Lorentzian Vacuum Transitions

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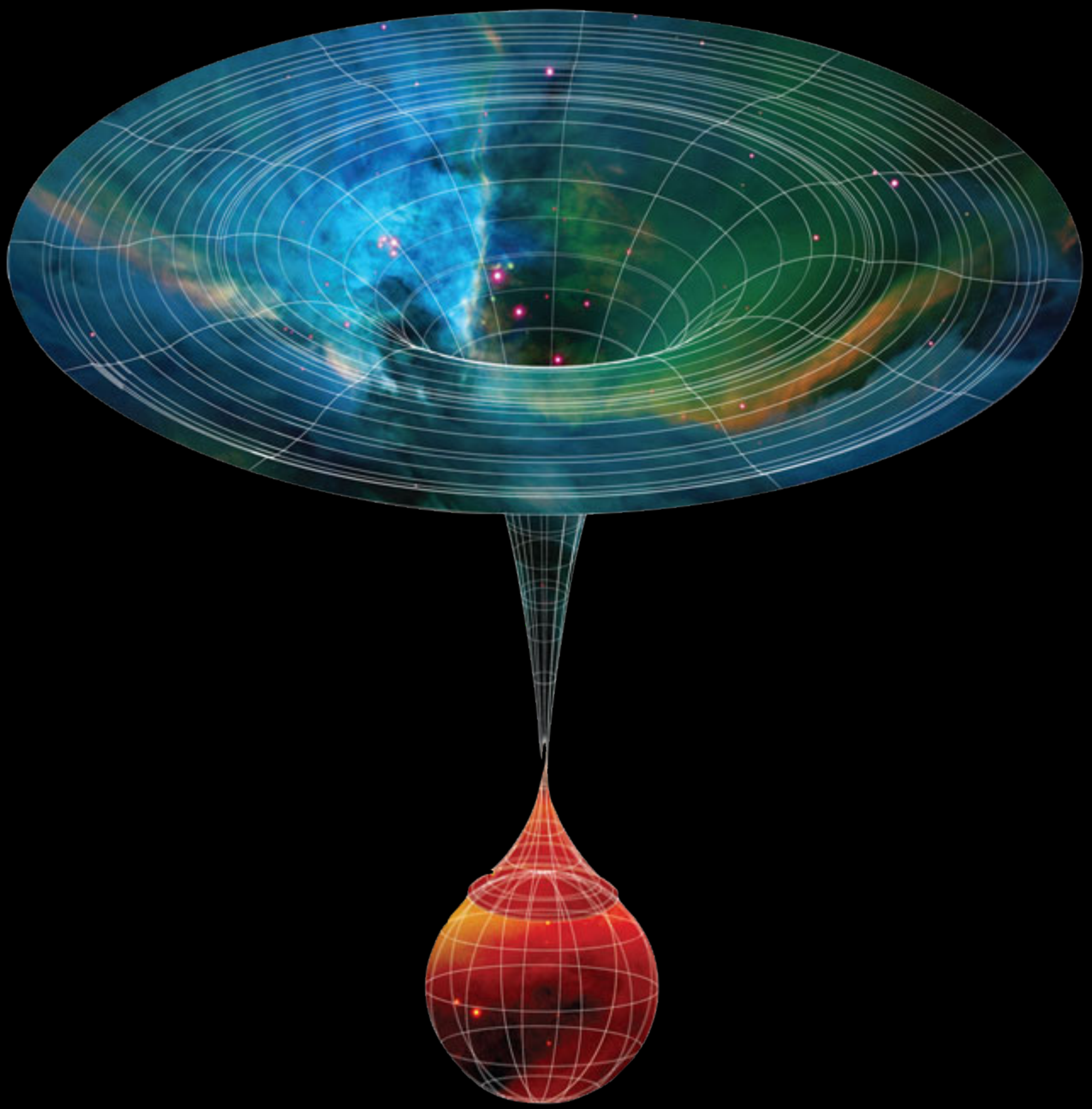

Francesco Muia

Cosmology 2021
05/01/2021

[S. de Alwis, S. Cespedes, F. Muia, F. Quevedo, 2020]

[S. de Alwis, F. Muia, V. Pasquarella, F. Quevedo, 2019]

The problem

$$\frac{\rho_{\rm vac}(\textbf{theory})}{\rho_{\rm vac}(\textbf{observed})} \simeq \left(\frac{10^{19}\ \textbf{GeV}}{10^{-3}\ \textbf{eV}}\right)^4 \approx 10^{122}$$

Anthropic selection

1. Microscopic theory provides a huge number of vacua, i.e. a Landscape.
2. Different regions of the universe sit in different vacua, and they are all populated.
3. Observers can only exist for a small range of vacuum energies.

"Worst solution to the CC problem, except for all the others."

[Fernando Quevedo]

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[Bousso, Polchinski, 2000]

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Populating the String Landscape

PHYSICAL REVIEW D VOLUME 15, NUMBER 10 15 MAY 1977

Fate of the false vacuum: Semiclassical theory*

Sidney Coleman
Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts 02138
(Received 24 January 1977)

It is possible for a classical field theory to have two homogeneous stable equilibrium states with different energy densities. In the quantum version of the theory, the state of higher energy density becomes unstable through barrier penetration; it is a false vacuum. This is the first of two papers developing the qualitative and quantitative semiclassical theory of the decay of such a false vacuum for theories of a single scalar field with nonderivative interactions. In the limit of vanishing energy density between the two ground states, it is possible to obtain explicit expressions for the relevant quantities to leading order in h; in the more general case, the problem can be reduced to solving a single nonlinear ordinary differential equation.

PHYSICAL REVIEW D VOLUME 21, NUMBER 12 15 JUNE 1980

Gravitational effects on and of vacuum decay

Sidney Coleman*
Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305

Frank De Luccia
Institute for Advanced Study, Princeton, New Jersey 88548
(Received 4 March 1980)

It is possible for a classical field theory to have two stable homogeneous ground states, only one of which is an absolute energy minimum. In the quantum version of the theory, the ground state of higher energy is a false vacuum, rendered unstable by barrier penetration. There exists a well-established semiclassical theory of the decay of such false vacuums. In this paper, we extend this theory to include the effects of gravitation. Contrary to naive expectation, these are not always negligible, and may sometimes be of critical importance, especially in the late stages of the decay process.

Motivations

- Populating the String Landscape.
  - How is it populated?

    Eternal inflation is not enough.

[Coleman, De Luccia, '80]	CDL
[Brown, Teitelboim, '88]	BT
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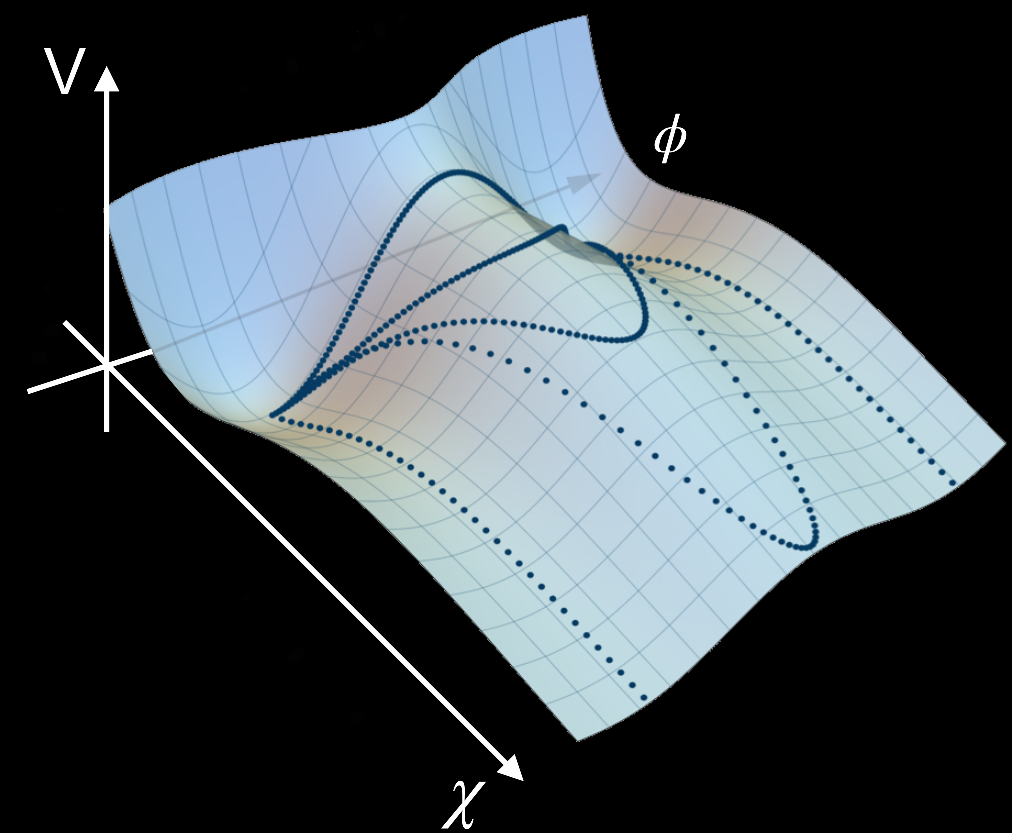

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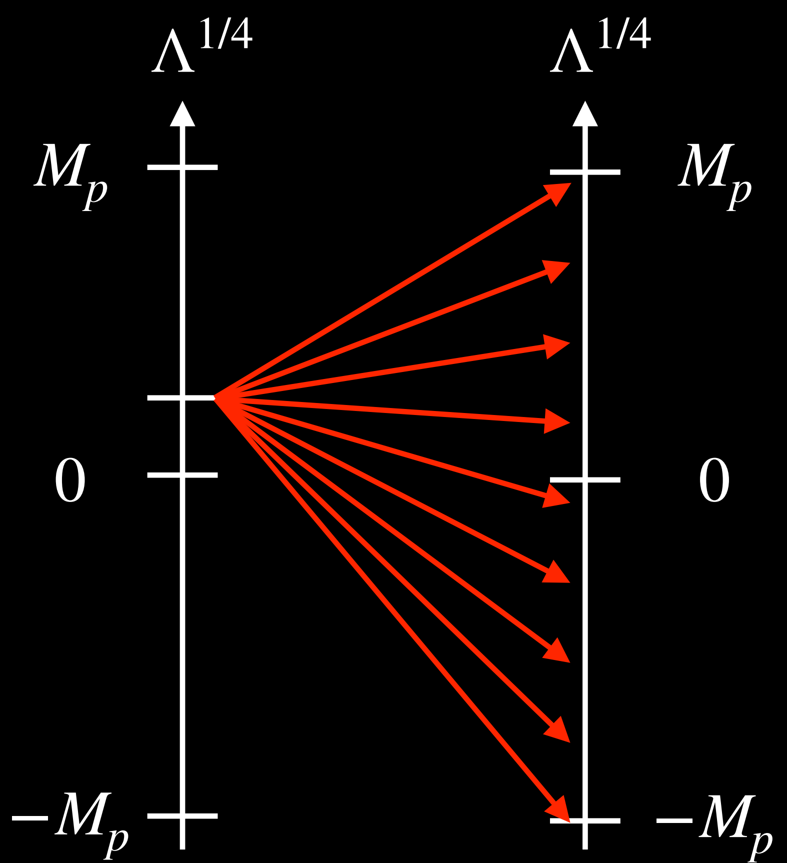

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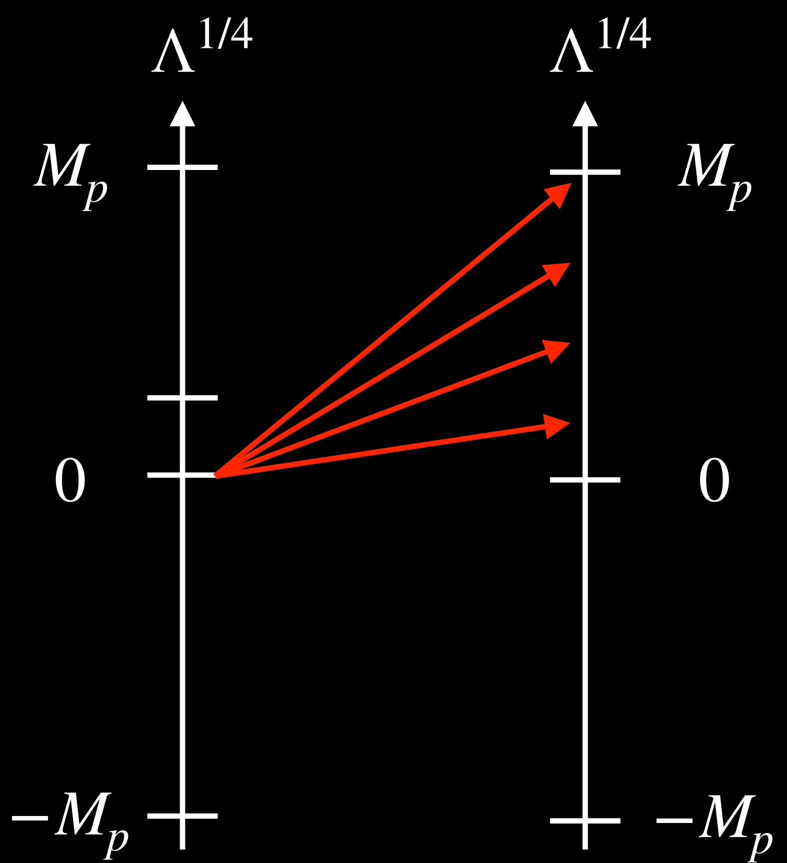

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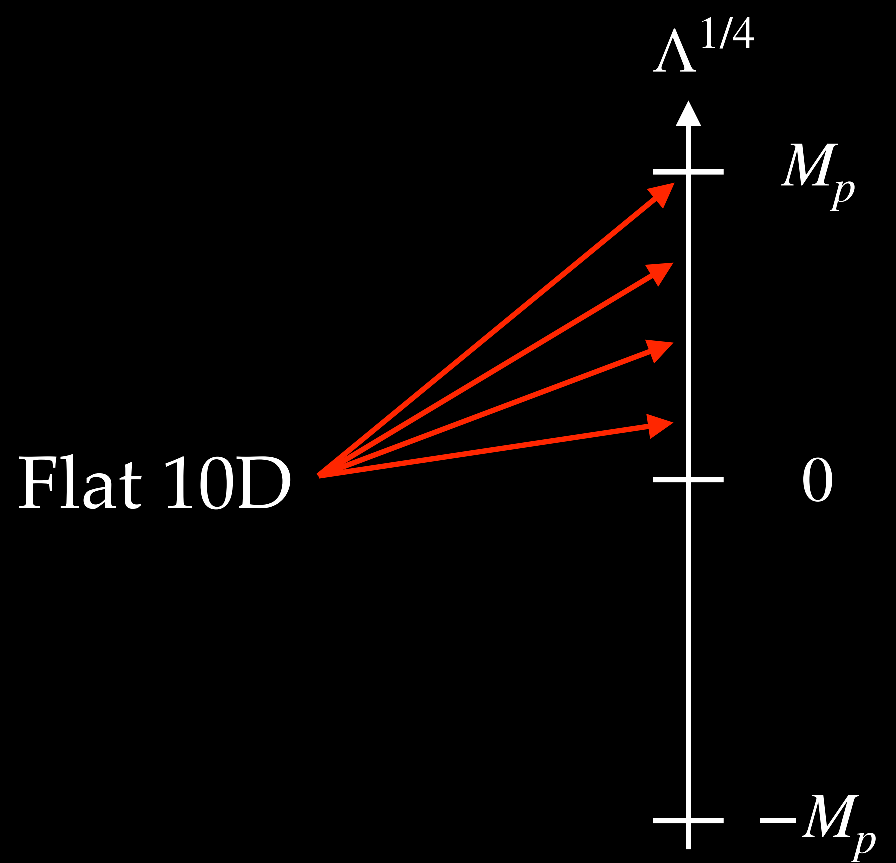

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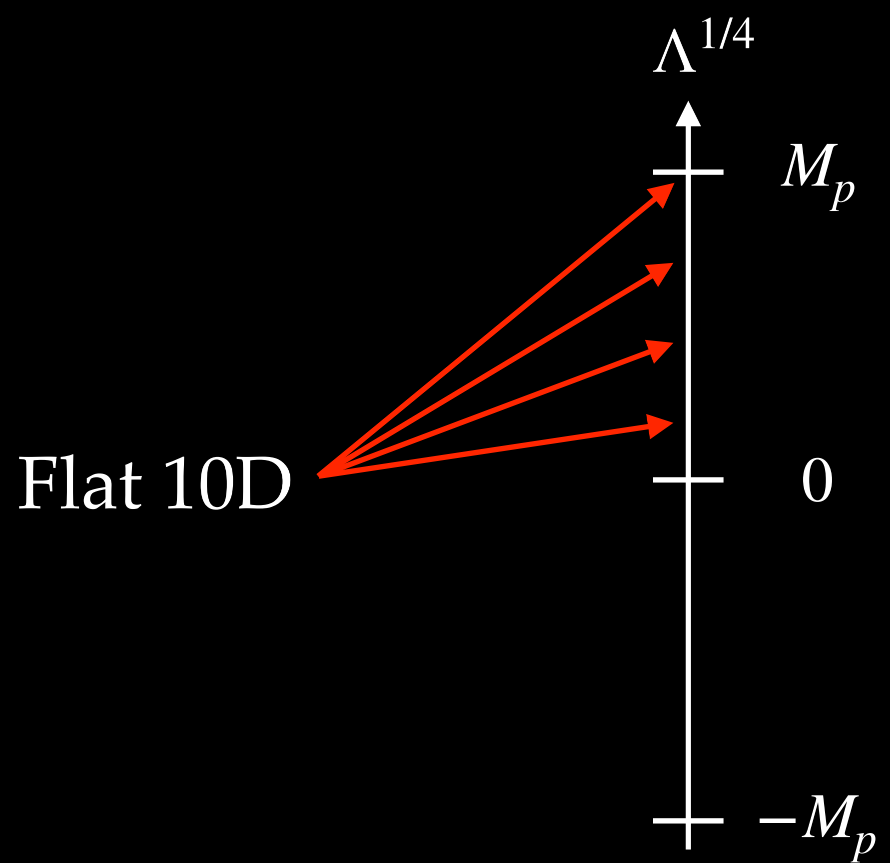

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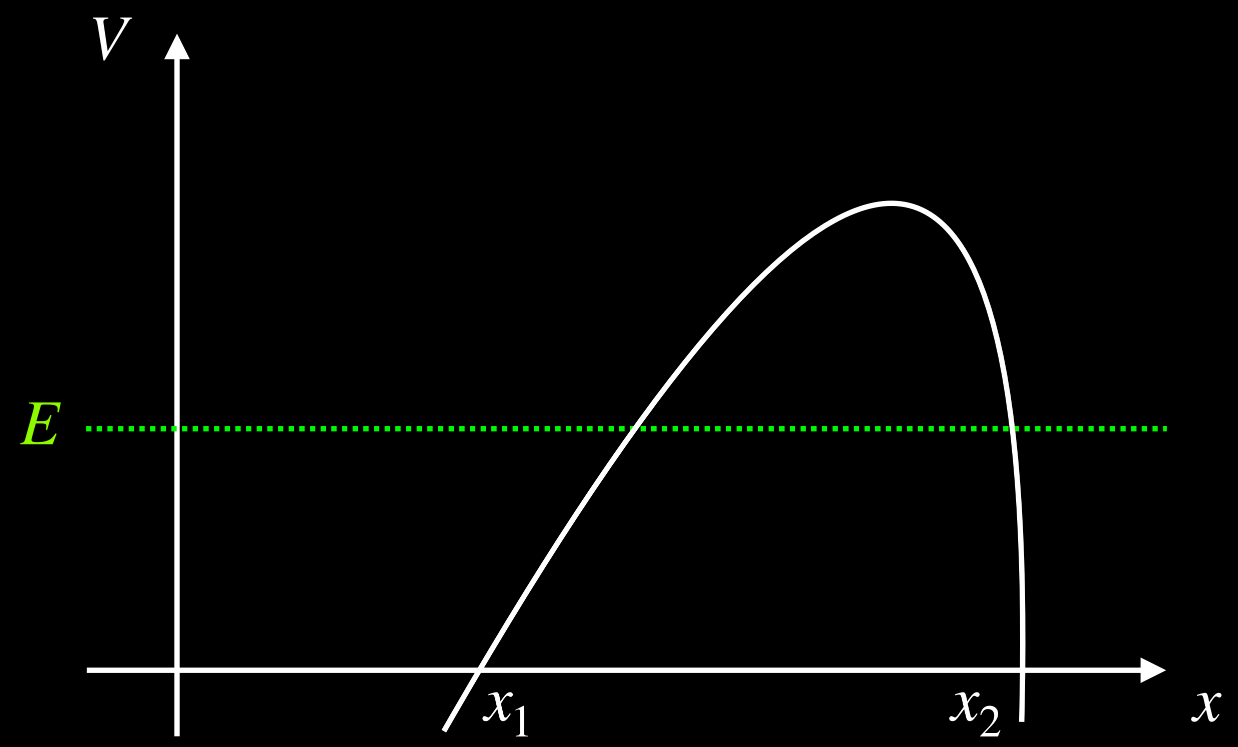
- Spontaneous Compactification from 10D Minkowski Vacuum of ST.
- Validity of Coleman-De Luccia.

WKB Approximation

- Schrödinger equation

$$\left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V\right)\psi(t,x) = i\hbar\frac{\partial}{\partial t}\psi(t,x)$$



  - Ansatz: $\psi(t,x) = e^{iS(x,t)/\hbar}$

$$\downarrow$$

$$-\frac{\partial S}{\partial t}\psi = \left(\frac{(S')^2}{2m} - \frac{i\hbar}{2m}S'' + V\right)\psi$$

- Semiclassical expansion

$$S(x,t) = S_0(x,t) + \hbar S_1(x,t) + \hbar^2 S_2(x,t) + \ldots$$

$$-\frac{\partial S_0}{\partial t} = \frac{\left(S_0'\right)^2}{2m} + V$$ Hamilton-Jacobi equation

solve order by order

$$-\frac{\partial S_1}{\partial t} = \frac{1}{2m}\left(-iS_0'' + 2S_0'S_1'\right)$$

WKB Approximation

- Energy eigenstates: $\psi(x) \propto e^{-iEt/\hbar} \longrightarrow S_0(x,t) = S_0(x) - Et$

$$E = \frac{\left(S_0'\right)^2}{2m} + V$$

$$\boxed{S_0(x) = \eta \int^x dx' \sqrt{2m\left(E - V(x')\right)} \equiv \eta \int^x p(x')\, dx'}$$

  - Always two solutions: $\eta = \pm$.
  - In the under the barrier region:

$$E - V(x) < 0 \longrightarrow S_0 \text{ is imaginary}$$

$$1/\tau \equiv \Gamma = A\, e^{-B}$$

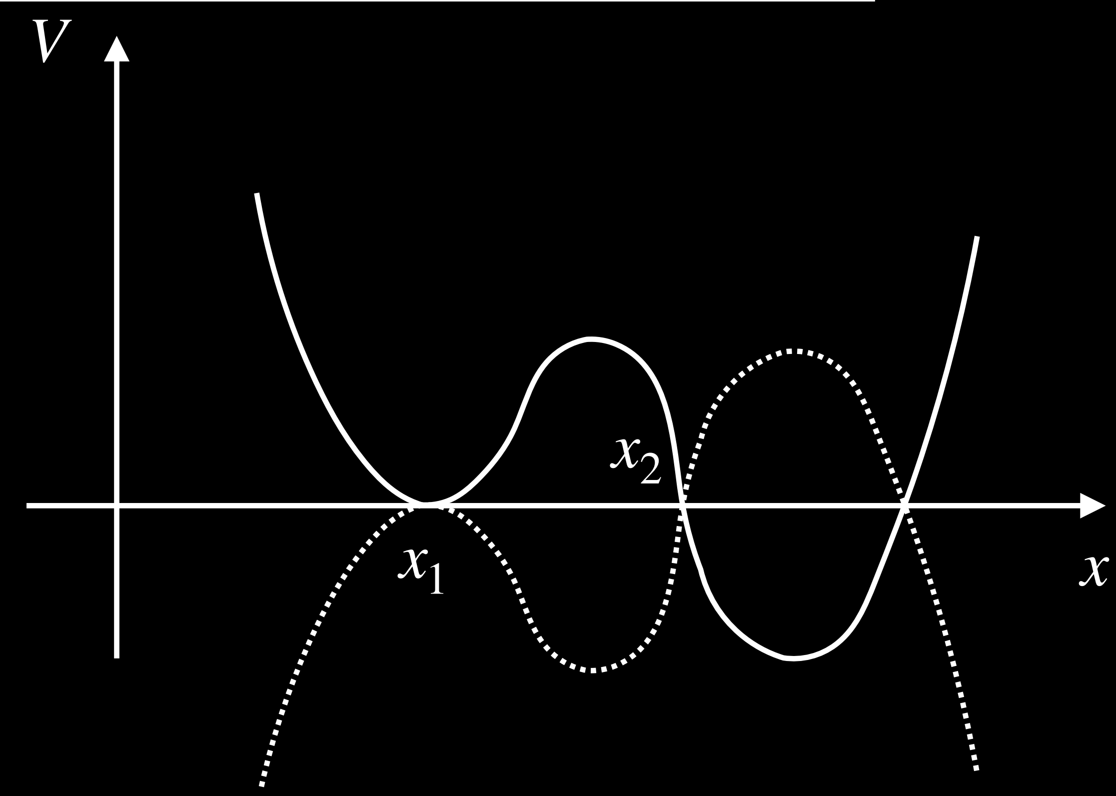
[Coleman, ’77]

[Andreassen, Farhi, Frost, Schwartz, ’16]

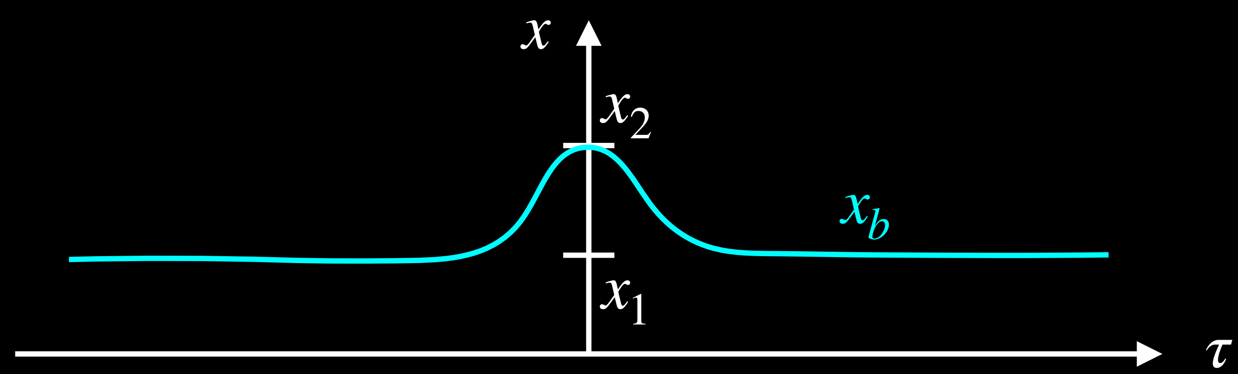
$$\boxed{B = iS_0 = 2i \int_{x_1}^{x_2} dx' \sqrt{2m(V(x') - E)}}$$

Decay of a Metastable State in QM

- Imaginary $p \sim \dot{x} \quad \Leftrightarrow \quad t \to -i\tau$



WKB result is equivalent to the Euclidean action evaluated on the bounce



$$S_E = \int d\tau \left(\frac{m}{2}\dot{x}^2 + V \right) \qquad \mathscr{E} = 0 = \frac{m}{2}\dot{x}^2 - V$$

$$\boxed{S_E(x_b) = \int_{-\infty}^{\infty} d\tau \left(\frac{m}{2}\dot{x}_b^2 + V(x_b) \right) = 2\int_{-\infty}^{0} d\tau\, 2V(x_b) = B} \qquad d\tau = \sqrt{\frac{m}{2V(x_b)}}dx_b$$

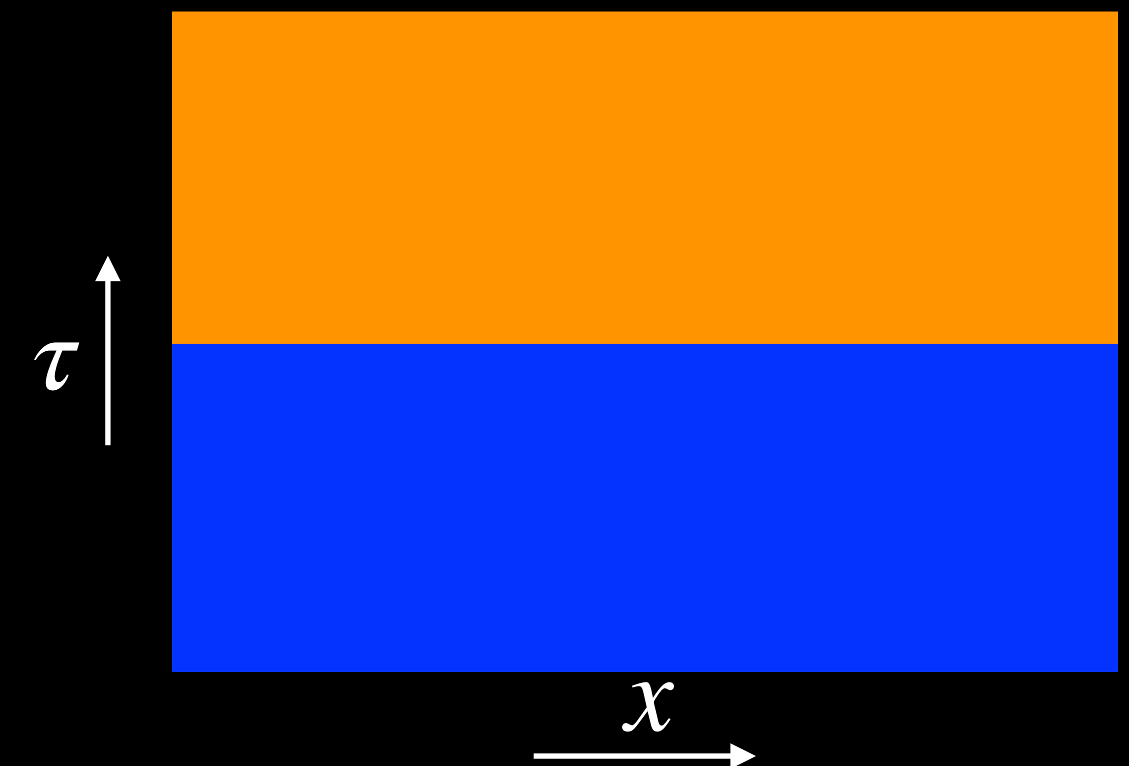
- If $S_E(x_1) \neq 0$, then $B = S_E - S_E(x_1)$. $\quad S_E(x_1) \equiv$ background

Quantum Field Theory

[Coleman, ’77]
[Callan, Coleman, ’77]

- Scalar field theory $V = \int d^4x \left(-\frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - V(\varphi)\right)$ infinite dimensional space of field configurations
- The corresponding potential energy is $U[\varphi(x)] = \int d^3x \left(\frac{1}{2}\left(\nabla\varphi\right)^2 + V(\varphi)\right)$

homogeneous tunnelling would correspond to go beyond an infinitely high barrier



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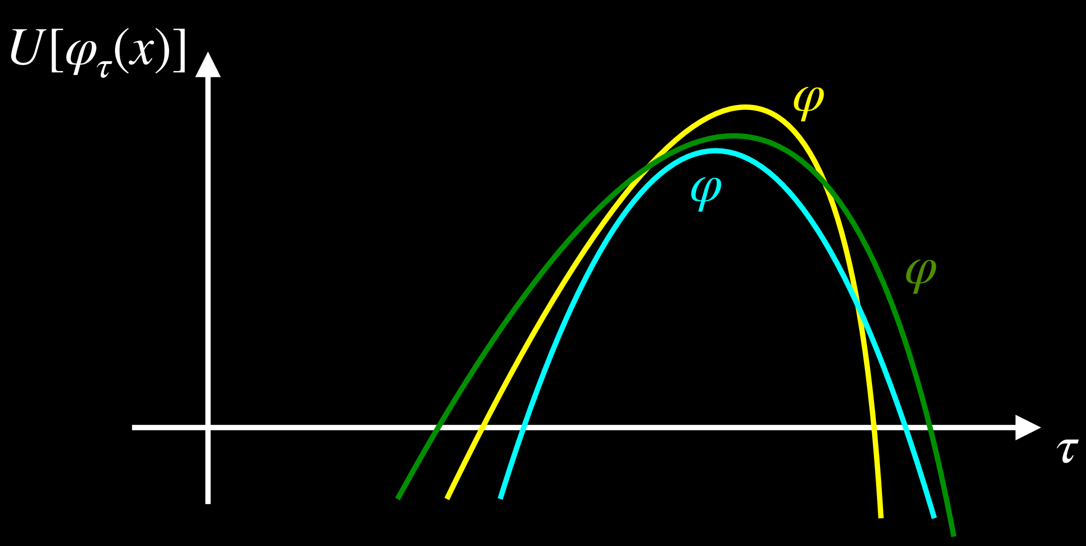
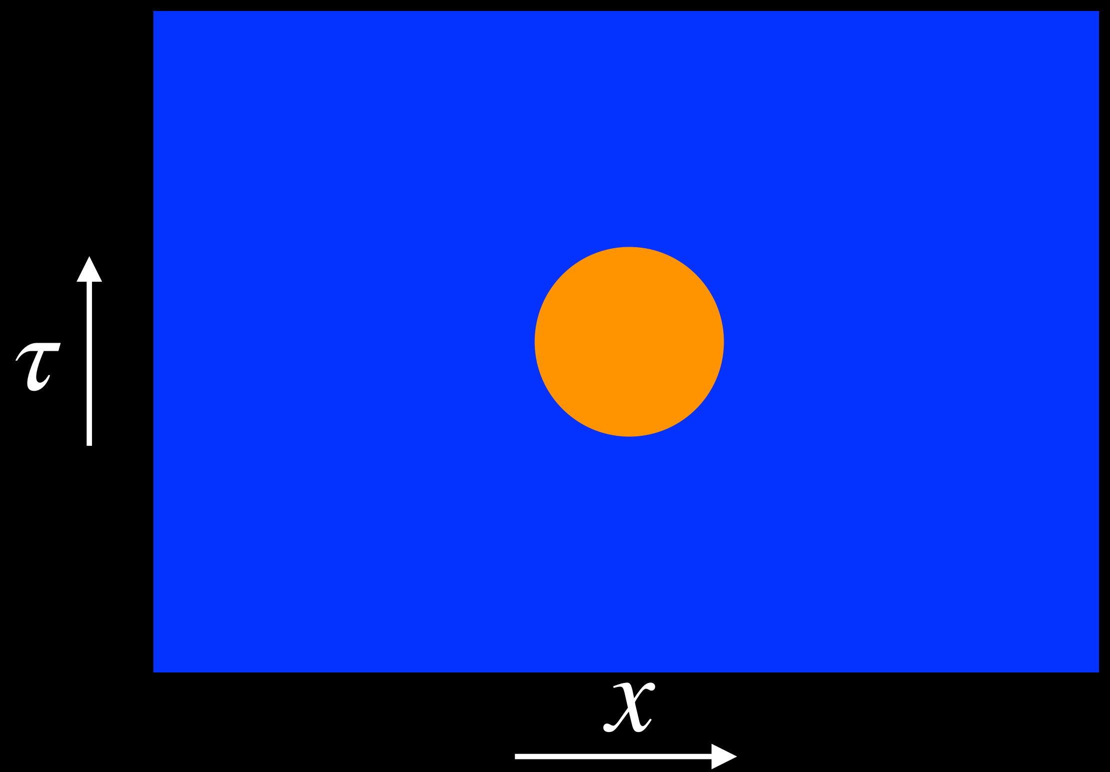
tunnelling is possible only locally

- Infinite many ways of intepolating

  $\longrightarrow$ bounce minimizes the integral of U





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$\boxed{\text{tunnelling is possible only locally}}$

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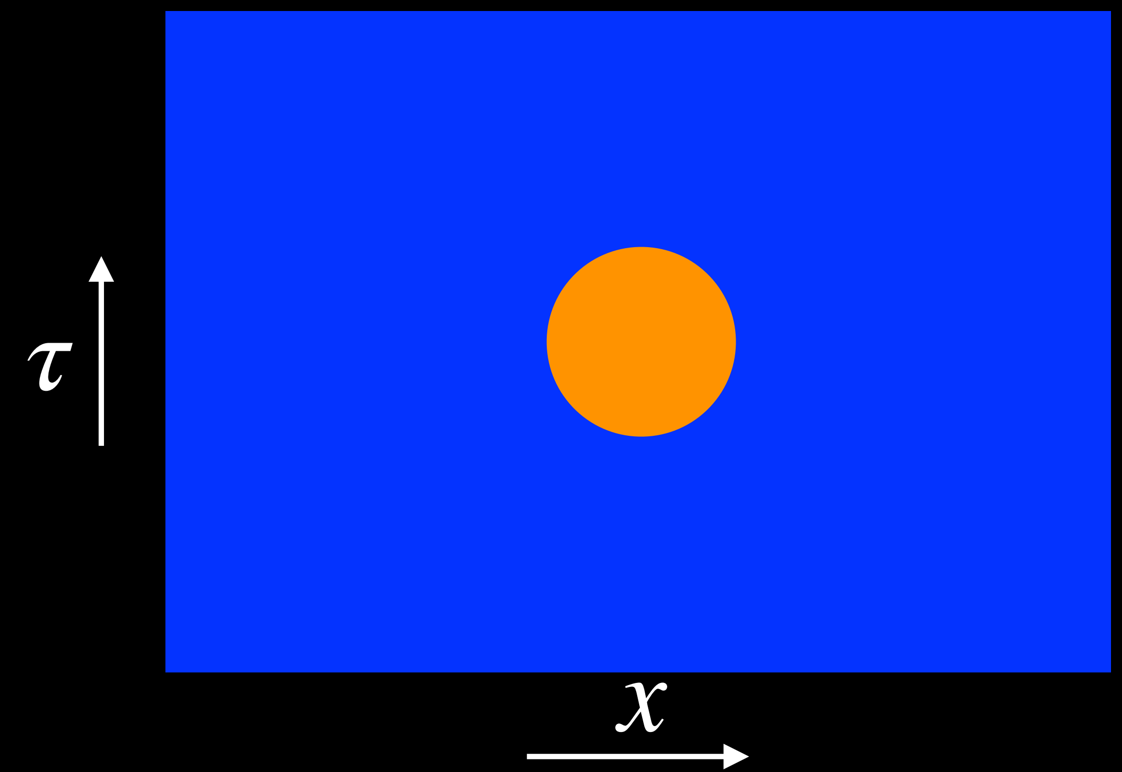
- Quantum tunnelling conserves energy

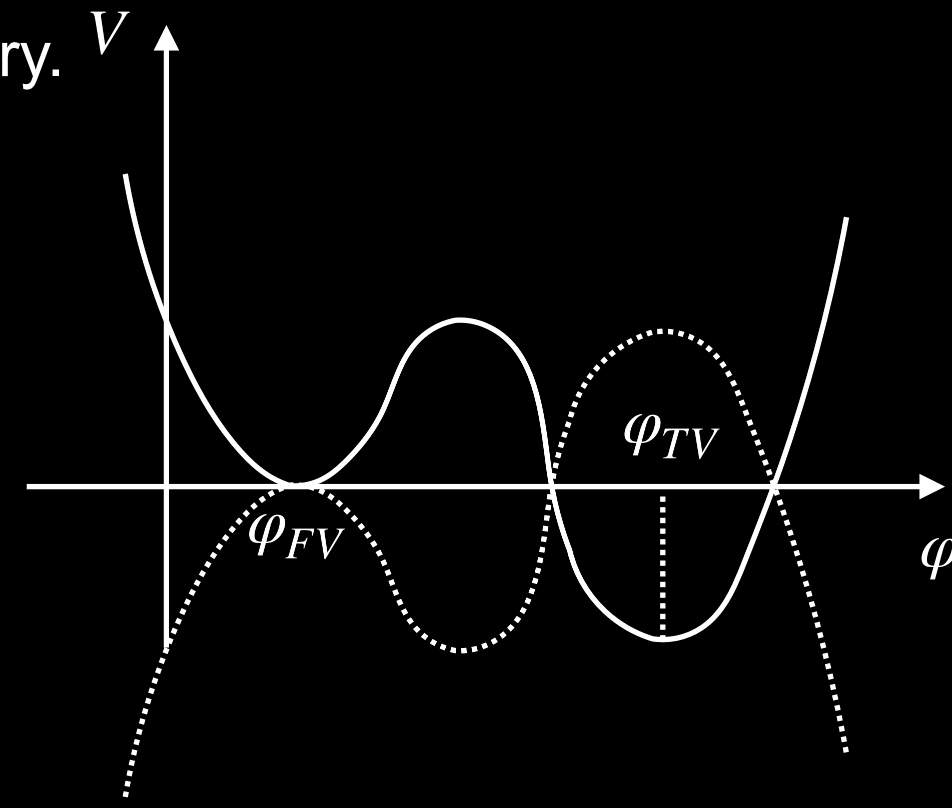
  $\longrightarrow$ up-tunnelling is forbidden



Tunnelling in Flat Space

[Coleman, '77]
[Callan, Coleman, '77]

- The bounce with the lowest action has $SO(4)$ symmetry.



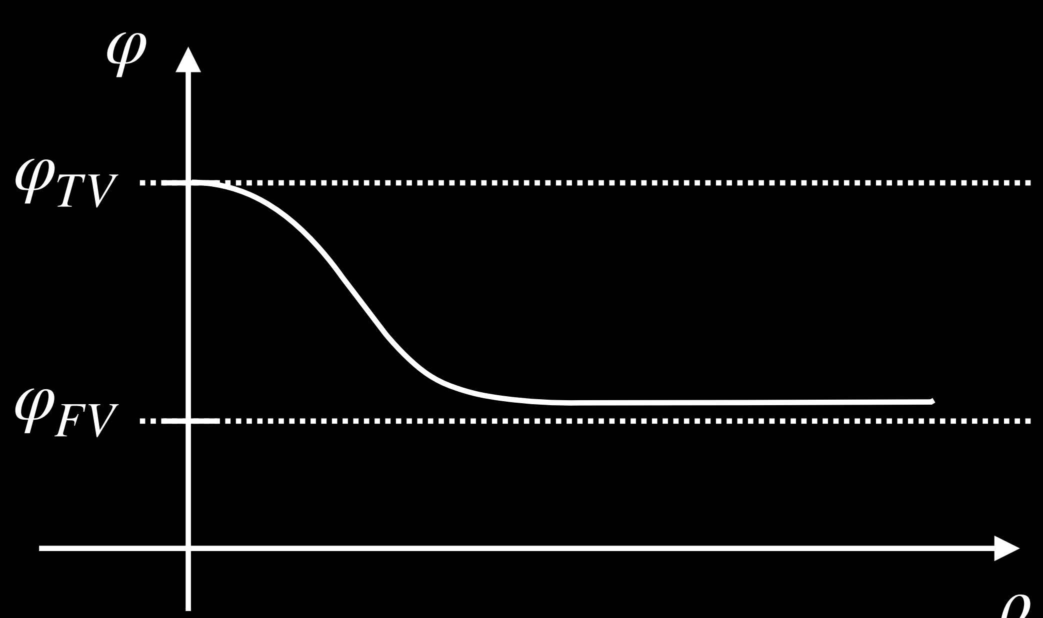
$$\rho = \sqrt{\tau^2 + x^2}$$

$$\varphi'' + \frac{3}{\rho}\varphi' - \frac{dV}{d\varphi} = 0$$

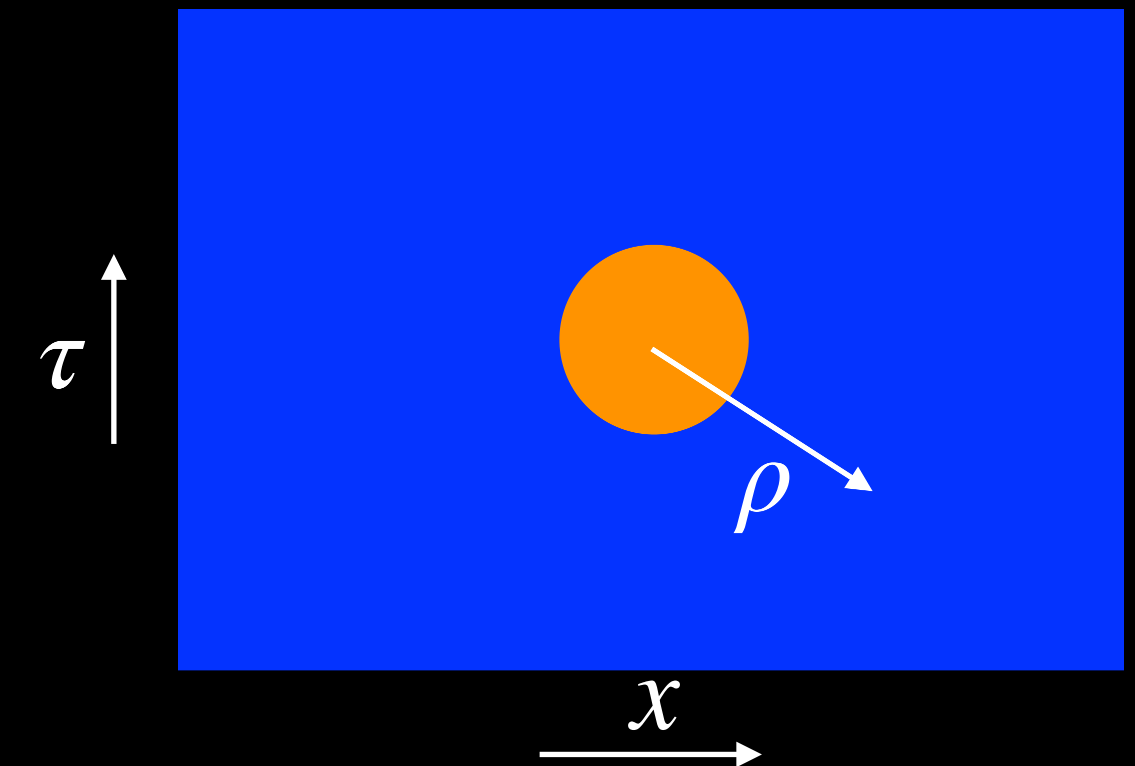
classical particle with
friction in inverted potential

BCs:
$$\varphi_b(0) = \varphi_{TV}$$
$$\varphi_b(\infty) = \varphi_{FV}$$
$$\varphi_b'(0) = 0$$

this solution always exists (overshoot/undershoot argument)



Euclidean space



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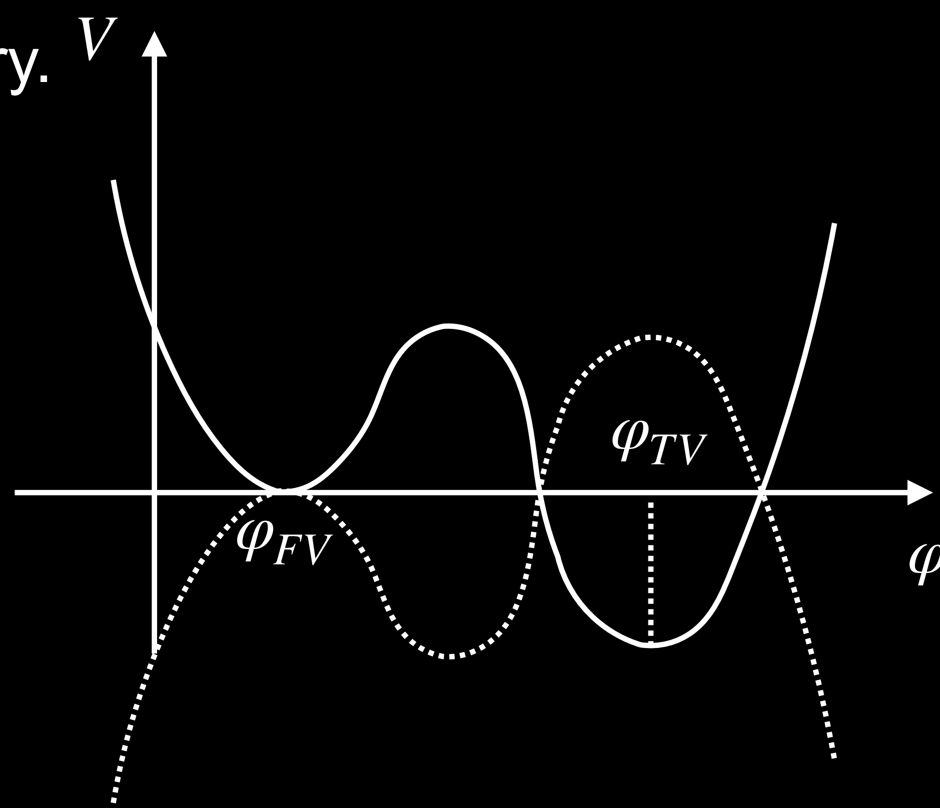

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- If pressure wins, the bubble expands.

- Evolution of the bubble after nucleation

$$\rho = \sqrt{|x|^2 + \tau^2} \xrightarrow{\tau \to it} r = \sqrt{|x|^2 - t^2}$$

$$\varphi_E = f(x, \tau) \longrightarrow \varphi_M = f(x, it) \text{ at } |x|^2 > t^2$$

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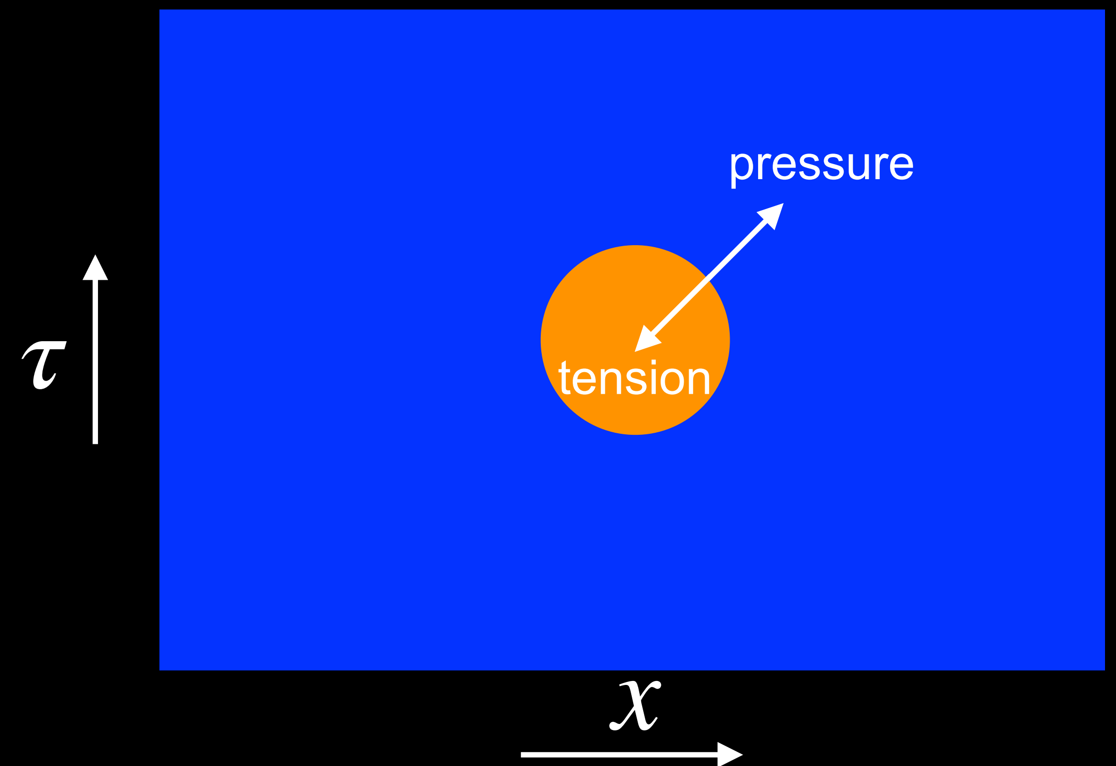

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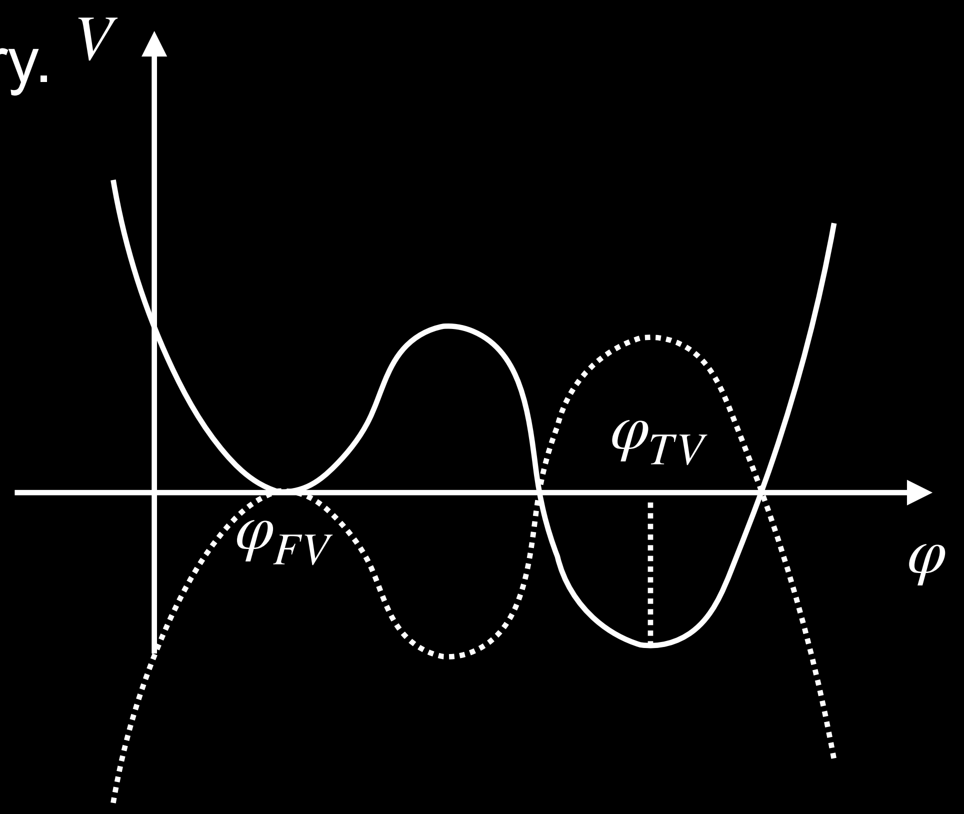

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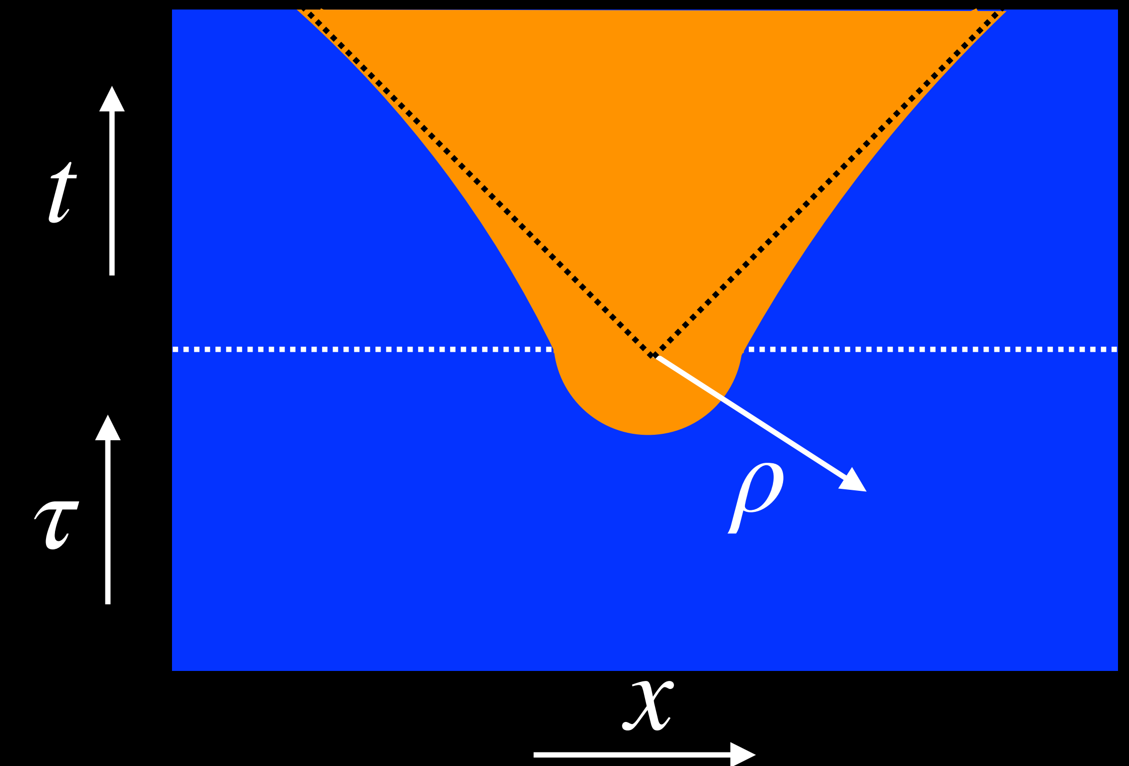


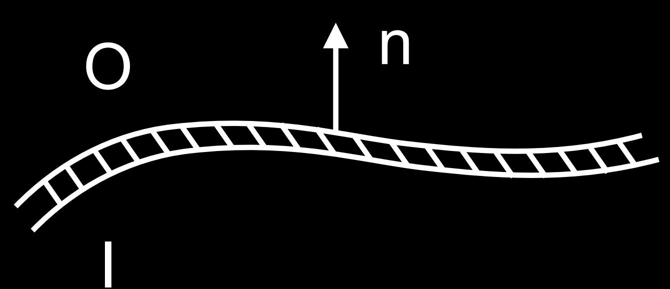
Coleman-De Luccia

[Coleman, De Luccia, '80]

- Including gravity?
  - Scales close to the Planck mass.
  - Radius of the bubble comparable to the horizon.
  - Spacetime and topology change.

[Israel, '67]

- Need to patch different spacetimes together $\longrightarrow$ Junction conditions



Σ (2+1)D timelike surface

$$S^\alpha_\beta = \lim_{\epsilon \to 0} \left[\int_{-\epsilon}^{\epsilon} T^\alpha_\beta dn \right]$$

- Pill-box integration of the Einstein equations

$$\lim_{\epsilon \to 0} \left[\int_{-\epsilon}^{\epsilon} G^\alpha_\beta dn \right] = 8\pi S^\alpha_\beta \longrightarrow \begin{array}{l} S^n_n = 0 \\ S^\alpha_n = 0 \end{array}$$

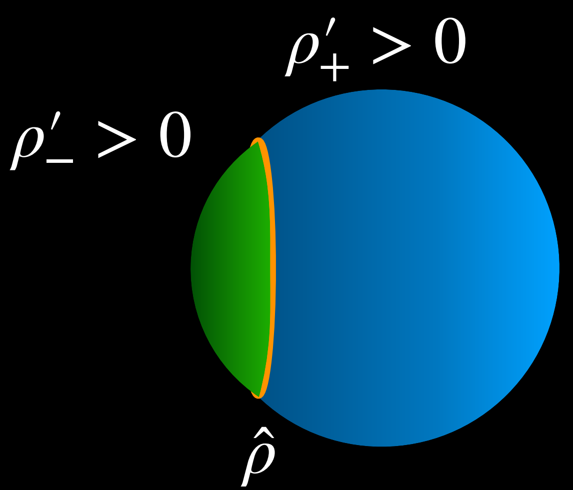
no momentum associated with the wall flows out of

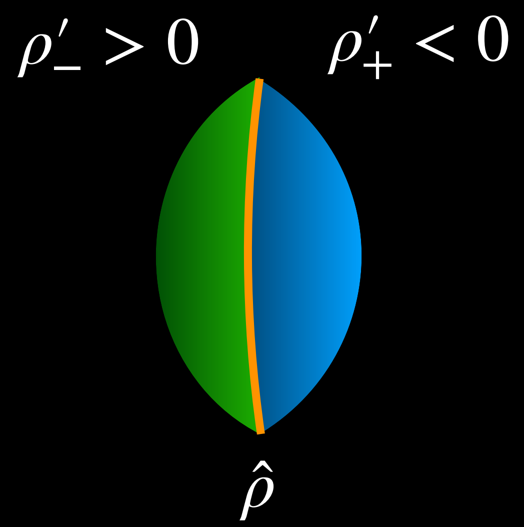
$$\underline{S^\alpha_\beta = \Delta K^\alpha_\beta - \Delta K \delta^\alpha_\beta} \longrightarrow K_{\alpha\beta} = \text{extrinsic curvature}$$

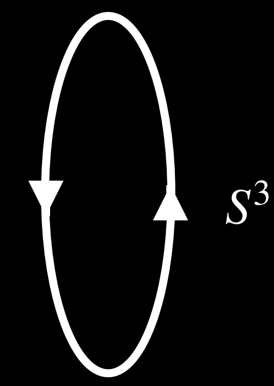
Israel junction conditions

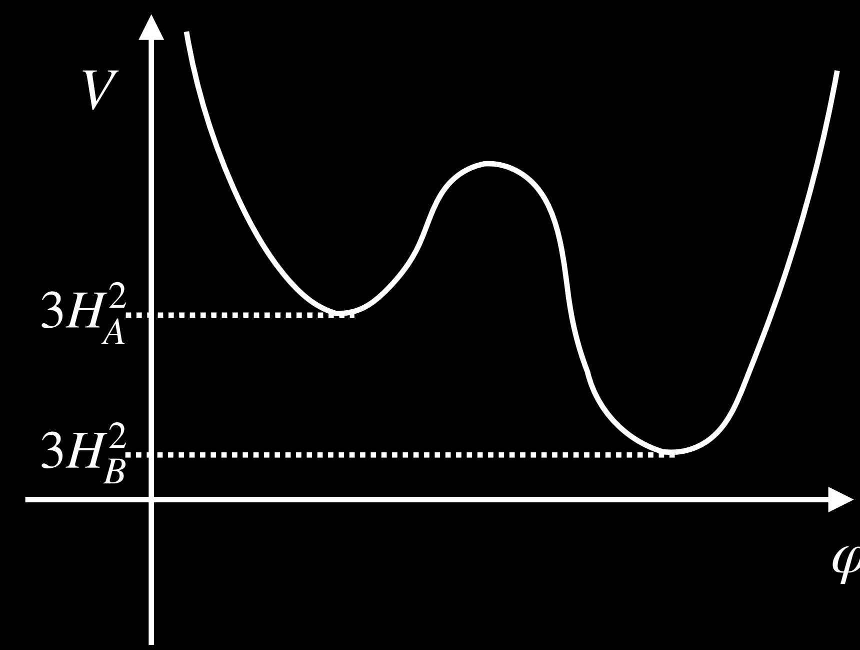
CDL: de Sitter to de Sitter

[Lee, Weinberg, '87]
[Brown, Teitelboim, '88]









Assume that the most relevant configuration is $SO(4)$ symmetric

$\kappa = 4\pi G\sigma$
$\hat{\rho} \equiv$ position of the wall

$$B = \pm 8\pi^2 \left[\frac{\left[(H_A^2 - H_B^2)^2 + \kappa^2(H_A^2 + H_B^2)\right]\hat{\rho}}{4\kappa H_A^2 H_B^2} - \frac{1}{2}\left(\frac{1}{H_B^2} - \frac{1}{H_A^2}\right)\right]$$

$$\hat{\rho}^2 = \frac{4\kappa^2}{(H_O^2 - H_I^2)^2 + 2\kappa^2(H_O^2 + H_I^2) + \kappa^4}$$

CDL: de Sitter to de Sitter

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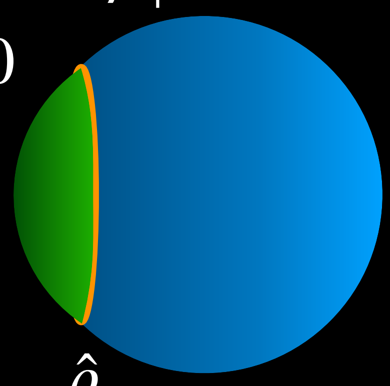

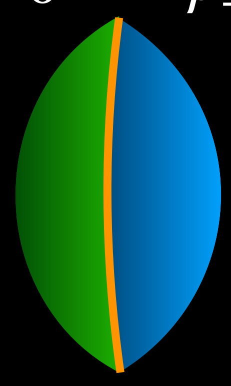

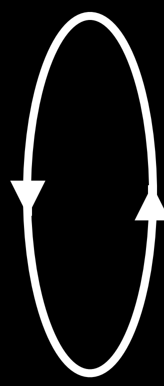

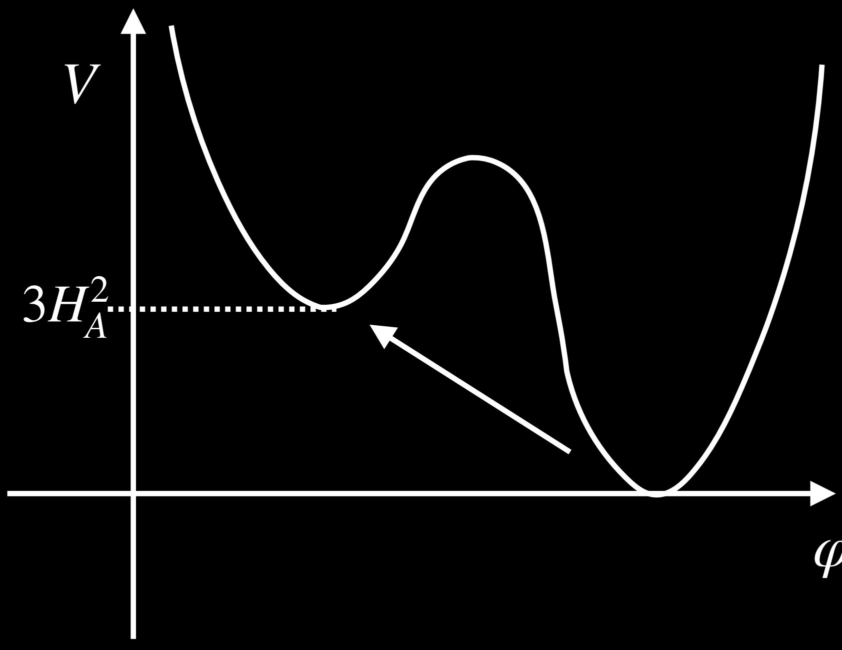

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$$\lim_{H_B \to 0} B = \infty \quad \longrightarrow \quad \mathrm{P} \sim e^{-B} \to 0$$

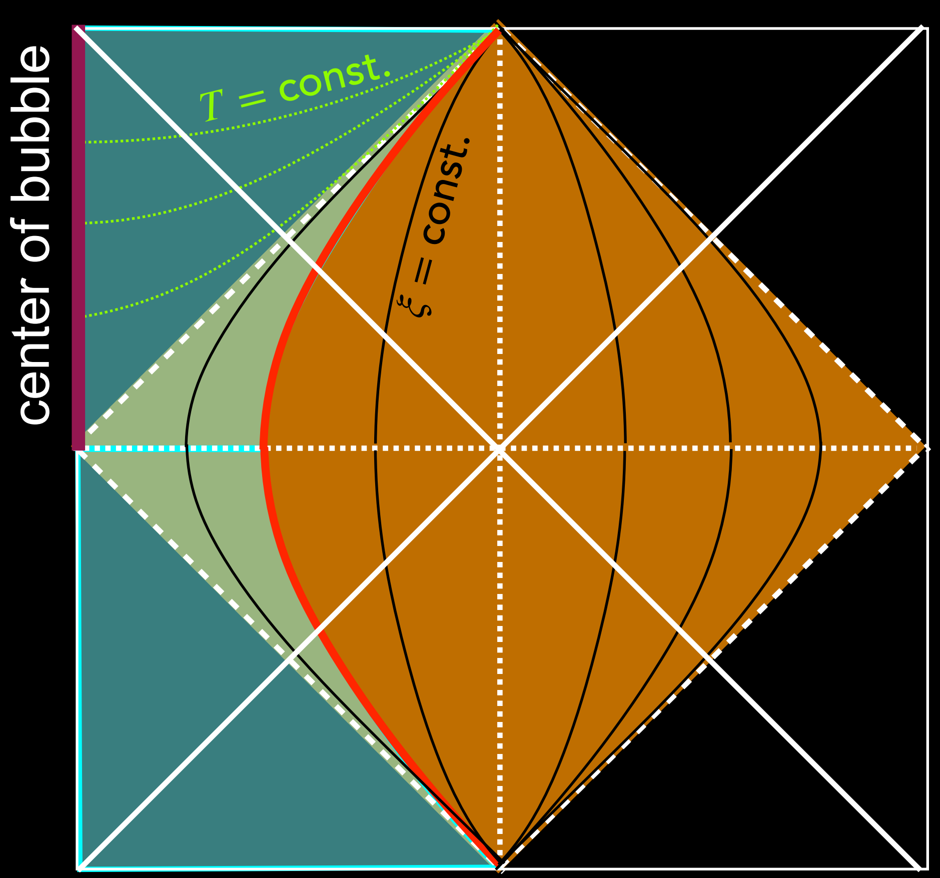
up-tunnelling from Minkowski is forbidden

CDL: Penrose Diagram

- $SO(4)$ symmetry $ds^2 = a^2(\xi)\left(d\xi^2 + d\Omega_3^2\right)$

$$d\Omega_3^2 = d\theta^2 + \sin^2\theta d\Omega_2^2$$

Scalar field only depends on ξ



- Analytic continuation of θ

$$\theta \to \frac{\pi}{2} + it$$

$$ds^2 = a^2(\xi)\left(d\xi^2 \underbrace{- dt^2 + \cosh^2 t\, d\Omega_2^2}_{dS_3}\right)$$

  - $SO(1,3)$ symmetry.
  - Describe orange diamond: it's not geodesically complete.
  - Black lines denote constant ξ surfaces.

- Analytic continuation $\xi \to T + i\frac{\pi}{2}$, $\theta \to i\rho$ $\longrightarrow$ $ds^2 = a^2(T)\left(-dT^2 + \underbrace{d\rho^2 + \sinh^2\rho\, d\Omega_2^2}_{dH_3}\right)$

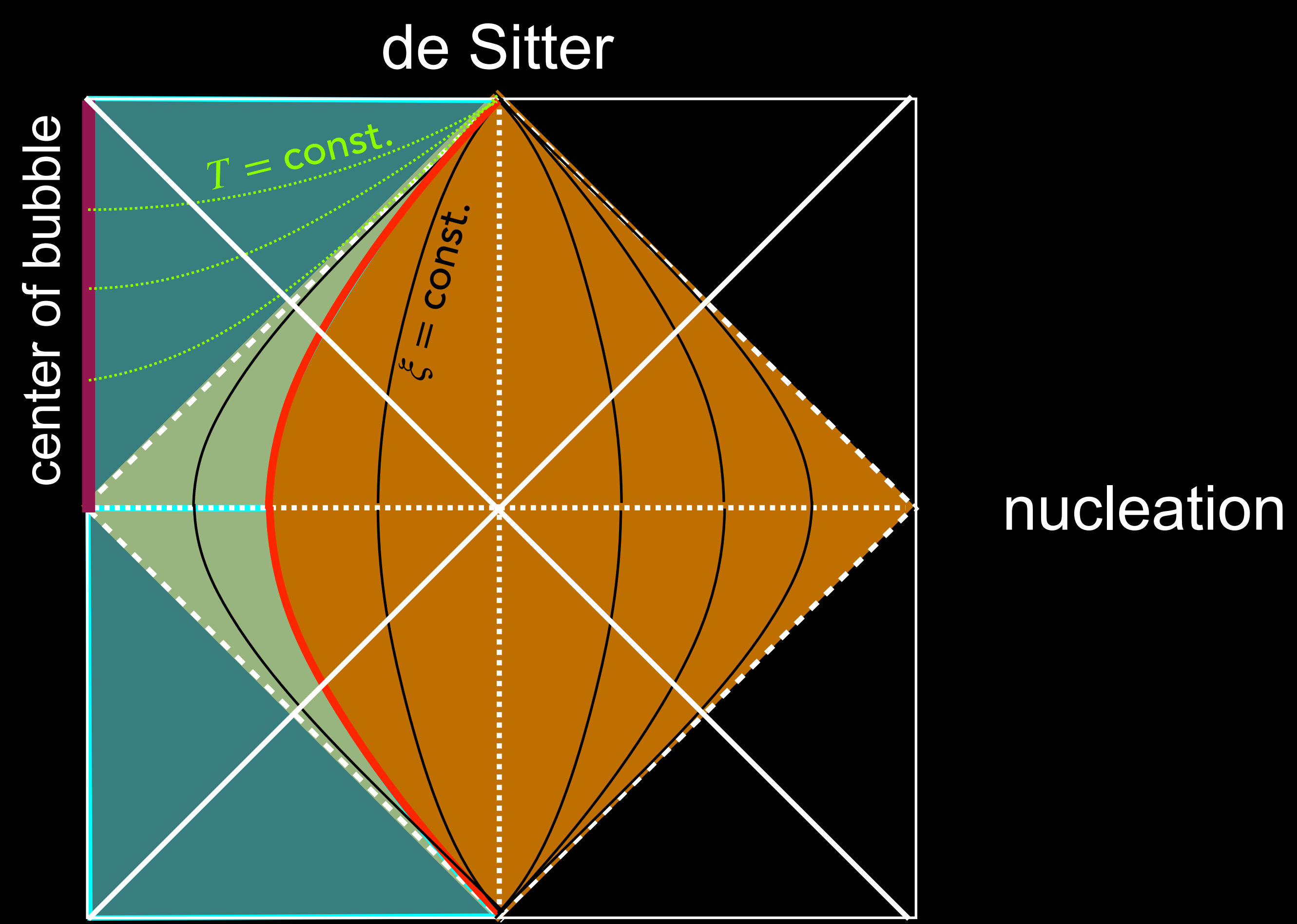
  - Describe upper left triangle.
  - Green lines denote constant T, open slices.

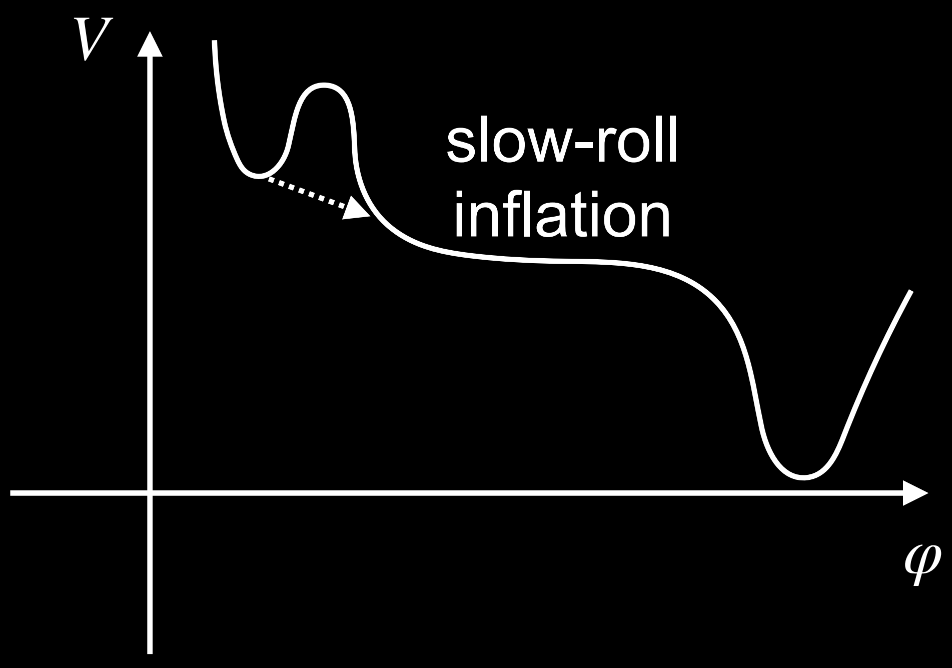
CDL: Open Universe

- Open slices → constant scalar field

↓

Observer on the west pole observes an open universe





[Freivogel, Kleban, Martinez, Susskind, ’06, ’14]
[Batra, Kleban, ’07] [Kleban, Schillo, ’12]

Observation of closed universe rules out the landscape and/or string theory?

Euclidean Techniques: Issues

- Is Coleman-De Luccia reliable in all cases and for all implications?

[Blanco-Pillado, Deng, Vilenkin, '19]

[11]. It should be noted, however, that while Coleman's flat space calculation was solidly based on first principles, the CdL formula (1) was proposed in [8] essentially by analogy with the flat space case, so its validity is open to question.

- $SO(4)$ symmetry $\longrightarrow$ is open universe a general consequence of tunnelling?

- Minkowski to de Sitter up-tunnelling is not possible.
  [Freivogel, Hubeny, Maloney, Myers, Rangamani, Shenker, '06]

- More general solutions using Euclidean techniques, e.g. Schwarzschild to de Sitter?
  [Farhi, Guth, Guven, '89]

[Guth's talk at string cosmology in '04]

Problem: The equal-time surface evolves in Euclidean space in a circular pattern, crossing itself and recovering some of the same spacetime points. The multiple coverings are not consistent. The Euclidean solution is not a manifold.

- Negative mode problem. [Lavrelashvili, Rubakov, Tinyakov, '85]

Hamiltonian Formalism for Tunnelling

- Wheeler-DeWitt equation: $$\mathscr{H}\Psi(\Phi) = \left[-\frac{\hbar^2}{2}G^{MN}(\Phi)\,\nabla_M\nabla_N + f(\Phi)\right]\Psi(\Phi) = 0$$ on a fixed time-slice
  [DeWitt, '67]

- Semiclassical expansion: $$\Psi(\Phi) = \exp\left(\frac{i}{\hbar}S\right) \longrightarrow S[\Phi] = S_0[\Phi] + \hbar S_1[\Phi] + \hbar^2 S_2[\Phi] + \ldots$$

- Hamilton-Jacobi: $$\frac{1}{2}G^{MN}\frac{\delta S_0}{\delta\Phi^M}\frac{\delta S_0}{\delta\Phi^N} + f(\Phi) = 0$$

- Action: $$\boxed{S_0(\Phi(s)) = \int^{\Phi(s)}\int d^3x\,\pi_M d\Phi^M}$$

- We compute $S_0[\Phi(s)] - S_0[\Phi(0)]$

  $\downarrow$
  background

$$\boxed{\mathrm{P} = \frac{|\Psi(\text{nucleated})|^2}{|\Psi(\text{background})|^2}}$$

compare wave functions of different spacetime configurations

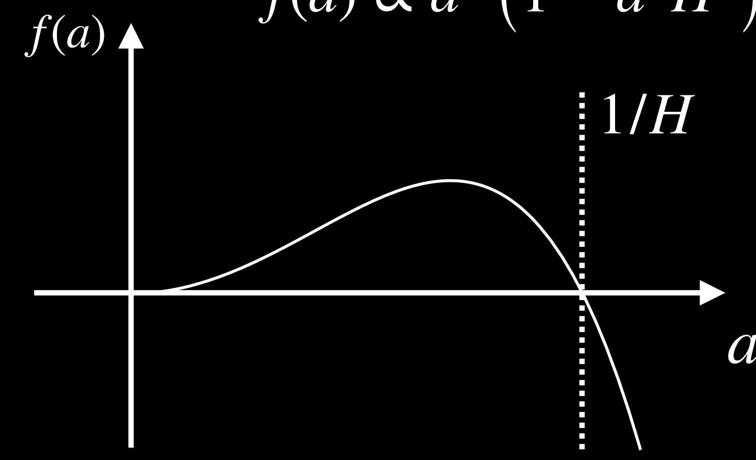
Vilenkin vs Hartle-Hawking

[Hartle, Hawking, Vilenkin et al., since '80s]

- Minisuperspace: $ds^2 = \ell^2 \left(-d\tau^2 + a^2(\tau) d\Omega_3^2\right)$ $\longrightarrow$ scale factor determines the metric

- Hamiltonian: $\mathscr{H} = \dfrac{p_a^2}{12a} + \underbrace{3a - 3a^3 H^2}_{f(a)}$

$f(a) \propto a^2 \left(1 - a^2 H^2\right)$



tunnelling
from "nothing"

- WDW: $\left[\dfrac{d^2}{da^2} - f(a)\right] \Psi(a) = 0$ $\qquad p_a \to -i \dfrac{\partial}{\partial a}$

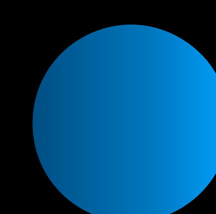
- Action:

$$S_0 = \eta\, 2\pi^2 \int_0^a p_a da = i\eta\, 12\pi^2 \int_0^a da\, a\sqrt{1 - a^2 H^2}$$

At $a = 1/H$ $\longrightarrow$ $\boxed{iS_0 = \dfrac{\eta\pi}{2GH^2}}$

$\Psi = c_1 e^{iS_0} + c_2 e^{-iS_0}$

Vilenkin/Hartle-Hawking
wave function



S^3

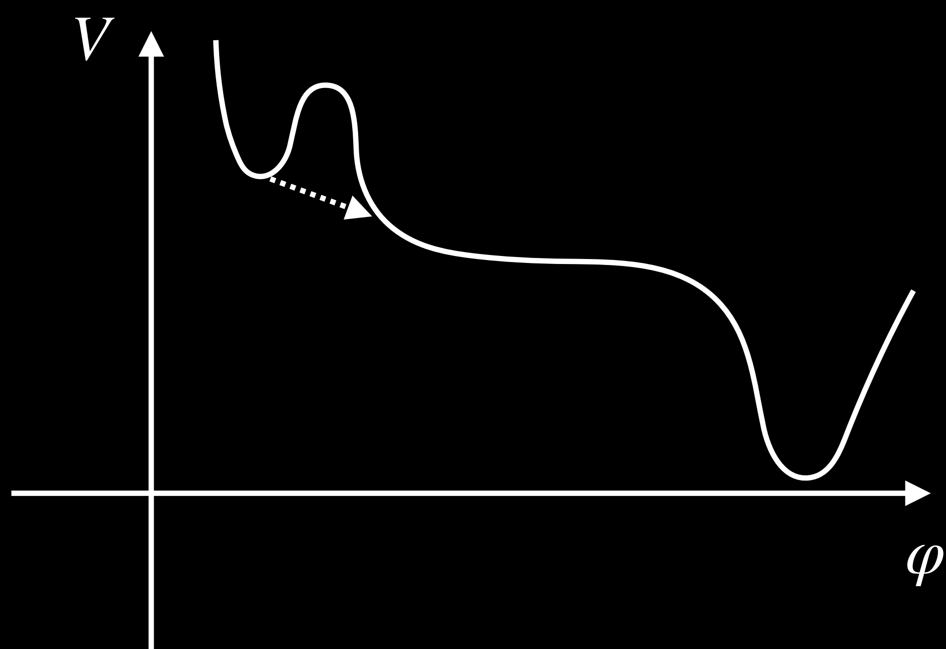
- Boundary conditions fix constants

Hartle-Hawking wave function requires $\Psi(0) = 0$ $\longrightarrow$ $\eta = +$

Vilenkin wave function requires only outgoing wave at $a \gg 1/H$ $\longrightarrow$ $\eta = -$

Summary of dS to dS Transitions

Transitions à la CDL



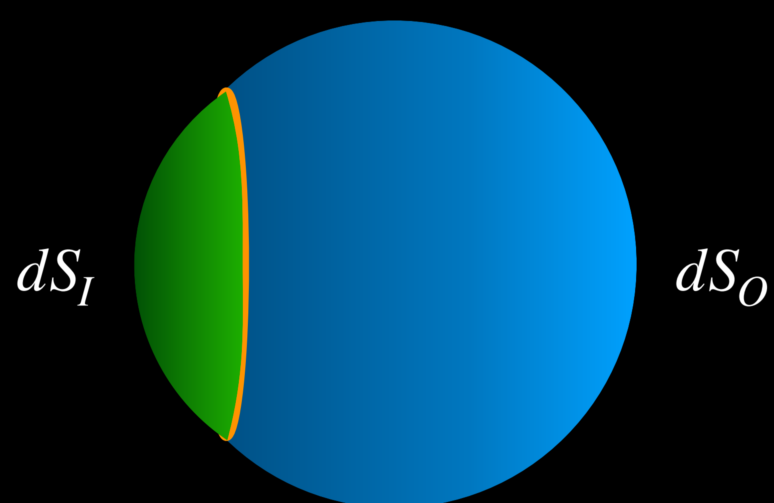
Transitions à la BT

$\text{EFT}_1 \longleftrightarrow \text{EFT}_2$

Λ_1 Λ_2

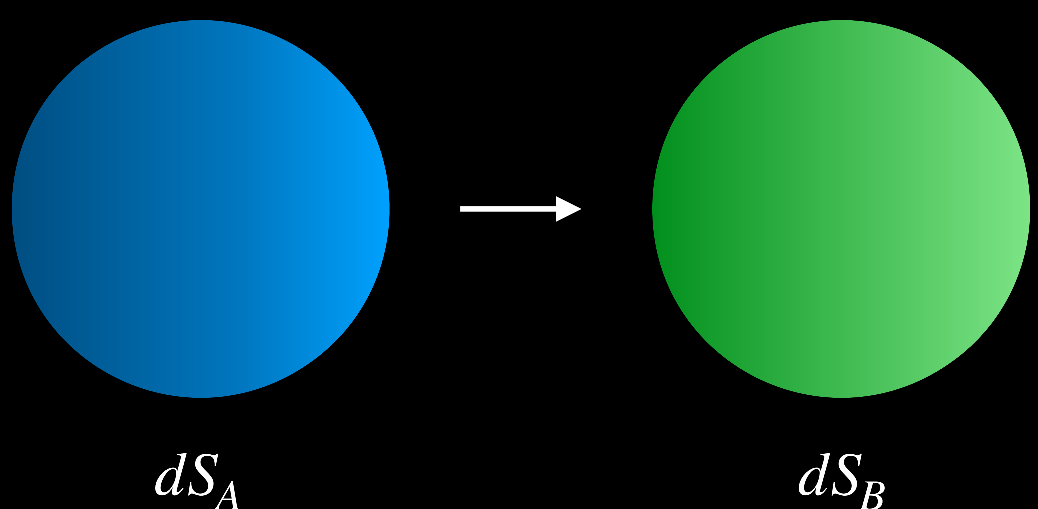
thin-wall approximation

Transitions with a wall



$dS_I / dS_O \oplus W$

Transitions in minisuperspace

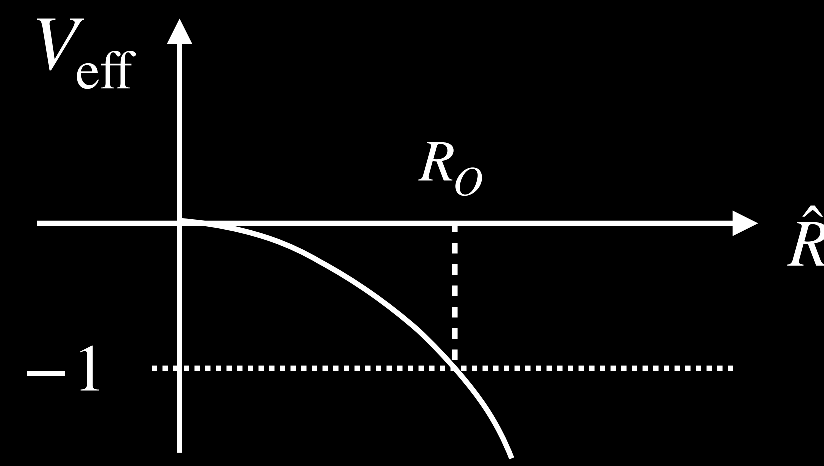


Dynamics of dS-dS Bubbles

[Blau, Guendelman, Guth, '86]
[Cespedes, de Alwis, Muia, Quevedo, '20]

Bubble trajectory for string landscape transitions

$$\dot{\hat{R}}^2 + V_{\text{eff}} = -1$$



tunnelling from 'nothing'

$\boxed{\text{Bubble trajectory}}$

$$\cos\hat{r} = \sqrt{1 - H^2R_0^2}\cos T$$

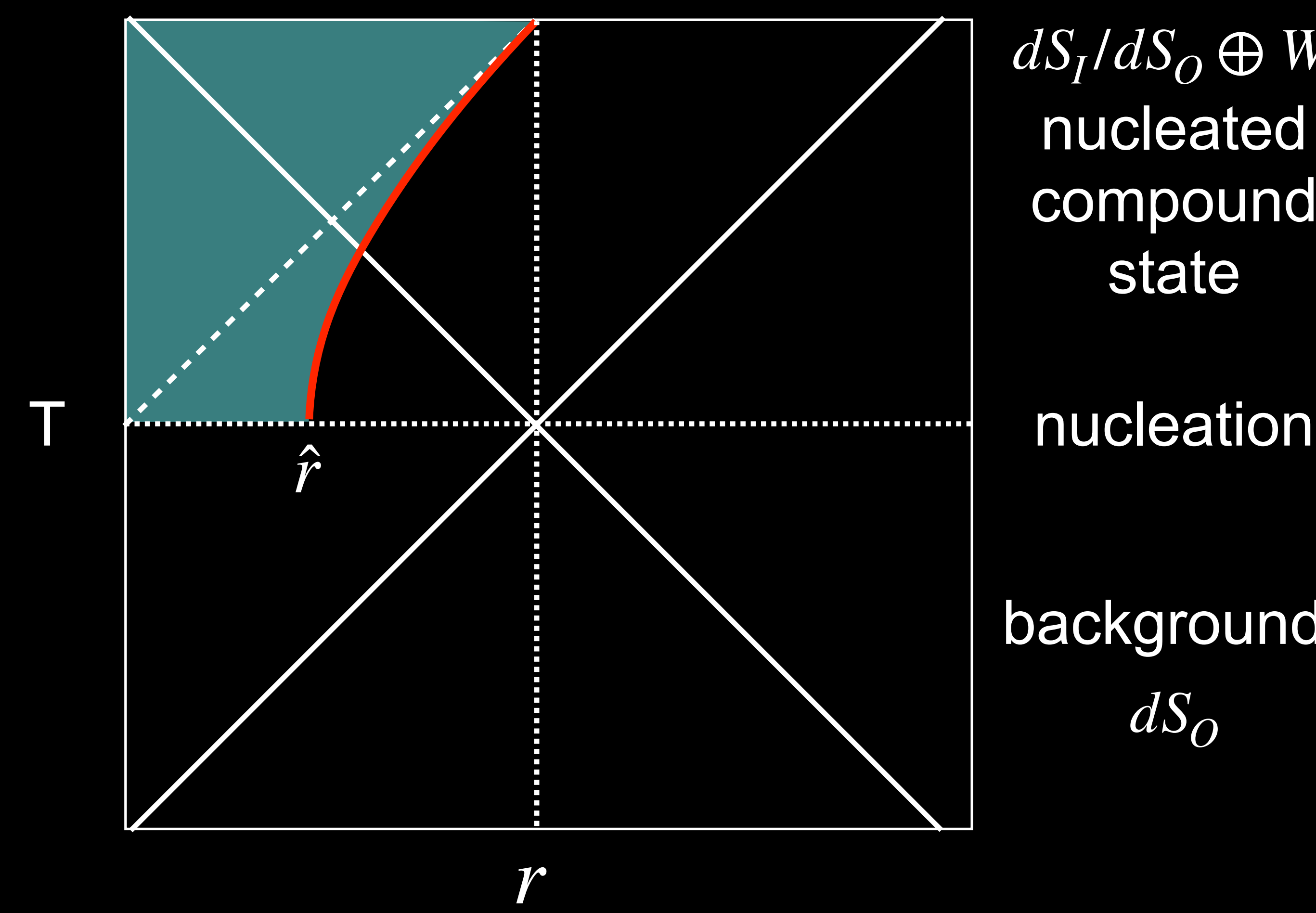
$$R_0 = H^{-1}\sin\hat{r}$$

- No reference to $SO(4)$ symmetry.
- Recover $SO(3,1)$ symmetry.

wall at $X_1 = \text{const.}$

$$-X_0^2 + X_2^2 + X_3^2 + X_4^2 = R_0^2$$

- Asymptotic speed smaller than c.



de Sitter to de Sitter

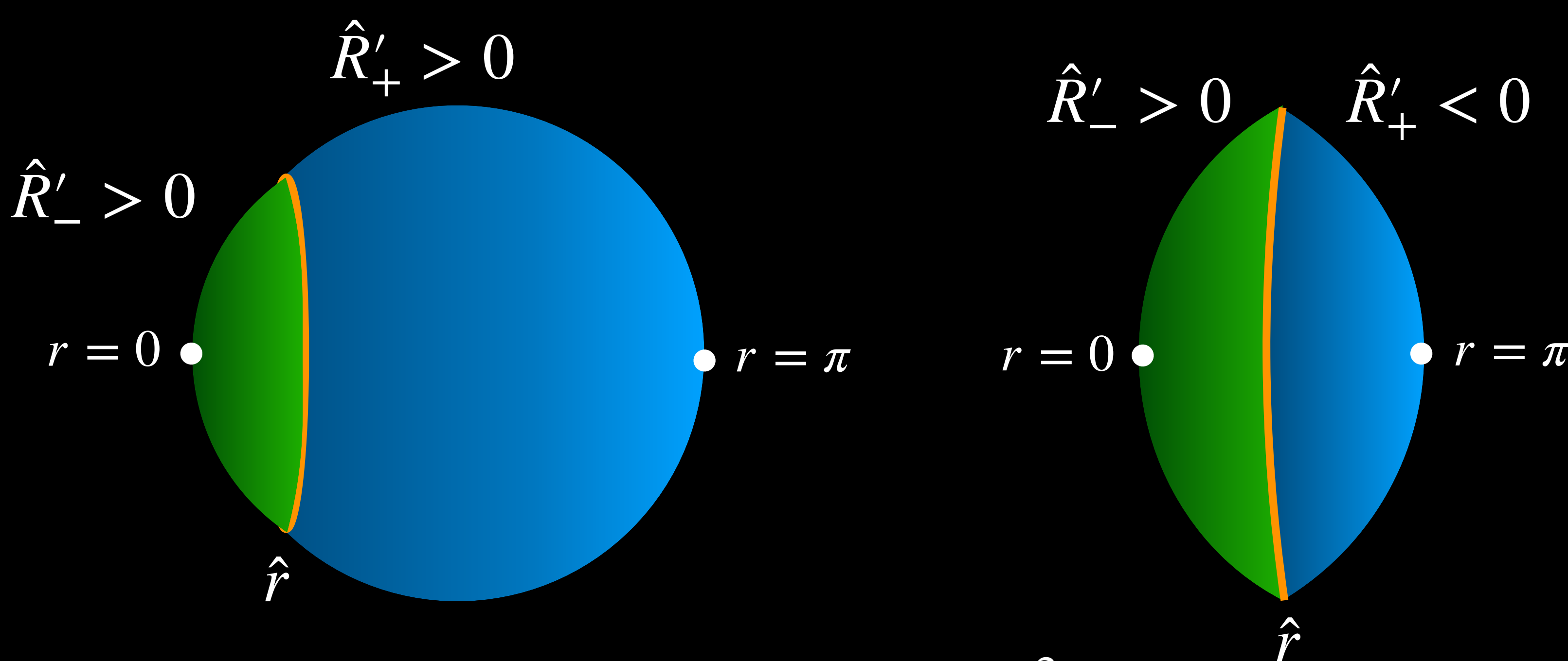
[Fischler, Morgan, Polchinski, ’90]
[de Alwis, Muia, Pasquarella, Quevedo, ’19]

- General $SO(3)$ symmetric solutions: $$ds^2 = -dt^2 + L^2(t,r)dr^2 + R^2(t,r)d\Omega_2^2$$

- Action: $$\delta S = \int dr\,\big[\underbrace{\pi_L \delta L + \pi_R \delta R}_{\text{bulk}} + \hat{p}\delta\hat{r}\big]$$ → wall

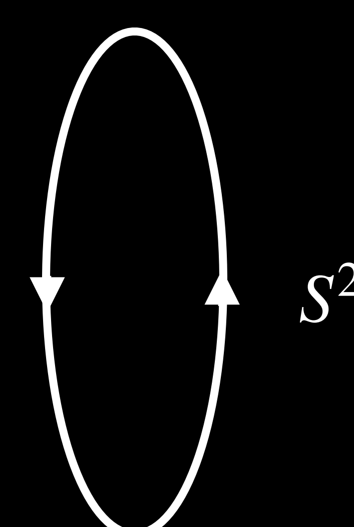
$$2iS_{\rm tot} = B_{tot} = B_B + B_w$$

bulk wall



portions of S^3

$SO(3)$ symmetry is preserved all the way through



The wall breaks $SO(4)$

de Sitter to de Sitter

[Fischler, Morgan, Polchinski, ’90]
[de Alwis, Muia, Pasquarella, Quevedo, ’19]

$$2iS_{\text{tot}} = B_{tot} = B_B + B_w$$

$$\frac{B_B}{2} = \frac{\eta}{G}\int_0^{\hat{r}-\epsilon} dr\, R\left[\sqrt{A_I L^2 - R'^2} - R' \arccos\left(\frac{R'}{L\sqrt{A_I}}\right)\right] + \int_{\hat{r}+\epsilon}^{\pi}\left[I \leftrightarrow O\right]$$

$$\frac{B_w}{2} = \frac{\eta}{G}\int d\hat{R}\,\hat{R}\arccos\left(\frac{R'}{L\sqrt{\hat{A}}}\right)\Bigg|_{\hat{r}-\epsilon}^{\hat{r}+\epsilon}$$

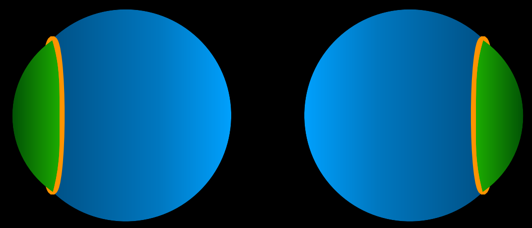
$$R_0^2 = \frac{4\kappa^2}{(H_O^2 - H_I^2)^2 + 2\kappa^2(H_O^2 + H_I^2) + \kappa^4}$$

$$\boxed{\frac{B_{tot}}{2} = -\frac{\eta\pi}{G}\left[\frac{\left(H_O^2 - H_I^2\right)^2 + \kappa^2\left(H_O^2 + H_I^2\right)R_0^2}{8\kappa H_O^2 H_I^2}\right] - \frac{1}{4H_I^2} - \frac{1}{4H_O^2}}$$

Symmetric under the exchange $I \leftrightarrow O$

$\downarrow$

Background subtraction breaks the symmetry



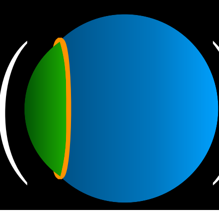
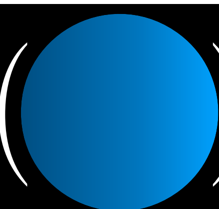
give the same result

Subtract Hartle-Hawking/Vilenkin wave function $\frac{\overline{B}}{2} = \frac{\eta\pi}{2GH_O^2}$

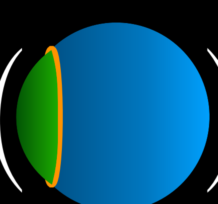
de Sitter to de Sitter

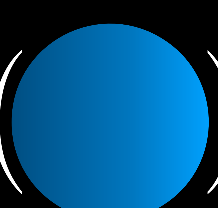
[de Alwis, Muia, Pasquarella, Quevedo, '19]

- The result is in agreement with CDL's final result, for $\eta = +1$.

$\mathrm{P}(dS_O \to dS_O/dS_I \oplus W) \equiv$ $\left|\Psi\left(\right.\right.$  $\left.\left.\right)\right|^2$ / $\left|\Psi\left(\right.\right.$  $\left.\left.\right)\right|^2$ $= \exp\left(-\eta B_{\mathrm{CDL}}\right)$

- Limit Minkowski to de Sitter: $H_O \to 0$.

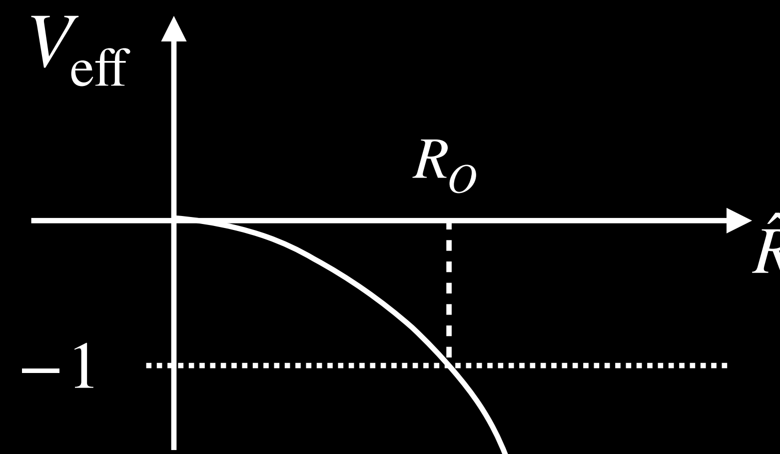
$\left|\Psi\left(\right.\right.$  $\left.\left.\right)\right|^2$ $\longrightarrow \dfrac{\eta\pi}{2G}\dfrac{H_I^2 + 2\kappa^2}{(H_I^2+\kappa^2)^2}$ finite

$\left|\Psi\left(\right.\right.$  $\left.\left.\right)\right|^2$ $\simeq e^{\frac{\pi}{GH^2}} \longrightarrow \infty$ blows-up

$\longrightarrow \mathrm{P} \to 0$ due to the background

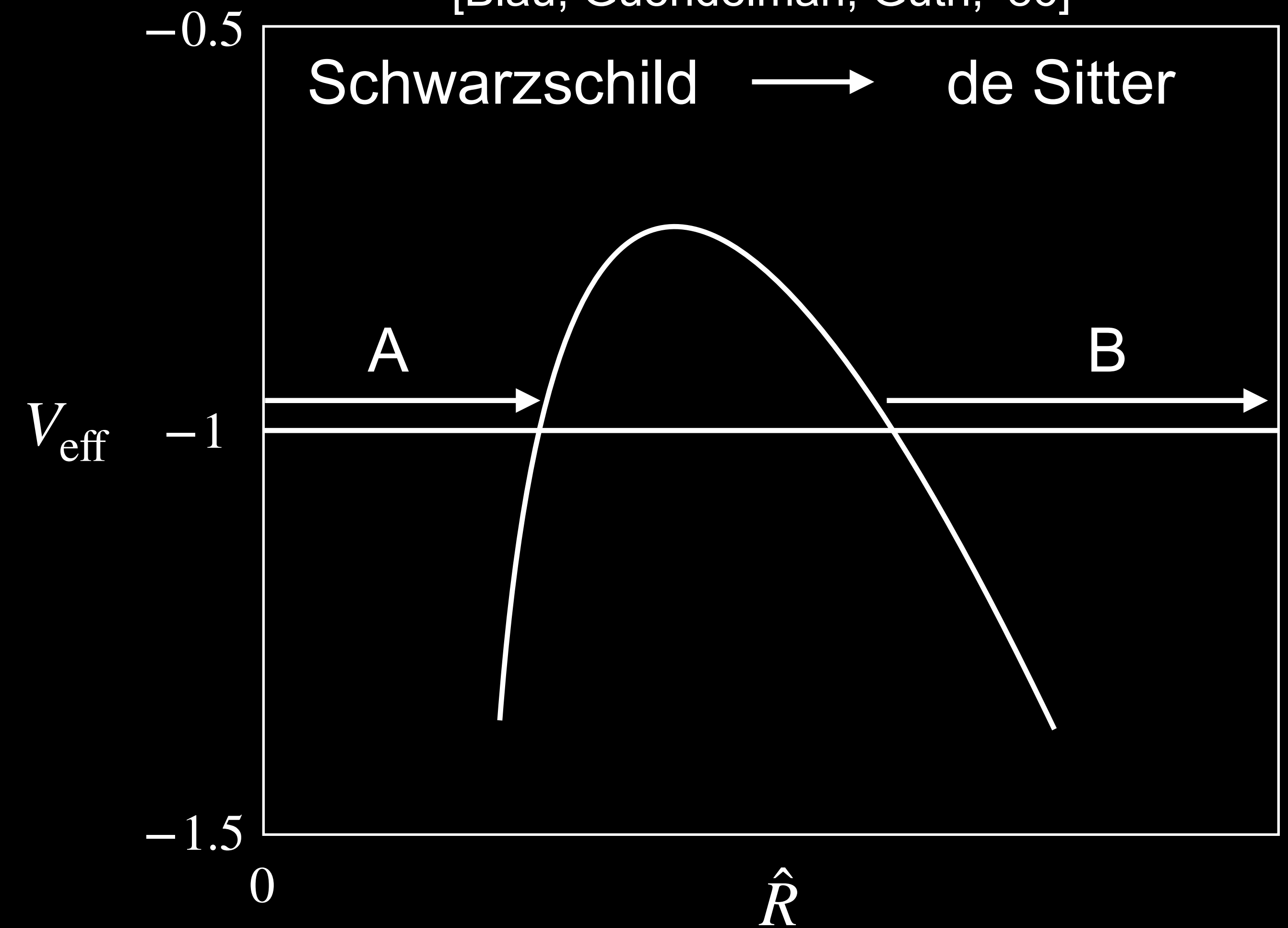
Dynamics of S-dS Bubbles

- dS to dS transitions as ‘tunnelling from nothing’.
- In general S-dS transitions initial state is not ‘nothing’.



tunnelling from ‘nothing’

[Blau, Guendelman, Guth, ’86]



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[Farhi, Guth, Guven, ’89]

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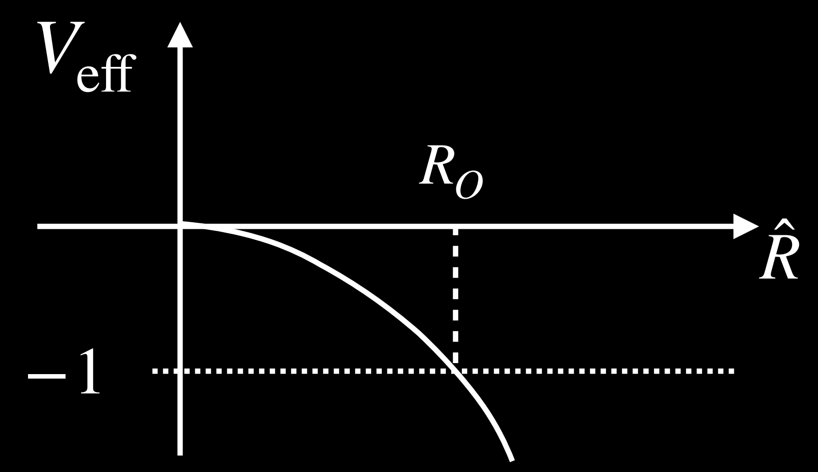
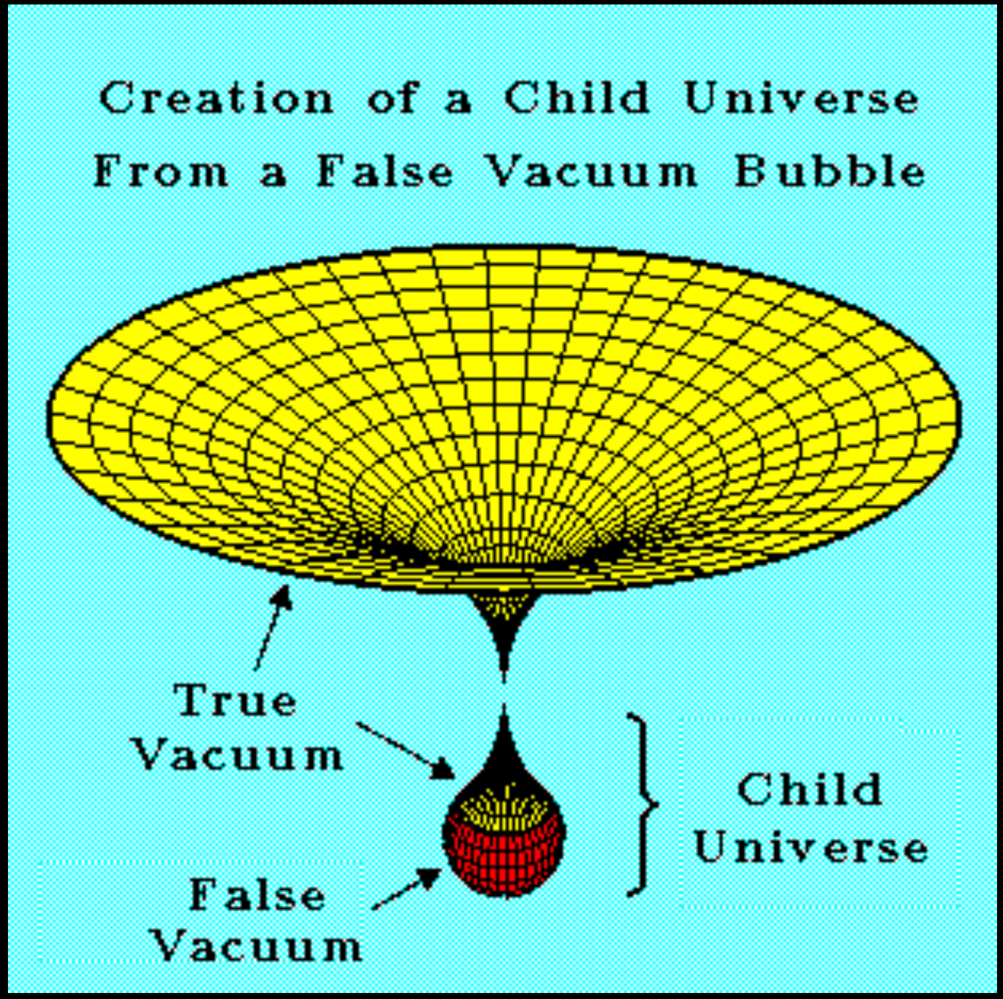
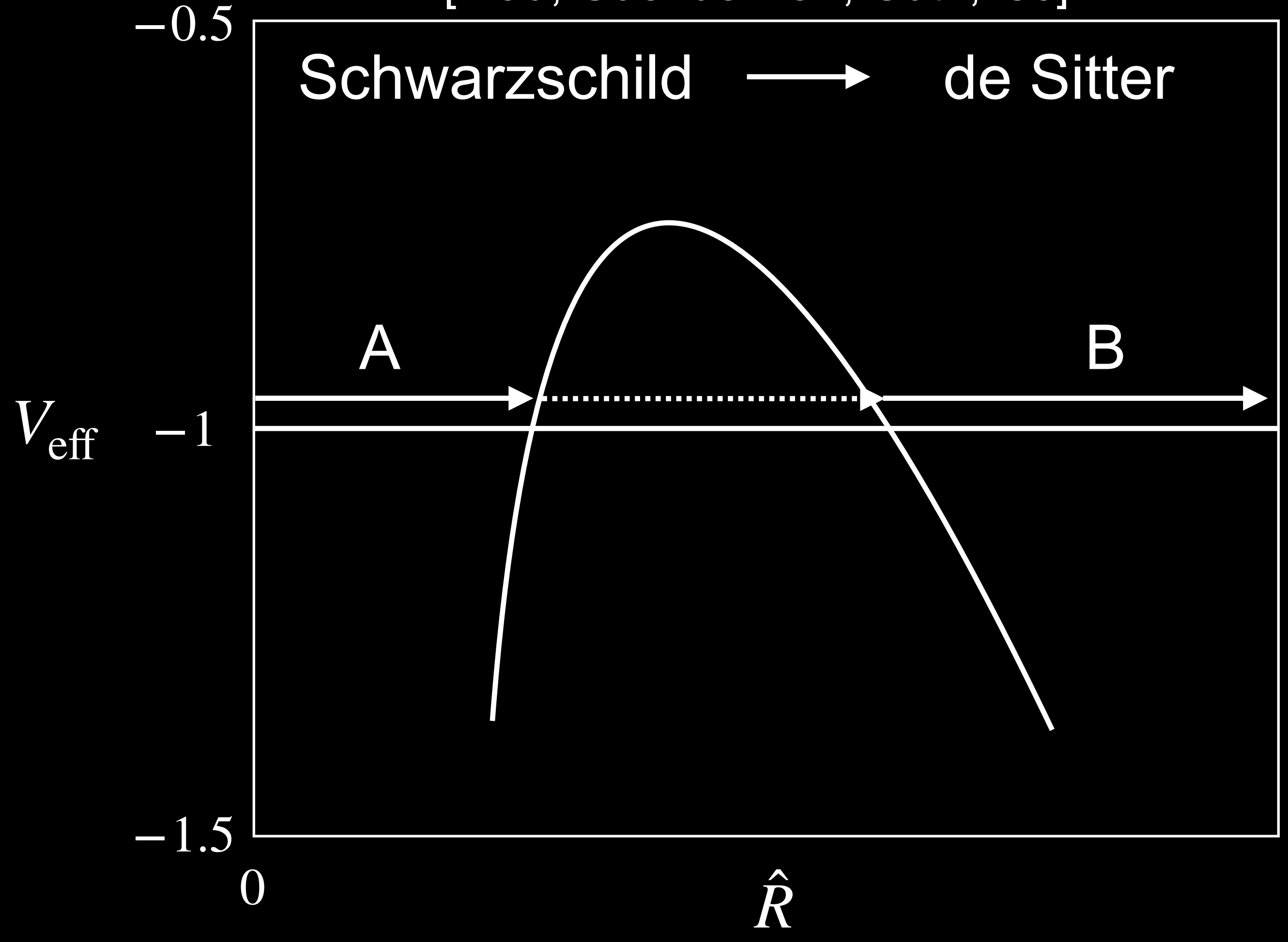


tunnelling from ‘nothing’



[Blau, Guendelman, Guth, ’86]

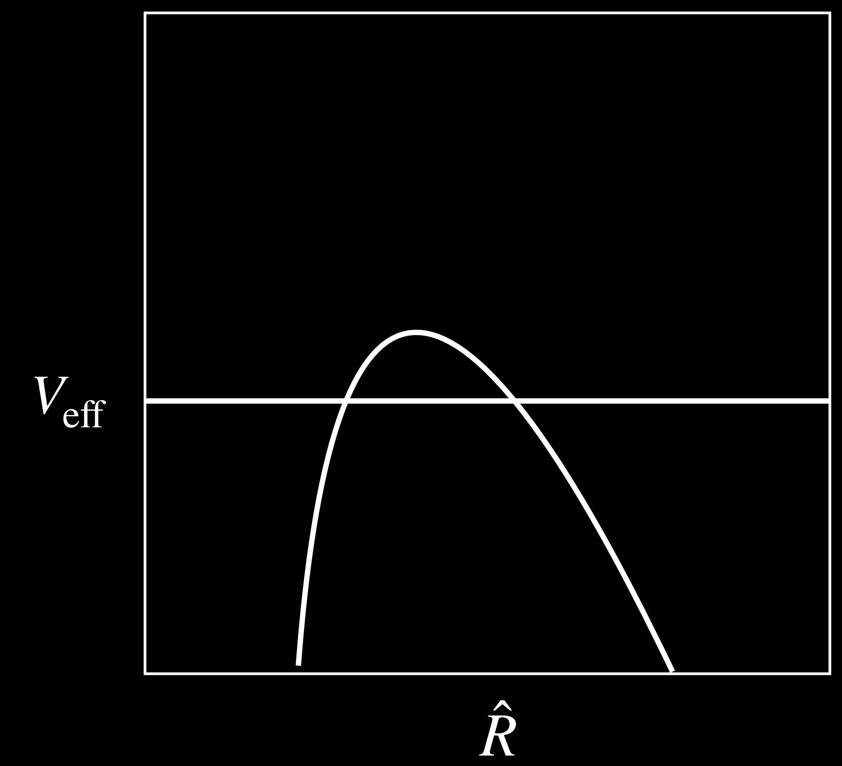


Minkowski to de Sitter

[de Alwis, Muia, Pasquarella, Quevedo, ’19]

- Mass of the bubble:

$$M = \frac{H^2 R^3}{2G} + 4\pi\sigma R^2 \mathsf{sign}(R'_-)\left(1 + \dot{R}^2\right)^{1/2} - 2\pi\sigma H^2 R^4 \mathsf{sign}(R'_-) - 8\pi^2 G\sigma^2 R^3$$



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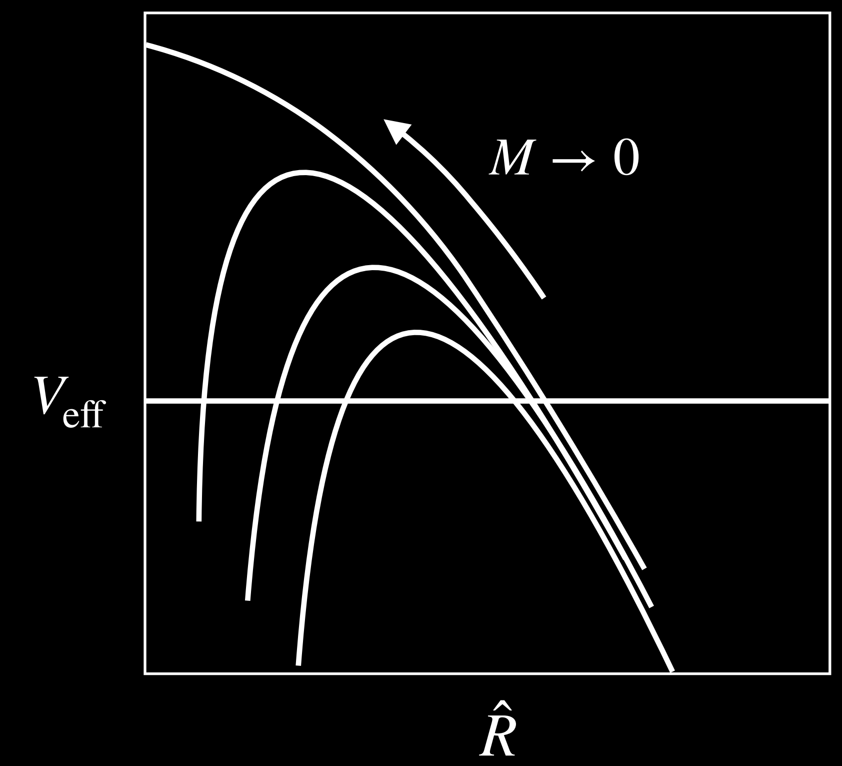
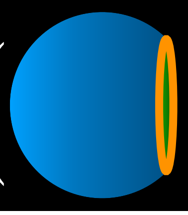
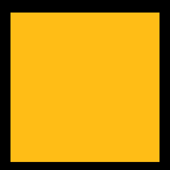
$\longrightarrow$ recover tunnelling from ‘nothing’ for $M \to 0$



Note: Minkowski does not decay completely.

$\longrightarrow$ de Sitter as a resonance? [Maltz, Susskind, ’17]

- Action $\quad \dfrac{B_{tot}}{2} = \dfrac{\eta\pi}{2G}\dfrac{H^2+2\kappa^2}{\left(H^2+\kappa^2\right)^2} \qquad \bar{B} = 0$

$\mathrm{P}(M \to M/dS \oplus W) \equiv \dfrac{\left|\Psi\left(\text{ }\right)\right|^2}{\left|\Psi\left(\text{ }\right)\right|^2}$ (numerator: , denominator: ) $= \exp\left[\dfrac{\eta\pi}{GH^2}\left(1 - \dfrac{\kappa^4}{\left(H^2+\kappa^2\right)^2}\right)\right]$ $\longrightarrow$ finite transition rate

Observations

$$\Psi = ae^{B} + be^{-B}$$

- Hartle-Hawking wave function always dominates at the turning point, unless the coefficient a is set to 0 imposing some boundary conditions.
- Detailed balance works with $\eta = +1$
  - Take two dS spacetimes A and B

$$\mathrm{P}(B \to B/A \oplus W) = \frac{|\Psi(B/A \oplus W)|^2}{|\Psi(B)|^2} = \frac{|\Psi(A/B \oplus W)|^2}{|\Psi(B)|^2}$$

$$\longrightarrow \quad \frac{\mathrm{P}(A \to A/B \oplus W)}{\mathrm{P}(B \to B/A \oplus W)} = \frac{|\Psi(B)|^2}{|\Psi(A)|^2} \underset{\eta = +1}{\approx} \frac{e^{s_B}}{e^{s_A}} \qquad s = \frac{\pi}{GH^2}$$

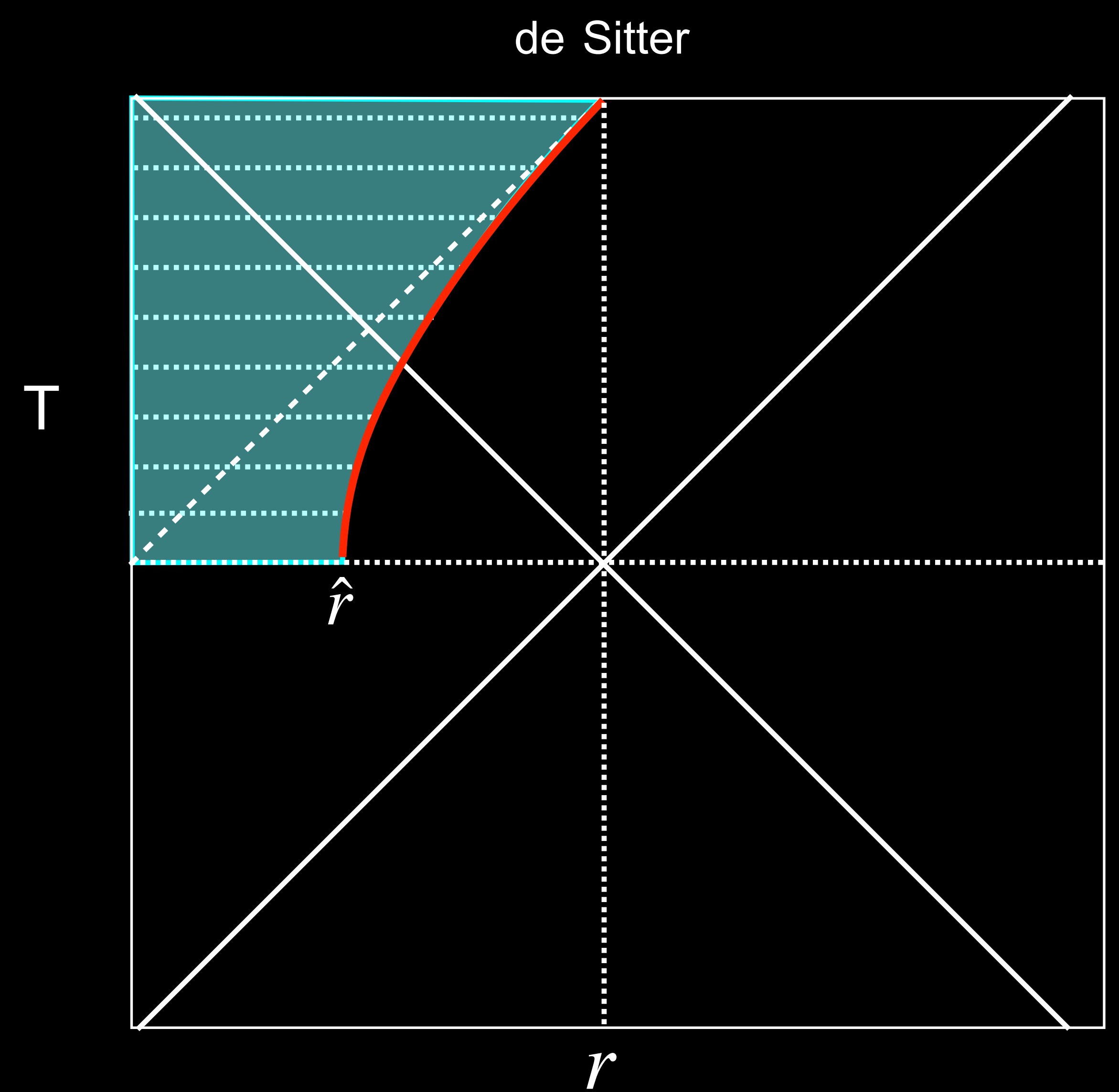
  - In the Minkowski to de Sitter case

$$\frac{\mathrm{P}(M \to M/dS \oplus W)}{\mathrm{P}(dS \to dS/M \oplus W)} \overset{\eta = +1}{=} e^{s_{dS}}$$

Open or Closed Universe?

[Cespedes, de Alwis, Muia, Quevedo, '20]

- Landscape transitions: closed universe?



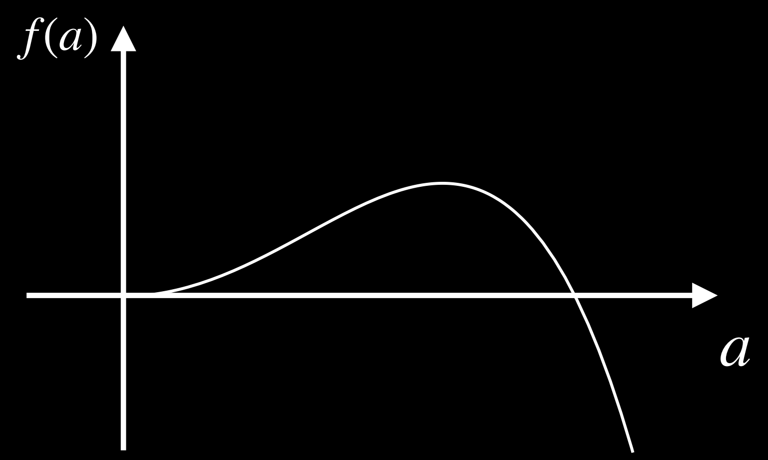
- Open question: how does the picture change when matter is added?

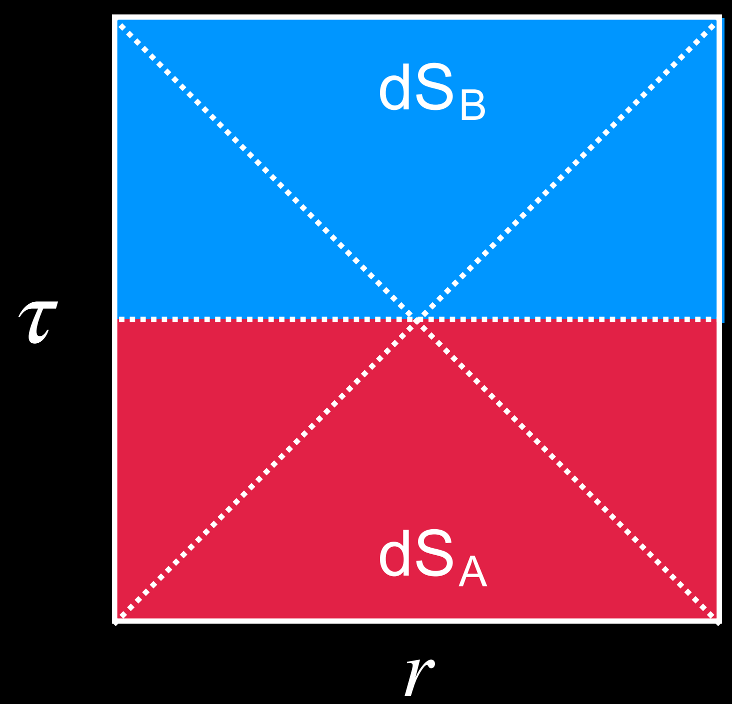
Minisuperspace transitions

[Cespedes, de Alwis, Muia, Quevedo, '20]

- Minisuperspace: $ds^2 = \ell^2 \left(-d\tau^2 + a^2(\tau) d\Omega_3^2\right) \longrightarrow$ no wall
  $SO(4)$ symmetry
- dS $\longrightarrow$ 'nothing' $\longrightarrow$ dS $\quad B = 24\pi^2 \left(\mp \frac{1}{V_B} \pm \frac{1}{V_A}\right)$

contribution larger than CDL





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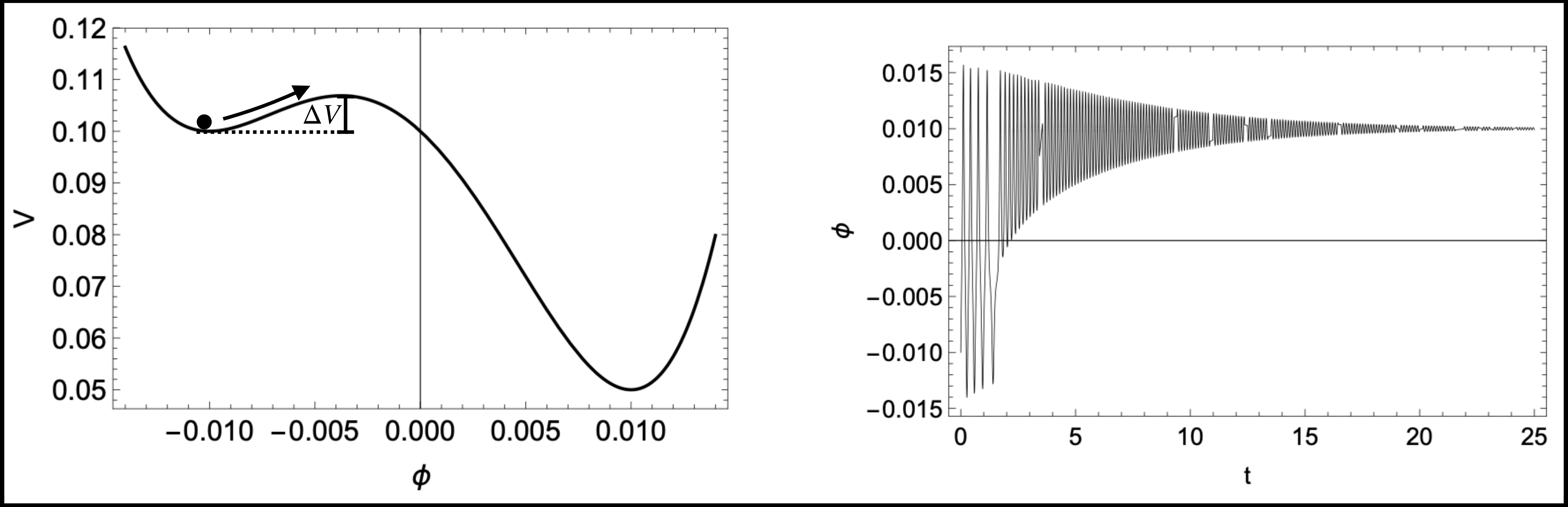
- dS $\longrightarrow$ ‘nothing’ $\longrightarrow$ dS $\quad B = 24\pi^2 \left(\mp \frac{1}{V_B} \pm \frac{1}{V_A}\right)$ contribution larger than CDL
- Standard classical path, eg. fly-over. $\longrightarrow$ kinetic energy $> \Delta V$

[Blanco-Pillado, Deng, Vilenkin, ’19]



see also Hawking-Moss [Hawking, Moss, ’82]

Minisuperspace transitions

[Cespedes, de Alwis, Muia, Quevedo, ’20]

- Contracting universe: $\dot{H} = -4\pi G(\rho + p) + \frac{k}{a^2}$

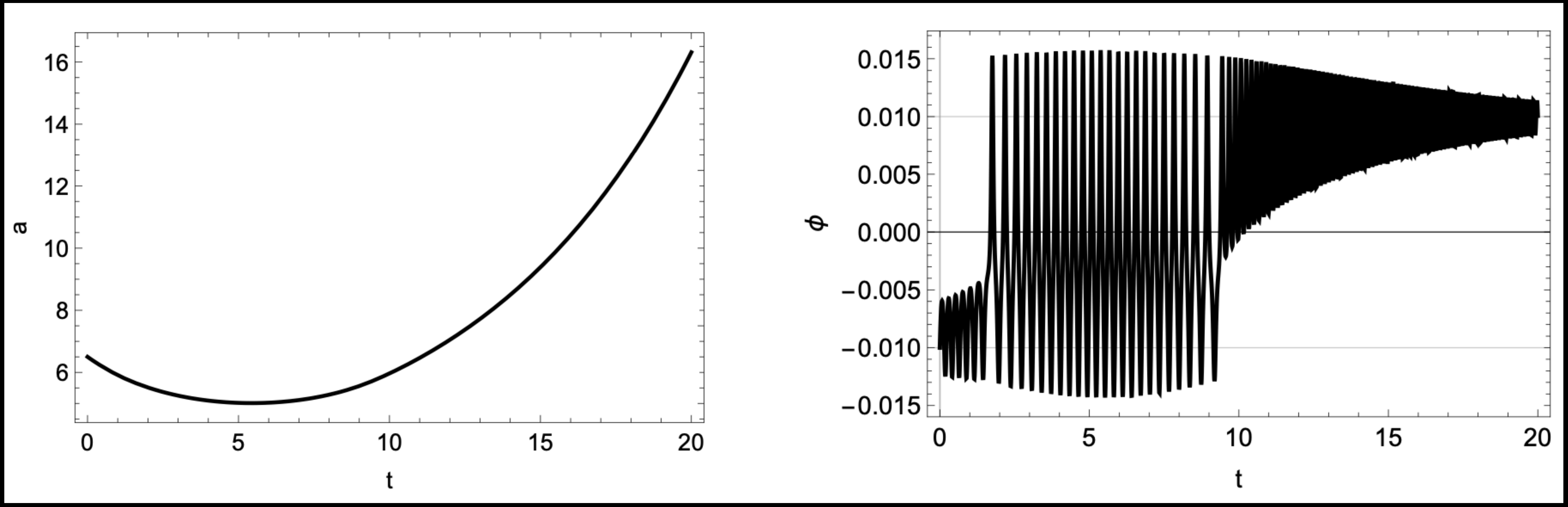
  At bounce $\dot{H} > 0$ $\longrightarrow$ if $k \leq 0$ need $\rho < -p$ $\longrightarrow$ phantom matter

  Phantom matter not required if $k > 0$

- Friedman equation: $\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\left(\frac{\dot{\phi}^2}{2} + V\right) - \frac{1}{a^2}$ $\longrightarrow$ smaller kinetic energy needed to overcome the barrier

[Starobinski, ’78] [Güngör, Starkman, ’20]

- Non-standard classical path. $\longrightarrow$ kinetic energy $< \Delta V$, initially contracting universe



Conclusions

- Main points:
  - We have tried to recover de Sitter to de Sitter transitions from a purely Lorentzian computation.
  - The final result agrees with CDL, but there are subtleties to be understood.
  - In this formalism Minkowski to de Sitter transitions are allowed, in the limit of vanishing black hole mass, while are not allowed in the limit of vanishing cosmological constant.
  - We find that for BT transitions the open Universe is not compelling.
    How the result changes if matter is added is an open question.
  - We observed non-standard classical transitions with an initially contracting Universe that need further investigation.

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- Lot of work left to do:
  - Extend FMP to include scalar fields.
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  - Phenomenological consequences of vacuum transitions.

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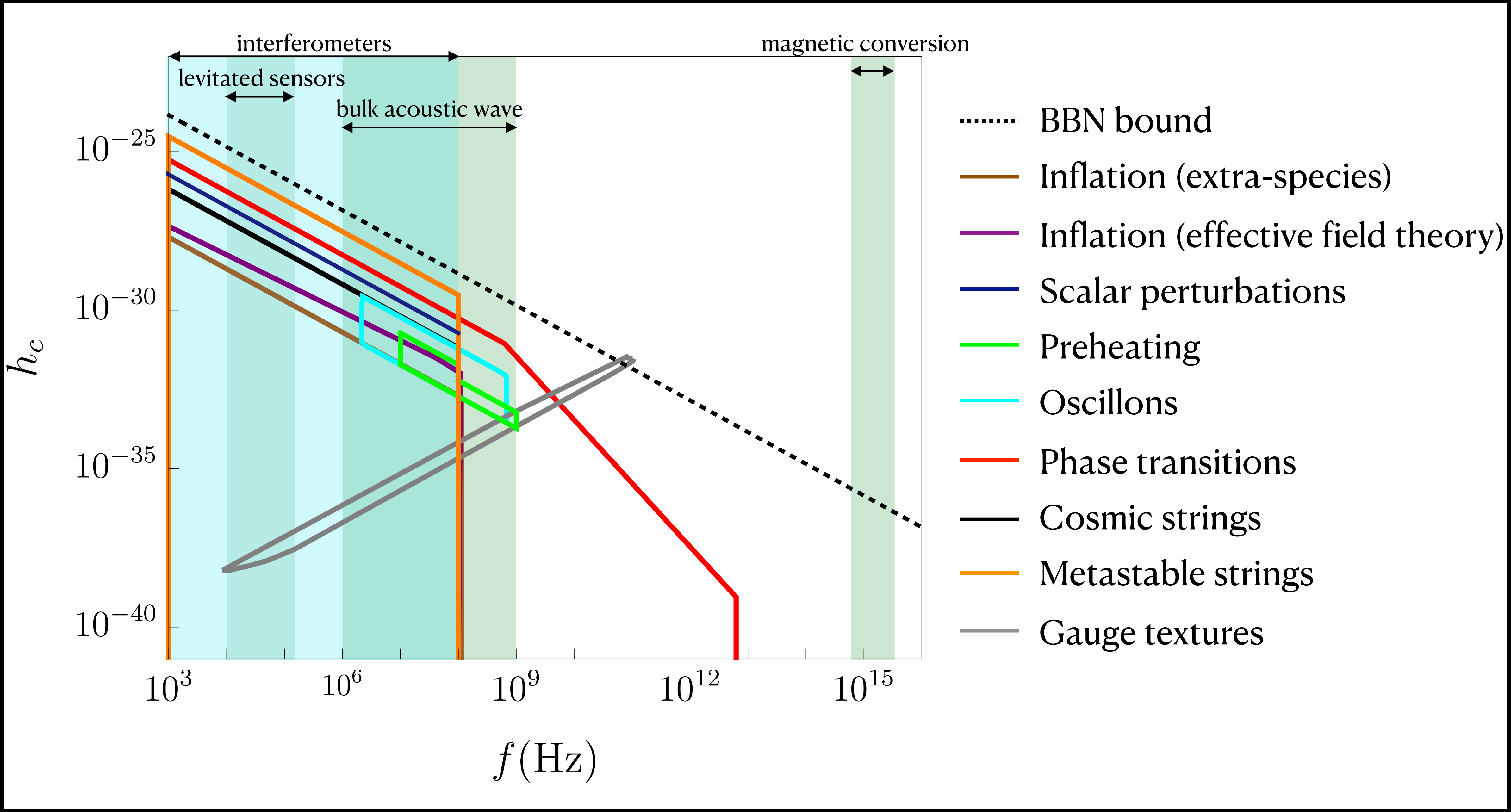
HFGW



interferometers
levitated sensors
bulk acoustic wave
magnetic conversion
10^{-25}
10^{-30}
10^{-35}
10^{-40}
h_c
10^3
10^6
10^9
10^{12}
10^{15}
$f\,(\mathrm{Hz})$
BBN bound
Inflation (extra-species)
Inflation (effective field theory)
Scalar perturbations
Preheating
Oscillons
Phase transitions
Cosmic strings
Metastable strings
Gauge textures

Challenges and Opportunities of Gravitational Wave Searches at MHz to GHz frequencies

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Abstract

The first direct measurement of gravitational waves by the LIGO/Virgo collaboration has opened up new avenues to explore our Universe. This white paper outlines the challenges and gains expected in gravitational wave searches at frequencies above the LIGO/Virgo band, with a particular focus on the MHz and GHz range. The absence of known astrophysical sources in this frequency range provides a unique opportunity to discover physics beyond the standard model operating both in the early and late Universe, and we highlight some of the most promising gravitational sources. We review several detector concepts which have been proposed to take up this challenge, and compare their expected sensitivity with the signal strength predicted in various models. This report is the summary of the workshop *Challenges and opportunities of high-frequency gravitational wave detection* held at ICTP Trieste, Italy in October 2019.

Technical concept	Frequency	Proposed sensitivity (dimensionless)	Proposed sensitivity $\sqrt{S_n(f)}$
Spherical resonant mass, Sec. 4.1.3 [277]			
Mini-GRAIL (built) [284]	2942.9 Hz	10^{-20} 2.3×10^{-23} (*)	$5 \times 10^{-20}\,\mathrm{Hz}^{-1/2}$ $10^{-22}\,\mathrm{Hz}^{-1/2}$ (*)
Schenberg antenna (built) [281]	3.2 kHz	2.6×10^{-20} 2.4×10^{-23}(*)	$1.1 \times 10^{-19}\,\mathrm{Hz}^{-1/2}$ $10^{-22}\,\mathrm{Hz}^{-1/2}$ (*)
Laser interferometers			
NEMO (devised), Sec. 4.1.1 [25, 268]	$[1-2.5]$ kHz	9.4×10^{-26}	$10^{-24}\,\mathrm{Hz}^{-1/2}$
Akutsu's detector, Sec. 4.1.2 [272, 323]	100 MHz	7×10^{-14} 2×10^{-19} (*)	$10^{-16}\,\mathrm{Hz}^{-1/2}$ $10^{-20}\,\mathrm{Hz}^{-1/2}(*)$
Holometer, Sec. 4.1.2 [274]	$[1-13]$ MHz	8×10^{-22}	$10^{-21}\,\mathrm{Hz}^{-1/2}$
Optically levitated sensors, Sec. 4.2.1 [59]			
1-meter prototype (under construction)	$(10-100)$ kHz	$2.4 \times 10^{-20} - 4.2 \times 10^{-22}$	$(10^{-19} - 10^{-21})\,\mathrm{Hz}^{-1/2}$
100-meter instrument (devised)	$(10-100)$ kHz	$2.4 \times 10^{-22} - 4.2 \times 10^{-24}$	$(10^{-21} - 10^{-23})\,\mathrm{Hz}^{-1/2}$
Inverse Gertsenshtein effect, Sec. 4.2.2			
GW-OSQAR II (built) [292]	$[200-800]$ THz	$h_{\mathrm{c,n}} \simeq 8 \times 10^{-26}$	$\times$
GW-CAST (built) [292]	$[0.5-1.5] \times 10^6$ THz	$h_{\mathrm{c,n}} \simeq 7 \times 10^{-28}$	$\times$
GW-ALPs II (devised) [292]	$[200-800]$ THz	$h_{\mathrm{c,n}} \simeq 2.8 \times 10^{-30}$	$\times$
Resonant polarization rotation, Sec. 4.2.4 [302]			
Cruise's detector (devised) [303]	(100 MHz – 100 THz)	$h \simeq 10^{-17}$	$\times$
Cruise & Ingley's detector (prototype) [304, 305]	100 MHz	8.9×10^{-14}	$10^{-14}\,\mathrm{Hz}^{-1/2}$
Enhanced magnetic conversion (theory), Sec. 4.2.5 [306]	5 GHz	$h \simeq 10^{-30} - 10^{-26}$	$\times$
Bulk acoustic wave resonators (built), Sec. 4.2.6 [311, 312]	(MHz – GHz)	$4.2 \times 10^{-21} - 2.4 \times 10^{-20}$	$10^{-22}\,\mathrm{Hz}^{-1/2}$
Superconducting rings, (theory), Sec. 4.2.7 [313]	10 GHz	$h_{0,\mathrm{n,mono}} \simeq 10^{-31}$	$\times$
Microwave cavities, Sec. 4.2.8			
Caves' detector (devised) [315]	500 Hz	$h \simeq 2 \times 10^{-21}$	$\times$
Reece's 1st detector (built) [316]	1 MHz	$h \simeq 4 \times 10^{-17}$	$\times$
Reece's 2nd detector (built) [317]	10 GHz	$h \simeq 6 \times 10^{-14}$	$\times$
Pegoraro's detector (devised) [318]	$(1-10)$ GHz	$h \simeq 10^{-25}$	$\times$
Graviton-magnon resonance (theory), Sec. 4.2.9 [319]	$(8-14)$ GHz	$9.1 \times 10^{-17} - 1.1 \times 10^{-15}$	$(10^{-22} - 10^{-20})\mathrm{Hz}^{-1/2}$

Future prospects

"such detectors have so low sensitivity that they are of little experimental interest"

Interferometer	Arm Length [m]	Effective Optical Path Length [km]	Year Construction Started
Hughes Research Lab (HRL) [87, 137, 142]	2	0.0085 (N=4)	1966
MIT prototype [202]	1.5	0.075 (N=50)	1971
Garching 3 m prototype	3	0.012 (N=4)	1975
Glasgow 1 m prototype [210]	1	0.036 (N=36; in static test reached N=280)	1976
Glasgow 10 m prototype [210]	10	25.5 (F-P: F=4000)	1980
Caltech 40 m prototype	40	75	1980
Garching 30 m prototype	30	2.7 (N=90)	1983
ISAS Tenko 10 m prototype [112]	10	1 (N=100)	1986
U. Tokyo prototype [14, 111]	3	0.42 (F-P: F=220)	1987
ISAS Tenko 100 m prototype [114, 139–141]	100	10 (N=100)	1991
NAOJ 20 m prototype [16]	20	4.5 (F-P: F=350)	1991
Q&A 3.5 m prototype [55]	3.5	67 (F-P: F=30000)	1993
TAMA 300 m [184]	300	96 (F-P: F=500)	1995
GEO 600 m [91, 209]	600	1.2 (N=2)	1995
LIGO Hanford (2 km) [1, 124]	2000	143 (F-P: F=112)	1994
LIGO Hanford (4 km) [124, 130]	4000	1150 (F-P: F=450)	1994
LIGO Livingston (4 km) [124, 130]	4000	1150 (F-P: F=450)	1995
VIRGO [5, 191]	3000	850 (F-P: F=440)	1996
AIGO prototype [205, 206]	80	760/66 (F-P: east arm F=15000; south arm F=1300)	1997
LISM [168]	20	320 (F-P: F=25000)	1999
CLIO 100 m cryogenic [7]	100	190 (F-P: F=3000)	2000
Q&A 7 m [134]	7	450 (F-P: F=100000)	2008
LCGT/KAGRA [21, 109]	3000	2850 (F-P: F=1500)	2010
Q&A 9 m [208]	9	570 (F-P: F=100000)	2016
LIGO India [102]	4000	1150 (F-P: F=450)	2016
ET [99]	10000	3200 (F-P: F$\sim$500)	proposal under study

MTW book

50 years
23 attempts

first direct detection

Future prospects

Collaboration

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Technology roadmap

- Involve all interested groups to collect information about current/planned technologies.
- Discuss fundamental limitations and best routes to pursue.
- Clarify achievable goals in terms of sensitivities, with and without new technical developments, within a given timeframe and budget.

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New meeting

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Application for joining the GWIC organisation as the HFGW community

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Application for fundings

Thanks a lot for the attention!