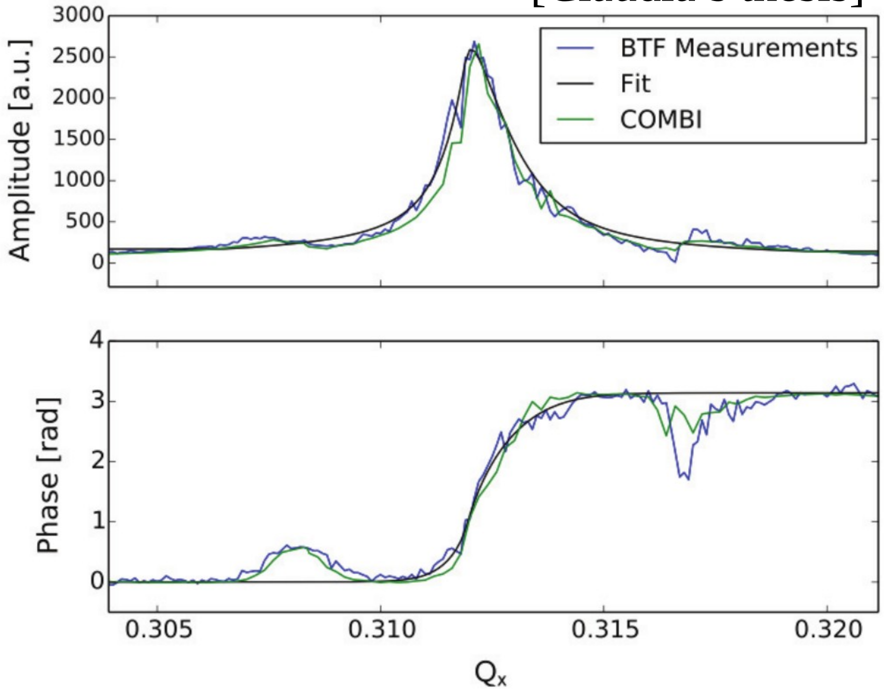
Theory of transverse beam transfer functions with chromaticity

X. Buffat

- Motivation
- Theory
  - Comparison with COMBI
  - Comparison to experimental data
- Application for chromaticity measurement
- Summary and outlook

Motivation

[Claudia's thesis]



- During MDs at the LHC, a significant response of the beam was observed at the synchrotron sidebands, with a dependence on chromaticity

[Claudia's thesis] C. Tambasco, Ph.D. thesis, EPFL, Lausanne, Switzerland (2017)

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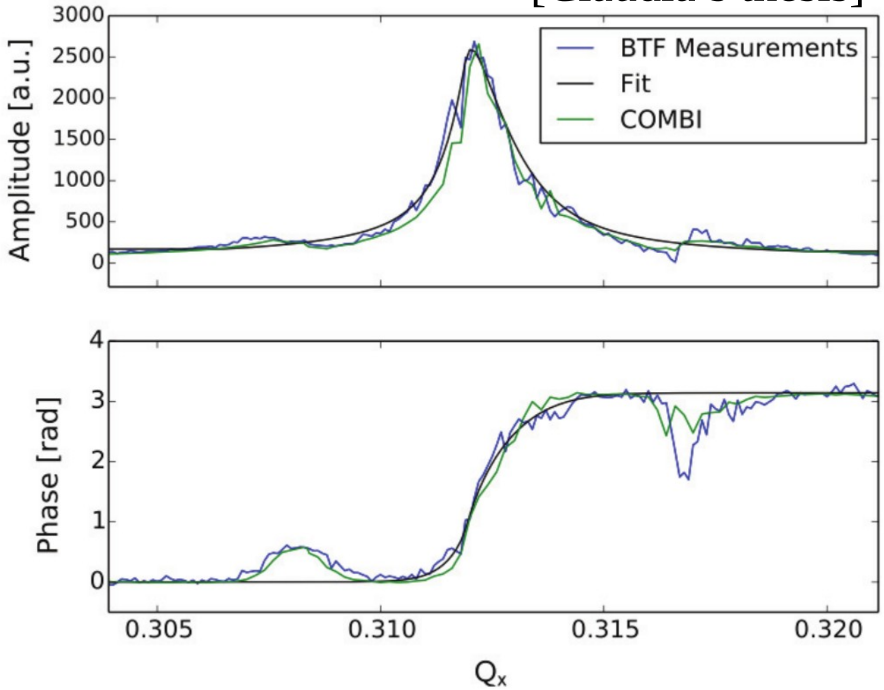

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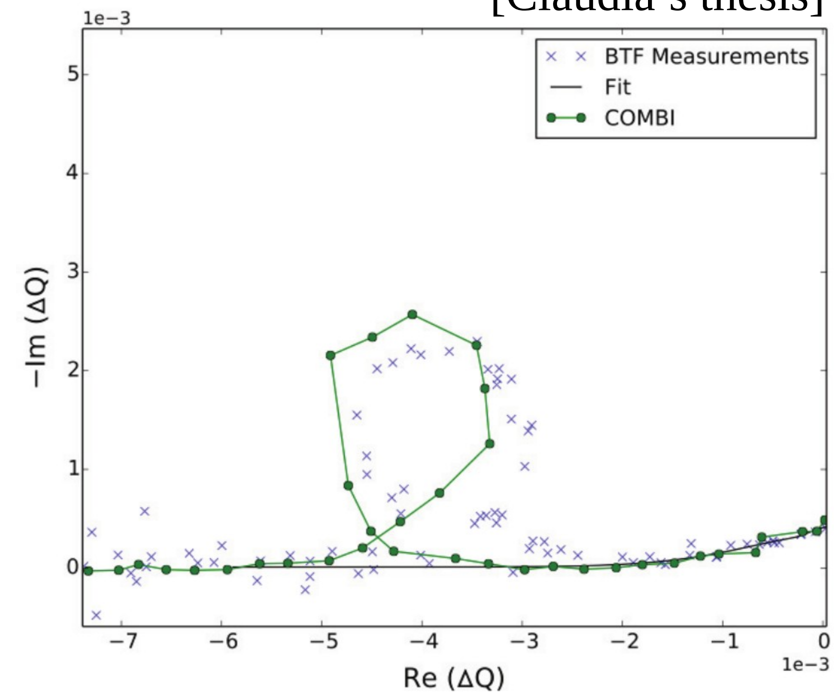

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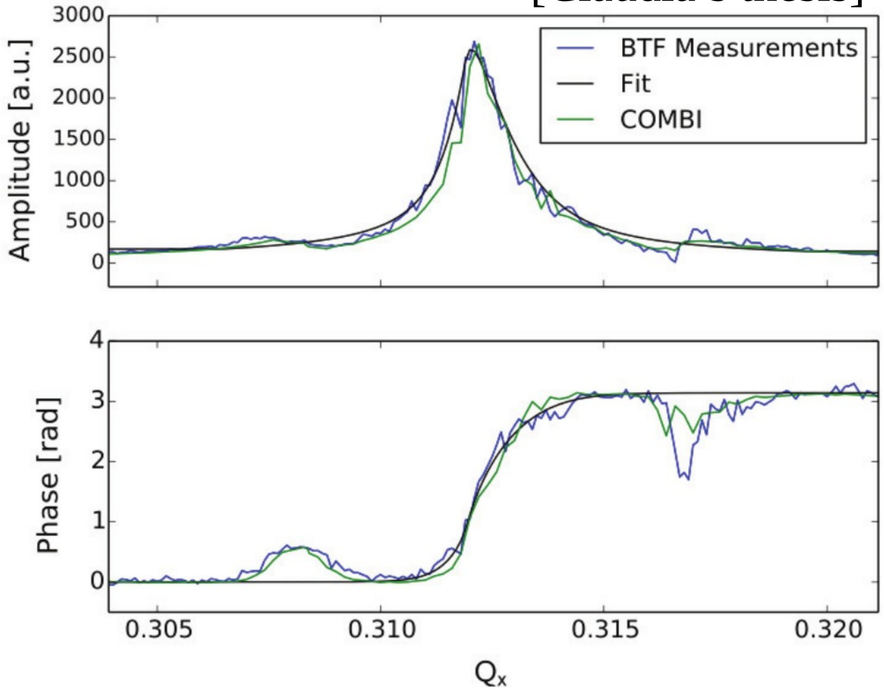

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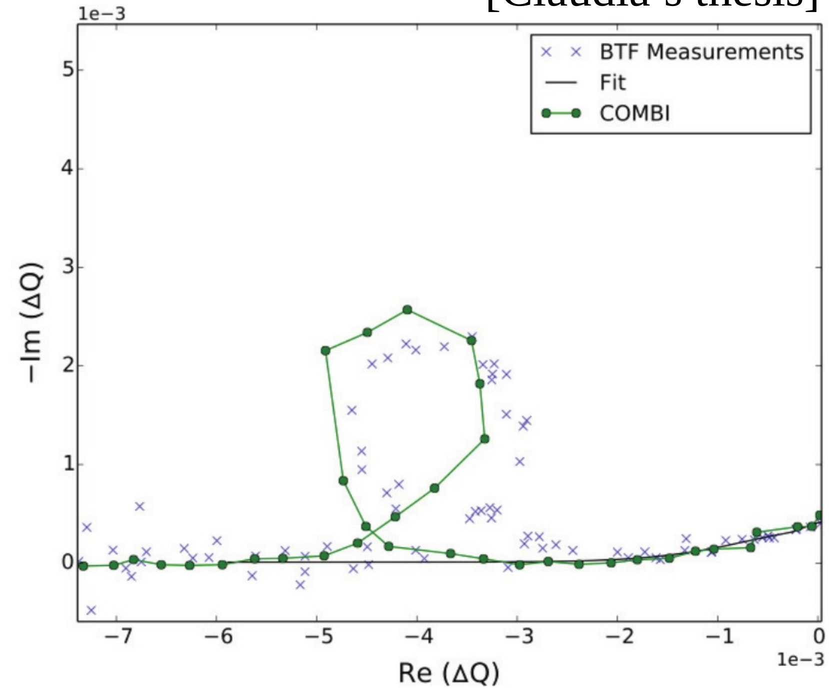

- During MDs at the LHC, a significant response of the beam was observed at the synchrotron sidebands, with a dependence on chromaticity
- When exploiting the relation between the beam response and the stability diagram, yet assuming no chromaticity, the reconstructed stability diagram featured loops arising from the response at the sidebands
- Questions following these observations (2015):
  - What is the origin of the loops ?
  - Do they represent Landau damping ?
  - Can we use this feature to measure chromaticity?

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- Following [Nicolas' lectures], we write the transverse modes of oscillation as (Eq. 68):

$$\Delta\psi(J_x, J_y, \theta, r, \phi, t) = e^{i\omega_0 Q_c t} e^{i\theta} \frac{df_0}{dJ_x} \sqrt{2J_x\beta}\, e^{-i\frac{Q' r \cos\phi}{\eta R}} \sum_l R_l(r) e^{-il\phi}$$

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- Identifying the terms in *l*, i.e. assuming no coupling between the azimuthal modes, we can write their longitudinal part:

$$R_l(r)e^{-il\phi} = \frac{A}{2p_0\omega_0}\frac{g_0(r)}{Q_c - Q(J_x, J_y) - lQ_s} i^{-l} e^{-il\phi} J_{-l}\left(\frac{Q'r}{\eta R}\right)$$

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- Injecting into the initial expression for the mode (using Jacobi-Anger again):

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Theory

- Playing a bit, we can write:

$$\frac{f_{rev} p_0}{\beta} \frac{\langle x \rangle}{F_c} = \frac{1}{4\pi} \sum_l \int \frac{J \frac{df_0}{dJ}}{Q_c - Q(J_x, J_y) - lQ_s} dJ \int g_0(r) J_l \left(\frac{Q' r}{\eta R} \right)^2 dr$$

$$\equiv \frac{1}{4\pi} \sum_l D_l(Q_c) w_l(Q')$$

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$$\equiv \frac{1}{4\pi} \sum_l D_l(Q_c) w_l(Q')$$

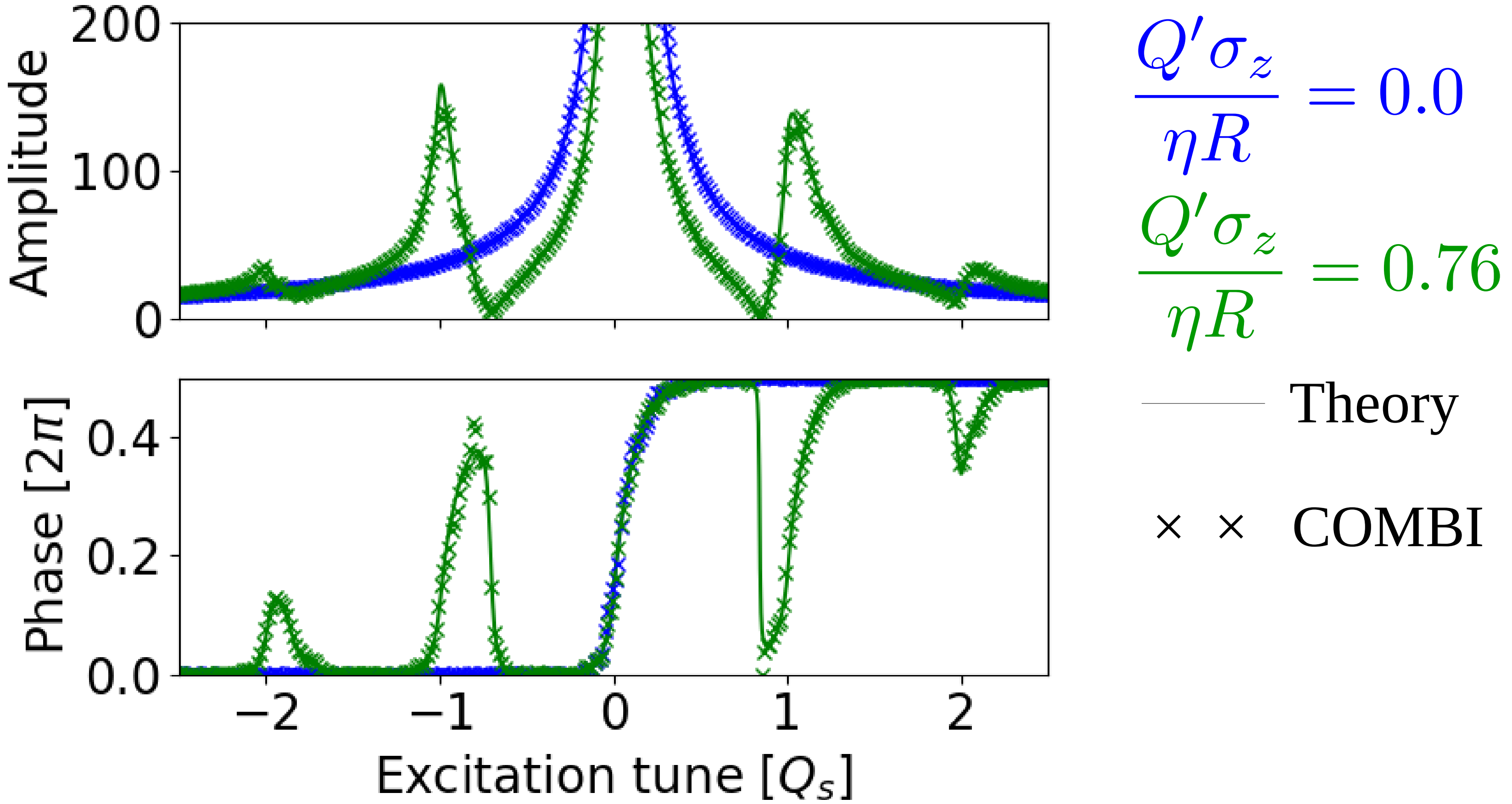
- For a Gaussian transverse distribution the dispersion integral can be expressed analytically [Scott Berg]
- For a Gaussian longitudinal distribution, the chromatic weight can be expressed (using Eq. 6.633 in [Gradshteyn]):

$$w_l^G(Q') = e^{-\left(\frac{Q'\sigma_z}{\eta R}\right)^2} I_l \left(\left(\frac{Q'\sigma_z}{\eta R} \right)^2 \right)$$

Chromatic phase
or head-tail phase

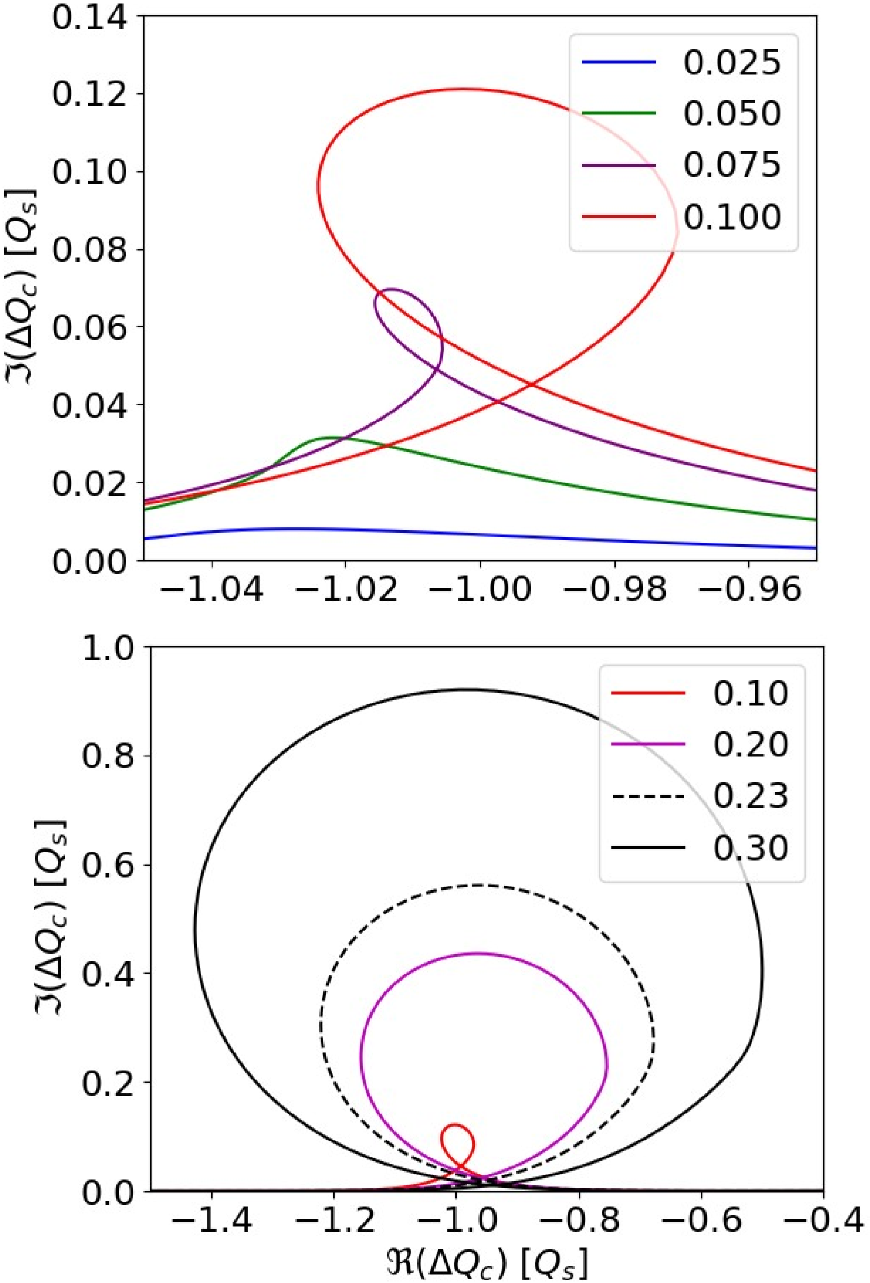
[Scott Berg] J. Scott Berg and F. Ruggiero, Landau damping with two-dimensional betatron tune spread, SL-96-071-AP
[Gradshteyn] I. S. Gradshteyn, I. M. Ryzhik, and A. Jeffrey, Table of integrals, series, and products; 6th ed. (Academic Press, San Diego, CA, 2000)

Comparison with COMBI



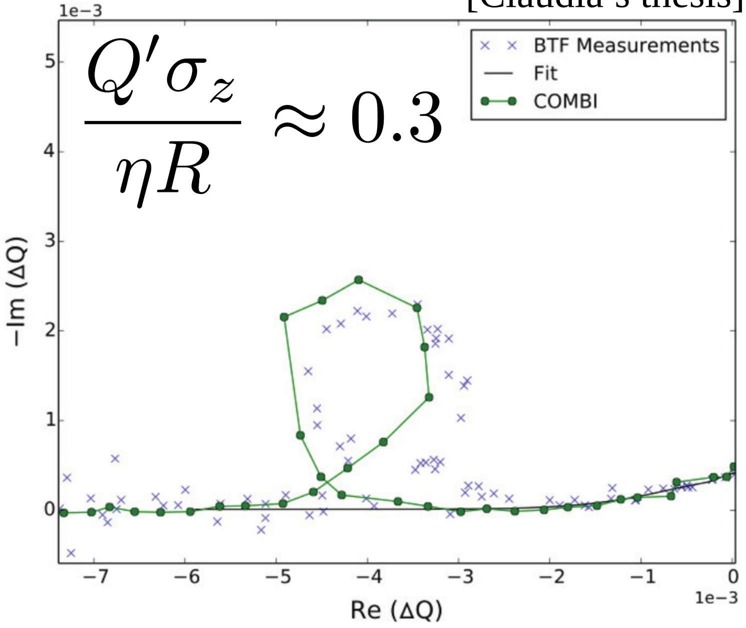
- White dipole noise is injected on the beam in COMBI, recording both the injected noise and the beam position at every turn. The beam transfer function is obtained with the ratio of the power spectral density of the beam oscillation to the one of the injected noise. (using Welch method for smoothing)
  - The agreement is stunning

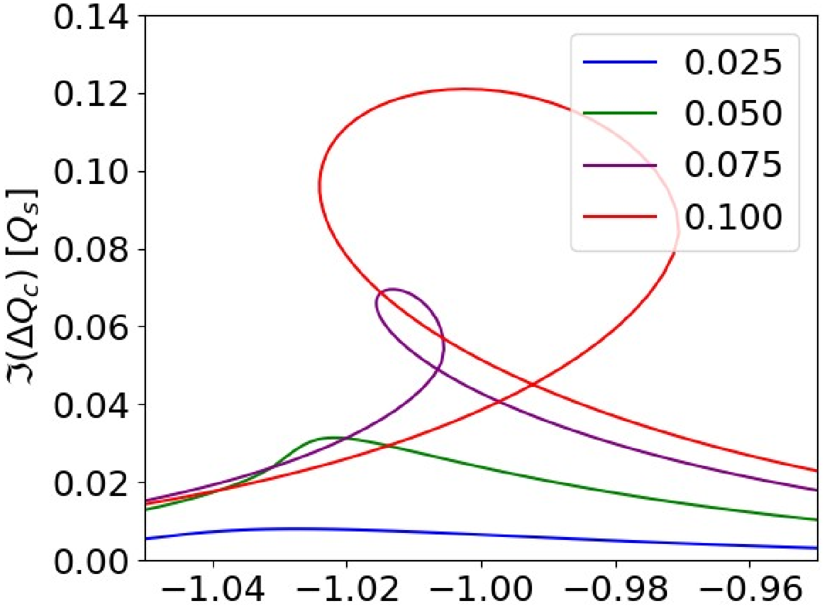

BTF
LOOPS

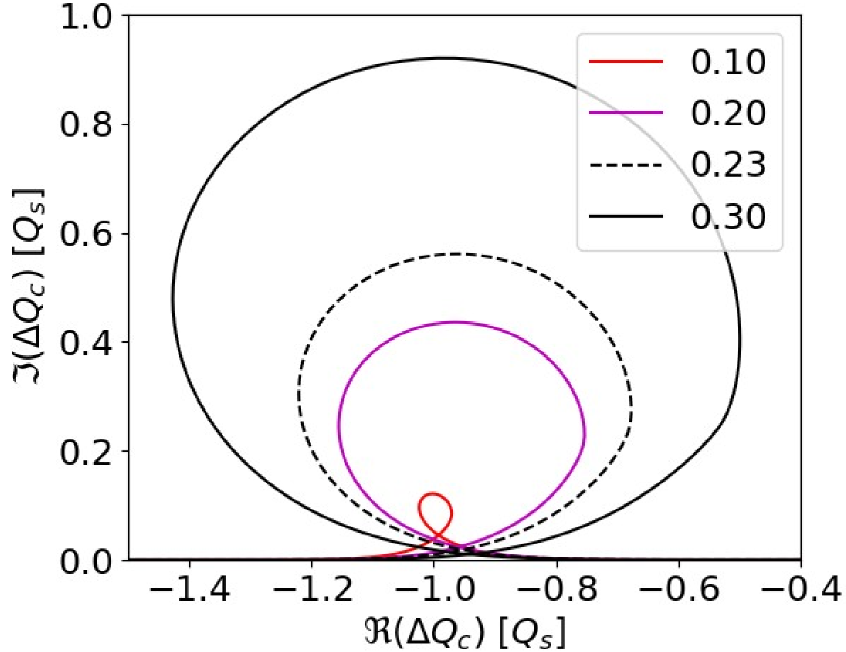

0.025
0.050
0.075
0.100
0.10
0.20
0.23
0.30
$\Im(\Delta Q_c)$ $[Q_s]$
$\Re(\Delta Q_c)$ $[Q_s]$
0.14
0.12
0.10
0.08
0.06
0.04
0.02
0.00
−1.04
−1.02
−1.00
−0.98
−0.96
1.0
0.8
0.6
0.4
0.2
0.0
−1.4
−1.2
−1.0
−0.8
−0.6
−0.4

- The loops in the reconstructed stability diagram originate from the response of the azimuthal modes, which acquire a dipole moment due to chromaticity
  - Comparsion to LHC data shows a reasonably good agreement

[Claudia's thesis]







The loops explained

- The loops in the reconstructed stability diagram originate from the response of the azimuthal modes, which acquire a dipole moment due to chromaticity
  - Comparsion to LHC data shows a reasonably good agreement

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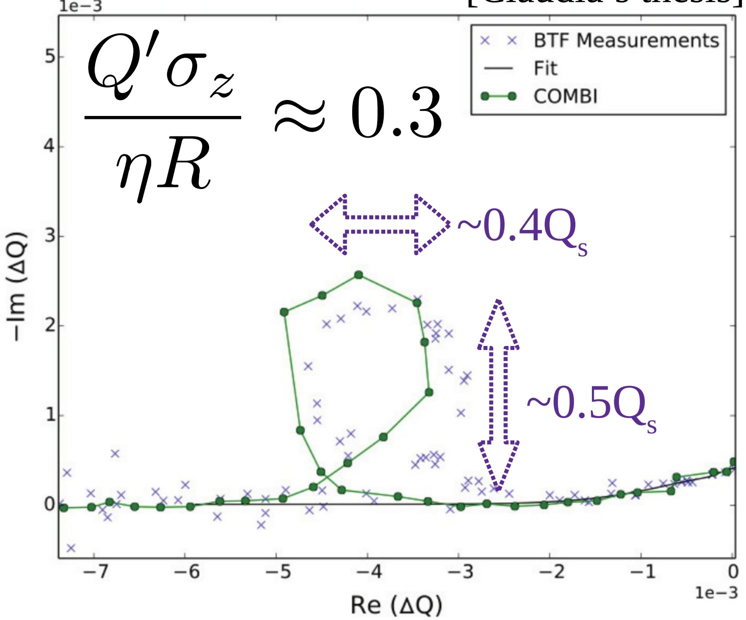
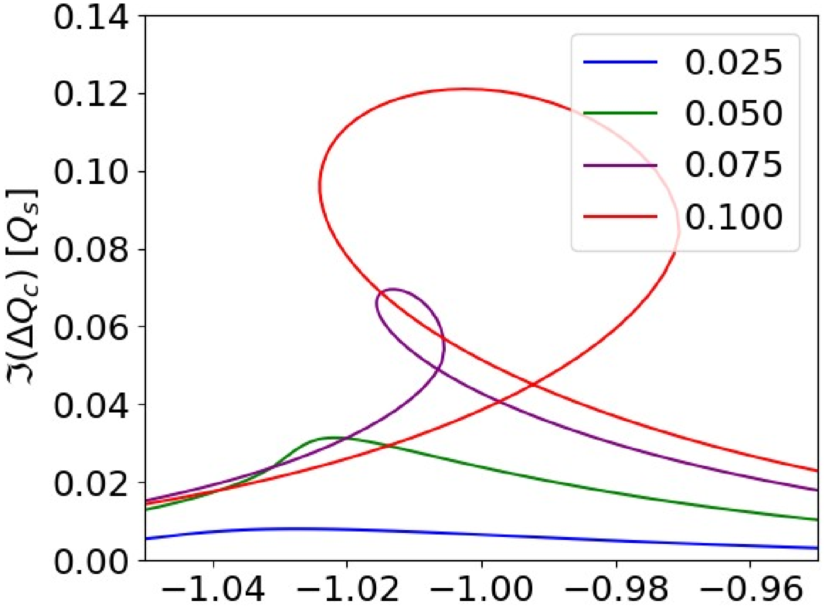
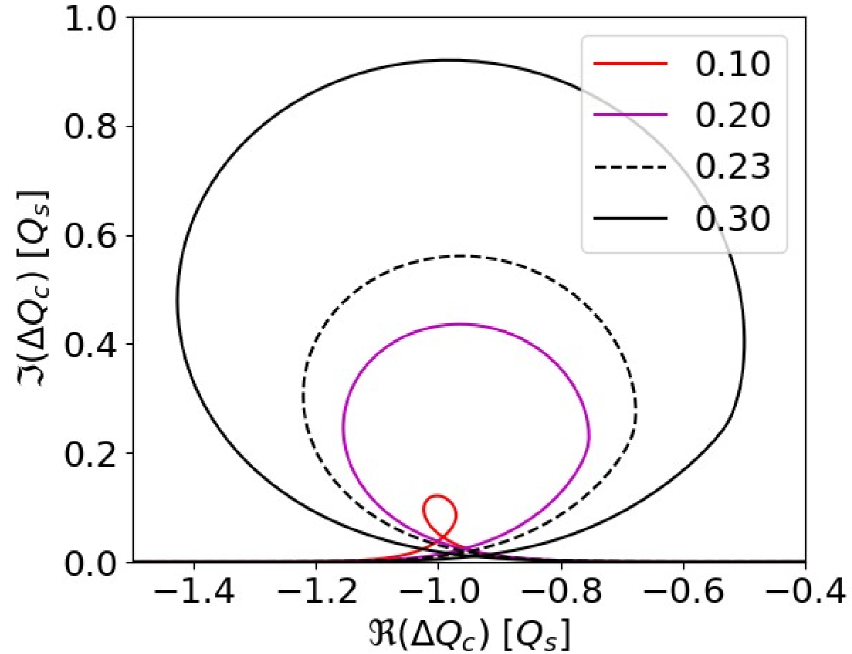
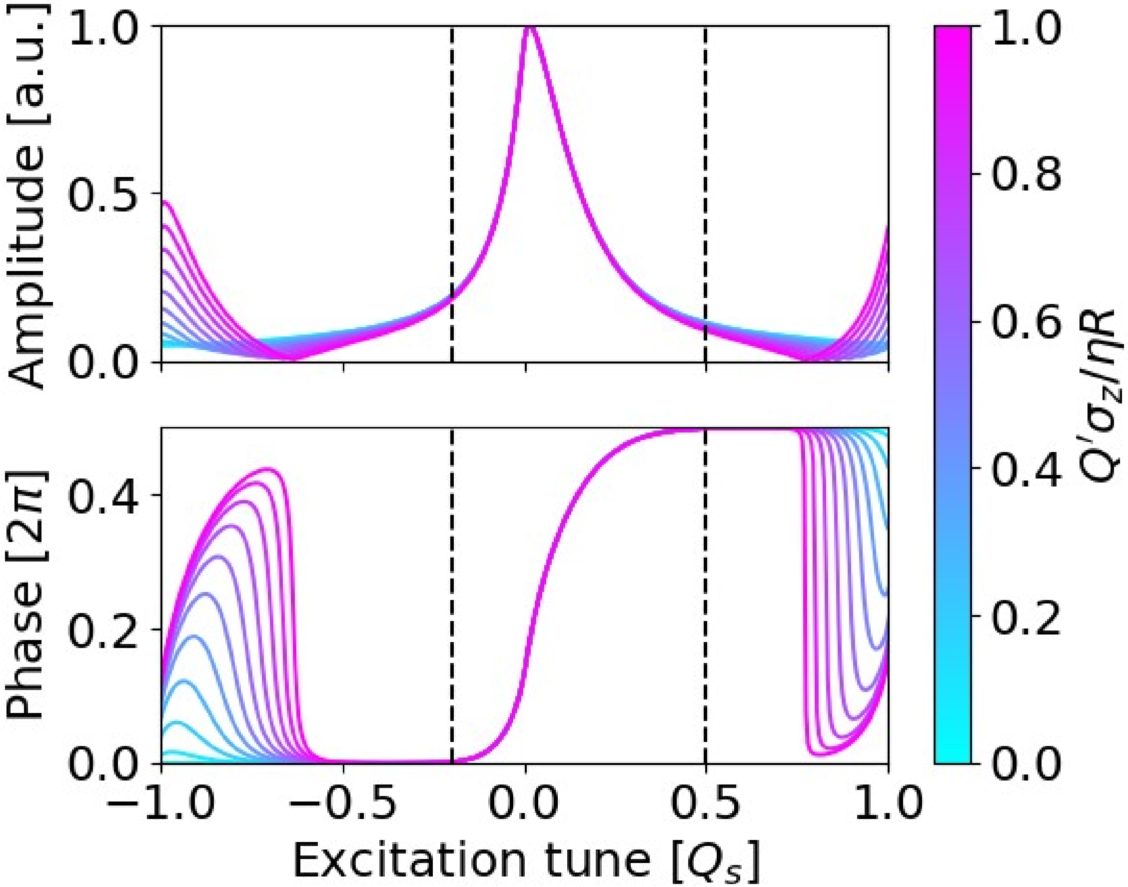






Amplitude scaling

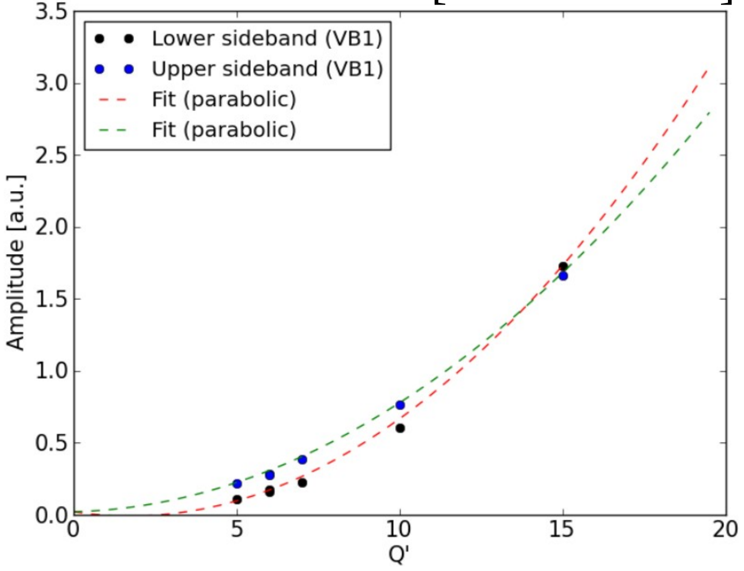


- Experimentally, the calibration of the amplitude of oscillation is not always trivial. For example in [Claudia's thesis], the stability diagram is reconstructed based on a fit on the amplitude and phase, normalising the amplitude of the BTF to its maximum

→ This method is well justified a posteriori since the region of interest is not affected by the chromaticity (if the tune spead is much smaller than Q_s)

Chromaticity measurement using the synchrotron sideband amplitude ratio

[BTF MD note]



[BTF MD note] C. Tambasco, et al., MD 382: Beam Transfer Function and diffusion mechanisms, CERN-ACC-NOTE-2016-0016

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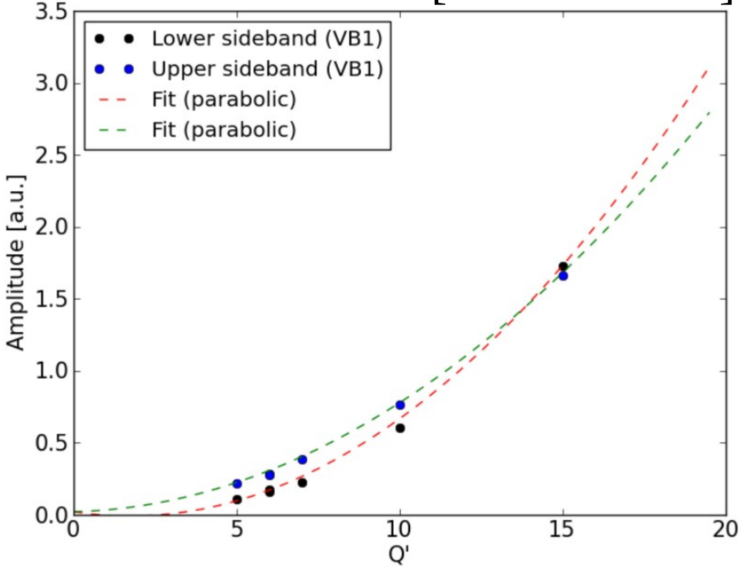

- Assuming that the amplitude of a given sideband is affected by two azimuthal modes only:

$$A_{\pm}(Q') = \frac{w_0(Q')D_0(Q_0 \pm Q_s) + w_{\pm 1}(Q')D_{\pm 1}(Q_0 \pm Q_s)}{w_0(Q')D_0(Q_0)}$$

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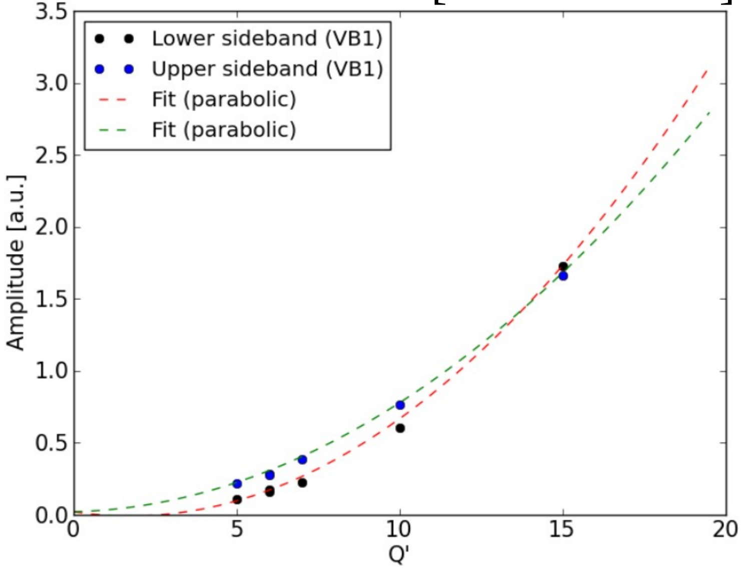

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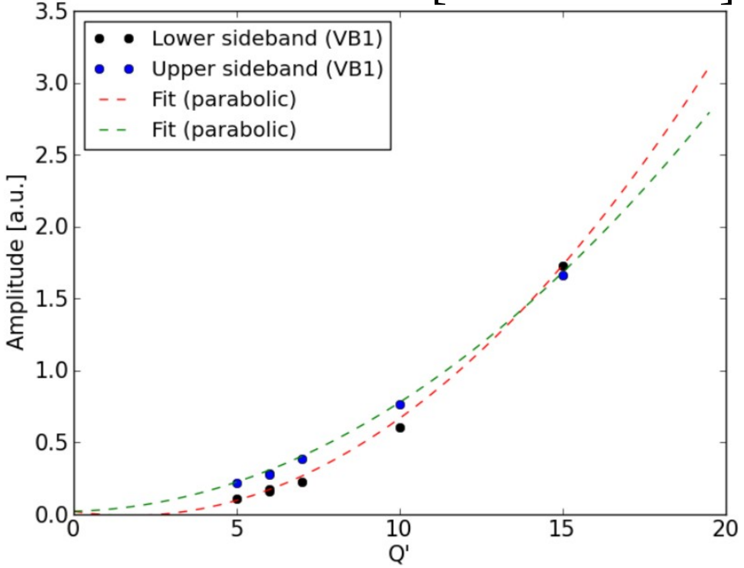

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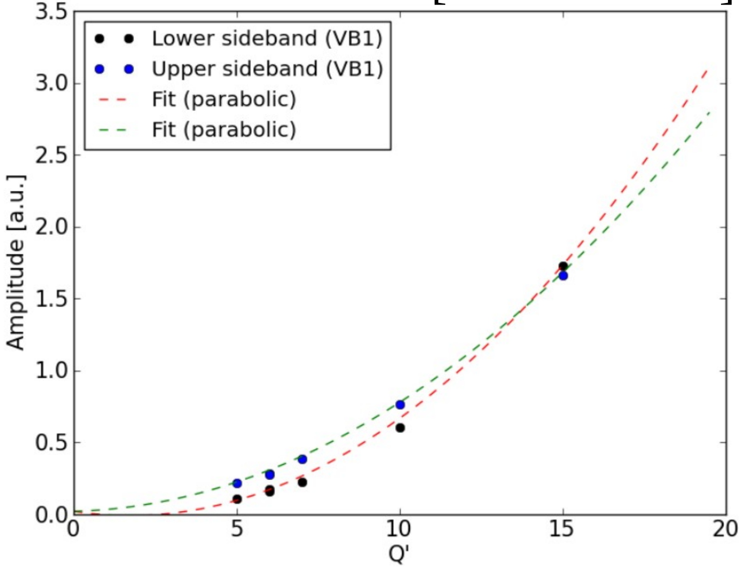

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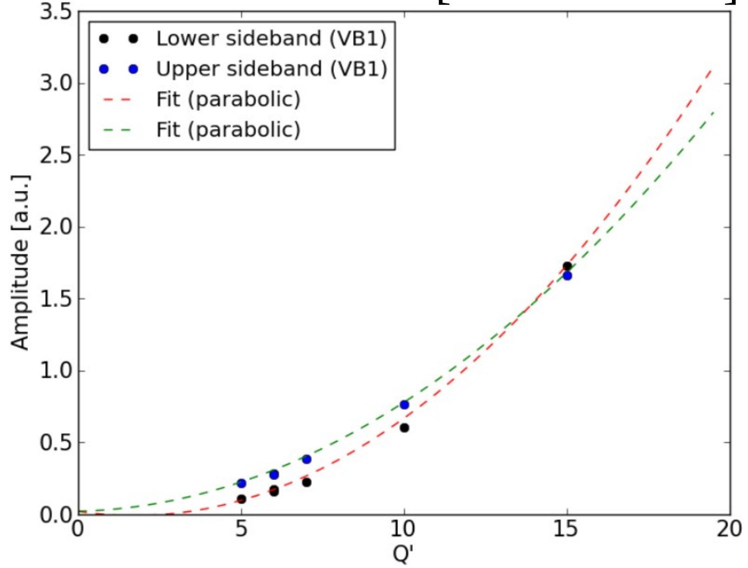

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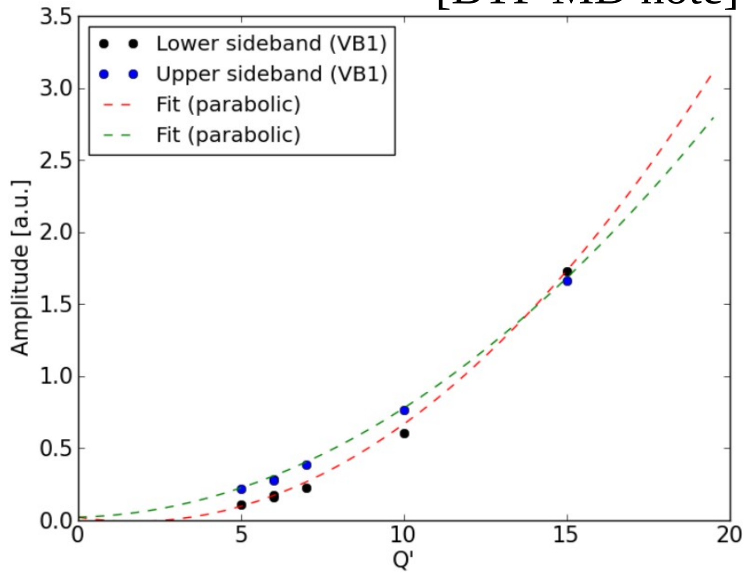

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→ Tune spread much smaller than Q_s

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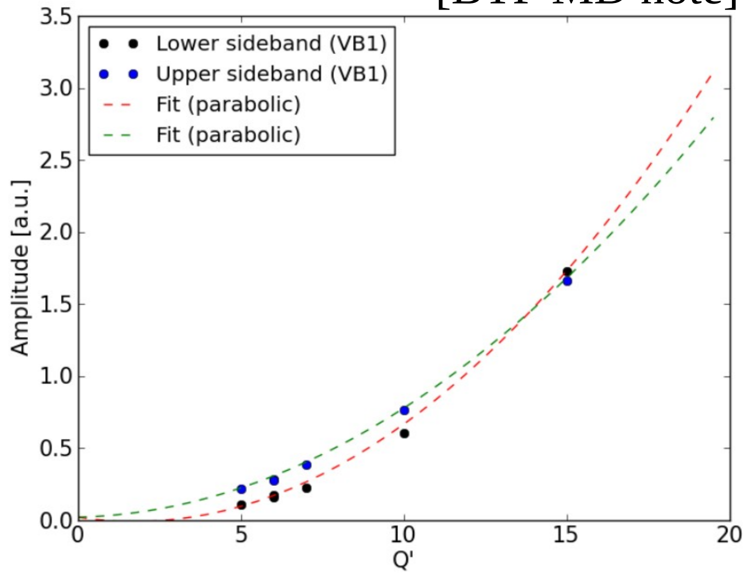

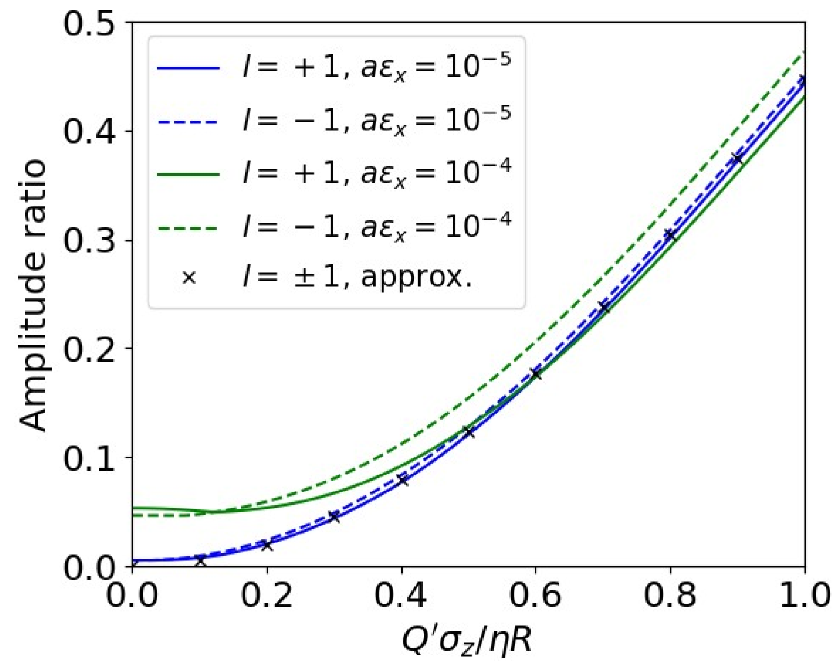

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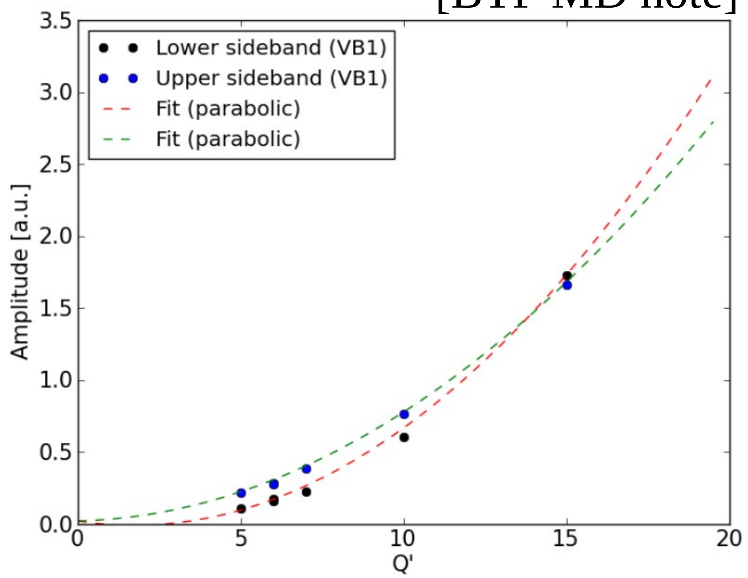

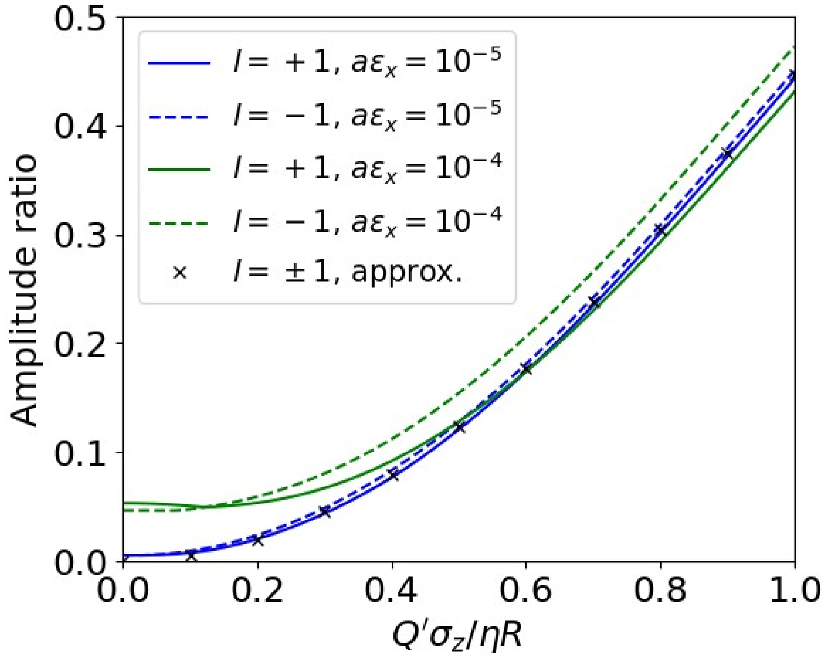

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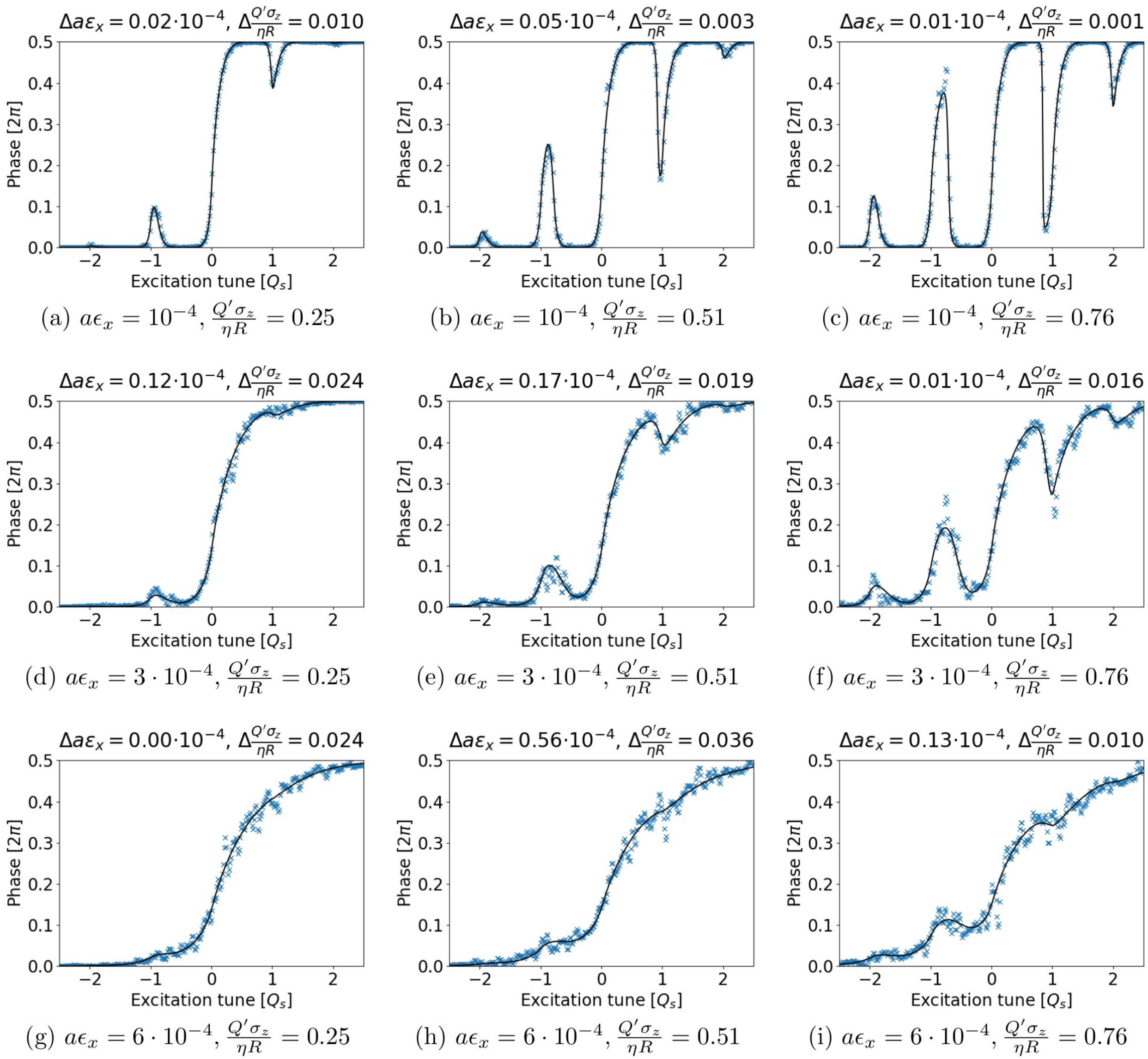
→ Tune spread much smaller than Q_s

- The amplitude ratio can indeed be used to measure the chromaticity

→ The dependence is a ratio of Bessel functions, not quadratic as claimed in [BTF MD note]

[BTF MD note] C. Tambasco, et al., MD 382: Beam Transfer Function and diffusion mechanisms, CERN-ACC-NOTE-2016-0016

Non-linear fit on COMBI output



(a) $a\epsilon_x = 10^{-4}$, $\frac{Q'\sigma_z}{\eta R} = 0.25$ (b) $a\epsilon_x = 10^{-4}$, $\frac{Q'\sigma_z}{\eta R} = 0.51$ (c) $a\epsilon_x = 10^{-4}$, $\frac{Q'\sigma_z}{\eta R} = 0.76$

(d) $a\epsilon_x = 3\cdot10^{-4}$, $\frac{Q'\sigma_z}{\eta R} = 0.25$ (e) $a\epsilon_x = 3\cdot10^{-4}$, $\frac{Q'\sigma_z}{\eta R} = 0.51$ (f) $a\epsilon_x = 3\cdot10^{-4}$, $\frac{Q'\sigma_z}{\eta R} = 0.76$

(g) $a\epsilon_x = 6\cdot10^{-4}$, $\frac{Q'\sigma_z}{\eta R} = 0.25$ (h) $a\epsilon_x = 6\cdot10^{-4}$, $\frac{Q'\sigma_z}{\eta R} = 0.51$ (i) $a\epsilon_x = 6\cdot10^{-4}$, $\frac{Q'\sigma_z}{\eta R} = 0.76$

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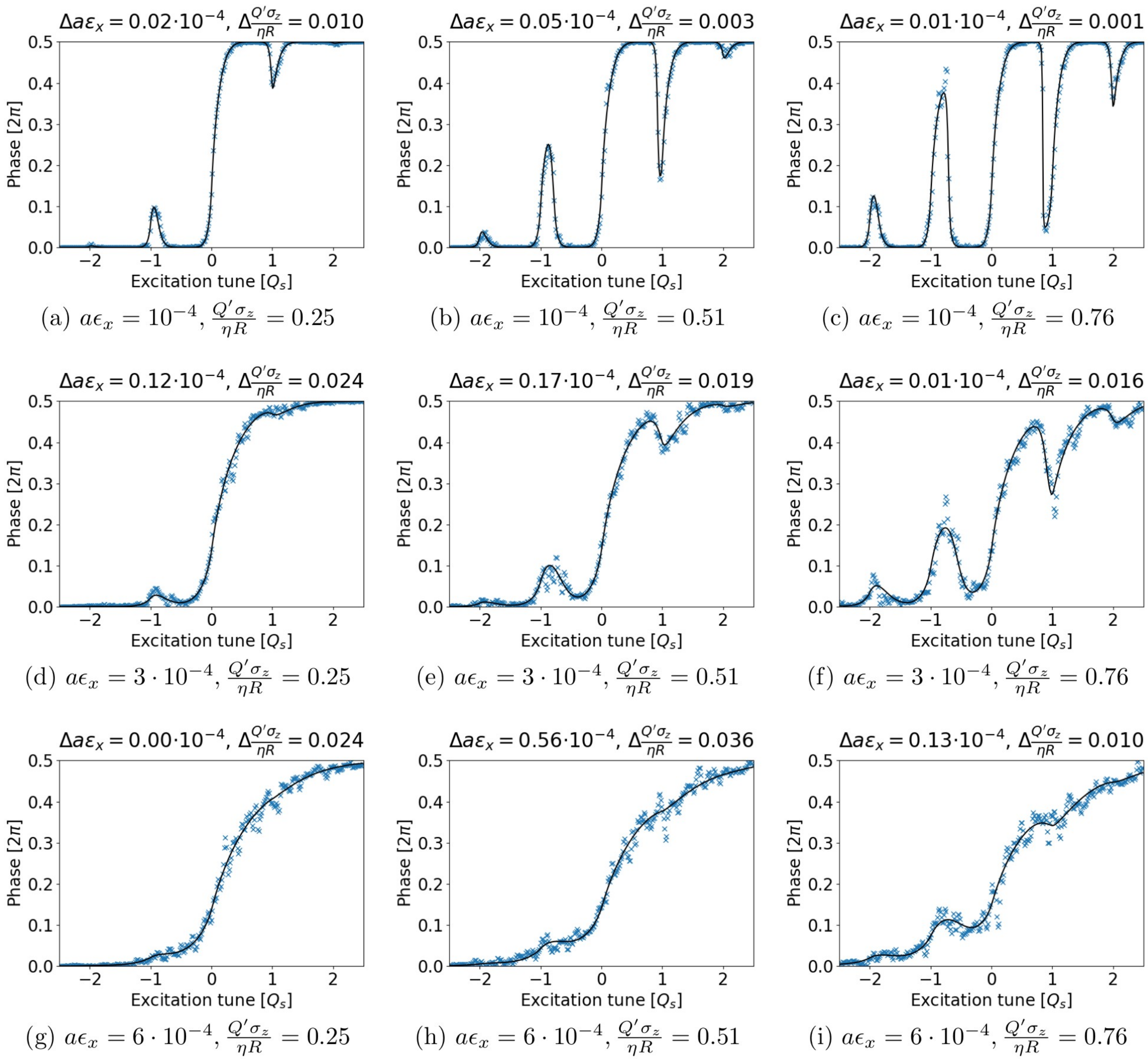

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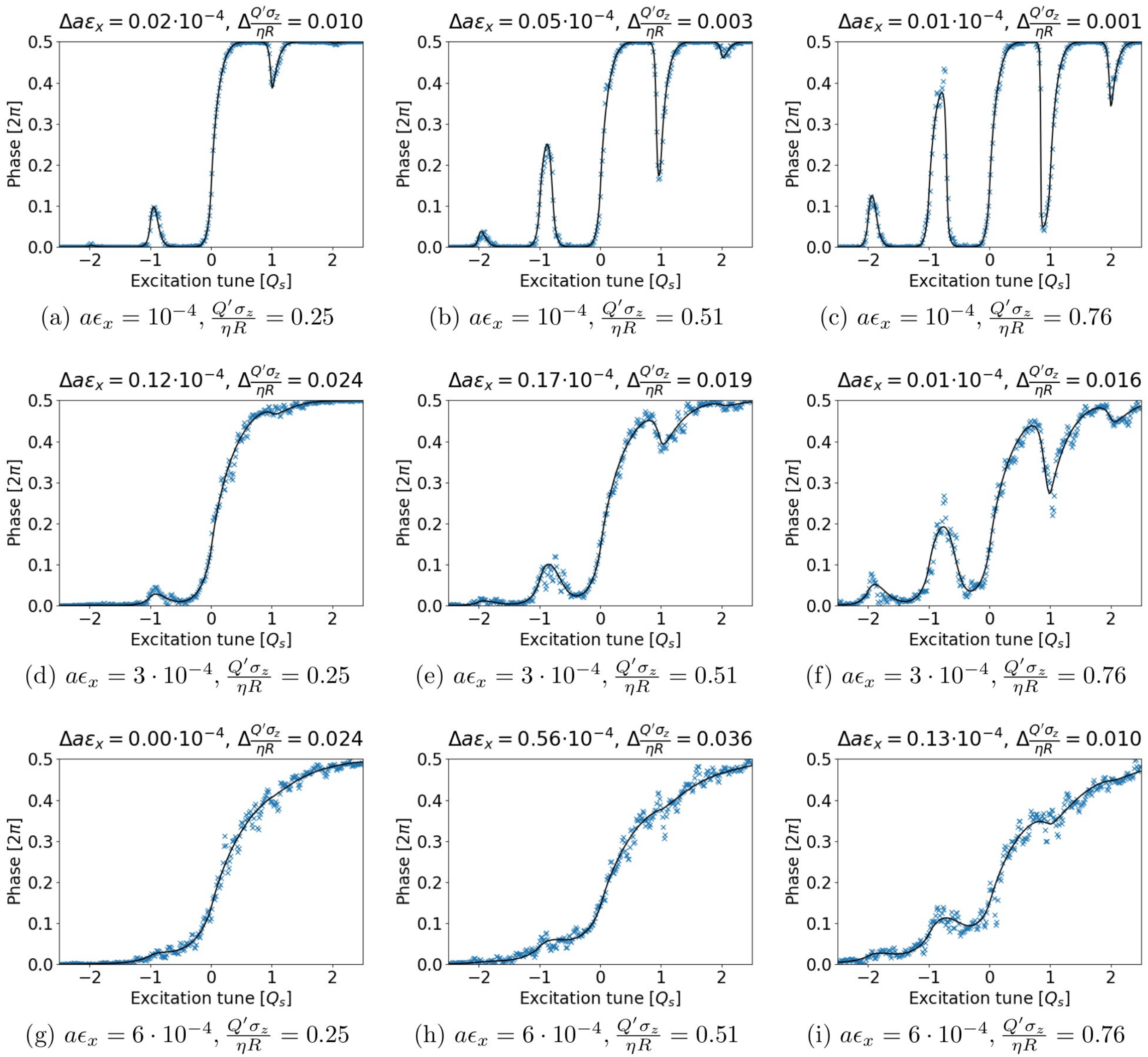
LHC flat top
I_{oct} ~ 570 A
Q' ~ 15

Non-linear fit on COMBI output

LHC flat top
I_{oct} = 570 A
Q' = 5



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Non-linear fit on COMBI output

LHC flat top
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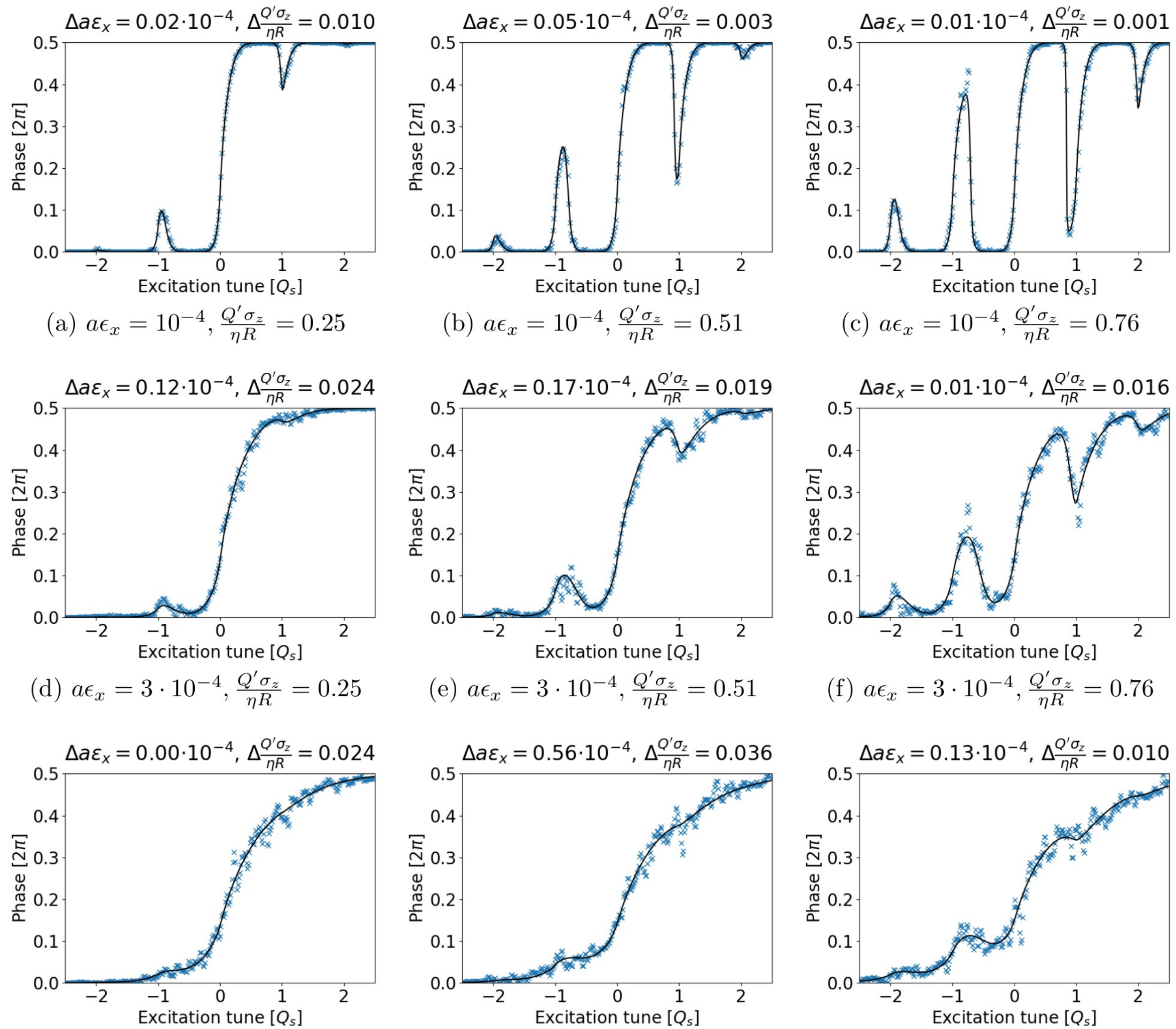
LHC flat top
I_{oct} ~ 570 A
Q' ~ 15

LHC injection
I_{oct} ~ 20 A
Q' ~ 4

LHC injection
I_{oct} ~ 20 A
Q' ~ 13



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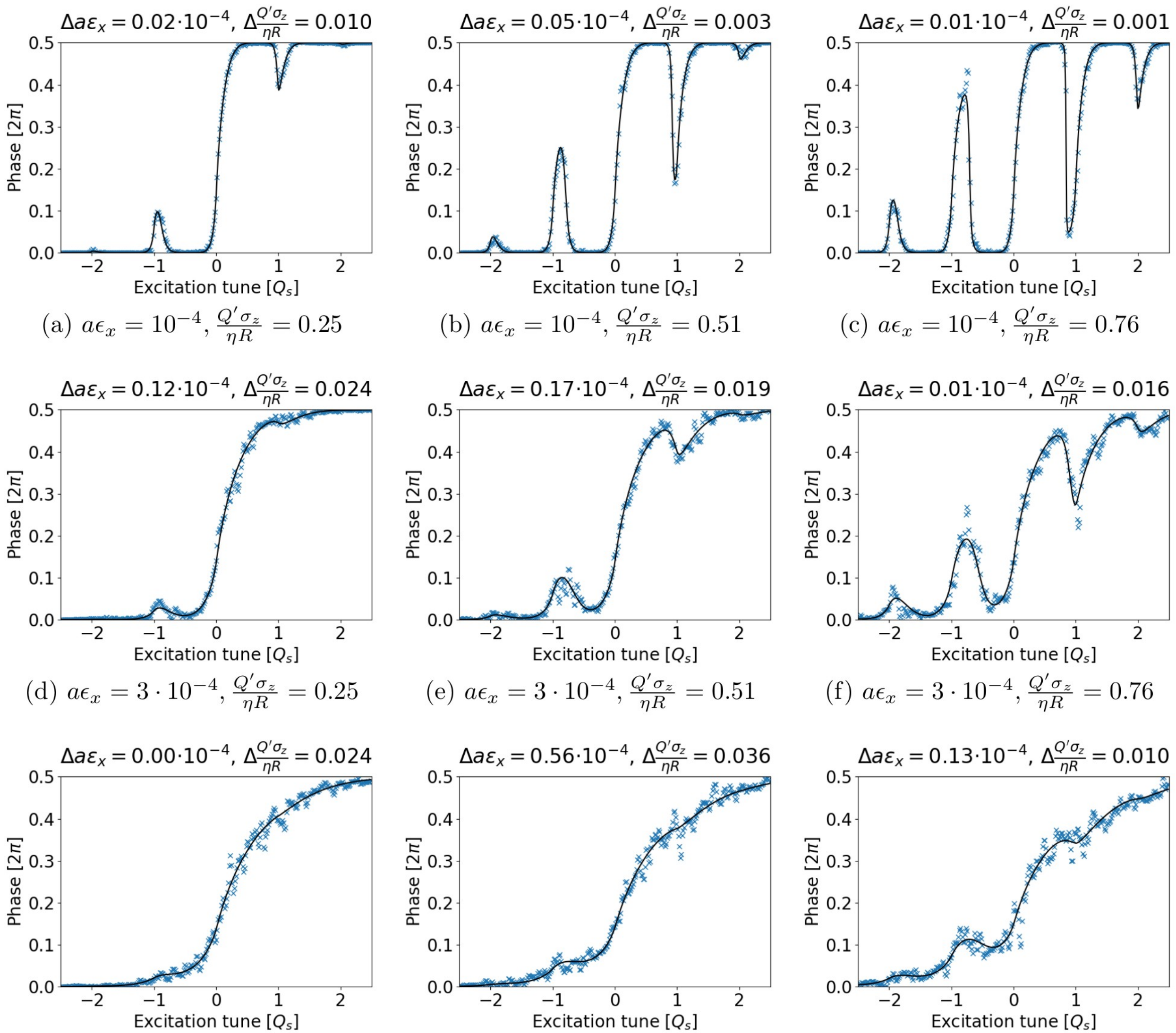

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- The BTF phase is particularly well suited to recover tune spread and chromatic phase from a non-linear fit
  - No dependence on calibration of the excitation / measurement amplitude
  - Strong sensitivity to the sidebands over a wide range of parameters

Summary and outlook

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- A simple expression for the BTF in the presence of chromaticity was derived in the framework of Vlasov perturbation theory
  - It was successfully benchmarked against macroparticle simulations
  - It explains the existence of loops in the stability diagram reconstructed from measured BTFs
  - This theory does not permit a link between the loops and Landau damping
  - It was shown that the ratio of the amplitude of the first synchrotron sidebands to the one of the central peak only depends on the chromatic phase in some conditions → potential for chromaticity measurement
  - The non-linear fit of the analytical model to measured BTF phase seems promising to measure chromaticity in a wide range of paramters

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- Next steps:
  - Include the impact of the damper (operationally, it would be easier to implement in the LHC if the ADT doesn't have to be switched off)
  - Include the impact of the wake fields

    → Important to understand whether such measurement remains reliable with high intensity beams

    → The BTF with wake fields is a key to make the link between the theories of diffusion driven by noise and wake fields in [Sondre] and [Lebedev].

[Sondre] S. V. Furuseth and X. Buffat. Loss of transverse landau damping by noise and wakefield driven diffusion. Phys. Rev. Accel. Beams, 23:114401, Nov 2020
[Lebedev] V. Lebedev, Transverse dampers with ultimate gain for suppression of instabilities in large hadron colliders, Proceedings of the ICFA mini-Workshop on Mitigation of Coherent Beam Instabilities in Particle Accelerators, Zermatt, 2019