Use and reuse of SMEFT: technical details

André David (CERN)
Giampiero Passarino (Torino)

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Two facets of this talk

Use and reuse means:

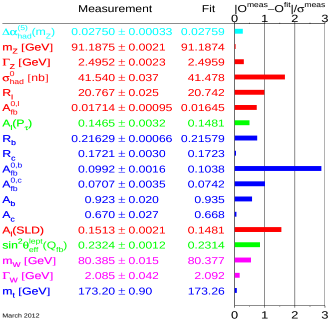
- Experiments cannot generate processes and reconstruct simulated event samples in every single BSM framework. SMEFT d.o.f. (the Wilson coefficients a_i) are being tested by experiments, are being expanded and improved upon, and are rather comprehensive as to the types of BSM deformations they can encode. SMEFT d.o.f. can be used as a bookkeeping tool in exploring the likelihood function, L, for (sub)sets of observables
- We can perform a fit of the SMEFT coefficients, $\{\mathrm{a}\}$, to a set of observables, $\{\mathrm{O}\}$. Take the best-fit results from the SMEFT fit to the data, $\{\hat{\mathrm{a}}\}$, and compute ,SMEFT-predicted observables $\{\hat{\mathrm{O}}\}_{\mathrm{SMEFT}} = \{\mathrm{O}\}_{\mathrm{SMEFT}}(\{\hat{\mathrm{a}}\})$. Within framework X, with parameters $\vec{p}$, we compute $\{\mathrm{O}\}_X(\vec{p})$ and compare with $\{\hat{\mathrm{O}}\}_{\mathrm{SMEFT}}$.
- André will give the general view (in a future meeting).

- Here be (un)happy with techical details, i.e. expect the SMEFT, hope for the BSM, be ready for whatever comes (it's a bit more than series truncation *Apologies*, i.e. series truncation is just one aspect of prediction accuracy that we want to address).

starting point

The starting point: it would be interesting to present plots similar to the ones produced at LEP with

observables *, measurements, SMEFT fits and pulls



*including e.g. fiducial cross-sections

Do we have a counter-argument to the *Warsaw is just too many parameters?* †

① Yes, physical observables are relevant (not Wilson coefficients) so .... IT'S LINEAR ALGEBRA with known equations (*but don't do linear algebra on something which is not a vector space while, at the same time, you violate gauge invariance*).

② Indeed, an alternative way of recording SMEFT fits has been introduced where a connection between Wilson coefficients and kappa-parameters was suggested: introducing generalized kappas ‡

† A. David, SMEFT bookkeeping, LHC EFT Working Group: preliminary open discussion, 2020

‡ Nowadays rebaptized PCA

BSM, EFT, loopy EFT

precision accuracy in general

Which SMEFT?

When using SMEFT Tools always check details of the implementation and make sure that we are all talking about the same SMEFT and the same road BSM $\to$ SMEFT.

① SMEFT representations, i.e. linear vs. partial quadratic vs complete quadratic

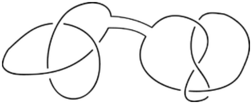
② QFT details (ask your local QFT expert ...)

③ SMEFT beyond LO, renormalization, scheme dependence, truncation error and gauge-dependence

④ SMEFT used in regions of phase-space where it breaks down

⑤ SMEFT NLO as a way to capture heavy-light contributions in BSM models (non-negligible kinematic logarithms)

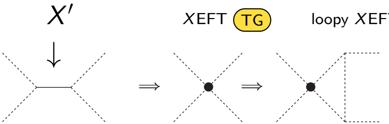
Here's an effect that is expected to exist and will only be properly dealt with if NLO is taken into account (LO/NLO truncation)



TG = tree generated

$X' \to X\text{EFT}$

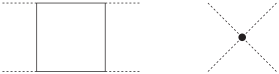
LG = loop generated



tree $\to$ loop

XEFT LG

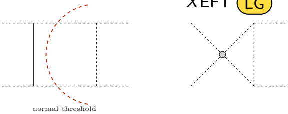
$s \to$



loop / local

XEFT LG

$s \to$



loop / non-local

normal threshold

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$X =$ **sigma-model** from Donoghue:2017pgk

- Low-energy behavior of $\mathrm{A}_{\mathrm{full}}(\pi^+\pi^0 \to \pi^+\pi^0)$

$$\mathrm{A}_{\mathrm{full}} \quad \mapsto \quad \frac{t}{\mathrm{v}^2} + \frac{1}{\mathrm{v}^4}\,\text{Polynomial}(s\,,\,t\,,\,u)$$
$$- \quad \frac{1}{96\,\pi^2\,\mathrm{v}^4}\left[3\,t^2\,\ln\frac{-t}{\mathrm{M}_\sigma^2} + s\,(s-u)\,\ln\frac{-s}{\mathrm{M}_\sigma^2} + u\,(u-s)\,\ln\frac{-u}{\mathrm{M}_\sigma^2}\right]$$

- $\mathrm{A}_{\mathrm{EFT}}$ computed using $\Sigma_{\mu\nu} = \partial_\mu \mathrm{U}\,\partial_\nu \mathrm{U}^\dagger$ and

$$\mathscr{L}_{\mathrm{EFT}} \quad = \quad \frac{\mathrm{v}^2}{4}\,\mathrm{Tr}\,\Sigma_\mu^\mu + \mathrm{a}_1\left(\mathrm{Tr}\,\Sigma_\mu^\mu\right)^2 + \mathrm{a}_2\left(\mathrm{Tr}\,\Sigma_{\mu\nu}\right)^2$$

- Match "full" and EFT, obtained by including one-loop bubbles (loopy EFT);

$$\mathrm{A}_{\mathrm{EFT}} \quad = \quad \frac{t}{\mathrm{v}^2} + \frac{1}{\mathrm{v}^4}\,\text{Polynomial}(s\,,\,t\,,\,u\,;\,\mathrm{a}_1\,,\,\mathrm{a}_2)$$
$$- \quad \frac{1}{96\,\pi^2\,\mathrm{v}^4}\left[3\,t^2\,\ln\frac{-t}{\mathrm{M}_\sigma^2} + s\,(s-u)\,\ln\frac{-s}{\mathrm{M}_\sigma^2} + u\,(u-s)\,\ln\frac{-u}{\mathrm{M}_\sigma^2}\right]$$

sigma-model Cont'd

- derive (renormalized) Wilson coefficients

$$a_1 = \frac{1}{8}\frac{v^2}{M_\sigma^2} + \frac{1}{384\,\pi^2}\left(\ln\frac{M_\sigma^2}{\mu_R^2} - \frac{35}{6}\right) \qquad a_2 = \frac{1}{192\,\pi^2}\left(\ln\frac{M_\sigma^2}{\mu_R^2} - \frac{11}{6}\right)$$

- Compare with the tree-level matching and conclude that we have taken into account an important kinematic feature,

  the logarithmic dependence upon the characteristic momentum transfer in the problem

  $\mathscr{L}_{EFT}$ and A_{EFT} have a different meaning;
  the Lagrangian is local (as it should), the amplitude generates long-distance kinematic logarithms. However, it is a question of language: nothing prevents us from introducing a one-loop, effective , non-local $\mathscr{L}$ including all processes up to a given order.


INTERPRETATION
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EXTENSION
ARBITRARY
GENERALITY
CONSTANT
AXIOM
SUBSTITUTION
EMPTY
STANDARD
REPRESENT
NATURAL
VALUES
ANALOGIES
LOGICALLY
EQUALITY
LETTER
FUNCTION
CONCEPTS
INFORMATION
RELATION
DISCOURSE
MEANING
SYNTACTIC
MODEL
DEFINABLE

Should we bother about interpretation? Not yet time?

- Well, if we (unintentionally) do it wrong now, interpretation will be damaged/impossible (too many examples in the past, SLD left-right asymmetry, deconvoluted HERA data etc)
- Comparing with the past: there was never a FermiEFT, BeyondFermi physics was pouring down hard (Fermi $\mapsto$ SM). Now we're in a BeyondSM desert (SM $\mapsto$ SMEFT $\cdots \mapsto \cdots$ the regulative ideal of an ultimate theory remains a powerful aesthetic ingredient)
- There is a hierarchy in the MHOU, ranging from 1% to 100%; for instance, LO PDFs and NLO PDFs in LO SMEFT give results differing by a large factor. We can say that there aspects of the problem which should be solved "today" but not at the price of forgetting aspects which will require our attention "tomorrow". For instance, while QCD corrections are dominant it would be inaccurate to say that EW corrections are negligible (i.e. well below 10%). Furthermore, there are QCD corrections in the SMEFT which are unrelated to the SM ones and can be sizeable.
- It is important to preserve data, not just the interpretation results. Interpretation can be subtle: more than ever "slightly bad predictions" $\mapsto$ "very bad interpretations".

Most BSM scenarios have extended scalar sectors

- **Higgses mix**, with mixing one gets pushed to the HEFT. There is no claim that the Higgs properties should be perturbed a lot since we are already almost at the point in measurements where it is counterindicated: it's going to be percent level precision (see also Cohen:2020xca).

- **Mixing is not only relevant for scalars**. Take a BSM model like $SU(3)_{\rm L} \otimes U(1)_X$; the non-zero VEVs break it into $SU(2)_{\rm L} \otimes U(1)_Y$ and induce a $\rm Z - Z'$ mixing.

  - common wisdom is EFT $=$ integrate out the heavy stuff
  - but low-energy behavior of any BSM model $=$ a considerable part of the deviation is due to *mixing* and not to *integration*. In presence of mixing "SMEFT truncation" becomes less relevant, i.e. truncation $\mapsto$ anything that leads to incomplete predictions.

Example of SMEFT breakdown

- In $\mathbf{Zh}$ production, for a wide choice of the value of the Wilson coefficients, the SMEFT(linear)/SM ratio becomes negative at a $\boldsymbol{p}_\perp$ around $\mathbf{350}$ GeV, while the SMEFT(partial quadratic)/SM ratio remains positive, but exploding for increasing values of $\boldsymbol{p}_\perp$.
- For $\boldsymbol{p}_\perp \approx \mathbf{500}$ GeV the ratios are $\mathbf{-2}$ for the linear representation and $\mathbf{+4}$ for the quadratic one, an evident sign of the breakdown of the SMEFT in those regimes.

$\overline{\mathrm{q}}\mathrm{q} \to \mathrm{HZ}$

There are 9 Wilson coeff. 3 of them are LG (divided by $16\,\pi^2$)

BSM For the singlet extension (SESM at LO) there is a simple rescaling of SM predictions

SMEFT **left** $\mathrm{a}_{\mathrm{AZ}}(\mathrm{LG}) = -1$, rest of Wilson coeff. is 0.

right $\mathrm{a}_{\phi\mathrm{dV}}(\mathrm{TG}) = 1$, rest of Wilson coeff. is -1.

$p_\perp$[GeV]	L/SM	Q/SM
10	−0.0259	−0.0239
59	−0.0259	−0.0239
108	−0.0260	−0.0240
157	−0.0261	−0.0240
206	−0.0262	−0.0241
255	−0.0263	−0.0242
304	−0.0263	−0.0242
353	−0.0264	−0.0243
402	−0.0265	−0.0243
451	−0.0265	−0.0243
500	−0.0266	−0.0244

$p_\perp$[GeV]	L/SM	Q/SM
10	−0.098	−0.083
59	−0.124	−0.099
108	−0.188	−0.129
157	−0.288	−0.149
206	−0.425	−0.127
255	−0.601	−0.017
304	−0.814	+0.235
353	−1.064	+0.692
402	−1.352	+1.426
451	−1.677	+2.518
500	−2.040	+4.056

linear is negative

Truncation uncertainty or breakdown? Or SESM-like?

quadratic exploding

Linear vs. Quadratic or **you cannot construct** S**-matrix elements at** $\mathscr{O}\left(1/\Lambda^4\right)$ **using a canonically transformed** $\mathscr{L}$ **truncated at** $\mathscr{O}\left(1/\Lambda^2\right)$.

$$\mathscr{L} = -\frac{1}{2}\,(1+\frac{M^2}{\Lambda^2}\,\delta Z_{\mathrm{H}}^6 + \boxed{\frac{M^4}{\Lambda^4}\,\delta Z_{\mathrm{H}}^8})\,\partial_\mu \mathrm{H}\,\partial_\mu \mathrm{H} + \ldots + \frac{1}{\Lambda^2}\Big[\overbrace{\mathrm{a}\,M^3\,\mathrm{H}\,\mathrm{Z}_\mu\,\mathrm{Z}_\mu}^{\text{pick at random}} + \ldots\Big] + \boxed{\frac{1}{\Lambda^4}\,\sum_i \mathrm{a}_i^8\,\mathscr{O}_i^{(8)}}$$

where the frame box indicates that the terms are not available.

$$\mathrm{H} = (1+\frac{M^2}{\Lambda^2}\,\eta_{\mathrm{H}}^6 + \frac{M^4}{\Lambda^4}\,\eta_{\mathrm{H}}^8)\,\hat{\mathrm{H}},$$

$$\eta_{\mathrm{H}}^6 = -\frac{1}{2}\,\delta Z_{\mathrm{H}}^6, \qquad \eta_{\mathrm{H}}^8 = \frac{3}{8}\,[\delta Z_{\mathrm{H}}^6]^2,$$

$$\hat{\mathscr{L}} = -\frac{1}{2}\,\partial_\mu \hat{\mathrm{H}}\,\partial_\mu \hat{\mathrm{H}} + \mathrm{a}\,\frac{M^3}{\Lambda^2}\,(1 - \boxed{\frac{1}{2}\,\frac{M^2}{\Lambda^2}\,\delta Z_{\mathrm{H}}^6}\,)\,\hat{\mathrm{H}}\,\mathrm{Z}_\mu\,\mathrm{Z}_\mu + \ldots$$

where the round box gives terms that are neglected in the "naive" quadratic approach, i.e. $\mid \mathrm{A}^{(6)} \mid^2$ and $\mathrm{A}^{(4)} \otimes \mathrm{A}^{(8)}$ are not the full $\mathscr{O}\left(1/\Lambda^4\right)$ answer.

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Other things which could go wrong (LO-NLO-$\cdots$ truncation uncertainty)

For the sake of simplicity we consider the decay $\mathrm{h} \to 4$ leptons. It follows:

❶ In the SMEFT we take the best available prediction for the $\dim = 4$ part and add tree diagrams containing one $\dim = 6$ operator

❷ In the BSM model we include tree diagrams and take the large Λ limit (once Λ has been identified).

❸ It follows that a comparison between ❶ and ❷ is not adequate since Loop-Generated (local) operators have been included in the SMEFT.

❹ Therefore, we consider the BSM model at one-loop and take the large Λ limit. However, in most cases, the result includes mixed heavy-light contributions which are not present in ❶. As a consequence

❺ we include loops with one $\dim = 6$ operator insertion in the SMEFT predictions. LG insertions call for a two-loop calculation in the BSM model. To be continued .....

❻ Keep calm, it is an asymptotic expansion (we are $\approx$ ❹)

https://mathworld.wolfram.com/AsymptoticSeries.html

Inclusion of $\dim = 8$ operators vs. BSM

It's about interpretation

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We consider SM′, an extension of the SM whose Lagrangian contains both SM and BSM parameters, and take the limit

$$\mathscr{L}_{\mathrm{SM}'} \mapsto \frac{a_\phi}{\Lambda_h^2}\,\mathscr{O}_\phi^{(6)} + \frac{b_{\phi\Box}}{\Lambda_h^2}\,\mathscr{O}_{\phi\Box}^{(6)} + \frac{c_{\phi D}}{\Lambda_h^2}\,\mathscr{O}_{\phi D;1}^{(6)} + \;\dots\; + \frac{d_\phi}{\Lambda_h^4}\,\mathscr{O}_\phi^{(8)} + \;\dots \qquad \boxed{E \ll \Lambda_h}$$

$$\mathscr{O}_{\phi D;1}^{(6)} = (\Phi^\dagger D_\mu \Phi)^\dagger (\Phi^\dagger D_\mu \Phi) \qquad\qquad \mathscr{O}_{\phi\Box}^{(6)} = (\Phi^\dagger \Phi)\,\Box\,(\Phi^\dagger \Phi)$$

where $a \dots d$ and the heavy scale Λ_h are functions of $\{p\}_{\mathrm{SM}}$ and $\{p\}_{\mathrm{BSM}}$.

However, imagine a situation where our EFT basis contains

$$\text{EFT basis} \qquad \ni \quad \mathscr{O}_\phi^{(6,8)},\ \mathscr{O}_{\phi D;1}^{(6)},\ \mathscr{O}_{\phi D;2}^{(6)}$$

$$\mathscr{O}_{\phi D;2}^{(6)} = (\Phi^\dagger \Phi)\,(D_\mu \Phi)^\dagger\,(D_\mu \Phi)$$

In order to compare the fitted (constrained) values of the a^6 and a^8 Wilson coefficients with the “computed” values of $a \dots d$ we have to perform a “translation” containing the following steps:

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1. neglecting total derivatives, we write

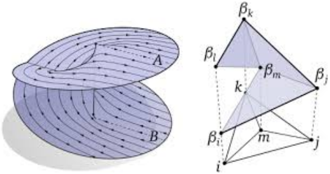
$$\mathscr{O}^{(6)}_{\phi \mathrm{D};2} = \frac{1}{2}\,\mathscr{O}^{(6)}_{\phi \Box} - (\Phi^\dagger \Phi)\left[\Phi^\dagger \mathrm{D}_\mu \mathrm{D}_\mu \Phi + (\mathrm{D}_\mu \mathrm{D}_\mu \Phi)^\dagger)\,\Phi\right]$$

2. The second term is eliminated performing the field transformation

$$\Phi \rightarrow \Phi - g^2\,\frac{\mathrm{a}^6_{\phi \mathrm{D};2}}{\Lambda^2}\,(\Phi^\dagger \Phi)\,\Phi$$

which will induce an higher order compensation (goto sl. 42).

- **Critical to note**: the higher order compensation is such that the Wilson coefficient of $\mathscr{O}^{(8)}_{\phi}$ (and others) will be a combination of a^8_{ϕ} (linear) and $\mathrm{a}^{(6)}_{\phi \mathrm{D};2}$ (quadratic).
  This combination is the one to be "compared" with d_{ϕ} (for the sake of simplicity we are neglecting the mixing among Wilson coefficients).



Time to introduce a metric

- i.e. to develop a precise distance between theories, which are treated as collections of events weighted by cross sections.

- When we work with a family of differential distributions (SM, SMEFT or BSM), there would seem to be an obvious way to proceed: calculate the distance between distributions. We ask how close we can come to guessing a BSM model, based on an observation. Given two distributions, $f(x)$ and $g(x)$ $(x \in X)$, where

$$\int_X dx\, f(x) \quad = \quad \int_X dx\, g(x) = 1,$$

we define "distances" (also definable for discrete distributions), e.g.

1. the Bhattacharyya distance

$$\rho(f\,,g) = \int_X dx\,\sqrt{f\,g}, \qquad\qquad \mathrm{dist}_{\mathrm{B}} = -\ln\big[\rho(f\,,g)\big].$$

2. the Hellinger distance

$$H^2(f\,,g) \quad = \quad \frac{1}{2}\int_X dx\,\big(\sqrt{f}-\sqrt{g}\big)^2.$$

To understand the next slides:

- start from taking SMEFT as an agnostic framework that is elastic enough to cover most BSM possibilities (if we reach the needed SMEFT precision accuracy)
- take care of SMEFT precision accuracy without particular assumptions (we don't need next-QCD to estimate QCD MHOU)
- use the data to constrain those possibilities
- if there will be *evidence* for a compatible, SM-contiguous BSM signal compute its low-energy limit (LEBSM) and see how good is the truncated expansion (a post-SMEFT phase)

𝒢

Using a geometric framework we will discuss the interplay between **SM, SMEFT and (SMEFT-constrained) BSM models**.

- First of all we analyze the impact of the BSM signal : for that we maximize the distance $\mathrm{dist}_{\mathrm{B}}(\mathrm{D}_{\mathrm{BSM}}, \mathrm{D}_{\mathrm{SM}})$, varying the BSM parameters;
  - this should be done under the condition that the BSM model remains a weakly coupled theory, e.g. the running coupling constants do not exceed some critical value and the conditions of vacuum stability are satisfied for each value of the high scale Λ. Therefore, conditions are necessary to single out parameter regions in the BSM model which cannot be treated perturbatively.
  - If the maximal distance is less than some, preselected, value then $\mathrm{D}(x)$ or x are not a good choice. The maximum H distance 1 is achieved when $\mathrm{D}_{\mathrm{BSM}}$ assigns probability zero to every set to which D_{SM} assigns a positive probability, and vice versa.

- Next we want to discuss **BSM-SMEFT compatibility** :
  - for that we minimize the distance $\mathrm{dist}_{\mathrm{B}}(\mathrm{D}_{\mathrm{BSM}},\mathrm{D}_{\mathrm{SMEFT}})$ by varying both the BSM parameters and the Wilson coefficients under the following condition:
  - we define a radius in the space of Wilson coefficients, $r^2 = \sum_i a_i^2$ and require that $r \geq \hat{r}$ where $\hat{r}^2 = \sum_i \hat{a}_i^2$. If the distance is greater than some, preselected, value then there will be a tension between the BSM model and the SMEFT.
  - At this point we can compare $\mathrm{D}_{\mathrm{BSM}}$ and $\mathrm{D}_{\mathrm{SMEFT}}$ at the minimum of their distance with the band corresponding to $\mathrm{D}_{\mathrm{SMEFT}}$ reconstructed and derive informations on the goodness of the BSM model.

Let Λ_{max} be the highest scale which can be tested at LHC. For a given differential distribution $D(x)$ we define $\hat{\mathrm{D}}_{\mathrm{SMEFT}}$ as the D-distribution as the D-distribution obtained by fitting the SMEFT to data. Given

$$\mathrm{dist}_{\mathrm{BSM}} = \mathrm{dist}_{\mathrm{B}}(\mathrm{D}_{\mathrm{BSM}}, \hat{\mathrm{D}}_{\mathrm{SMEFT}}),$$

we minimize w.r.t. the BSM parameters (including the heavy scale Λ_{BSM}, e.g. M_s in the SESM) Given a reference value for the distance, d_{ex} we have the following situations:

1. $\min \mathrm{dist}_{\mathrm{B}} > \mathrm{d}_{ex}$, the BSM model is excluded ,
2. $\min \mathrm{dist}_{\mathrm{B}} < \mathrm{d}_{ex}$ and $\Lambda_{\mathrm{BSM}} > \Lambda_{max}$. The BSM model and the SM are not contiguous (i.e. isolation of the SM).
3. $\min \mathrm{dist}_{\mathrm{B}} < \mathrm{d}_{ex}$ and $\Lambda_{\mathrm{BSM}} < \Lambda_{max}$: the BSM model and the SM are contiguous .

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- **Truncation**: for a given process and a given distribution we want to compute the "distance" between $f = \mathrm{D}_{\mathrm{BSM}}$, i.e. distribution D computed in the full BSM model, and its truncated, low-energy, expansion, i.e.

$$g = \mathrm{D}_{\mathrm{SM}}\left(1 + \Delta\right) \qquad \Delta = \Delta_v \frac{\mathrm{v}^2}{\Lambda^2} + \Delta_e \frac{\mathrm{E}^2}{\Lambda^2}$$

where Δ_v and Δ_e are, process dependent, kinematic factors and we have separated the "scale"-growing contribution, i.e. E can be any scale describing the process while v is the Higgs VEV. We obtain that

$$H^2\left(\mathrm{D}_{\mathrm{BSM}}, \mathrm{D}_{\mathrm{LEBSM}}\right) = H^2\left(\mathrm{D}_{\mathrm{BSM}}, \mathrm{D}_{\mathrm{SM}}\right) - \int_X dx \left(\sqrt{\mathrm{D}_{\mathrm{BSM}}} - \sqrt{\mathrm{D}_{\mathrm{SM}}}\right)\sqrt{\mathrm{D}_{\mathrm{SM}}}\,\Delta$$

is as a quantity (function of E and of Λ) which can "measure" uncertainties associated to the truncation at $\mathscr{O}(1/\Lambda^2)$, i.e. what is the difference between interpreting data with effective operators and with UV completion? goto sl. 47


truncation

SMEFT precision accuracy: summary

- There are many sources for the $\mathcal{O}\left(1/\Lambda^4\right)$ terms, including **canonical normalization** (at this order the canonical normalization procedure induces changes to the shape of differential distributions, not only to their integral goto sl. 44) **finite renormalization** (make sure that extraction of the the parameters from different experimental results is done using SMEFT and do not expand denominators goto sl. 50) and **double insertions**
- You cannot make something positive by assuming it
- Use the difference Linear-Quadratic as a rough estimate of the TH uncertainty when it is "small" (central value?)
- Avoid using EFT in regions where the difference is large (and eventually Linear is negative)
- Statements on the uncertainty are process dependent and it would be nice to start using a metric
- The results of Hays:2020scx put a new perspective on the "linear vs. quadratic" option, discussing a comparison between partial $\mathcal{O}(1/\Lambda^4)$ and full $\mathcal{O}(1/\Lambda^4)$.

- When the energy transferred in the hard scattering process Q is such that $Q \approx \Lambda$ then the approach is, at least, incomplete: the EFT expansion does not make much sense if we do not know the coefficients or, the predictive power is lost since the EFT requires more and more renormalized parameters, Preskill:1990.
- Therefore, we need to quantify what is the fraction of events used in the analysis that makes sense for EFT.
- The removal of events in the high Q region would be a simple way of obtaining conservative limits for an EFT, e.g. evaluate how much percentage of the initially accepted number of events survives after truncation. The underlying assumption is that the UV complete cross-sections go very rapidly to zero for Q above Λ.

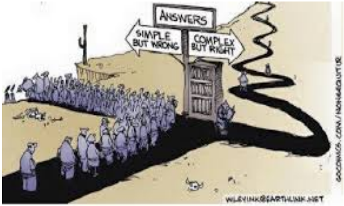
Are you willing to accept theoretical input/bias

- Unitarity?
- Causality?
- Anomaly cancellation?
- The heavy particles in any BSM model are unstable and their description requires the introduction of the corresponding complex poles, therefore what is Λ? goto sl. 49
- etc.

It is also possible that part of the observable parameter (Wilson coefficients) space will be inconsistent with the above, i.e.

Conclusions:

- The "fitted" Wilson coefficients (in the Warsaw basis) can be used to derive SMEFT-predicted observables which become the pseudo-data and we may take any specific BSM model, compute the corresponding low-energy limit and confront the BSM parameters with the pseudo-measurements. Consistently set SMEFT limits to be reinterpreted in any specific underlying model.



Thank you for your attention


Backup Slides

for $\boldsymbol{EFT\ representations}$

Linear vs. quadratic representation

- To summarize, the proper definition of *quadraticEFT* proceeds as follows: given a "truncated" Lagrangian

$$\mathscr{L} \quad = \quad \mathscr{L}^{(4)} + \frac{1}{\Lambda^2}\,\mathscr{L}^{(6)} + \frac{1}{\Lambda^4}\,\mathscr{L}^{(8)}$$

- we distinguish between redundant and non-redundant operators (select between $\mathscr{O}^{(n)}_i - \mathscr{O}^{(n)}_j = \mathrm{F}(\phi)\,\delta\mathscr{L}/\delta\phi$);

$$\mathscr{L}^{(6,8)} \quad = \quad \mathscr{L}^{(6,8)}_{\mathrm{NR}} + \sum_{i\in\mathrm{R}} \mathscr{O}^{(6,8)}_i\,\frac{\delta\mathscr{L}^{(4)}}{\delta\phi}$$

- redefine fields according to

$$\phi \quad \to \quad \phi - \sum_{n=2,4}\frac{1}{\Lambda^n}\sum_{i\in\mathrm{R}}\mathscr{O}^{(n+4)}_i$$

→ The corresponding shift in $\mathscr{L}$ will eliminate redundant operators leaving a (neglected) term

$$\Delta\mathscr{L} \quad = \quad -\frac{1}{\Lambda^4}\Big[\frac{\delta\mathscr{L}^{(4)}}{\delta\phi}\sum_{i\in\mathrm{R}}\mathscr{O}^{(8)}_i + \frac{\delta\mathscr{L}^{(6)}}{\delta\phi}\sum_{i\in\mathrm{R}}\mathscr{O}^{(6)}_i + \frac{1}{2}\,\frac{\delta^2\mathscr{L}^{(4)}}{\delta\phi^2}\sum_{i,j\in\mathrm{R}}\mathscr{O}^{(6)}_i\,\mathscr{O}^{(6)}_j\Big]$$

- We could do without elimination (overcomplete basis), indeed the S-matrix cannot distinguish between two equivalent operators $(\mathscr{O}\,,\mathscr{O}')$. Quadratic then means the full $\mathscr{O}\left(1/\Lambda^4\right)$ S-matrix. Different story is if we ask whether or not the underlying theory can generate $\mathscr{O}'$.

Linear vs. quadratic representation

- Once again, $\Delta\mathscr{L}$ will never generate terms that are not present in $\mathscr{L}^{(8)}$ (symmetry). however, we will see a difference when interpreting "fitted" Wilson coefficients in terms of the low-energy behavior of some X'. Furthermore, assembling the used terms

$$\mathscr{L} = -\frac{1}{2} Z_{\phi}^{ij}\, \partial_\mu \phi_i \, \partial_\mu \phi_j - \frac{1}{2} Z_m^{ij}\, \phi_i \phi_j + \mathscr{L}_{\text{rest}}$$

$$Z_{\phi}^{ij} = \delta^{ij} + \frac{1}{\Lambda^2}\, \delta Z_{\phi}^{(6);ij} + \frac{1}{\Lambda^4}\, \delta Z_{\phi}^{(8);ij}$$

$$Z_m^{ij} = m_i^2\, \delta^{ij} + \frac{1}{\Lambda^2}\, \delta Z_m^{(6);ij} + \frac{1}{\Lambda^4}\, \delta Z_m^{(8);ij}$$

- We rescale fields and masses (and possibly couplings) in order to reestablish canonical normalization.
  - ▷ This additional transformation will affect $\mathscr{L}_{\text{rest}}$
- Actually, this is not the end of the story since we have to link the Lagrangian parameters to a given set of experimental data.
  - ▷ These relations will, once again, change $\mathscr{L}_{\text{rest}}$

Linear vs. quadratic representation

- Furthermore, a given $\mathrm{A}_{\mathrm{tree}}$ containing terms up to $\mathscr{O}\left(1/\Lambda^4\right)$ has single and double insertions of $\dim = 6$ operators in the tree diagrams $\in \mathscr{L}^{(4)}$ (plus set of diagrams having new structures, $\notin \mathscr{L}^{(4)}$).
- Once we have the Lagrangian (up to $\mathscr{O}\left(1/\Lambda^4\right)$) we can obtain Feynman rules and amplitudes. Given

$$\mathrm{A} \quad = \quad \mathrm{A}^{(4)} + \frac{1}{\Lambda^2}\,\mathrm{A}^{(6)} + \frac{1}{\Lambda^4}\,\mathrm{A}^{(8)}$$

- **linear** means including the interference between $\mathrm{A}^{(4)}$ and $\mathrm{A}^{(6)}$,
- **quadratic** "currently" means including the square of $\mathrm{A}^{(6)}$ and **Not**
- the **complete inclusion** of all terms giving $1/\Lambda^4$ (before considering $\mathrm{A}^{(8)}$).

N.B. heavy-light turn on $\mathrm{Im}\,\mathrm{A}^{(6)}$, i.e. π^2 terms in "quadratic"

EOI: towards a descendant of the blue -band?

for $\dim = 8$

$\mathscr{G}$

An example (Einhorn, Wudka) is as follows:

$$\mathscr{L} \;=\; -\frac{1}{2}\,\partial_\mu\phi\,\partial_\mu\phi - \frac{1}{2}\,m^2\phi^2 - \frac{1}{4}\,g\phi^4 - \overline{\psi}(\partial\!\!\!/ - \lambda\,\phi)\,\psi + \frac{\mathrm{a}^1_{\mathrm{PTG}}}{\Lambda^2}\,\phi^3\,\overline{\psi}\psi + \frac{\mathrm{a}^2_{\mathrm{PTG}}}{\Lambda^2}\,(\overline{\psi}\psi)^2$$

where the $\dim = 6$ operators are PTG.

- Imagine that the underlying theory generates $\mathscr{O}_{\mathrm{LG}} = (\overline{\psi}\psi)\,\Box\phi$ which is LG.
- However, $\mathscr{O}_{\mathrm{LG}}$ and $-g\,\mathscr{O}^1_{\mathrm{PTG}} + \lambda\,\mathscr{O}^2_{\mathrm{PTG}}$ are equivalent. Furthermore, there are 10 $\dim = 6$ operators and 5 independent equivalence classes, so there must be 5 independent basis operators and we need the set of transformations and the set of equivalence relations.
- The field transformation needed to eliminate $\mathscr{O}_{\mathrm{LG}}$ is of the form

$$\phi \;\to\; \phi + \eta\,\frac{\mathrm{a}_{\mathrm{LG}}}{\Lambda^2}\,\overline{\psi}\psi$$

generating $\dim = 8$ compensations, e.g. $\phi^2\,(\overline{\psi}\psi)^2$.

for *Canonical normalization*

Canonical normalization is more than "normalization"

- field normalization, $\mathrm{H} \to \left[1 - \frac{1}{4}\,\frac{M^2}{\Lambda^2}\,(\mathrm{a}_{\phi\mathrm{D}} - 4\,\mathrm{a}_{\phi\Box})\right]\mathrm{H}$
- Process, $\overline{\mathrm{u}}(x_1\,p_1) + \mathrm{u}(x_2\,p_2) \to \mathrm{H}(-p_{\mathrm{H}}) + \mathrm{Z}(-p_{\mathrm{Z}})$
- Invariants, $\hat{s} = -2\,x_1 x_2\,p_1 \cdot p_2$ and $\hat{t} = 2\,x_1\,p_1 \cdot p_{\mathrm{Z}} + \mathrm{M}_{\mathrm{Z}}^2$
- Look for $\mathrm{a}_{\phi\Box}$ effects:

$$\sum_{\mathrm{spin}} |\,\mathrm{A}\,|^2 = \frac{3}{4}\,\frac{g^4\,\mathrm{v}_{\mathrm{u}}^2}{\mathrm{c}_{\mathrm{W}}^4}\,\mathrm{M}_{\mathrm{Z}}^2\,\hat{s}\,|\,\Delta_{\mathrm{Z}}\,|^2\,\left(1(=\text{LO SM}) + 2\,\frac{g_6}{\sqrt{2}}\,\mathrm{a}_{\phi\Box}\right)$$

$$+ \frac{g^4}{\mathrm{c}_{\mathrm{W}}^4}\,g_6^2\,\mathrm{a}_{\phi\Box}\,\left[\mathrm{A}_2(\hat{s},\hat{t})\,|\,\Delta_{\mathrm{Z}}\,|^2 + \mathrm{A}_1(\hat{s},\hat{t})\,\mathrm{Re}\,\Delta_{\mathrm{Z}}\right]$$

$$\mathrm{v}_{\mathrm{u}} = 1 - \frac{8}{3}\,\mathrm{s}_{\mathrm{W}}^2 \qquad \Delta_{\mathrm{Z}} = \frac{1}{\hat{s} - \mathrm{M}_{\mathrm{Z}}^2} \qquad g_6 = \frac{1}{\sqrt{2}\,G_{\mathrm{F}}\,\Lambda^2}$$

- At $\mathscr{O}(1/\Lambda^2)$ the Wilson coefficient $\mathrm{a}_{\phi\Box}$ modifies the normalization of the $\hat{s}$ distribution; at $\mathscr{O}(1/\Lambda^4)$ the shape of the $\hat{t}$-distribution is modified.

for ***Hellinger distance***

$\mathscr{G}$

$$
\begin{aligned}
\mathscr{L} &= -\frac{1}{2}\,\partial_\mu \mathrm{h}\partial_\mu \mathrm{h} - \frac{1}{2}\,m^2\,\mathrm{h}^2 + \mathrm{V}(\mathrm{h}) - \frac{1}{2}\left(1+\frac{\mathrm{h}}{\mathrm{v}}\right)\partial_\mu \pi\partial_\mu \pi \\
\mathrm{A}_{\pi\pi\to\pi\pi} &= -\frac{\lambda}{8\,m^2}\left(\frac{s^2}{s-m^2}+\frac{t^2}{t-m^2}+\frac{u^2}{u-m^2}\right) \\
\overline{\mathrm{A}}_{\pi\pi\to\pi\pi} &= \frac{\lambda}{8\,m^2}\left(\frac{s^2+t^2+u^2}{m^2}+\mathscr{O}\left(m^{-4}\right)\right) \\
\Sigma(s) &= \frac{\sigma(s)}{\int_{s_0}^{s_1} ds\sigma(s)} \\
\mathrm{H}^2 &= 1-\int_{s_0}^{s_1}\left[\Sigma\,\overline{\Sigma}\right]^{1/2}
\end{aligned}
$$

–

$\sqrt{s_1}$	$\mathrm{H}\mathscr{O}\left(m^{-10}\right)$	$\mathrm{H}\mathscr{O}\left(m^{-8}\right)$
150	$0.69\,10^{-4}$	$0.69\,10^{-3}$
300	$0.12\,10^{-2}$	$0.41\,10^{-2}$
500	$0.99\,10^{-2}$	$0.17\,10^{-1}$
700	$0.43\,10^{-1}$	$0.56\,10^{-1}$
900	0.17	0.19
950	0.28	0.29
975	0.38	0.40

–

$m = 1\,\mathrm{TeV}$ $\qquad \sqrt{s_0} = 100\,\mathrm{GeV}$

for ***complex poles***

$\mathscr{G}$

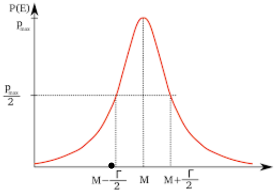
$$\Delta_\phi(s) = \Big[s - \mathrm{M}_\phi^2 + \Sigma_\phi(s)\Big]^{-1} \qquad s_\phi - \mathrm{M}_\phi^2 + \Sigma_\phi(s_\phi) = 0 \qquad s_\phi = \mu_\phi^2 - i\,\gamma_\phi\,\mu_\phi$$

$$\overline{\mathrm{M}}_\phi^2 = \mu_\phi^2 + \gamma_\phi^2, \qquad \mu_\phi\,\overline{\Gamma}_\phi = \overline{\mathrm{M}}_\phi\,\gamma_\phi$$

$$\frac{1}{s - s_\phi} = \left(1 + i\,\frac{\overline{\Gamma}_\phi}{\overline{\mathrm{M}}_\phi}\right)\left(s - \overline{\mathrm{M}}_\phi^2 + i\,\frac{\overline{\Gamma}_\phi}{\overline{\mathrm{M}}_\phi}\,s\right)^{-1}$$

$$\mu_\phi^2 = \mathrm{M}_{\phi\mathrm{OS}}^2 - \Gamma_{\phi\mathrm{OS}}^2 + \mathrm{h.o.}\,, \qquad \gamma_\phi = \Gamma_{\phi\mathrm{OS}}\left[1 - \frac{1}{2}\left(\frac{\Gamma_{\phi\mathrm{OS}}}{\mathrm{M}_{\phi\mathrm{OS}}}\right)^2\right] + \mathrm{h.o.}\,,$$

- The low-energy limit of the propagator is controlled by barred parameters and not by the on-shell mass.
- For perturbatively small values of the BSM parameters the ratio $\Gamma_{\mathrm{H}}/\mathrm{M}_{\mathrm{H}}$ (here H is a generic heavy field) remains small, the difference between barred and on-shell parameters is negligible, i.e. $s/\mathrm{M}_{\mathrm{H}}^2$ and $\Gamma_{\mathrm{H}}/\mathrm{M}_{\mathrm{H}}$ can be taken as the correct expansion parameters.



○ Being M the heavy scale, would you trust a truncated expansion (in powers of $1/\mathrm{M}^2$) up to the $\bullet$ $(\Gamma \neq 0)$?

Production, decay and Higgs propagator

$$\sigma_{i\to \mathrm{h}\to f} = \frac{1}{\pi}\,\sigma_{i\to \mathrm{h}}(s)\,\frac{s^2}{\mid s-s_{\mathrm{h}}\mid^2}\,\frac{\Gamma_{\mathrm{h}\to f}}{\sqrt{s}}$$

$$\frac{1}{s-s_{\mathrm{h}}} = \left(1+i\,\frac{\overline{\Gamma}_{\mathrm{h}}}{\overline{\mathrm{M}}_{\mathrm{h}}}\right)\left(s-\overline{\mathrm{M}}^2_{\mathrm{h}}+i\,\frac{\overline{\Gamma}_{\mathrm{h}}}{\overline{\mathrm{M}}_{\mathrm{h}}}\,s\right)^{-1}$$

$\mathrm{M}_{\mathrm{hos}}$	exp. data
Γ_{hos}	from SMEFT theory

- "best practice" is to expand wave function factors only