On the Question of Validity and Interpretation of EFT Analyses

While an EFT offers a practical and broadly model-independent way to analyse experimental data in search of new physics evidence, the question of EFT validity cannot be addressed in the EFT framework. There is no way of estimating the errors associated with neglecting higher order operators (of, say, $d > 6$). The magnitude of these operators cannot be estimated based on the magnitude of those (of, say, $d = 6$) that have been retained in the truncated EFT used to analyse the data. Estimates are possible only relying on assumptions on the UV dynamics that underlies the EFT, or on the explicit knowledge of the UV model. However the results of EFT experimental analyses should be independent of UV assumptions as much as possible. Therefore they should not rely on the estimate of the higher order operators in order to be interpretable in a class of models as broad as possible.

This observation does not impact the EFT interpretation of "low" energy measurements, such as Flavor data or LEP results. However it does impact the EFT interpretation of the LHC data and particularly of the high-p_T measurements of differential distributions at scales much above the EW scale. We illustrate this issue below and propose a simple solution, following Ref. [1] (see in particular Chapter II.2) and other articles. These recommendations are essential for the experimental constraints to be interpretable under a class of UV assumptions as broad as possible, and are in line with those given in Ref. [2] for the EFT interpretation of top-quark LHC measurements.

EFT Validity

Consider a very simple example: a single BSM particle of mass M that contributes to a $2 \to 2$ SM-particle scattering. The amplitude of the process takes the schematic form

$$\mathcal{A} = g_{\rm SM}^2 + \frac{g_{\rm BSM}^2 E^2}{E^2 - M^2} \overset{E \ll M}{=} \underbrace{g_{\rm SM}^2}_{\mathcal{A}_{\rm SM}} \underbrace{- \frac{g_{\rm BSM}^2}{M^2} E^2}_{\mathcal{A}^{(6)}} \underbrace{- \frac{g_{\rm BSM}^2}{M^4} E^4}_{\mathcal{A}^{(8)}} + \dots\,, \qquad (1)$$

where $g_{\rm SM}$ generically denotes some SM coupling and $g_{\rm BSM}$ is the coupling between the BSM particle and a pair of SM particles. We denoted as "E" the characteristic energy scale of the process. For instance, $E = \sqrt{s}$ if the heavy particle is exchanged in the s-channel.

The second and the third terms in the right-hand side of Eq. (1) correspond, in the EFT description of the process, to the contribution from operators of dimension 6 and 8, respectively. The Wilson coefficients of these operators are $G_{(6)} = g_{\rm BSM}^2/M^2\,,\ G_{(8)} = g_{\rm BSM}^2/M^4 = G_{(6)}/M^2$. The dots in Eq. (1) correspond to operators of even higher energy dimension.

Only the $d = 6$ operator is retained in the truncated EFT that is used to analyze the data, and its coefficient $G_{(6)}$ is the only free parameter. The operator of dimension 8 (and higher) is not included. Therefore its contribution $\mathcal{A}^{(8)}$ to the amplitude should be regarded as an error of the truncated EFT prediction. The magnitude of $G_{(8)}$ relative to $G_{(6)}$ depends on M, which is not a parameter of the EFT but a parameter of the underlying UV theory the EFT emerges from. We see that estimating $\mathcal{A}^{(8)}$ requires some input from the UV theory. In this extremely simple setup with a single BSM particle coupled to the SM with a trilinear vertex, the required UV input is the mass M (or, equivalently, $g_{\rm BSM}$). In many BSM scenarios, the estimate of the Wilson coefficients is provided by a much richer power-counting rule than the one above and much more UV input is required for the estimate. Several compelling broad classes new physics models, such as composite Higgs and some SUSY scenarios, as the simple example above, can be broadly described at energies accessible at LHC and future colliders, by a single mass scale and a single coupling, $g_{\rm BSM}$, with appropriate selection rules characteristic of the UV dynamics. They offer interesting EFT benchmarks where the validity of the experimental EFT analyses can be studied.

Since it requires UV theory input, the validity of the EFT in each experimental analysis can only be assessed a posteriori, when the analysis is interpreted in a concrete BSM scenario and a (model-dependent) estimate of $G_{(8)}$ becomes available. However the estimate of $G_{(8)}$ can be turned into an estimate of $\mathcal{A}^{(8)}$ (governing the truncation error) only provided we know the energy "E" of the measurement. The experimental analyses should thus report information on the energy scale of the measurements that drive the sensitivity. In Flavor Physics, for LEP data, or even at the LHC for measurements that are performed in a very narrow range of energy like Higgs branching ratios, this information is implicitly available. For processes measured over a wide energy range, like the high-p_T measurements at the LHC, it should be reported explicitly. In particular, since the importance of $\mathcal{A}^{(8)}$ relative to the truncated EFT prediction ($\mathcal{A}^{(6)}$) grows with the energy, one should report the maximal energy scale of the data that are relevant in the analysis.

Suggestions for how to report bounds

Bounds as function of maximal scale $M_{\rm cut}$. In high-p_T EFT analyses, the sensitivity to the EFT emerges from the combination of measurements performed at different energies, possibly including the extreme tail of the kinematical distributions. However, it is easy to remove from the likelihood fit the measurements at E larger than a sliding scale $M_{\rm cut}$, and report the experimental result as a function of $M_{\rm cut}$. For instance, if no deviations from the SM are observed, the experimental results can be expressed as limits of the form

$$G_{(6)} < \delta^{\rm exp}(M_{\rm cut})\,. \tag{2}$$

The monotonically decreasing function $\delta^{\rm exp}$ depends on the upper value, collectively denoted by $M_{\rm cut}$, of suitably designed kinematic variables, e.g., transverse momenta or invariant masses, that set the typical energy scale characterising the process. In general, the bound $\delta^{\rm exp}$ is obtained by imposing cuts on these variables and making use of the differential kinematic distributions.

The limit $\delta^{\rm exp}(M_{\rm cut})$ is obtained by only employing EFT predictions for observables at an energy $E < M_{\rm cut}$. Therefore they can be consistently applied to UV models where the EFT predictions are sufficiently accurate at $E = M_{\rm cut}$ or larger. Inspecting $\delta^{\rm exp}$ as a function of $M_{\rm cut}$ greatly facilitates the interpretation of the limit in specific UV models or scenarios. For instance in the UV model of Eq. (1) for a given value of M, the experimental constraint $G_{(6)} = g^2_{\rm BSM}/M^2 < \delta^{\rm exp}(M_{\rm cut})$ can be consistently applied only for $M_{\rm cut}$ smaller (or much smaller) than M. Effectively, this is equivalent to remove from the analyses the data at $E \geq M$, where the EFT description does not apply.

Having to report bounds as a function of $M_{\rm cut}$, targeting the best possible sensitivity in the entire $M_{\rm cut}$ range, encourages accurate experimental measurements at low energy, even in cases where the growing-with-energy behavior of the EFT contribution would allow to probe $G_{(6)}$ effectively (for $M_{\rm cut} = \infty$) by simple event-counting in the extreme high-energy tail. This is important because accurate experimental measurements at low energy are more valuable theoretically since they can be interpreted, as we have seen, in a larger set of UV models (i.e., at smaller M).

Theoretical uncertainties. Physical observables are computed from the truncated Lagrangian in a perturbative expansion according to the usual rules of effective field theories. The perturbative order to be reached depends on the experimental precision and on the aimed theoretical accuracy. Theoretical predictions obtained in this way are functions of the effective coefficients $G_{(6)}$ and can be used to perform a fit to the experimental data. The impact of loop corrections on the fit can be estimated a posteriori based on the extracted values of (or limits on) the effective coefficients. The fit to the coefficients $G_{(6)}$ should be performed by correctly including the effect of all the theoretical uncertainties (such as those from the PDFs and missing SM loop contributions not originating from the EFT perturbative expansion). The errors due to the truncation at the dim-6 level and higher-loop diagrams involving insertions of different effective operators, on the other hand, are not quantifiable in a model-independent way and thus can not be included in the likelihood fit. They can be quantified a posteriori as previously explained.

Linear vs Quadratic. When taking the square of the amplitude in Eq. (1) a quadratic term $\left|\mathcal{A}^{(6)}\right|^2 = G_{(6)}^2 E^4 = (g_{\rm BSM}^4/M^4)E^4$ emerges. This scales with the fourth power of the energy like the term $\sim \mathcal{A}_{\rm SM}\mathcal{A}^{(8)} = g_{\rm SM}^2 G_{(8)} E^4 = (g_{\rm SM}^2 g_{\rm BSM}^2/M^4)E^4$ from the interference with the SM of the dim-8 term, which however is not present in the truncated EFT. This might raise the question of whether the quadratic term should be retained in the calculation or not. The answer is affirmative. If $g_{\rm SM} < g_{\rm BSM}$, the quadratic term is larger than the dim-8 contribution. We can thus accurately predict the E^4 term of the cross-section in the truncated EFT, provided of course the quadratic term is retained in the prediction. If $g_{\rm SM} > g_{\rm BSM}$, the quadratic term is smaller than the dim-8 contribution but this is not a good reason to remove it from the prediction. Eliminating the quadratic term means setting to zero the E^4 term of the cross-section, which is not a better approximation of the true UV theory prediction. Also notice that for $g_{\rm SM} > g_{\rm BSM}$, the interference between $\mathcal{A}_{\rm SM}$ and $\mathcal{A}^{(6)}$ (the linear term), of order $g_{\rm SM}^2 G_{(6)} E^2 = (g_{\rm SM}^2 g_{\rm BSM}^2/M^2)E^2$ is larger than the quadratic term in the entire range of validity of the EFT $E < M$. Provided the EFT is consistently used only in its range of validity, the presence of the quadratic term will not affect considerably the EFT prediction for $g_{\rm SM} > g_{\rm BSM}$. This scaling of the different EFT contributions have been obtained in the one-mediator toy model, and even if rather generic in all one-scale-one-coupling scenarios, it is certainly not universal.

Still, it is useful to accompany the proper EFT results, where the quadratic terms is duly included, with limits where only the linear term is retained in the prediction. Indeed if removing the quadratic term is found to affect the sensitivity significantly, it means that the reach is dominated by high enough energy measurements where $G_{(6)}E^2$ is of order $g_{\rm SM}^2$ or larger. Much more detailed and rigorous information about the relevant energy scale is obtained by analyzing the data with the sliding $M_{\rm cut}$ as previously explained.

Contributors: This proposal results from some discussions among R. Contino, A. Falkowski, F. Goertz, C. Grojean, F. Maltoni, G. Panico, F. Riva, A. Wulzer…

References

[1] LHC Higgs Cross Section Working Group, D. de Florian et al., *Handbook of LHC Higgs Cross Sections: 4. Deciphering the Nature of the Higgs Sector*, 1610.07922.

[2] J. A. Aguilar-Saavedra et al., *Interpreting top-quark LHC measurements in the standard-model effective field theory*, 1802.07237.