Resummation benchmark: scale variations

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How low in q_T?

SMALL TRANSVERSE MOMENTUM DISTRIBUTIONS IN HARD PROCESSES

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The effects of soft gluon brehmsstrahlung on the $k_\perp$ distributions of μ pairs produced in hadron–hadron collisions are studied using the Block–Nordsiek method. At moderate energies we obtain a good fit to present experimental data by adjusting the values of two phenomenological parameters. At quite large energies the predictions are independent of specific values assigned to the parameters and the whole $p_\perp$ distribution, including $p_\perp \simeq 0$, turns out to be computable.

How low in q_T?

Momentum-space resummation for transverse observables and the Higgs $p_\perp$ at N^{3}LL+NNLO

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The resummation of the p_t spectrum of a heavy colour singlet was first analysed in the seminal work by Parisi and Petronzio [13], where it was shown that in the low-p_t region the spectrum vanishes as $d\sigma/dp_t \sim p_t$, instead of vanishing exponentially as suggested by Sudakov suppression. This power-law behaviour is due to configurations in which p_t vanishes due to cancellations among the non-vanishing transverse momenta of all emissions. Around and below the peak of the distribution, this mechanism dominates with respect to kinematical configurations where p_t becomes small due to all the emissions having a small transverse momentum, i.e. the configurations which would yield an exponential suppression. In order to properly deal with these two competing mechanisms, in ref. [14] it was proposed to perform the resummation in the impact-parameter (b) space, where both effects leading to a vanishing p_t are handled through a Fourier transform.

- To rephrase it: the $q_T \simeq 0$ region (below the peak) is governed by finite-k_T emissions that cancel to give $q_T \simeq 0$. Therefore the perturbative component in this region is *computable* with around the same perturbative accuracy of the peak region.

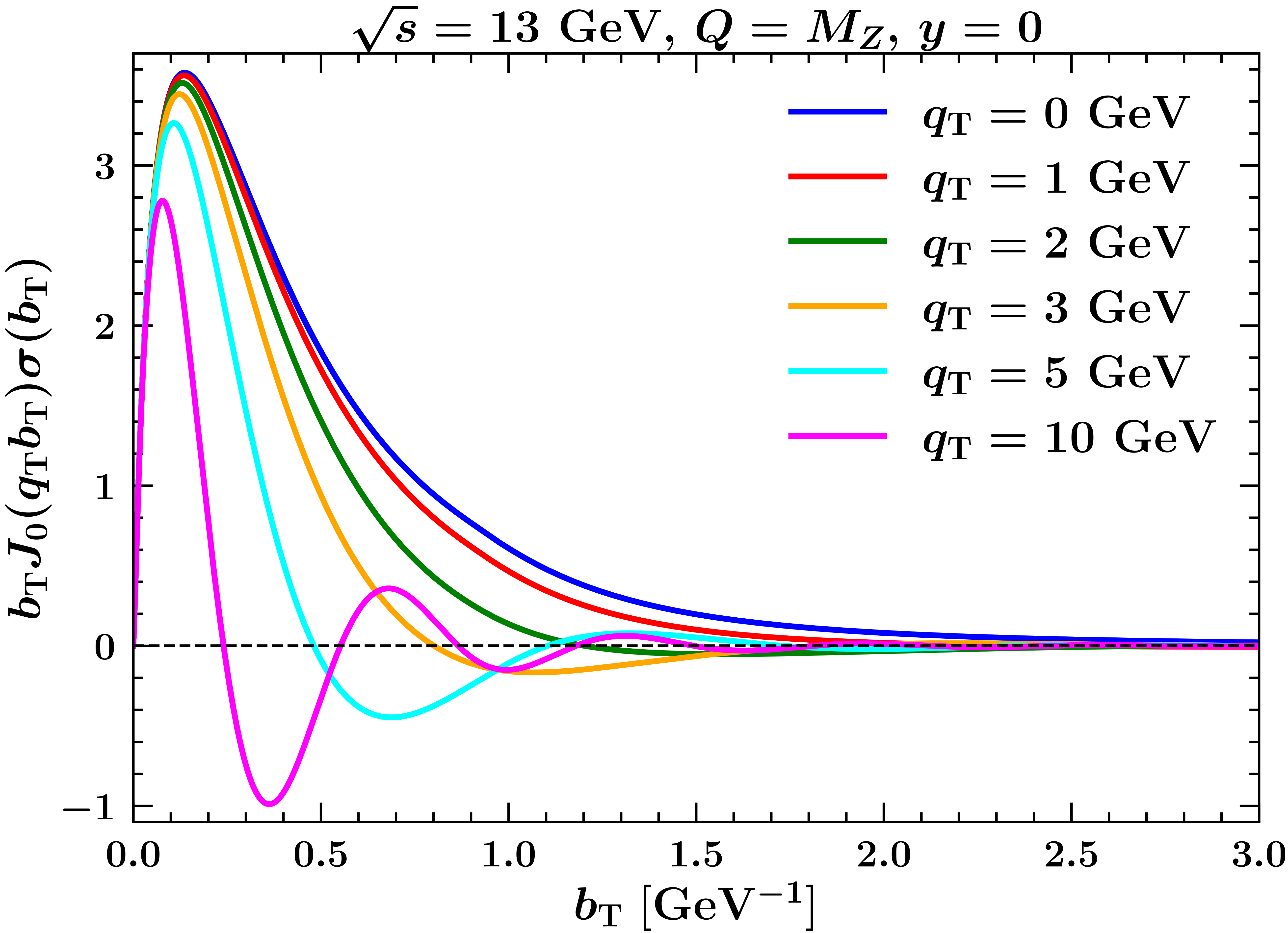
- Put differently: the perturbative uncertainty **should not** explode for $q_T \to 0$.

The cross section

- In impact parameter space:

$$\frac{d\sigma}{dy\,dM\,dq_T^2} \propto \int_0^\infty db_T\, b_T J_0(q_T b_T)\widetilde{\sigma}(b_T)$$

- For fixed y and $M \gg \Lambda_{QCD}$ the q_T dependence is **entirely** driven by the modulation of the integrand given by the Bessel function.



Integrating these curves between 0 and ∞ gives the distribution in dq_T^2.

The cross section

- In impact parameter space:

$$\frac{d\sigma}{dy\,dM\,dq_T^2} \propto \int_0^\infty db_T\, b_T J_0(q_T b_T)\tilde{\sigma}(b_T)$$

- For fixed y and $M \gg \Lambda_{QCD}$ the q_T dependence is **entirely** driven by the modulation of the integrand given by the Bessel function.
- The resummation pattern implies that at small q_T the perturbative uncertainty of the integrand scales as:

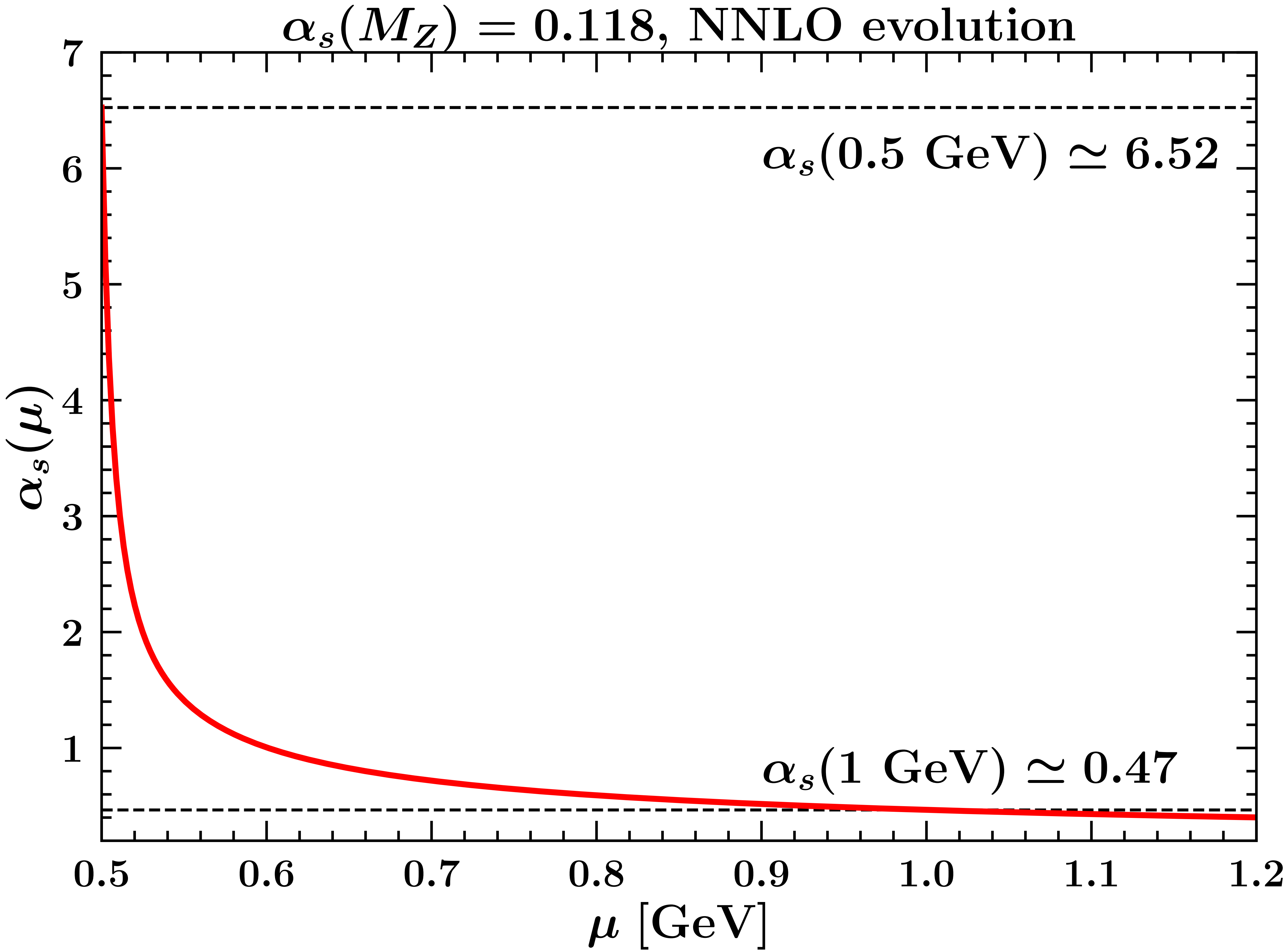
$$\text{LL}: \qquad \tilde{\sigma}(b_T)\left\{1 + \mathcal{O}\left[\alpha_s\left(\frac{b_0}{b_*(b_T)}\right)/\alpha_s(M)\right]\right\}$$

$$\text{N}^k\text{LL}: \qquad \tilde{\sigma}(b_T)\left\{1 + \mathcal{O}\left[\alpha_s^k\left(\frac{b_0}{b_*(b_T)}\right) - \alpha_s^k(M)\right]\right\}$$

- Since $\alpha_s(M) \ll 1$, for the computation to be asymptotically convergent:

$$\alpha_s\left(\frac{b_0}{b_*(b_T)}\right) \ll 1 \quad \forall\, b_T \quad \Rightarrow \quad \boxed{\lim_{b_T\to\infty}\frac{b_0}{b_*(b_T)} \gg \Lambda_{QCD}} \quad \Rightarrow \quad b_{\max} \ll \frac{1}{\Lambda_{QCD}}$$

For *all* scale choices



- If the argument of α_s is allowed to take too small values, any estimate of the perturbative uncertainty is hardly meaningful.

Exercise

- Based of this pattern:

$$\text{LL}: \qquad \widetilde{\sigma}(b_{\rm T})\left\{1+\mathcal{O}\left[\alpha_s\left(\frac{b_0}{b_*(b_T)}\right)/\alpha_s(M)\right]\right\}$$

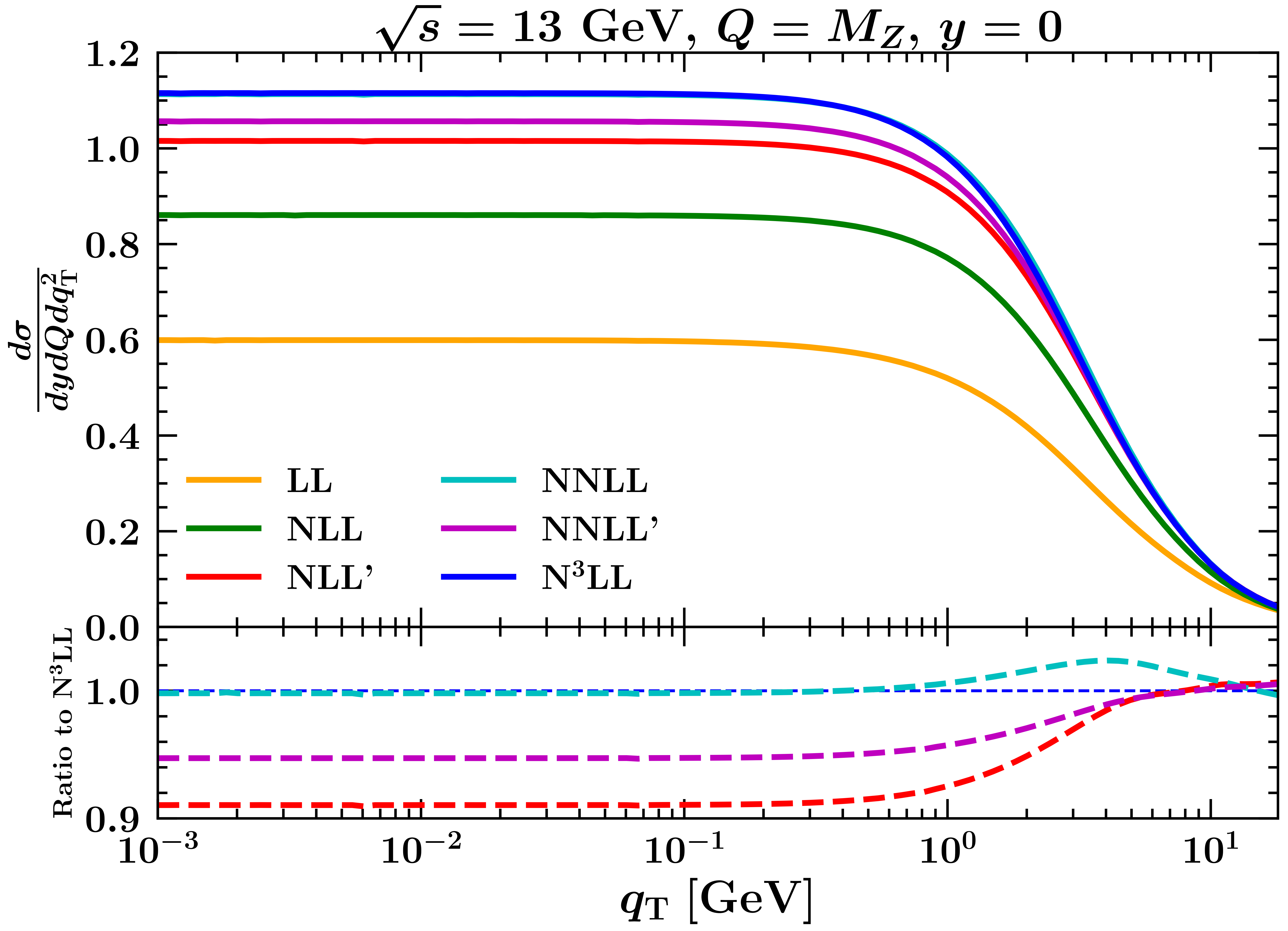
$$\text{N}^k\text{LL}: \qquad \widetilde{\sigma}(b_{\rm T})\left\{1+\mathcal{O}\left[\alpha_s^k\left(\frac{b_0}{b_*(b_T)}\right)-\alpha_s^k(M)\right]\right\}$$

- We chose a kinematic configuration ($Q = M$, $y = 0$) and made the shift:

$$\text{LL}: \qquad \widetilde{\sigma}(b_{\rm T})\left\{1+K\left[\alpha_s\left(\frac{b_0}{b_*(b_T)}\right)/\alpha_s(M)\right]\right\}$$

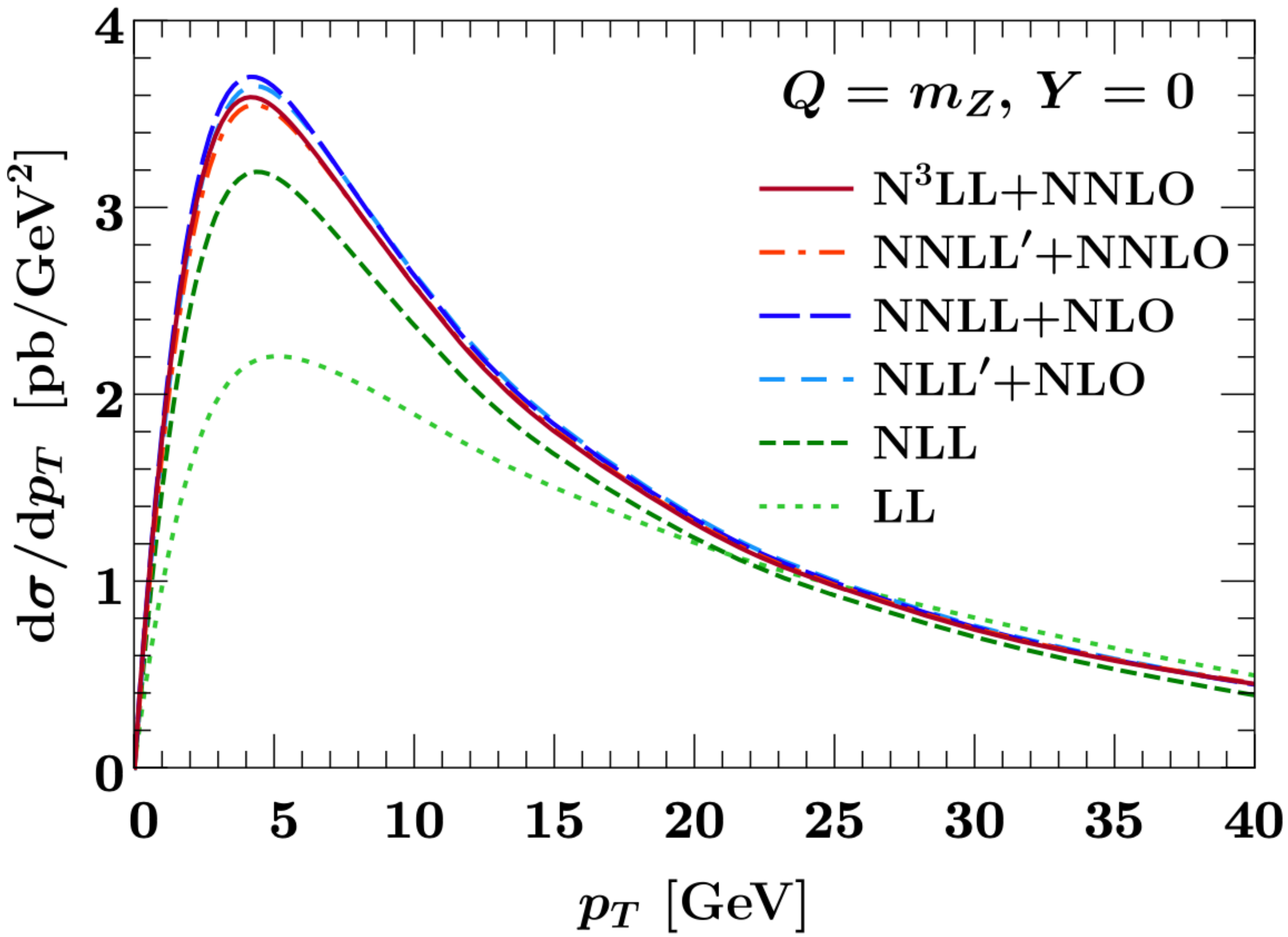
$$\text{N}^k\text{LL}: \qquad \widetilde{\sigma}(b_{\rm T})\left\{1+K\left[\alpha_s^k\left(\frac{b_0}{b_*(b_T)}\right)-\alpha_s^k(M)\right]\right\}$$

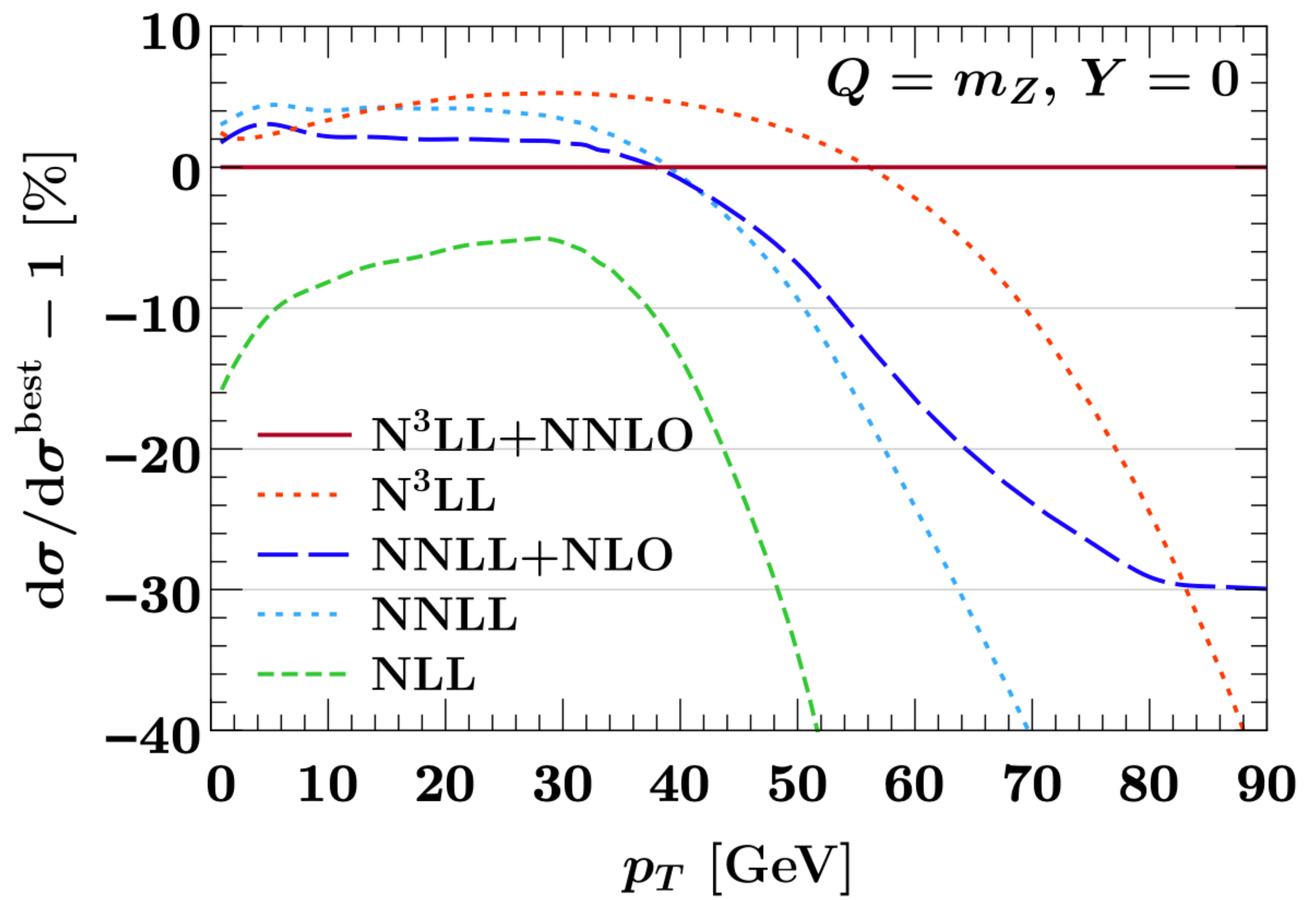
- K is computable but it should be around **one** at all orders:
  - unless **renormalon** effects (factorially growing perturbative coefficients) are getting large, but this does not seem the case up to N^{3}LL given the observed perturbative convergence when moving from LL to N^{3}LL.
  - Given the perturbative convergence at low $q_{\rm T}$, how comes that perturbative uncertainties remain so large?



$\sqrt{s} = 13$ GeV, $Q = M_Z$, $y = 0$
$\frac{d\sigma}{dy dQ dq_T^2}$
1.2
1.0
0.8
0.6
0.4
0.2
0.0
LL
NLL
NLL'
NNLL
NNLL'
N^{3}LL
Ratio to N^{3}LL
1.0
0.9
10^{-3}
10^{-2}
10^{-1}
10^{0}
10^{1}
q_T [GeV]

[F. Tackmann, EWWG meeting 12/11/2020]





Exercise

- Based of this pattern:

$$\text{LL}: \qquad \widetilde{\sigma}(b_{\text{T}})\left\{1+\mathcal{O}\left[\alpha_s\left(\frac{b_0}{b_*(b_T)}\right)/\alpha_s(M)\right]\right\}$$

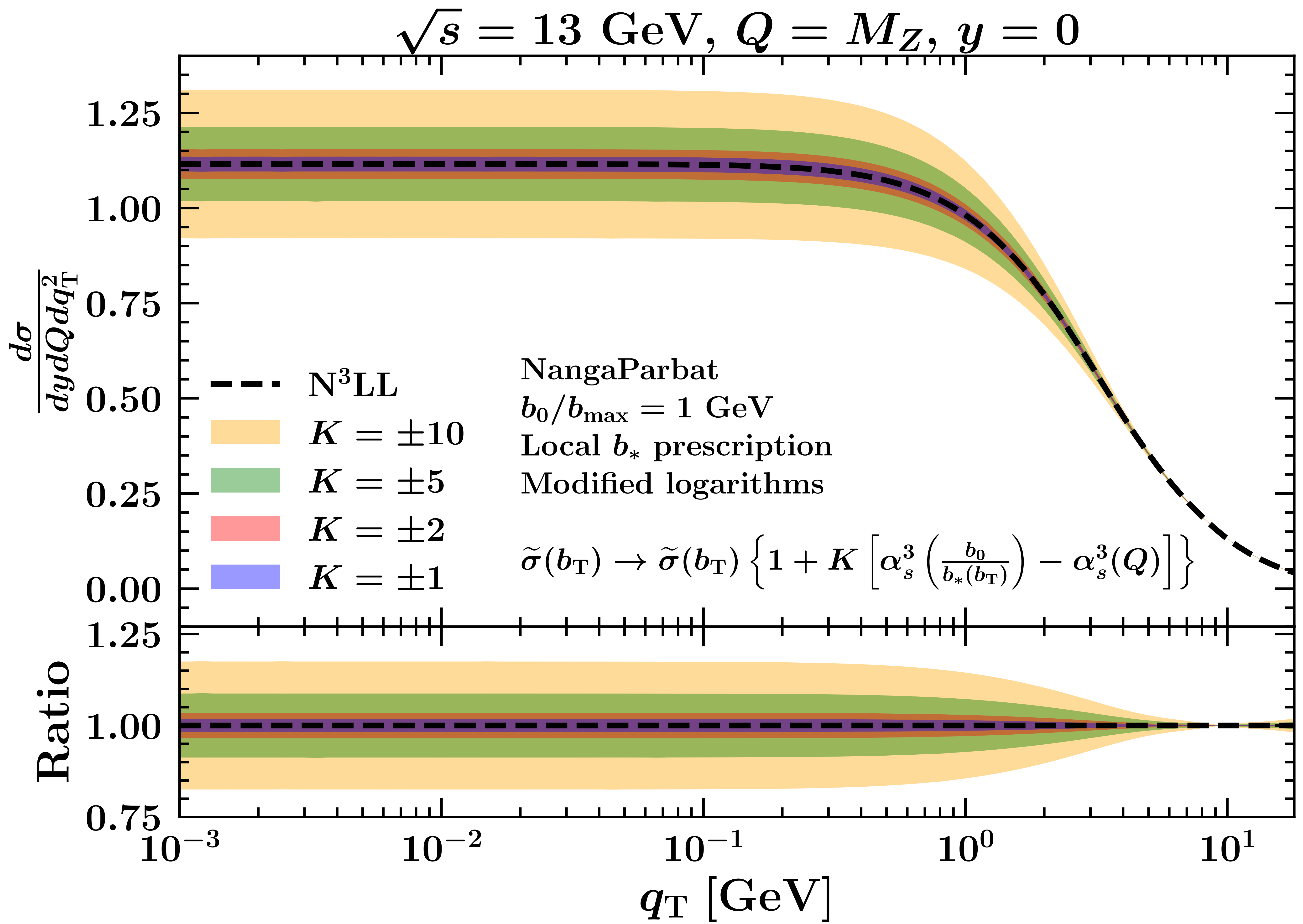
$$\text{N}^k\text{LL}: \qquad \widetilde{\sigma}(b_{\text{T}})\left\{1+\mathcal{O}\left[\alpha_s^k\left(\frac{b_0}{b_*(b_T)}\right)-\alpha_s^k(M)\right]\right\}$$

- We chose a kinematic configuration ($Q = M$, $y = 0$) and made the shift:

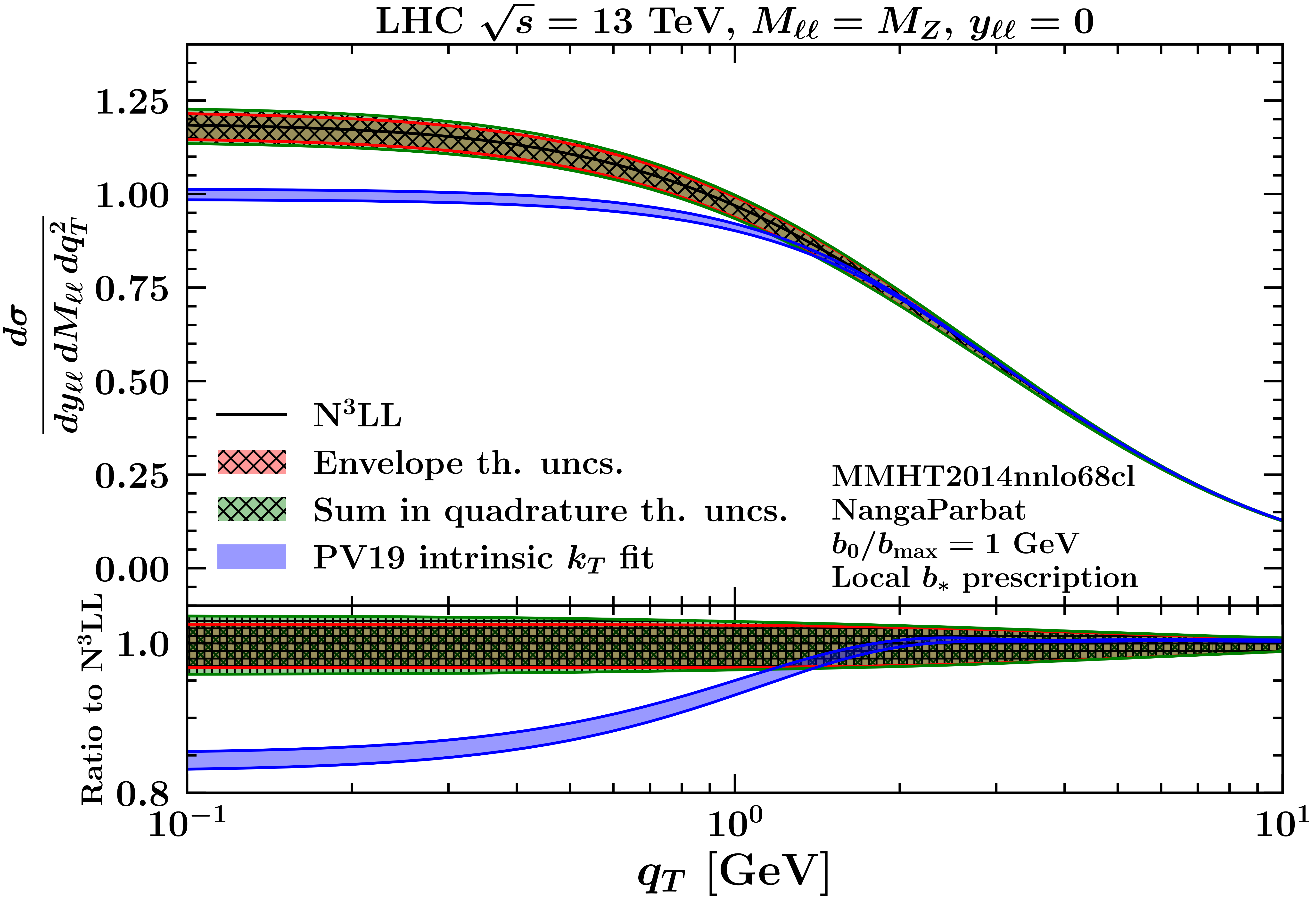
$$\text{LL}: \qquad \widetilde{\sigma}(b_{\text{T}})\left\{1+K\left[\alpha_s\left(\frac{b_0}{b_*(b_T)}\right)/\alpha_s(M)\right]\right\}$$

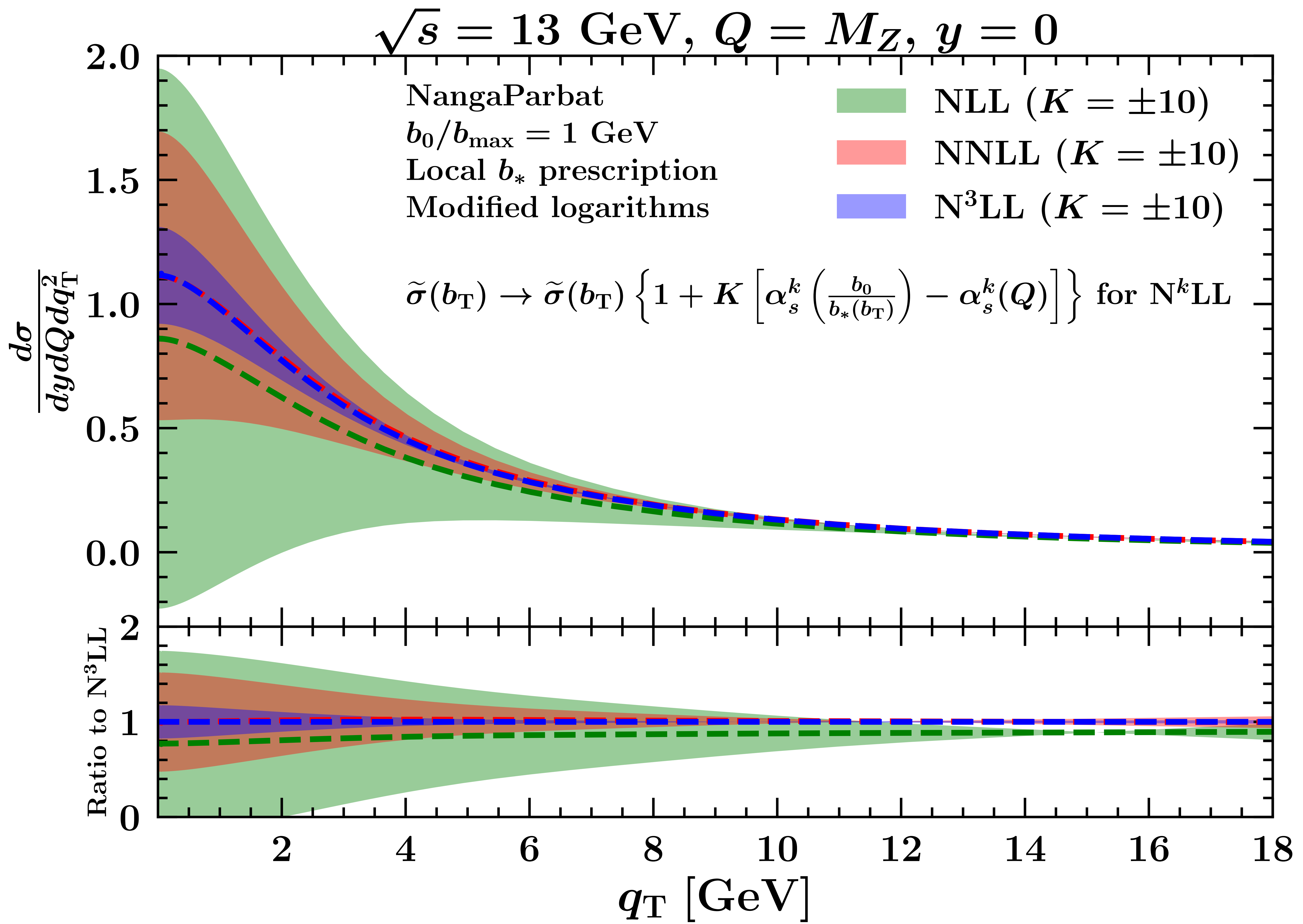
$$\text{N}^k\text{LL}: \qquad \widetilde{\sigma}(b_{\text{T}})\left\{1+K\left[\alpha_s^k\left(\frac{b_0}{b_*(b_T)}\right)-\alpha_s^k(M)\right]\right\}$$

- ***K*** is computable but it should be around **one** at all orders:
  - unless **renormalon** effects (factorially growing perturbative coefficients) are getting large, but this does not seem the case up to N^3LL given the observed perturbative convergence when moving from LL to N^3LL.
- The main goal of this exercise is not to estimate ***K***, but to study the **behaviour** of perturbative uncertainties on the cross section as $q_{\text{T}} \to 0$.

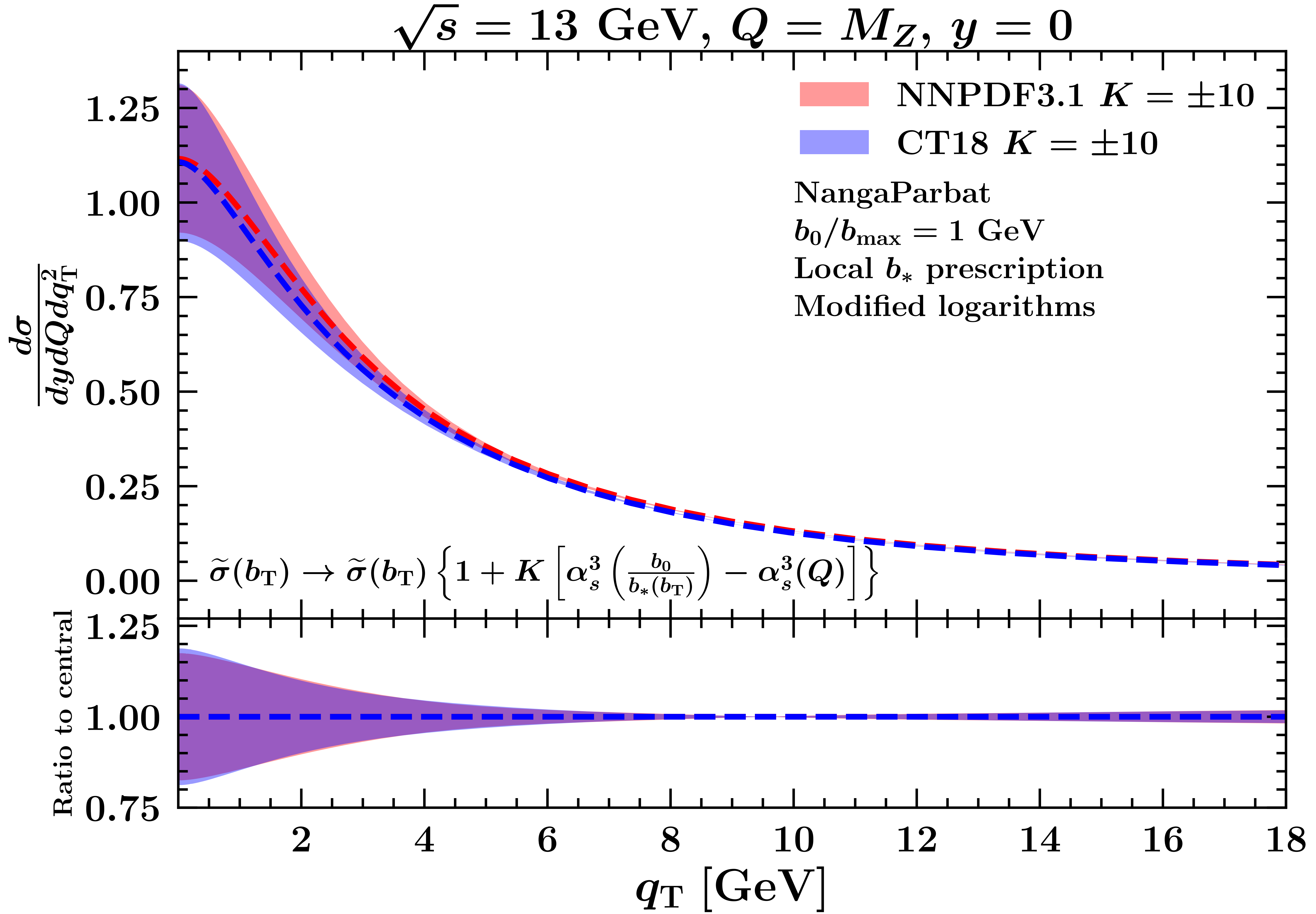


$\sqrt{s} = 13$ GeV, $Q = M_Z$, $y = 0$
$\frac{d\sigma}{dy dQ dq_T^2}$
Ratio
q_T [GeV]
N^{3}LL
$K = \pm 10$
$K = \pm 5$
$K = \pm 2$
$K = \pm 1$
NangaParbat
$b_0/b_{\max} = 1$ GeV
Local b_* prescription
Modified logarithms
$\widetilde{\sigma}(b_T) \to \widetilde{\sigma}(b_T)\left\{1 + K\left[\alpha_s^3\left(\frac{b_0}{b_*(b_T)}\right) - \alpha_s^3(Q)\right]\right\}$


LHC $\sqrt{s} = 13$ TeV, $M_{\ell\ell} = M_Z$, $y_{\ell\ell} = 0$
$\frac{d\sigma}{dy_{\ell\ell}\, dM_{\ell\ell}\, dq_T^2}$
1.25
1.00
0.75
0.50
0.25
0.00
N^3LL
Envelope th. uncs.
Sum in quadrature th. uncs.
PV19 intrinsic k_T fit
MMHT2014nnlo68cl
NangaParbat
$b_0/b_{\max} = 1$ GeV
Local b_* prescription
Ratio to N^3LL
1.0
0.8
10^{-1}
10^{0}
10^{1}
q_T [GeV]



$\sqrt{s} = 13$ GeV, $Q = M_Z$, $y = 0$
NangaParbat
$b_0/b_{\max} = 1$ GeV
Local b_* prescription
Modified logarithms
NLL ($K = \pm 10$)
NNLL ($K = \pm 10$)
N^{3}LL ($K = \pm 10$)
$\widetilde{\sigma}(b_T) \to \widetilde{\sigma}(b_T)\left\{1 + K\left[\alpha_s^k\left(\frac{b_0}{b_*(b_T)}\right) - \alpha_s^k(Q)\right]\right\}$ for N^{k}LL
$\frac{d\sigma}{dy dQ dq_T^2}$
Ratio to N^{3}LL
q_T [GeV]



$\sqrt{s} = 13$ GeV, $Q = M_Z$, $y = 0$
NNPDF3.1 $K = \pm 10$
CT18 $K = \pm 10$
NangaParbat
$b_0/b_{\max} = 1$ GeV
Local b_* prescription
Modified logarithms
$\widetilde{\sigma}(b_T) \to \widetilde{\sigma}(b_T)\left\{1 + K\left[\alpha_s^3\left(\frac{b_0}{b_*(b_T)}\right) - \alpha_s^3(Q)\right]\right\}$
$\frac{d\sigma}{dy dQ dq_T^2}$
Ratio to central
q_T [GeV]
1.25
1.00
0.75
0.50
0.25
0.00
1.25
1.00
0.75
2
4
6
8
10
12
14
16
18

Summary and suggestions

- Our numerical simulations seem to meet the theoretical expectation according to which perturbative uncertainties at $q_T \lesssim 1$ GeV are as under control as at $q_T \lesssim 2$ - 3 GeV.

- This pattern becomes particularly clear when looking at cross sections **differential in q_T^2** and not in q_T because the spectrum goes to constant (of course, relative differences should not depend of this):

  - we suggest to look at the cross section differential in q_T^2 possibly in **logarithmic** scale down to $q_T = 0.1$ (this was one of the settings of the benchmark).

  - It would be interesting to see if all other codes agree on this and if not why.

  - For the record, our estimate of the uncertainties at N^3LL for $Q = M$ and $y = 0$ suggests a value $\boldsymbol{K \simeq 2 - 4}$.

  - We observe **no instability** at the level of the perturbative uncertainty due to a different choice of the **PDF** set.