Scattering Amplitude Methods for Effective Field Theories

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Effective Field Theory (EFT)

- Every physics experiment is probing a certain energy scale, E.
- Degrees of freedom with much smaller or much bigger energy than E are not excited and, therefore, can be removed (integrated out) from our theory, obtaining a simpler effective theory for the relevant degrees of freedom.
- For example we can use hydrodynamics to describe the dynamics of water on macroscopic distances, without knowing the details of QCD and QED.

$$e^{iS[\phi]_{eff}} = \int D\chi e^{iS[\phi,\chi]}$$

Building an Effective Field Theory from Bottom Up

When the full theory is not known we can build the effective action by writing all the possible operators, compatible with the symmetries, multiplied by arbitrary coefficients. For example and EFT of quantum gravity should look like

$$S[g]_{eff} = \frac{M_{pl}}{2} \int \sqrt{-g}(R + \frac{a}{\Lambda^2} R^2 + \frac{b}{\Lambda^2} R_{\mu\nu} R^{\mu\nu} + \frac{c}{\Lambda^2} R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} + ...)$$

Constraining the EFT coefficients

The coefficients in effective action must obey certain conditions in the EFT to be compatible with the axioms of QFT:

1) Unitarity
2) Causality
3) Locality
4) Poincare invariance

i.e. in order for the effective theory to have a standard UV completion.

Constraining the EFT coefficients from 2-2 scattering amplitude

Each of these axioms gives a precise mathematical condition of 2-2 scattering amplitude $A(s,t)$:

1) Unitarity $\rightarrow \text{Im}(A(s,0)) > 0$
2) Causality $\rightarrow$ Analytic structure of $A(s,t)$
3) Locality $\rightarrow \lim_{s\rightarrow\infty} A(s,t) < Cs^2$
4) Poincare invariance $\rightarrow$ Crossing symmetry

Relating high energy and low energy

We can relate the values of the amplitude at low energies (where the EFT is valid) to the high energy values using the previous axioms.

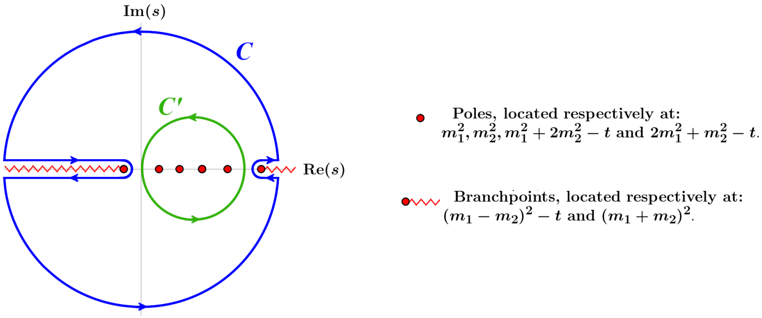


Figure: Figure from 1910.11799.

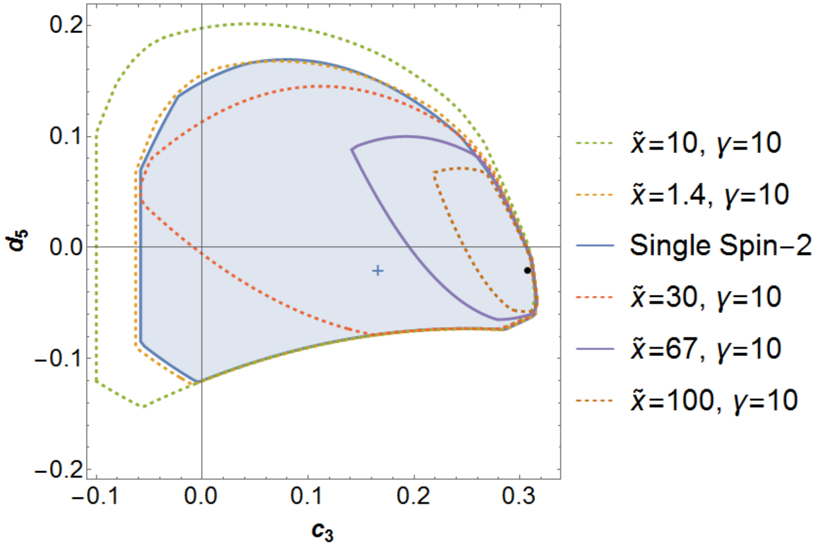
Positivity bounds

Then we can show that

$$\begin{aligned}
f =& \frac{1}{2}\frac{d^2}{ds^2}(A(s,0) - \text{poles}) \\
=& \frac{1}{2\pi i}\oint ds' \frac{(A(s,0) - \text{poles})}{(s'-s)^2} \\
f =& \frac{1}{\pi}\int_{(m_1+m_2)^2}^{\infty} ds' \left(\frac{Abs_s A(s',0)}{s'-s} + \frac{Abs_u A(s',0)}{s'-u} \right) > 0,
\end{aligned}$$

Applications

We applied positivity bounds to constrain an allowed parameter space of EFT of interacting massive spin-2 fields.



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Another feature of scattering amplitudes is new relations between two theories that are not apparent from their actions:

$$S_{YM} = -\frac{1}{2g^2}\int d^D x \, \mathrm{tr}(F_{\mu\nu}F^{\mu\nu}) \tag{1}$$

and

$$S_{adGR} = S = \int d^D x \sqrt{-g}\left[\frac{2}{\kappa^2}R - \frac{D-2}{2}\partial^\mu\phi\partial_\mu\phi - \frac{1}{6}e^{-2\kappa\phi}H^{\lambda\mu\nu}H_{\lambda\mu\nu}\right], \tag{2}$$

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The n-point scattering amplitudes of Yang-Mills theory, A_n, and axion-dilaton gravity, M_n, can be written as

$$A_n = g^{n-2} \sum_i \frac{c_i n_i}{D_i}, \tag{3}$$

with $c_i + c_j + c_k = 0 \to n_i + n_j + n_k = 0$,

$$M_n = \left(\frac{\kappa}{2}\right)^{n-2} \sum_i \frac{n_i n_i}{D_i}. \tag{4}$$

Double Copy

Currently trying to find more examples of double copy, in particular we are considering

- massive states
- more EFT operators
- different spacetime dimensions

Further reading

Positivity bounds and EFT of spin-2 fields:

[Alberte, de Rham, Momeni, Rumbutis, Tolley]

- [arXiv: 1910.05285]
- [arXiv: 1910.11799]
- [arXiv: 1912.10018]

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[Momeni, Rumbutis, Tolley]

- [arXiv: 2004.07853]
- [arXiv: 2012.09711]