EFT truncation, uncertainty, validity

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Building up on the EFT note of the LHC TOP WG [1], we make the following proposal regarding EFT truncation, uncertainty, and validity.

1. The SMEFT is meant to provide a parametrization of heavy new-physics scenarios in a low-energy limit.

2. The absolute normalization of the operators vs. their coefficients bears no physical consequence. For convenience, dimensionless operator coefficients are normalized by a dimensionful quantity Λ which bears no physical meaning and can be chosen arbitrarily ($\Lambda = 1\,\text{TeV}$ is a common choice). Only the C/Λ^{d-4} combination can be probed experimentally and distinctions between couplings and scales require UV assumptions.

3. The relative magnitude of operator coefficients (dimension six vs. dimension six, or dimension six vs. dimension eight) is model dependent. Assumptions about the type of physics taking place in the UV lead to different power counting rules that can be used to estimate the size of the operator coefficients. They allow to account for classes of models and may be tied to a specific operator basis.

   One naively expects that power countings would suppress dimension-eight contributions with respect to dimension-six ones. In practice, one therefore commonly omits the former altogether. There are, however, cases where this is not justified. The dimension-six truncation is then not valid (see item 5).

   A well-known example of power counting is that of SILH, for strongly interacting scenarios [2]. In weakly interacting cases, one can also distinguish tree- and loop-level generated operators [3]. Allowing for a single scale Λ (both at dimension six and eight) and positing all operator coefficients to be of the same order is also a power counting. Depending on the choice of basis and operator normalization, this choice may or not match to a family of consistent new-physics scenarios.

4. The linear dimension-six truncation of amplitudes calculated with canonically normalized Lagrangians (or S-matrix elements via the LSZ theorem) can be translated exactly from one operator basis to the other, as stated in the equivalence theorem. The squares of such truncated amplitudes are thus unambiguous and well-defined. They include a partial set of $1/\Lambda^4$ contributions that excludes diagrams with two or more operator insertions, and contributions to the field redefinitions beyond $1/\Lambda^2$. Note that, if, as is customary, a given set of observables is singled out and employed to fix SM input parameters, an exact translation between two bases requires the same input set.

5. The linear dimension-six truncation at the amplitude level is deemed *valid* if it provides the leading contribution to the considered observables in the low-energy limit of a given new-physics scenario.

   Without power countings or explicit models, assessing the validity and the truncation uncertainty is not possible. This should be performed when interpreting EFT results under definite new-physics assumptions.

6. Once the amplitude truncated to the $1/\Lambda^2$ order is squared, the relative importance of linear and quadratic terms in a specific observable may be used to give indications on a number of aspects of the EFT interpretation. For example, in some cases it can give a sense of the region of convergence of the EFT expansion in the space of operator coefficients.

   One should however bear in mind that interferences can be suppressed in interesting cases (e.g. due to symmetries, non-overlaping helicity configurations, see e.g. ref. [4]), and especially at dimension-six where the diversity of possible Lorentz structures is limited. Providing results with and without quadratic contributions should therefore be recommended (see Figure 1).

7. Restricting the energy probed in a measurement with a sliding upper cut (E_{cut}) on an appropriate kinematic variable [5, 6] can allow for the interpretation of EFT results in new-physics scenarios of lower scales. Providing limits as functions of this sliding cut should be recommended (see Figure 1).

   The quantities probed in a dimension-six EFT analysis are C_i/Λ^2. These can only be related to couplings and physical mass scales (M) with additional assumptions about the underlying new-physics scenario. The EFT only provides an appropriate parametrization when the energy probed in a measurement is lower than the physical new-physics mass scale, $E_{\text{cut}} \ll M$.

8. Unitarity also limits the range of validity of the dimension-six truncation, by bounding combinations like CE^2/Λ^2 where E is a characteristic energy scale of the process considered and C is a (linear combination of) dimension-six operator coefficient(s). In the simplest case of two-to-two scatterings, perturbative unitarity can be imposed on

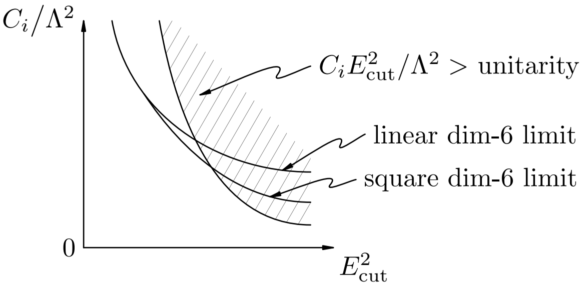

Figure 1: Limits set on dimension-six operator coefficients as functions of an upper cut on the characteristic energy scale of the process considered. Results obtained using either linear and squared dimension-six truncations start to diverge before unitarity is lost, in the hatched region.

each partial wave. Unitarity constraints on CE^2/Λ^2 (see Figure 1) are independent of the underlying new-physics scenario.

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