KKMC-hh: Update on A_{FB} and Comments on Quark Mass Dependence

S.A. Yost

The Citadel

December 18, 2020

KKMC-hh is a collaboration with S. Jadach, B.F.L. Ward and Z. Wąs.

Computational resources provided by IFJ-PAN, Kraków.

Results Presented in the Following Tables

- All results are for muon pair final states with proton collisions with $\sqrt{s} = 8000\ \mathrm{GeV}$. Comparisons are made for DIZET 6.21 and 6.45.
- Our tabulated results all include a dilepton mass cut in all cases:
$$60\ \mathrm{GeV} < M_{ll} < 150\ \mathrm{GeV}.$$
- The following table shows only A_4 calculated from A_{FB} in the full phase space without fermion cuts.
- We also calculated A_{FB} with lepton cuts $P_T > 25\ \mathrm{GeV}, |\eta| < 2.5$ on both muons. The corresponding table is on the following page. All results use NNPDF3.1 NLO and include FSR corrections.
- After the tables, I have included 9-10 billion event distributions for A_{FB} and A_4 with 1 GeV binning for the full range 60 GeV < M_{ll} < 150 GeV. These are not new.

Comparisons of A_4 for Dizet 6.21 and 6.45

$A_4 = \frac{8}{3} A_{\mathrm{FB}}$ is calculated in the full phases space with complete (KKMC best) photonic corrections. $\Delta\mathrm{ISR}$ is the difference in A_4 with ISR on minus ISR off, with IFI off in both cases. $\Delta\mathrm{IFI}$ is the difference in A_4 with IFI on minus IFI off. The numbers are based on a sample of 9G – 10G muon events.

	DIZET	$60 < M_{ll} < 81$	$81 < M_{ll} < 101$	$101 < M_{ll} < 150$	$60 < M_{ll} < 120$	$89 < M_{ll} < 93$
A_4 (best)	6.45	$-0.28892(8)$	$0.07785(3)$	$0.5836(1)$	$0.05606(3)$	$0.08329(2)$
	6.21	$-0.28892(8)$	$0.07697(3)$	$0.5828(1)$	0.05522(3)	$0.08246(5)$
Difference		$\pm 1.1 \times 10^{-4}$	$(8.8 \pm 0.5) \times 10^{-4}$	$(7.8 \pm 1.4) \times 10^{-4}$	$(8.4 \pm 04) \times 10^{-4}$	$(8.4 \pm 0.6) \times 10^{-4}$
Δ ISR	6.45	$(0.2 \pm 1.1) \times 10^{-4}$	$-(4.9 \pm 4.6\,) \times 10^{-5}$	$-(0.8 \pm 0.2) \times 10^{-3}$	$-(6.8 \pm 4.2) \times 10^{-5}$	$-(1.0 \pm 0.6) \times 10^{-4}$
	6.21	$(0.2 \pm 1.1) \times 10^{-4}$	$(3.6 \pm 3.9) \times 10^{-5}$	$-(1.1 \pm 0.1) \times 10^{-3}$	$(2.6 \pm 3.4) \times 10^{-5}$	$(8.5 \pm 6.0) \times 10^{-5}$
Δ IFI	6.45	$(3.4 \pm 0.9) \times 10^{-4}$	$(\,3.1 \pm 0.2) \times 10^{-4}$	$-(6.19 \pm 0.08) \times 10^{-3}$	$(1.3 \pm 0.4) \times 10^{-4}$	$(2.0 \pm 0.3) \times 10^{-4}$
	6.21	$(3.4 \pm 0.9) \times 10^{-4}$	$(3.1 \pm 0.2) \times 10^{-4}$	$-(6.18 \pm 0.06) \times 10^{-3}$	$(1.3 \pm 0.3) \times 10^{-4}$	$(1.8 \pm 0.4) \times 10^{-4}$

Comparisons of A_{FB} for Dizet 6.21 and 6.45

A_{FB} is calculated with fermion cuts $P_T > 25$ GeV, $|\eta| < 2.5$ with complete (KKMC best) photonic corrections. ΔISR is the difference in A_4 with ISR on minus ISR off, with IFI off in both cases. ΔIFI is the difference in A_4 with IFI on minus IFI off. The numbers are based on a sample of 9G – 10G muon events.

	DIZET	$60 < M_{ll} < 81$	$81 < M_{ll} < 101$	$101 < M_{ll} < 150$	$60 < M_{ll} < 120$	$89 < M_{ll} < 93$
A_{FB} (best)	6.45	$-0.04136(5)$	$0.02158(2)$	$0.10120(6)$	$0.01116(2)$	$0.01290(2)$
	6.21	$-0.04160(4)$	$0.01249(2)$	$0.10127(5)$	0.01108(2)	$0.01283(3)$
Difference		$(2.3 \pm 0.7) \times 10^{-4}$	$(8.8 \pm 0.5) \times 10^{-4}$	$-(7.0 \pm 8.2) \times 10^{-5}$	$(8.4 \pm 2.3) \times 10^{-5}$	$(7.3 \pm 0.6) \times 10^{-4}$
Δ ISR	6.45	$-(2.7 \pm 0.5) \times 10^{-4}$	$-(4.9 \pm 4.6) \times 10^{-5}$	$(4.9 \pm 1.0) \times 10^{-4}$	$-(0.3 \pm 2.3) \times 10^{-5}$	$-(0.8 \pm 3.3) \times 10^{-5}$
	6.21	$-(3.8 \pm 0.5) \times 10^{-4}$	$(3.6 \pm 3.9) \times 10^{-5}$	$(6.1 \pm 0.7) \times 10^{-4}$	$(8.1 \pm 1.9) \times 10^{-5}$	$(1.4 \pm 0.3) \times 10^{-4}$
Δ IFI	6.45	$(3.4 \pm 0.9) \times 10^{-4}$	$(3.1 \pm 1.0) \times 10^{-5}$	$-(7.5 \pm 0.3) \times 10^{-4}$	$(2.1 \pm 0.9) \times 10^{-5}$	$(2.3 \pm 1.3) \times 10^{-5}$
	6.21	$(2.7 \pm 0.9) \times 10^{-4}$	$(3.3 \pm 0.9) \times 10^{-5}$	$-(7.8 \pm 0.3) \times 10^{-4}$	$(2.0 \pm 0.8) \times 10^{-5}$	$(2.0 \pm 1.4) \times 10^{-5}$

The KKMC-hh Approach to ISR

- TheCrude MC integral has the (simplified) form

$$\sigma_{\mathrm{Cr}} = \int_0^1 dx_1 \int_0^1 dx_2 \sum_{q=1}^{N_q} \mathrm{PDF}_q(s_X, x_1)\mathrm{PDF}_q(s_X, x_2) \int_0^1 \frac{dv}{v}\gamma_q v^{\gamma_q}\sigma_{q\,\mathrm{Cr}}\big((1-v)s_X\big)$$

where $\gamma_q = \frac{2\alpha}{\pi} Q_q^2\big(\ln\big(s_X/m_q^2\big) - 1\big)$ and $\sigma_{\mathrm{q\,Cr}}$ is a crude partonic Born CS, $s_X \equiv s x_1 x_2$

- The presence of the quark mass m_i in the logarithm $\ln(s_X/m_q^2)$ governing ISR introduces dependence on the value of m_q typically at the fractional per-mil level.
- We use the current masses from PDG for the light quarks.
- Specific values of quark masses used:

$$m_d = 4.7\ \mathrm{MeV},\ \ m_u = 2.2\ \mathrm{MeV},$$
$$m_s = 150\ \mathrm{MeV}, \qquad m_c = 1.2\ \mathrm{GeV}, \qquad m_b = 4.6\ \mathrm{GeV}$$

Sensitivity to Light Quark Masses

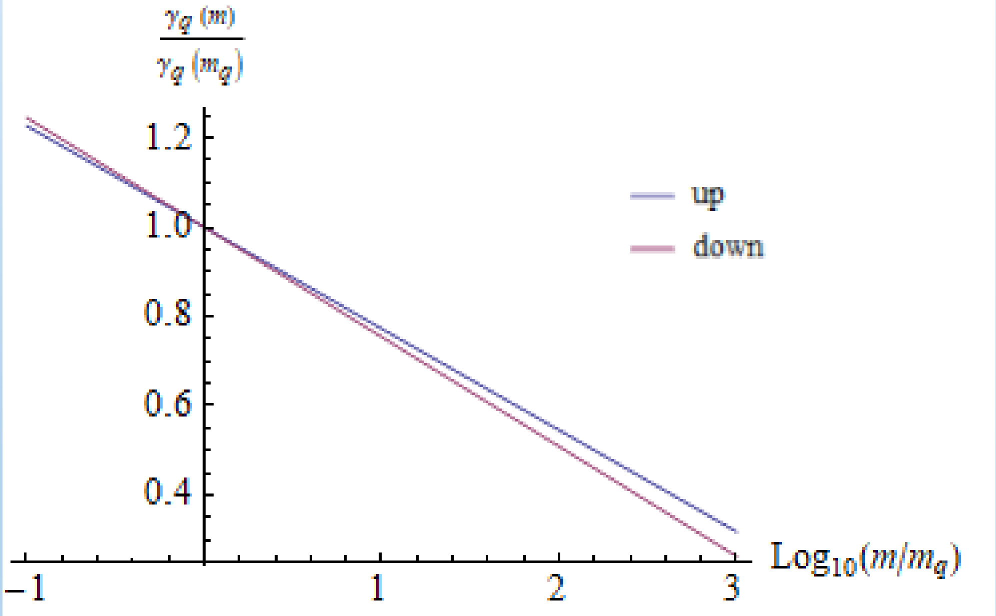
The sensitivity to the light quark masses can be tested by varying them by an order of magnitude. Recall that the cross section dependence is logarithmic via the factor

$$\gamma_i v^{\gamma_i} \sigma_{i\,\mathrm{Cr}}\big((1-v)s_X\big) \quad \text{with} \quad \gamma_i = \frac{2\alpha}{\pi} Q_i^2\big(\ln\big(s_X/m_i^2\big) - 1\big).$$

If $m_i \to 10\, m_i$, then $\Delta\gamma_i = -\frac{4\alpha}{\pi} Q_i^2 \ln 10$. For u and d quarks at $s_X = M_Z^2$,

$$\gamma_u = 0.0418, \qquad \Delta\gamma_u = -0.0095, \qquad \frac{\Delta\gamma_u}{\gamma_u} = -0.23$$

$$\gamma_d = 0.00968, \qquad \Delta\gamma_d = -0.0024, \qquad \frac{\Delta\gamma_d}{\gamma_d} = -0.25$$



In quantities where ISR barely detectable, this shift is less so. It could provide an observable sensitivity to the quark masses in sufficiently exclusive observables.

Sensitivity to Light Quark Masses

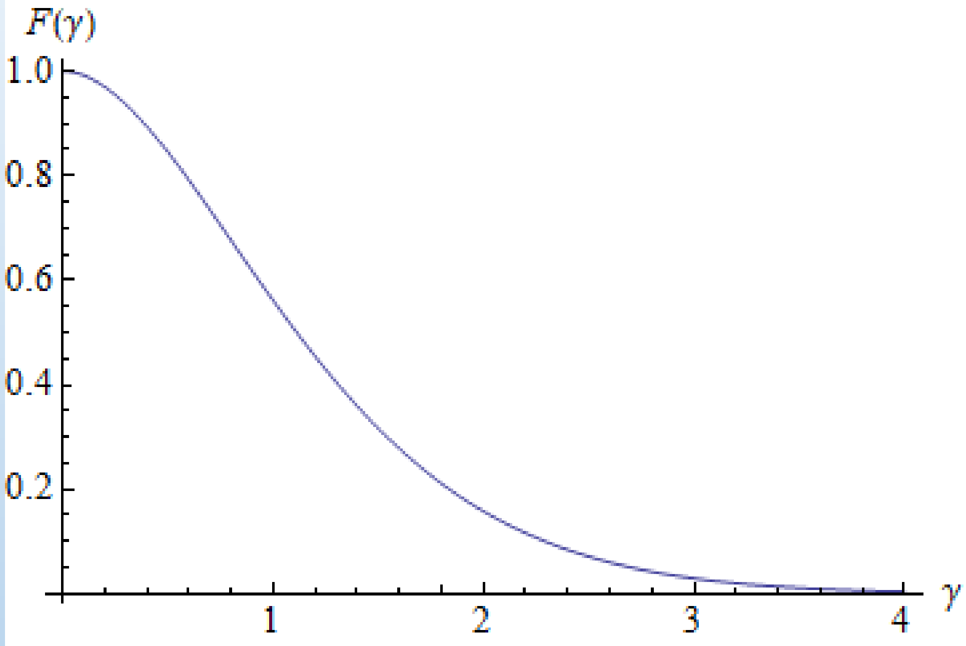
Inclusive observables have far less mass sensitivity. When $\gamma_q > 0$, as is the case for all cuts of interest here, the ISR distribution is integrable and always leads to a finite integral. In a semi-soft approximation, the quark-level cross section can be expressed in terms of a YFS form factor

$$F(\gamma) = \frac{e^{-\gamma C_E}}{\Gamma(1+\gamma)}$$

where C_E is the Euler constant. After integrating over angles and summing over all photons, with total photon energy K^0, the result is a multiplicative ISR factor

$$\int_0^{\sqrt{s_X}} \frac{dK^0}{K_0} \gamma F(\gamma) \left(\frac{K^0}{\sqrt{s_X}}\right)^{\gamma} = \int_0^1 dv\, \gamma v^{\gamma-1} F(\gamma) = F(\gamma)$$

with $\gamma = \frac{2\alpha}{\pi} Q^2(\ln(s_X/m^2) - 1)$.



Sensitivity to Light Quark Masses

The YFS form factor

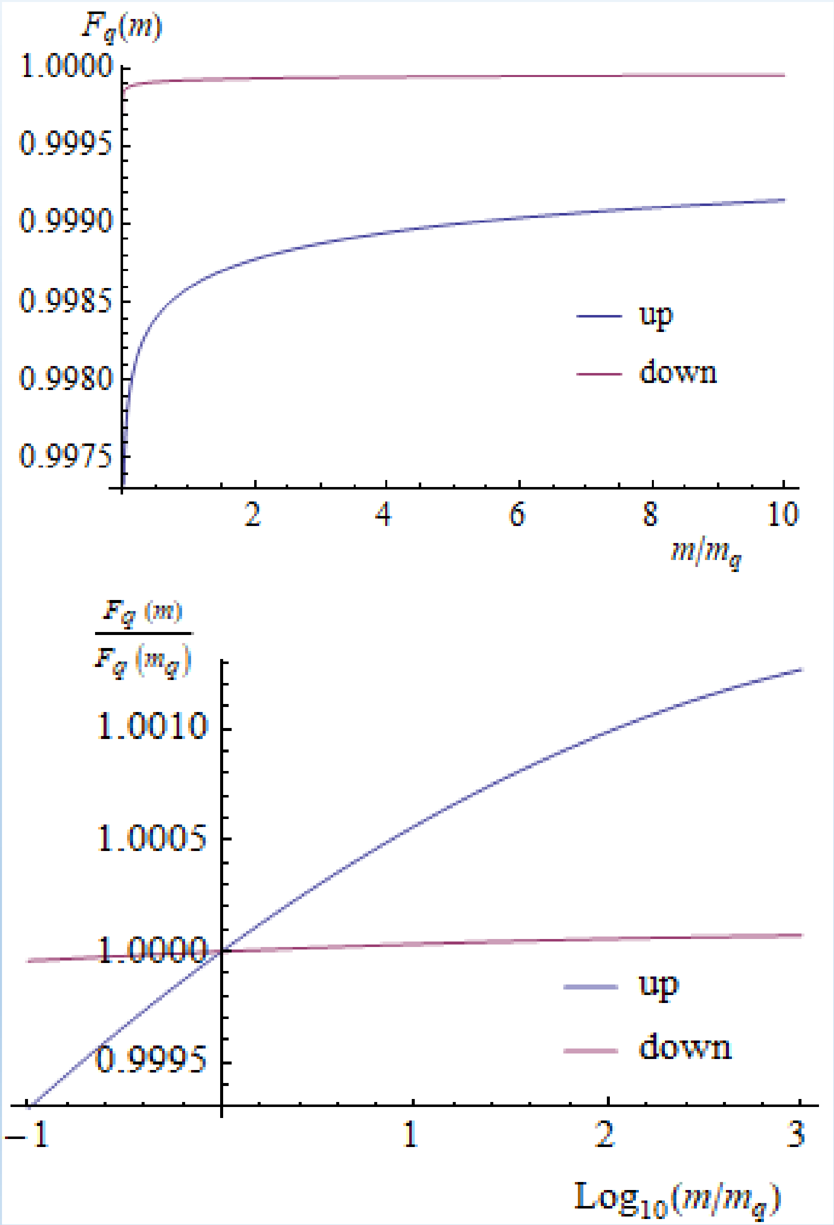
$$F(\gamma) = \frac{e^{-\gamma C_E}}{\Gamma(1+\gamma)}$$

has very weak sensitivity (fractional per-mil) to the mass in

$$\gamma_q = \frac{2\alpha}{\pi} Q_q^2\left(\ln\left(s_X/m_q^2\right) - 1\right)$$

and it is finite for $m_i = 0$. The graph at the right shows the ratio of $F\left(\gamma_q(m)\right)$ at $s_X = M_Z$ for arbitrary mass m to the value $F\left(\gamma_q(m_q)\right)$ for the current mass m_q for $q = u, d$.

In addition, there is a set of real and virtual corrections to the soft approximation that, to leading log, depend on $\alpha L \sim \gamma_q,\ \alpha^2 L^2 \sim \gamma_q^2, \ldots$ Calculating the complete set of these is best done in a MC run since there are real-virtual cancelations that depend on the cuts.



Comment on IFI

KKMC-hh includes exponentiated IFI, for which a similar analysis to ISR and FSR leads to a "big logarithm" factor

$$\gamma_X = \frac{2\alpha}{\pi} Q_i Q_f \ln\left(\frac{1-\cos\theta_{\mathrm{CS}}}{1+\cos\theta_{\mathrm{CS}}}\right) = -\frac{4\alpha}{\pi} Q_i Q_f\, \eta_{CS}$$

which depends on the CS angle but not on the quark or lepton masses. Thus, the IFI predictions should be unaffected by the choice of quark masses.

Plans for Future Results

Once the Krakow farm is back up (soon?) I will run tests of KKMC-hh ISR vs the PDF approximation using MMHT2014 and MMHT2015qed PDFs. This will give another view of the comparison previously made to NNPDF3.1 and NNPDF3.1-LuxQED.

KKMC-hh (and KKMC-ee) has been transcoded entirely to C++. The new version of KKMC-hh is more modular, permitting multiple EW-correction objects to be initialized, in principle, for tests against models other than DIZET. This could be useful for cross checks, but is likely to require some testing. It may be relatively simple to run a test against a leading order setup without weak corrections, and I plan to try that.

I also plan to save multi-dimensional histograms next time so that different projections can be made, for example the M_{ll} dependence of A_{FB} or A_4 in different rapidity ranges.

I have attached the A_{FB} and A_4 plots from the last meeting in case of questions.

Summary

The shifts in A_{FB} and A_4 predicted using both DIZET 6.21 and DIZET 6.45 were compared. Changing to DIZET 6.45 introduces shifts on the order of a per mil (or less in some regions).

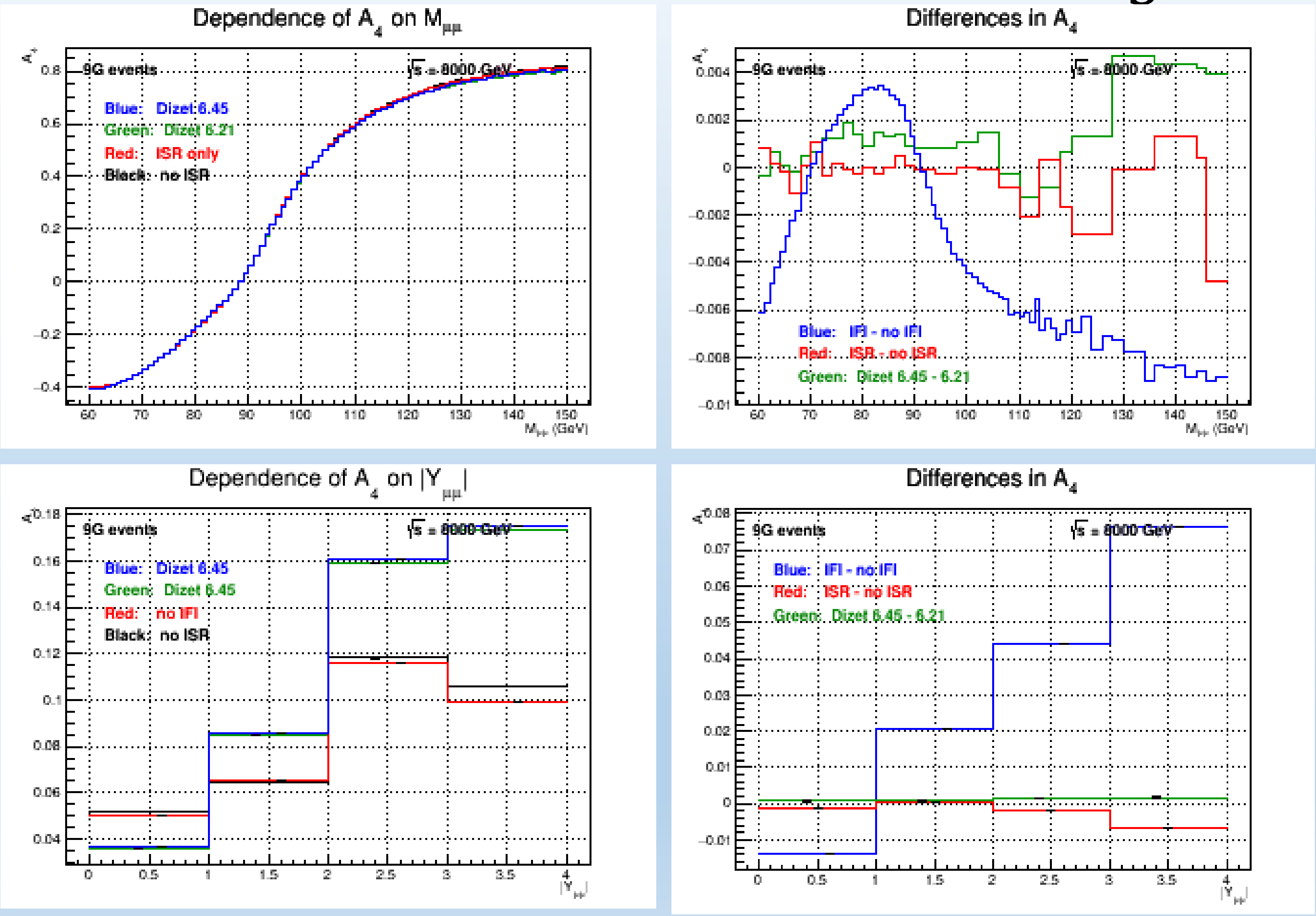
The IFI correction is independent of the version of DIZET used.

The ISR corrections are more affected by statistical errors, but the two DIZET versions are consistent here to a few times 10^{-5}.

KKMC's ISR depends on the quark masses in a calculable way. The effect on two key factors γ_q and $F_{\rm ISR}(\gamma_q)$ was described, but factors similar to γ_q also appear in the higher-order real and virtual corrections, where cancelations can depend strongly on the observable. The dependence for $A_{\rm FB}$ and A_4 will be checked via $m_q \to 10\ m_q$ for the $q = u, d$ when the Krakow farm is available again.

IFI is affected by the CS angle, but not the quark masses.

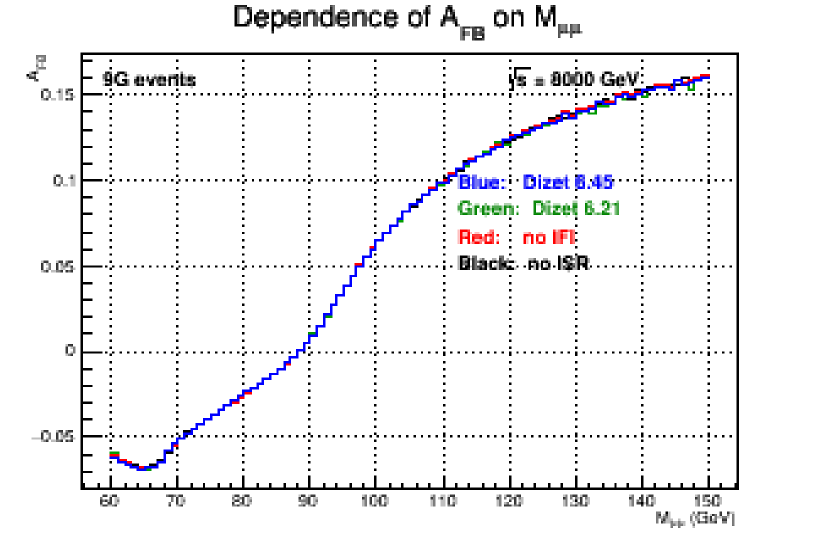
ISR and IFI contributions to A_4 ($\frac{8}{3}A_{\mathrm{FB}}$, no lepton cuts)

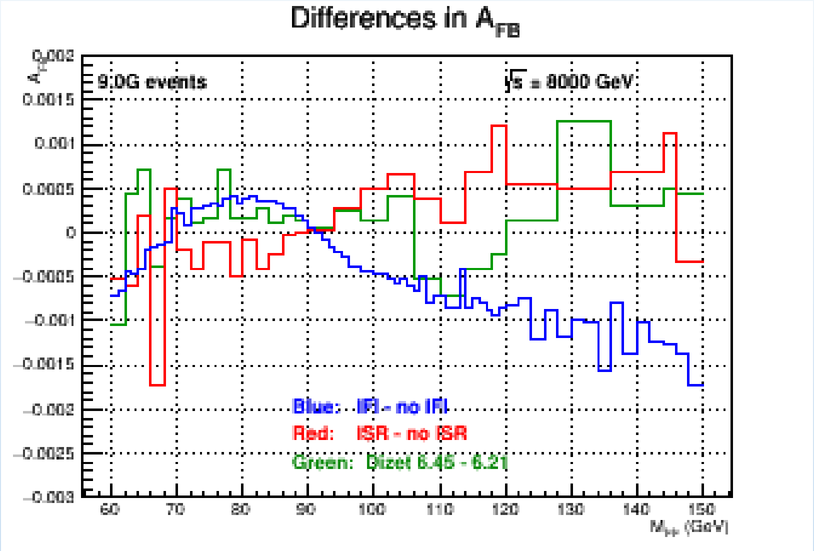


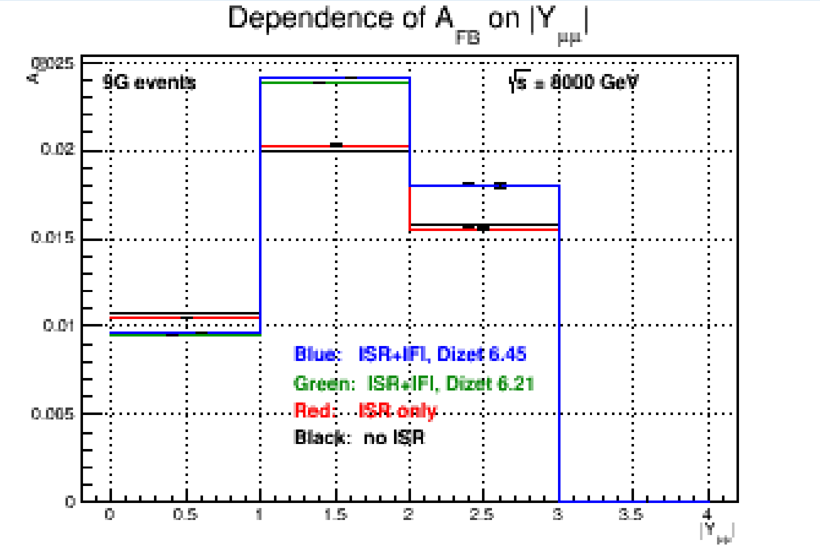
The IFI contribution has a clear structure, but the ISR contribution is near zero and flat up to statistical fluctuations. The Dizet shift is about a per-mil upward, with fluctuations.

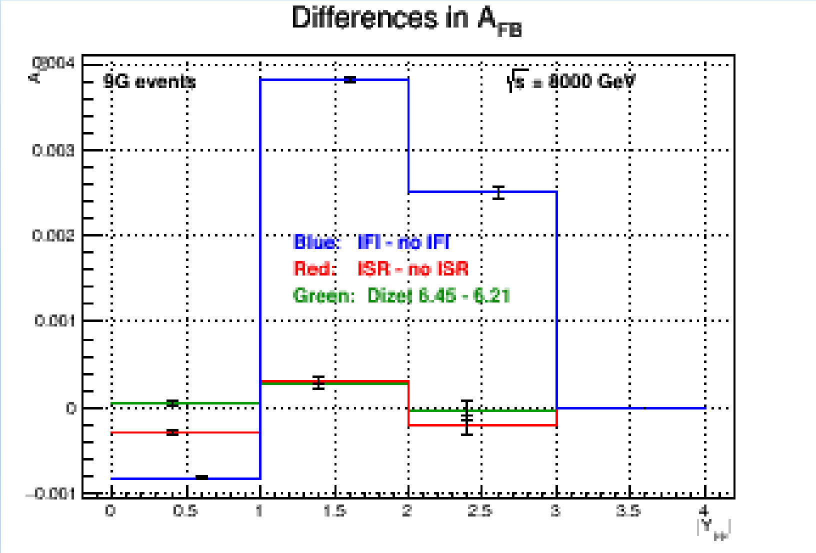
When binned in Y, the ISR contribution and Dizet shift are much smaller than IFI.

ISR and IFI contributions to A_{FB} (with lepton cuts)







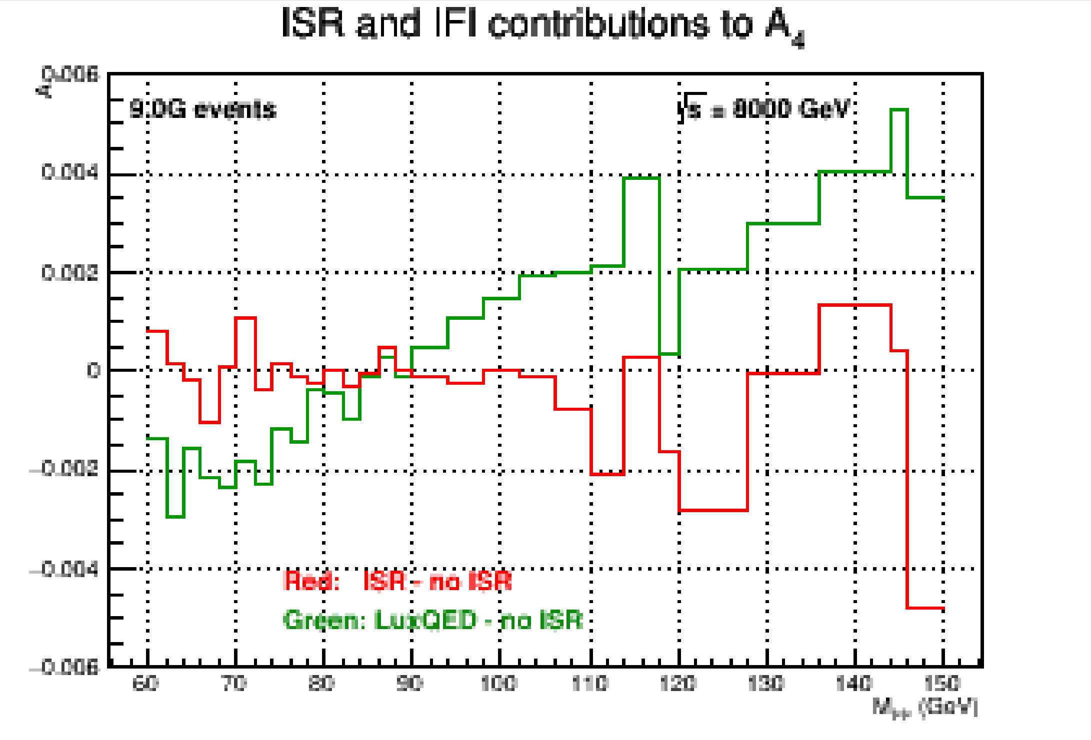


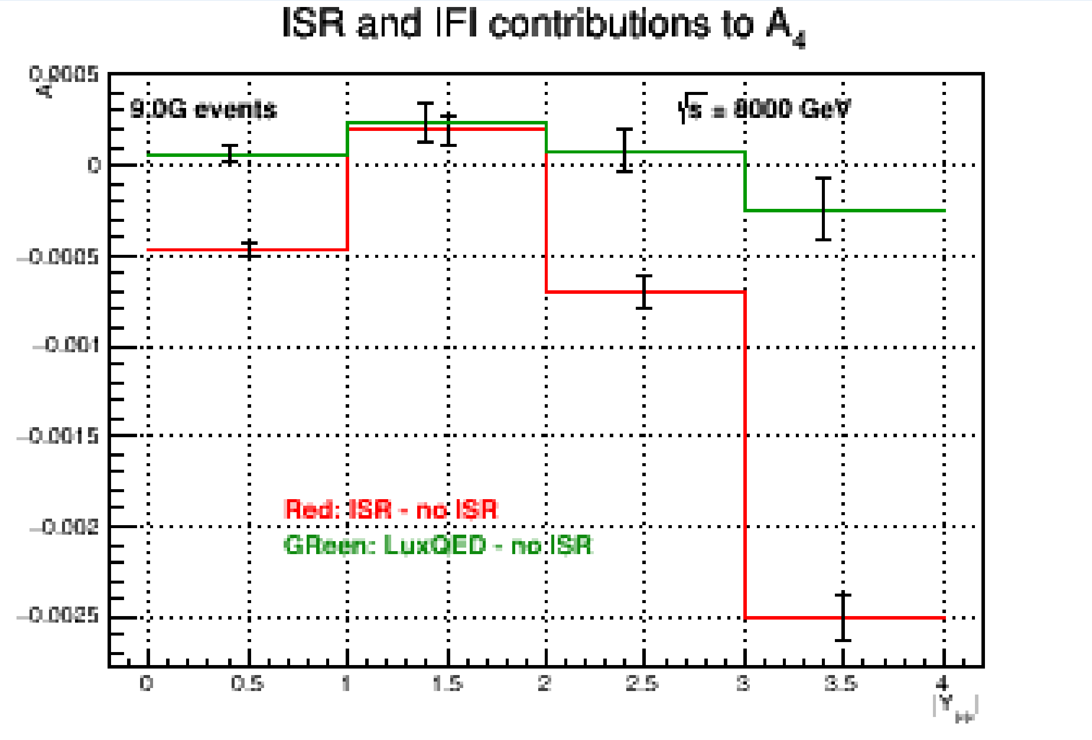
These plots show A_{FB} calculated with the lepton cuts. When binned in M_{ll}, all three shifts, IFI, ISR, and the change in DIZET are typically below 1 per-mil.

When binned in Y_{ll}, the IFI shift is again the largest, with the others being a fraction of a per-mil.

ISR Comparisons for $A_4 = \frac{8}{3} A_{\mathrm{FB}}$ (full phase space)

This is a similar comparison for the ISR contribution to A_4. The red curves were shown earlier for KKMC-hh ISR. The green curve compares the effect of turning on QED in the PDF set instead. There effects are at the per-mille level, but clearly distinct on the right.





ISR Comparisons for A_{FB} with Lepton Cuts

With the lepton cuts, the KKMC-hh ISR effect is indistinguishable from turning on QED in the PDF set. The two corrections track very closely when binned in $M_{\ell\ell}$. The scale on the right is in tenths of a per-mille, so the agreement there is also within a fraction of a per-mille.