Gauge-invariance issues in Drell–Yan production and solutions implemented in RADY

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NLO + leading higher order:
S. Dittmaier and M. Krämer, PRD 65 (2002), 073007 [hep-ph/0109062];
S. Brensing et al., PRD 77 (2008), 073006 [arXiv:0710.3309];
S. Dittmaier and M. Huber, JHEP 01 (2010), 060 [arXiv:0911.2329];

Mixed NNLO QCD $\times$ EW:
S. Dittmaier, A. Huss and C. Schwinn, NPB 885 (2014), 318 [arXiv:1403.3216] and NPB 904 (2016), 216 [arXiv:1511.08016];
S. Dittmaier, T. Schmidt and J. Schwarz, arXiv:2009.02229, to appear in JHEP.

Features of RADY:

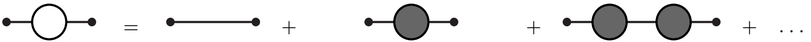
- Processes: $\mathrm{pp} \to \ell^+\ell^- + X$ and $\mathrm{pp} \to \ell^+\nu_\ell/\ell^-\bar{\nu}_\ell + X$
- Corrections:
  - NLO EW+QCD
  - universal EW corrections beyond NLO
  - higher-order FSR via structure functions
  - dominant $\mathcal{O}(\alpha_\mathrm{s}\alpha)$ corrections in resonance regions
  - off-shell $\mathcal{O}(N_f\alpha_\mathrm{s}\alpha)$ corrections
- Models: SM, MSSM, THDM, SESM

- Special features:
  - NLO corrections to $\gamma\gamma \to \ell^+\ell^-$ channel
  - various EW input schemes:
    $\{G_\mu, M_\mathrm{W}, M_\mathrm{Z}\}$, $\{\alpha(0), M_\mathrm{W}, M_\mathrm{Z}\}$, $\{\alpha(M_\mathrm{Z}), M_\mathrm{W}, M_\mathrm{Z}\}$,
    new: $\{G_\mu, \sin^2\theta^{\mathrm{lept}}_{\mathrm{eff}}, M_\mathrm{Z}\}$, $\{\alpha(0), \sin^2\theta^{\mathrm{lept}}_{\mathrm{eff}}, M_\mathrm{Z}\}$, $\{\alpha(M_\mathrm{Z}), \sin^2\theta^{\mathrm{lept}}_{\mathrm{eff}}, M_\mathrm{Z}\}$
  - different gauge-invariant resonance schemes:
    complex-mass scheme, pole scheme, factorization scheme
  - optional: leading pole expansion

Gauge invariance and treatment of resonances

for more details, see Denner, SD, 1912.06823 and refs. therein

Dyson summation of propagators mixes perturbative orders.



$$G(p^2) \;=\; \frac{\mathrm{i}}{p^2-M^2} \;+\; \frac{\mathrm{i}}{p^2-M^2}\,\mathrm{i}\Sigma_{\mathrm{R}}(p^2)\,\frac{\mathrm{i}}{p^2-M^2} \;+\; \dots$$

$$= \frac{\mathrm{i}}{p^2 - M^2 + \Sigma_{\mathrm{R}}(p^2)}, \qquad \Sigma_{\mathrm{R}}(M^2) = \mathrm{i}M\Gamma$$

But:

Consistency of pert. calculations often requires complete fixed orders.

$\hookrightarrow$ Consistency jeopardized if no special care is taken!

Gauge-invariance requirements:

- proper cancellation of gauge-parameter dependences
  (relations between self-energies, vertex corrections, boxes, etc.)
- validity of (internal) Ward identities
  (e.g. ruling cancellations for forward scattering of $\mathrm{e}^{\pm}$ or at high energies)

Required: schemes to introduce width Γ

- without breaking gauge invariance
- maintaining (at least) NLO accuracy everywhere in phase space

Some incidental remarks:

The issue of gauge invariance goes

- beyond the definition of M and Γ and also
- beyond the question of parametrizing the resonance!

It is about the consistency of amplitudes everywhere in phase space, i.e.

- on resonance,
- in off-shell regions, and
- in the transition region between on-/off-shell domains.

Fixed versus running widths:

Both deliver possible phenomenological parametrizations of resonances, which can be translated into each other. Bardin et al. '88; Beenakker et al. '96

But in quantum field theory:

- Fixed widths appear as imaginary parts of (gauge-invariant) pole locations in propagators (complex pole mass). Gambino, Grassi '99; Grassi et al. '01
- Running widths appear in the context of (non-gauge-invariant) real on-shell definitions of masses (e.g. LEP M_{Z} mass). Sirlin '91

Width schemes for corrections in a nutshell:

for more details, see Denner, SD, 1912.06823 and refs. therein

- Naive schemes

$$\frac{1}{p^2 - M^2} \to \frac{1}{p^2 - M^2 + \mathrm{i}M\Gamma(p^2)} \qquad \text{in all or at least in resonant props.}$$

Fixed-width scheme: $\Gamma(p^2) = \text{const.}$

$\hookrightarrow$ breaks gauge invariance only "mildly",
but partial inclusion of Γ in loops screws up consistency

Running-width scheme: $M\Gamma(p^2) = \mathrm{Im}\{\Sigma_\mathrm{R}(p^2)\} \neq \text{const.}$

$\hookrightarrow$ crude breaking of gauge invariance in off-shell regions,
often completely wrong results

- "Factorization Scheme" (FS) SD, Krämer '01; SD, Huber '09

Global correction factor (limit $\Gamma \to 0$) times gauge-invariant LO XS, e.g.:

$$\mathrm{d}\hat{\sigma}^{q\bar{q}\to\mathrm{Z}/\gamma\to\ell\ell}_{\mathrm{weak}} = \delta_{\mathrm{weak}}|_{\Gamma_\mathrm{Z}=0} \times \mathrm{d}\hat{\sigma}^{q\bar{q}\to\mathrm{Z}/\gamma\to\ell\ell}_{\mathrm{LO}}|_{\Gamma_\mathrm{Z}\neq 0},$$
$$\mathrm{d}\hat{\sigma}^{q\bar{q}'\to\mathrm{W}\to\ell\nu}_{\mathrm{virt}} = \delta_{\mathrm{virt}}|_{\Gamma_\mathrm{W}\to 0} \times \mathrm{d}\hat{\sigma}^{q\bar{q}'\to\mathrm{W}\to\ell\nu}_{\mathrm{LO}}|_{\Gamma_\mathrm{W}\neq 0}$$

$\hookrightarrow$ gauge invariant, simple for DY,
but problematic for radiation, not simple (impossible?) beyond NLO

Note: NLO on and off resonance, but transition region interpolated.

Width schemes for corrections in a nutshell: (continued)

- **Pole Scheme (PS)** Stuart '91; Aeppli et al. '93, '94; etc.

  Isolate resonance in a gauge-invariant way and introduce Γ only there:

$$\mathcal{M} = \frac{R(p^2)}{p^2 - M^2} + N(p^2) = \frac{R(M^2)}{p^2 - M^2} + \frac{R(p^2) - R(M^2)}{p^2 - M^2} + N(p^2)$$

$$\to \underbrace{\frac{\tilde{R}(M^2 - \mathrm{i}M\Gamma)}{p^2 - M^2 + \mathrm{i}M\Gamma}}_{\text{resonant}} + \underbrace{\frac{R(p^2) - R(M^2)}{p^2 - M^2} + \tilde{N}(p^2)}_{\text{non-resonant}}.$$

  $\hookrightarrow$ consistent, gauge invariant, NLO everywhere possible, but subtle and cumbersome in practice (complex kinematics, pole location is branch point rather than pole, IR structure of radiation)

- **Leading pole approximation (PA)**

  Take term with highest resonance enhancement of pole expansion
  = leading term of Pole Scheme

  $\hookrightarrow$ consistent, gauge invariant, straightforward,
  but valid only in resonance neighbourhood,
  rel. uncertainty for EW corrections $= \frac{\alpha}{\pi} \times \mathcal{O}(\Gamma/M)$

  $\hookrightarrow$ in general not sufficient at NLO for high-precision DY,
  but e.g. good basis for higher-order corrections such as $\mathcal{O}(\alpha_{\mathrm{s}}\alpha)$

Width schemes for corrections in a nutshell: (continued)

- Complex-Mass Scheme (CMS) Denner et al. '99,'05; Denner, SD '19

  Complex masses for $V = Z, W$ from

  $$\mu_V^2 = M_V^2 - \mathrm{i} M_V \Gamma_V = \text{location of complex poles in } V \text{ propagators.}$$

  Complex (on-shell) weak mixing angle via

  $$c_\mathrm{W} = \mu_\mathrm{W} / \mu_\mathrm{Z}.$$

  Perturbative calculation as usual (with complex on-shell renormalization).

  All algebraic relations expressing gauge invariance hold exactly (gauge-parameter cancellation, Ward identities).

  $\hookrightarrow$ General and systematic, gauge invariant, NLO everywhere, but all one-loop integrals with complex masses needed (known!)

- Further schemes (not in use for DY processes):

  Effective Field Theories Beneke et al. '03,'04; Hoang, Reisser '04

  $\hookrightarrow$ related to Pole Scheme and Leading Pole Approximation

  Schemes based on resummations of propagator corrections

  $\hookrightarrow$ still not fully gauge invariant or no full inclusion of NLO corrections

Gauge invariance and DY amplitudes

Z production:

- LO and photonic corrections:

  Gauge invariance ok if $\mathcal{M}_{\mathrm{LO}}$ is parametrized in terms of α, s_{W}, and $\mu_{\mathrm{Z}}^2 = M_{\mathrm{Z}}^2 - \mathrm{i}M_{\mathrm{Z}}\Gamma_{\mathrm{Z}}$ (dependent: $c_{\mathrm{W}}^2 = 1 - s_{\mathrm{W}}^2$, $M_{\mathrm{W}} = c_{\mathrm{W}}\mu_{\mathrm{Z}}$, but M_{W} does not appear).

  $\hookrightarrow$ PS/FS definition of $\hat{\sigma}_{\mathrm{LO}}^{q\bar{q}\to\mathrm{Z}/\gamma\to\ell\ell}$ and $\delta_{\mathrm{phot}}^{q\bar{q}\to\mathrm{Z}/\gamma\to\ell\ell}$ in RADY.

- Weak corrections: more complicated!

  $\hookrightarrow$ Dedicated CMS, PS, and FS implementations in RADY

W production:

- LO:

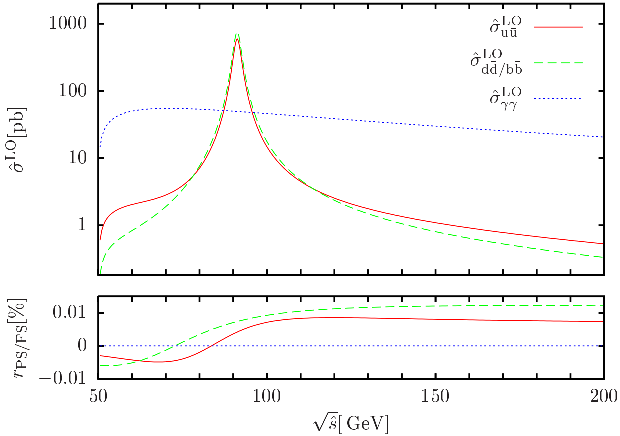
  Gauge invariance ok if $\mathcal{M}_{\mathrm{LO}}$ is parametrized in terms of α, s_{W}, and $\mu_{\mathrm{W}}^2 = M_{\mathrm{W}}^2 - \mathrm{i}M_{\mathrm{W}}\Gamma_{\mathrm{W}}$ (dependent: $c_{\mathrm{W}}^2 = 1 - s_{\mathrm{W}}^2$, $M_{\mathrm{Z}} = \mu_{\mathrm{W}}/cw$, but M_{Z} does not appear).

  $\hookrightarrow$ FS definition of $\hat{\sigma}_{\mathrm{LO}}^{q\bar{q}'\to\mathrm{W}/\gamma\to\ell\nu}$ in RADY.

- No gauge-invariant decomposition of EW corrections into photonic and weak parts!

  $\hookrightarrow$ Dedicated CMS and FS implementations in RADY

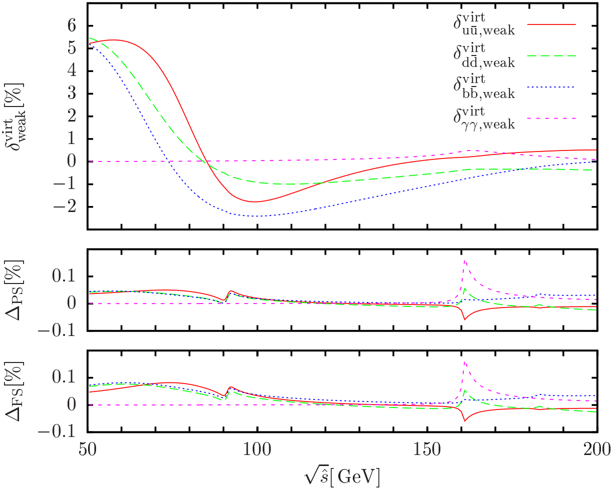
Comparison of width schemes – partonic LO Drell–Yan cross sections



SD, Huber
arXiv:0911.2329

Rel. difference: $r_X = \frac{\hat\sigma^{\rm LO}|_X}{\hat\sigma^{\rm LO}|_{\rm CMS}} - 1 \lesssim 0.01\%$ $\quad X = \text{PS/FS}$

Comparison of width schemes – weak corrections to partonic LO DY XS



SD, Huber
arXiv:0911.2329

Difference: $\Delta_X = \delta^{\rm virt}_{\rm weak}\Big|_X - \delta^{\rm virt}_{\rm weak}\Big|_{\rm CMS} \lesssim 0.1\%, \quad X = \mathrm{PS/FS}$

Slide from D.Froidevaux, LHCEWWG, Nov 11, 2020

Status of EW virtual correction comparisons

- Cross sections (pb) and ratios: NLO/LO, (NLO+HO)/LO with different codes in EW scheme G_mu
  - Excellent agreement at all levels to about 10^-3 relative. LO QCD numbers using the same PDF set

Table 51: Cross-sections (pb) and ratios: NLO/LO, (NLO+HO)/LO, predictions with different codes in EW scheme G_μ. Show are also corrections estimated with TauSpinner+DIZET, needed to match (NLO+HO) predictions calculated in EW $\alpha(0)$ v0 scheme.

Programs	$89 < m_{ee} < 93$ GeV	$60 < m_{ee} < 81$ GeV	$81 < m_{ee} < 101$ GeV	$101 < m_{ee} < 150$ GeV
	$\sigma(LO)$ (pb)			
MCSANC	612.531(5)	46.870(2)	880.527(6)	-
Powheg_ew	612.529(8)	46.870(1)	880.513(9)	-
WZGRAD2	612.521(7)	46.868(4)	880.520(10)	-
RADY(FS)	612.526(1)	46.871(1)	880.520(2)	-
	$\sigma(NLO)/\sigma(LO)$			
MCSANC	0.99167(2)	1.02865(7)	0.99206(1)	-
Powheg_ew	0.99155(1)	1.02863(2)	0.99196(1)	-
WZGRAD2	0.99198(1)	1.02913(4)	0.99239(1)	-
RADY(FS)	0.99148(1)	1.02864(4)	0.99189(1)	-
	$\sigma(NLO+HO)/\sigma(LO)$			
MCSANC	0.99232(2)	1.02614(7)	0.99268(1)	-
Powheg_ew	0.99218(1)	1.02592(2)	0.99255(1)	-
WZGRAD2	-	-	-	-
RADY(FS)	0.99179(1)	1.02589(1)	0.99216(1)	-
TauSpinner+DIZET (estimated)	0.99211(0)	1.02321(0)	0.99264(0)	0.98884

Comparison of width schemes in RADY

Cross sections:

Scheme:	$89 < M_{\ell\ell}$[GeV]$ < 93$	$60 < M_{\ell\ell}$[GeV]$ < 81$	$81 < M_{\ell\ell}$[GeV]$ < 101$	$101 < M_{\ell\ell}$[GeV]$ < 150$
		$\sigma(\mathrm{LO})[\mathrm{pb}]$		
RADY CMS	612.456(1)	46.8732(1)	880.420(2)	30.86266(6)
RADY PS	612.526(1)	46.8708(1)	880.520(2)	30.86835(6)
RADY FS	612.526(1)	46.8708(1)	880.520(2)	30.86835(6)
rel. diff.	0.01%	0.005%	0.01%	0.02%
		$\sigma(\mathrm{NLO})/\sigma(\mathrm{LO})$		
RADY CMS	0.99102(1)	1.02786(1)	0.99143(1)	0.98908(1)
RADY PS	0.99131(1)	1.02843(1)	0.99172(1)	0.98917(1)
RADY FS	0.99148(1)	1.02864(1)	0.99189(1)	0.98924(1)
diff.	0.05%	0.08%	0.05%	0.02%
		$\sigma(\mathrm{NLO+HO})/\sigma(\mathrm{LO})$		
RADY CMS	0.99131(1)	1.02508(1)	0.99168(1)	0.98898(1)
RADY PS	0.99161(1)	1.02568(1)	0.99198(1)	0.98907(1)
RADY FS	0.99179(1)	1.02589(1)	0.99216(1)	0.98915(1)
diff.	0.05%	0.08%	0.05%	0.02%

$\hookrightarrow$ Width scheme dependence $\lesssim$ 0.1%

A_{FB} in NC DY: RADY FS versus results from other programs

Slide from D.Froidevaux, LHCEWWG, Nov 11, 2020

Status of EW virtual correction comparisons

Table 53: Forward-backward asymmetry A_{fb} and differences: NLO/LO, (NLO+HO)/LO, predictions with different codes in EW scheme G_μ. Show are also corrections estimated with TauSpinner+DIZET, needed to match (NLO+HO) predictions calculated in EW $\alpha(0)$ v0 scheme.

Programs	$89 < m_{ee} < 93$ GeV	$60 < m_{ee} < 81$ GeV	$81 < m_{ee} < 101$ GeV	$101 < m_{ee} < 150$ GeV
	$A_{fb}(LO)$			
MCSANC	0.04654(1)	-0.20299(4)	0.04481(1)	-
Powheg_ew	0.04655(1)	-0.20298(2)	0.04481(1)	-
WZGRAD2	0.04654(1)	-0.20299(8)	0.04482(1)	-
RADY(FS)	0.04655(1)	-0.20295(1)	0.04481(1)	-
	$A_{fb}(NLO) - A_{fb}(LO)$			
MCSANC	-0.01688(2)	-0.01170(8)	-0.01688(2)	-
Powheg_ew	-0.01740(2)	-0.01214(3)	-0.01738(2)	-
WZGRAD2	-0.01716(2)	-0.01186(11)	-0.01715(2)	-
RADY(FS)	-0.01717(1)	-0.01199(1)	-0.01716(1)	-
	$A_{fb}(NLO+HO) - A_{fb}(NLO)$			
MCSANC	0.00137(2)	0.00111(8)	0.00137(2)	-
Powheg_ew	0.00137(2)	0.00112(3)	0.00136(2)	-
WZGRAD2	-	-	-	-
RADY(FS)	0.00122(1)	0.00103(1)	0.00122(1)	-
	$A_{fb}(NLO+HO) - A_{fb}(LO)$			
MCSANC	-0.01551(2)	-0.01059(8)	-0.01551(1)	-
Powheg_ew	-0.01603(2)	-0.01102(3)	-0.01602(2)	-
WZGRAD2	-	-	-	-
RADY(FS)	-0.01595(1)	-0.01096(1)	-0.01594(1)	-
TauSpinner+DIZET (estimated)	-0.01507(0)	-0.01104(0)	-0.01514(0)	0.00684

Comparison of width schemes in RADY

FB asymmetry A_{FB}:

Scheme:	$89 < M_{\ell\ell}$[GeV]<93	$60 < M_{\ell\ell}$[GeV]<81	$81 < M_{\ell\ell}$[GeV]<101	$101 < M_{\ell\ell}$[GeV]<150
		$A_{\mathrm{FB}}(\mathrm{LO})$		
RADY CMS	0.046551(4)	−0.202894(5)	0.044817(4)	0.226101(5)
RADY PS	0.046547(4)	−0.202955(4)	0.044812(3)	0.226090(4)
RADY FS	0.046547(4)	−0.202955(4)	0.044812(3)	0.226090(4)
|X−CMS|	$<$ 0.00001	0.00006	$<$ 0.00001	0.00001
		$A_{\mathrm{FB}}(\mathrm{NLO}) - A_{\mathrm{FB}}(\mathrm{LO})$		
RADY CMS	−0.01736(1)	−0.01233(1)	−0.01735(1)	−0.00689(1)
RADY PS	−0.01735(1)	−0.01220(1)	−0.01734(1)	−0.00691(1)
RADY FS	−0.01717(1)	−0.01199(1)	−0.01716(1)	−0.00681(1)
|PS−CMS|	$\lesssim$ 0.00001	0.0001	$\lesssim$ 0.00001	$\lesssim$ 0.00002
|FS−CMS|	0.0002	0.0003	0.0002	0.0001
		$A_{\mathrm{FB}}(\mathrm{NLO+HO}) - A_{\mathrm{FB}}(\mathrm{LO})$		
RADY CMS	−0.01615(1)	−0.01131(1)	−0.01614(1)	−0.00656(1)
RADY PS	−0.01614(1)	−0.01118(1)	−0.01613(1)	−0.00657(1)
RADY FS	−0.01595(1)	−0.01096(1)	−0.01594(1)	−0.00647(1)
|PS−CMS|	$\lesssim$ 0.00001	0.0001	$\lesssim$ 0.00001	$\lesssim$ 0.00001
|FS−CMS|	0.0002	0.0003	0.0002	0.0001

$\hookrightarrow$ |PS-CMS| $\lesssim$ 0.00001 in resonance window

FS less accurate?

Outlook: further issues in high-precision DY production

- $\sin^2\theta_{\mathrm{eff}}^{\mathrm{lept}}$ as input parameter:
  tuned comparison of results from different codes for XS and A_{FB}
- $\gamma\gamma$ channels: separation of elastic and inelastic parts of photon PDF
  $\hookrightarrow$ influences EW input scheme and PDF factorization
  $\hookrightarrow$ required to control $\gamma\gamma$ contribution at the % level
- Fixed or running Z width in resonance parametrization
  $\hookrightarrow$ extrem care needed concerning gauge invariance (see above!), but consistent studies in leading pole approximation possible with RADY
- BSM effects on DY cross sections
  $\hookrightarrow$ could be studied in RADY for MSSM, THDM, SESM

A final commercial ...

A. Denner and SD,
"Electroweak Radiative Corrections for Collider Physics,"
Phys. Rept. 864 (2020), 1-163,
doi:10.1016/j.physrep.2020.04.001 [arXiv:1912.06823 [hep-ph]]

From the contents: