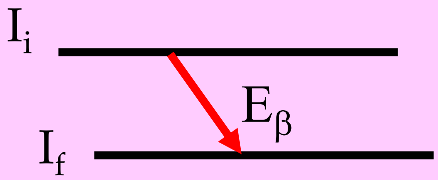
- 1 cm^3 Iron = 7.9 g
  - 85,000,000,000,000,000,000,000 atoms !
  - 99.9999999999% of matter is empty !
  - 1 cm^3 of nuclei = 300 million tons !

50/60's Accelerators➔ hadron zoo(100)
Hadrons combination of 2-3 quarks

Useful empirical rules

The fifth power beta decay rule:

The speed of a β transition increases approximately in proportion to the fifth power of the total transition energy (if other things are being equal, of course!)



$$\frac{1}{\tau} \propto [(M(Z)-M(Z\pm1))c^2]^5$$

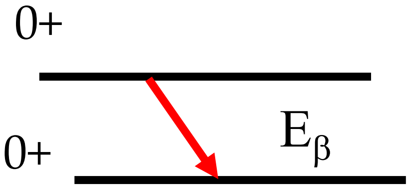
F (Z,W)= 1 & Q >>$m_e c^2$

- **Depends on spin and parity changes between the initial and final state**
- **Additional hindrance due to nuclear structure effects – isospin, "l-forbidden", "K-forbidden", etc.**

Classification of allowed β-transitions

$$(p_i p_f = +1)$$

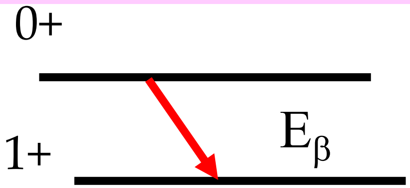
Fermi



$$\Delta I = \left| I_i - I_f \right| \equiv 0$$

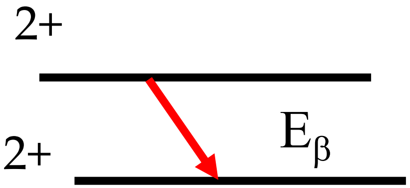
$$L_\beta = 0 \qquad S_\beta = 0 \downarrow\uparrow$$

Gamow-Teller



$$\Delta I = \left| I_i - I_f \right| \equiv 1$$

$$L_\beta = 0 \;\; S_\beta = 1 \uparrow\uparrow or \downarrow\downarrow$$



mixed Fermi & Gamow-Teller

$$\Delta I = \left| I_i - I_f \right| \equiv 0 \qquad I_i \neq 0$$

Classification of β-transitions

Type of transition	Order of forbiddenness	ΔJ	$\pi_i\pi_f$
Allowed		0,+1	+1
Forbidden unique	1	∓2	-1
	2	∓3	+1
	3	∓4	-1
	4	∓5	+1
	.	.	.
Forbidden	1	0, ∓1	-1
	2	∓2	+1
	3	∓3	-1
	4	∓4	+1
	.	.	.

The order of forbideness is given by the angular momentum carried by the electron and neutrino.

Log*ft* Values

$$\log ft = \log f + \log t$$

coming from
calculations

coming from experiment

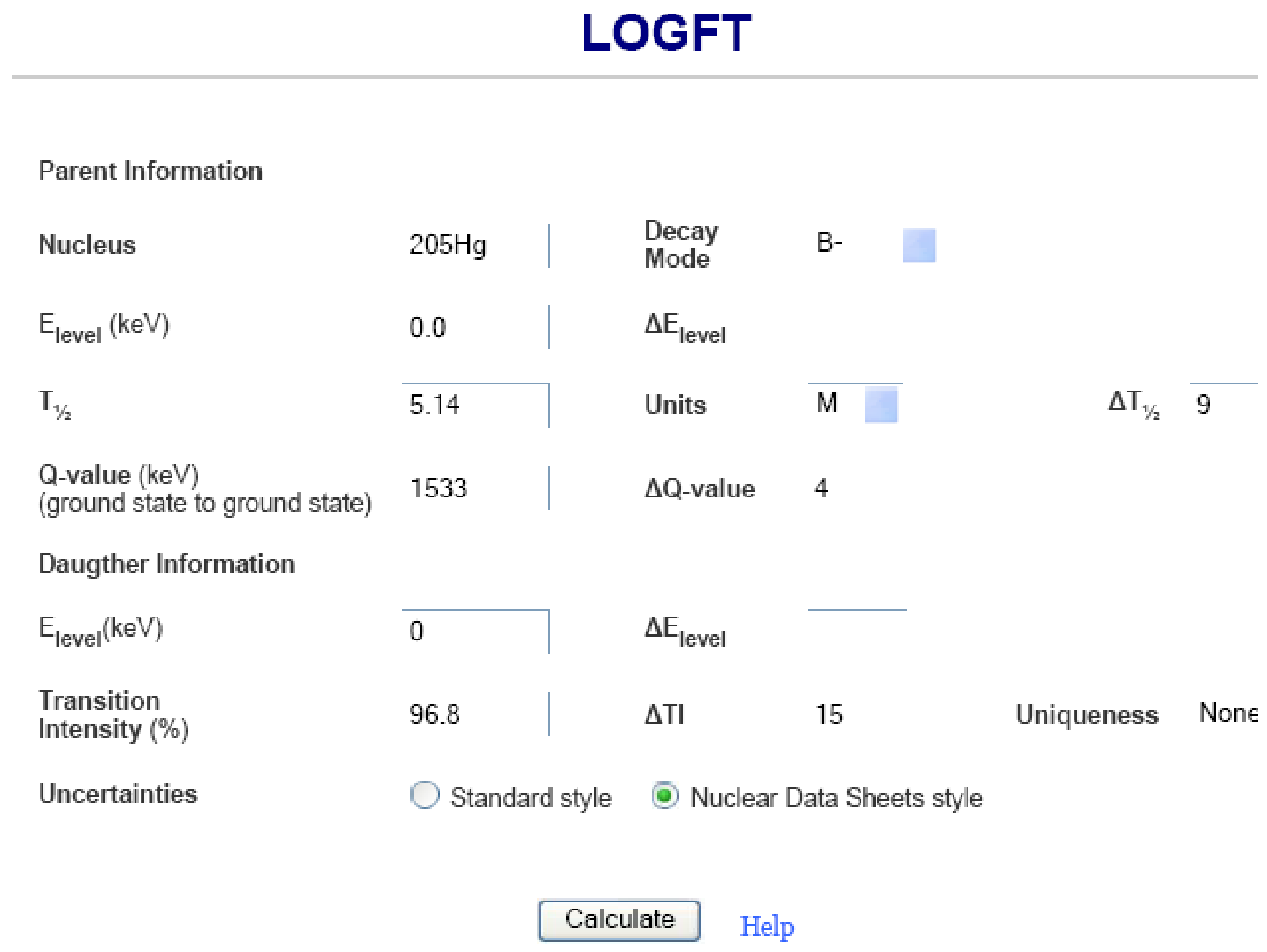
For allowed trans: Wilkinson & Macefield,
NPA232 (1974) 58

N.B. Gove and M. Martin, Nuclear Data Tables **10** (1971) 205

Decay Mode	Type	ΔI ($\pi_i\pi_f$)	log *f*
β– **EC + β+**	allowed	0, +1 (+)	$\log f_0^-$ $\log(f_0^{EC} + f_0^+)$
β– **EC + β+**	1st-forb unique	∓2 (-)	$\log f_0^- + \log(f_1^- / f_0^-)$ $\log[(f_1^{EC} + f_1^+)/(f_0^{EC} + f_0^+)]$

Logf for dummy’s

- ❑ ENSDF analysis program LOGFT – both Windows & Linux distribution http://www.nndc.bnl.gov/nndcscr/ensdf_pgm/analysis/logft/
- ❑ LOGFT Web interface at NNDC http://www.nndc.bnl.gov/logft/



LOGFT

Parent Information

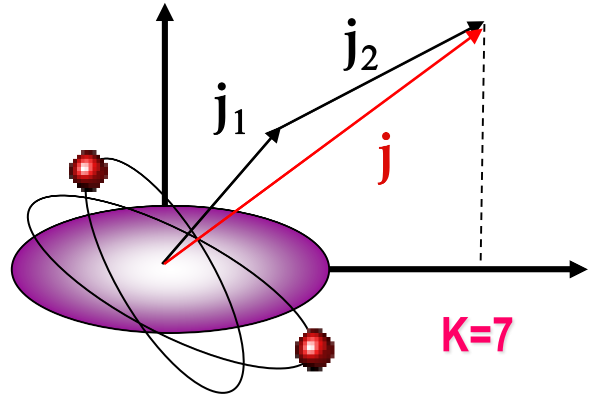
Nucleus	205Hg	Decay Mode	B-		
E_{level} (keV)	0.0	ΔE_{level}			
$T_{½}$	5.14	Units	M	$\Delta T_{½}$	9
Q-value (keV) (ground state to ground state)	1533	ΔQ-value	4		

Daugther Information

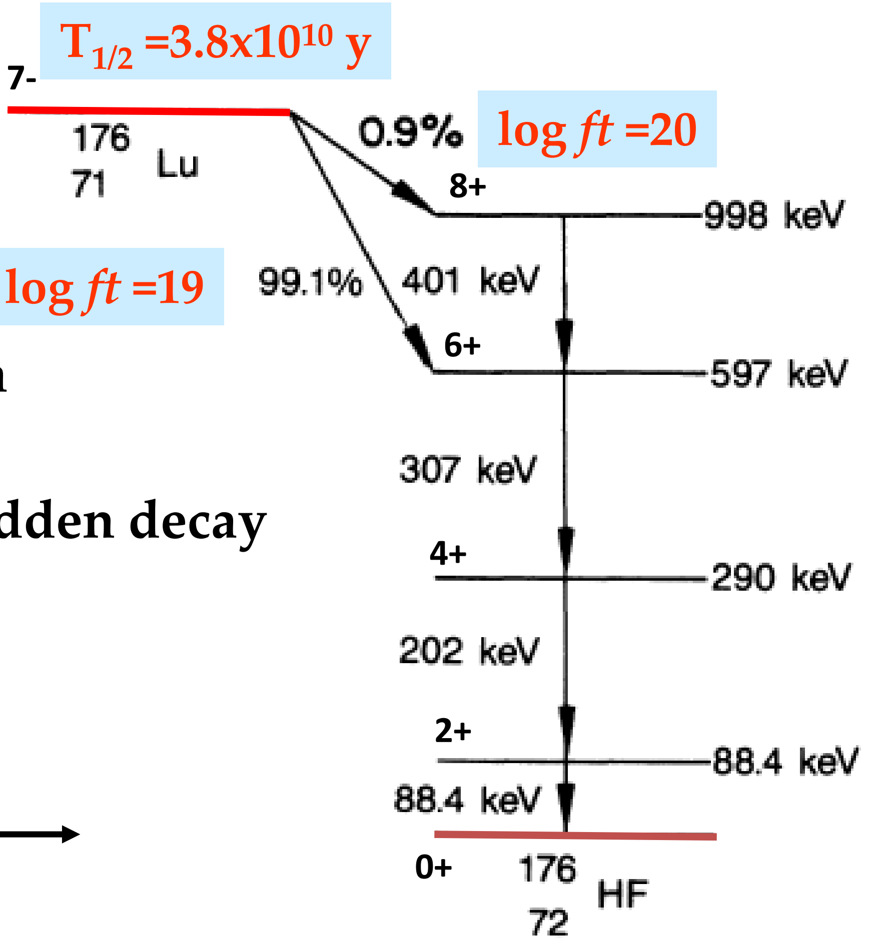
E_{level}(keV)	0	ΔE_{level}			
Transition Intensity (%)	96.8	ΔTI	15	Uniqueness	None

Uncertainties ○ Standard style ● Nuclear Data Sheets style

Be careful: Nuclear Structure is important



First forbidden → 5 < log *ft* <10



large angular momentum
re-orientation

K-forbidden decay