

# Precise theoretical predictions for VBS

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*in collaboration with*

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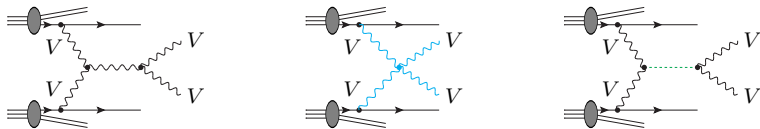
**Winter 2021 Topical Meeting on VBS: VBS at Snowmass, 25. January 2021**

- 1 Introduction
- 2 Full NLO corrections to vector-boson pair plus jet-pair production
- 3 Electroweak corrections to vector-boson scattering (VBS)
- 4 Quality of VBS approximation
- 5 Polarised VBS
- 6 Conclusion

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- Run 1:
  - Discovery of the Higgs boson
  - exclusion limits for new physics models
- Run 2:
  - Study of properties of the Higgs boson
  - precise measurements of standard-candle processes (Drell-Yan,  $t\bar{t}$ ,  $VV$ , ...)
  - measurement of new SM processes ( $t\bar{t}H$ , VBS,  $VVV$ , ...)
  - further exclusion limits for new physics models
- Run 3 and beyond:
  - Improved precision tests of SM processes and parameters
  - measurement of further new SM processes
  - discovery of New Physics?

Precise theoretical predictions needed to match improved experimental accuracy!



Physics issues of vector-boson scattering (VBS): ( $V = W, Z$ )

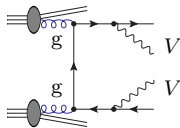
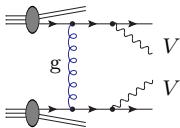
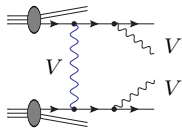
- key process to test electroweak symmetry breaking  
Higgs boson crucial for unitarity of process
- search for anomalous quartic-gauge-boson couplings  
sensitivity grows with energy of gauge bosons

Improvement of experimental precision

Integrated Luminosity	36 fb	150 fb	300 fb	3000 fb-
Year	2016	2019	2022	2038
EW(VBS) $W_{\pm}W_{\pm}$	20%	10%	7%	2%
EW (VBS) ZZ	35%	18%	13%	6%
EW (VBS) WZ	35%	18%	13%	6%

Jakob Salfeld-Nebgen in <https://indico.cern.ch/event/711256>

must be matched by theoretical calculations

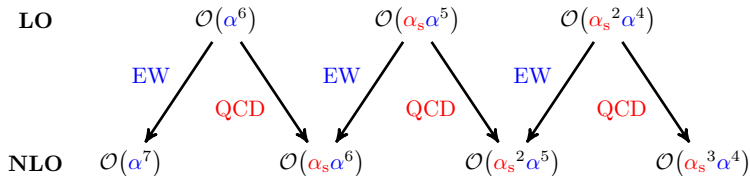
Final state:  $VV + 2j$  ( $4l + 2j$ )


- **Full electroweak (EW) process** [ $\mathcal{O}(\alpha^4)$  for stable  $V$ s]  
not separable from VBS
- **QCD process** [ $\mathcal{O}(\alpha_s^2 \alpha^2)$  for stable  $V$ s]  
gauge-invariant contribution
- **interferences** between EW and QCD contributions  
[ $\mathcal{O}(\alpha_s \alpha^3)$  for stable  $V$ s]  
appear only for channels with identical or weak-isospin partner quarks
- **gluonic channels** for neutral final states
- **irreducible background can be suppressed by cuts** on  $M_{jj}$  and  $|\Delta y_{jj}|$   
 $\sigma_{EW}^{W^+W^+} \sim 10 \sigma_{QCD}^{W^+W^+}$ ,  $\sigma_{EW}^{W^+Z} \sim 0.25 \sigma_{QCD}^{W^+Z}$ ,  $\sigma_{EW}^{ZZ} \sim 0.1 \sigma_{QCD}^{ZZ}$

LO: pure EW diagrams  $\mathcal{O}(e^6)$  and diagrams with gluons  $\mathcal{O}(e^4 g_s^2)$

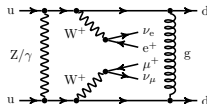
NLO: EW and QCD corrections to both types of diagrams

at level of cross section:



Virtual diagrams mix QCD and EW corrections:

- EW correction to LO QCD amplitude
- QCD correction to LO EW amplitude

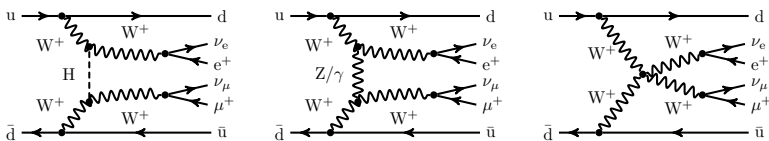


$\Rightarrow$  QCD and EW corrections mix at  $\mathcal{O}(\alpha_s \alpha^6)$  and  $\mathcal{O}(\alpha_s^2 \alpha^5)$

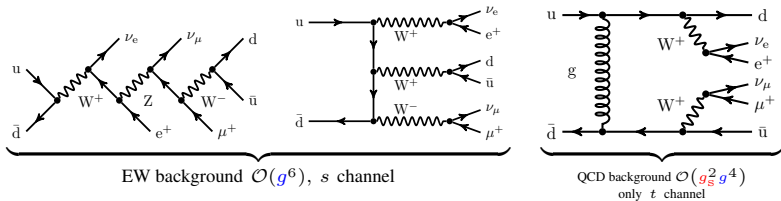
**QCD and EW corrections cannot be separated in general**

possible in VBS approximation (neglects interferences)

Vector-boson scattering (VBS) topologies:  $\mathcal{O}(g^6)$  all  $t$  channel



irreducible background to VBS:



- $t$  channel: incoming quarks/antiquarks connected to outgoing quarks/antiquarks
- $u$  channel: exchange identical quarks/antiquarks in final state
- $s$  channel: incoming quark and anti-quark connected, all boson propagators time like
- VBS approximation:** only  $t$  and  $u$  channel, no interferences



- full LO predictions: **Ballestrero, Franzosi, Maina '10 (PHANTOM)**

NLO QCD separately for EW ( $\mathcal{O}(\alpha^6)$ ) and QCD-induced production ( $\mathcal{O}(\alpha_s^2 \alpha^4)$ )

- NLO QCD corrections to EW production in VBS approximation:

**Jäger, Oleari, Zeppenfeld (+ Bozzi) '06, '07, '09 (VBFNLO);**

**Denner, Hošeková, Kallweit '12**

PS matching: **Jäger, Zanderighi '11, '13 + Karlberg '14** ( $W^+W^+$ ,  $W^+W^-$ ,  $ZZ$ )

**Rauch, Plätzer '16** ( $W^+W^-$ ), **Jäger, Karlberg, Scheller '18** ( $WZ$ )

- NLO QCD corrections to QCD production:

**Melia, Melnikov, Rötsch, Zanderighi '10, '11** ( $W^+W^+$ ); **Greiner et al. '12** ( $W^+W^-$ );

**Campanario, Kerner, Ninh, Zeppenfeld '13, '14 (VBFNLO)** ( $W^+W^+$ ,  $WZ$ ,  $ZZ$ )

PS matching: **Melia, Nason, Rötsch, Zanderighi '11** ( $W^+W^+$ )

- **EW corrections for complete processes**  $pp \rightarrow 4f + 2j$

- NLO EW and QCD corrections for  $W^\pm W^\pm$ ,  $WZ$  and  $ZZ$  final states

**Biedermann, Denner, Pellen '16; Denner, Dittmaier, Pellen, Schwan '19,**

**Denner, Franken, Pellen, Schmidt '20**

- full NLO corrections to  $W^\pm W^\pm$  **Biedermann, Denner, Pellen '17**

- NLO EW matched to EW PS and interfaced to QCD PS for  $W^\pm W^\pm$

**Chiesa, Denner, Lang, Pellen '19**

## Calculations for VBS within the SM

- all processes known at NLO QCD accuracy matched to PS
  - in VBS approximation (no  $s$  channel, no interferences)
  - for both QCD-/EW-induced process
  - all available in VBFNLO (apart from QCD-induced  $W^+W^-$ )
  - all available in POWHEG-BOX ( $\Rightarrow$  PS matching)
  - possible to generate in MG5\_AMC@NLO or SHERPA
- NLO EW corrections known for  $W^+W^+$ ,  $WZ$ , and  $ZZ$   
 ( $W^+W^-$  in progress)
- full NLO computation only available for  $W^+W^+$  ( $ZZ$  in progress)
- no NNLO results known

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Scale uncertainty reduced by factor 5:

Biedermann et al. '17

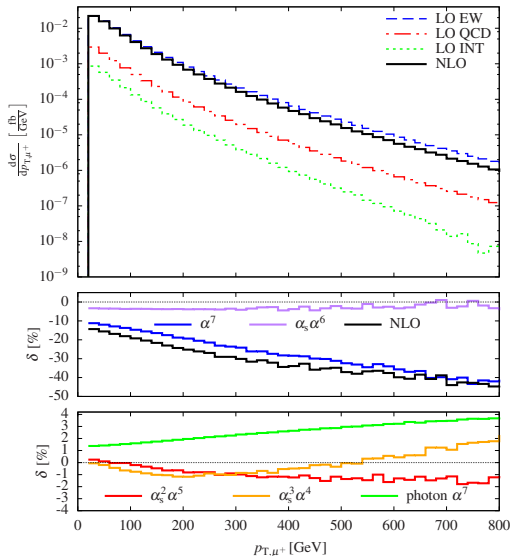
$$\sigma_{\text{LO}} = 1.6383(2)_{-9.44(2)\%}^{+11.66(2)\%} \text{ fb}, \quad \sigma_{\text{NLO}} = 1.3577(7)_{-2.7(1)\%}^{+1.2(1)\%} \text{ fb}$$

results for separate orders:

order	$\mathcal{O}(\alpha^6)$	$\mathcal{O}(\alpha_s \alpha^5)$	$\mathcal{O}(\alpha_s^2 \alpha^4)$	sum
$\sigma_{\text{LO}}$ [fb]	1.4178(2)	0.04815(2)	0.17229(5)	1.6383(2)
$\delta\sigma_{\text{LO}}/\sigma_{\text{LO}}$ [%]	86.5	2.9	10.5	100

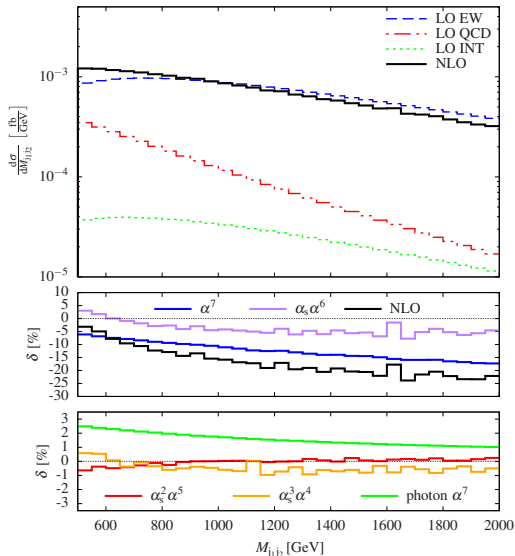
order	$\mathcal{O}(\alpha^7)$	$\mathcal{O}(\alpha_s \alpha^6)$	$\mathcal{O}(\alpha_s^2 \alpha^5)$	$\mathcal{O}(\alpha_s^3 \alpha^4)$	sum
$\delta\sigma_{\text{NLO}}$ [fb]	-0.2169(3)	-0.0568(5)	-0.00032(13)	-0.0063(4)	-0.2804(7)
$\delta\sigma_{\text{NLO}}/\sigma_{\text{LO}}$ [%]	-13.2	-3.5	0.0	-0.4	-17.1

- LO EW contribution dominates for  $W^+W^+jj$
- LO interference small but non-negligible
- surprisingly large EW corrections at  $\mathcal{O}(\alpha^7)$
- photon-induced contribution at NLO +1.5% (LUXqed Manohar et al. '16, '17)



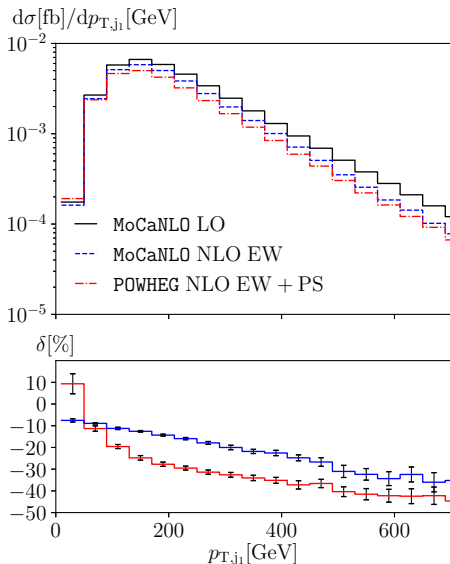
$$pp \rightarrow \mu^+ \nu_\mu e^+ \nu_e jj$$

- EW contribution dominates everywhere
- $\mathcal{O}(\alpha^7)$  –40% at 800 GeV (Sudakov logarithms) dominant correction
- $\mathcal{O}(\alpha_s \alpha^6)$  –4% – 0%
- $\mathcal{O}(\alpha_s^2 \alpha^5)$ ,  $\mathcal{O}(\alpha_s^3 \alpha^4)$  between –2% and +2% cancelling for large  $p_{T,\mu^+}$
- photon-induced corrections increase to 4% at  $p_{T,\mu^+} = 800$  GeV (photon PDF grows with energy)



$$pp \rightarrow \mu^+ \nu_\mu e^+ \nu_e jj$$

- Large cross section also for high  $M_{jj}$
- QCD-induced contrib. drops much faster
- $\mathcal{O}(\alpha^7)$  -6% - -17%
- $\mathcal{O}(\alpha_s \alpha^6)$  +5% - -5%
- $\mathcal{O}(\alpha_s^2 \alpha^5)$ ,  $\mathcal{O}(\alpha_s^3 \alpha^4)$  tiny
- photon-induced corrections decrease with  $M_{jj}$



Chiesa et al. '19

- Event generator based on POWHEG and RECOLA for  $pp \rightarrow \mu^\pm \nu_\mu e^\pm \nu_e jj$  and  $pp \rightarrow e^\pm \nu_e e^\pm \nu_e jj$  including EW corrections matched to QED parton shower and interfaced to QCD parton shower
- PS shifts events to smaller  $p_{T,j1}$ , partially out of acceptance

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## Large universal NLO EW corrections to VBS processes

process	$\sigma_{\text{LO}}^{\mathcal{O}(\alpha^6)}$ [fb]	$\sigma_{\text{NLO,EW}}^{\mathcal{O}(\alpha^7)}$ [fb]	$\delta_{\text{EW}}$ [%]
<b>Biedermann et al. '16</b> pp $\rightarrow \mu^+ \nu_\mu e^+ \nu_e jj$ ( $W^+ W^+$ )	1.5348(2)	1.2895(6)	<b>-16.0</b>
<b>Denner et al. '19</b> pp $\rightarrow \mu^+ \mu^- e^+ \nu_e jj$ ( $ZW^+$ )	0.25511(1)	2.142(2)	<b>-16.0</b>
<b>Denner et al. '20</b> pp $\rightarrow \mu^+ \mu^- e^+ e^- jj$ ( $ZZ$ )	0.097681(2)	0.08214(5)	<b>-15.9</b>

largely independent of cuts  $\Rightarrow$  intrinsic feature of VBS processes

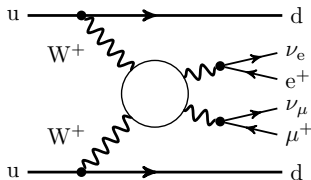
## Relative NLO EW corrections in logarithmic approximation

process	$\delta_{\text{EW}}$ [%]	$\delta_{\text{EW}}^{\text{log,int}}$ [%]	$\delta_{\text{EW}}^{\text{log,diff}}$ [%]	$\langle M_{4\ell} \rangle$ [GeV]
pp $\rightarrow \mu^+ \nu_\mu e^+ \nu_e jj$	<b>-16.0</b>	-16.1	-15.0	390
pp $\rightarrow \mu^+ \mu^- e^+ \nu_e jj$	<b>-16.0</b>	-17.5	-16.4	413
pp $\rightarrow \mu^+ \mu^- e^+ e^- jj$	<b>-15.9</b>	-15.8	-14.8	385

Double-pole approximation (DPA) for outgoing W bosons

Effective vector-boson approximation (EVBA) for incoming W bosons

- DPA and EVBA reduce discussion to  $V_1 V_2 \rightarrow V_3 V_4$
- DPA accurate for cross section within 1%
- EVBA crude approximation but sufficient to understand dominant effects



leading-logarithmic approximation for  $V_1 V_2 \rightarrow V_3 V_4$

Denner, Pozzorini '00

$$d\sigma_{LL} = d\sigma_{LO} \left[ 1 - \frac{\alpha}{4\pi} 4C_W^{ew} \log^2 \left( \frac{Q^2}{M_W^2} \right) + \frac{\alpha}{4\pi} 2b_W^{ew} \log \left( \frac{Q^2}{M_W^2} \right) \right]$$

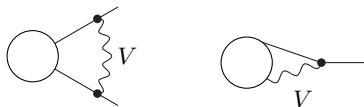
$$C_W^{ew} = \frac{2}{s_w^2}, \quad b_W^{ew} = \frac{19}{6s_w^2}$$

for transverse W bosons,  $Q \rightarrow M_{4\ell}$

(double EW logs, collinear single EW logs, and single logs from parameter renormalization included) (angular-dependent logarithms omitted)

large NLO EW corrections intrinsic feature of VBS

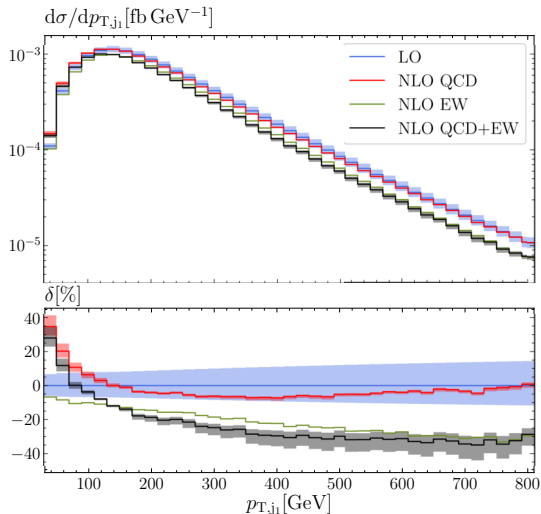
## Feynman diagrams leading to double- and single-logarithmic corrections



- Leading-logarithmic EW corrections depend only on gauge structure of model, external lines and polarisations of process.
- For all VBS processes, leading double logarithms and single logarithms universal for fixed polarisations ( $U_Y(1)$  does not contribute to VBS).
- Angular-dependent single logarithms (not included in approximation) turned out to be small so far ( $\sim 1-2\%$  owing to cancellations)
- Expect similarly large EW corrections for SMEFT or extended models that do not modify the  $SU(2) \times U(1)$  gauge structure of the SM.
- Expect smaller corrections for scattering of longitudinal vector bosons different coefficients in logarithmic corrections  
 $\Rightarrow$  very naively  $\sim 40\%$  of those for transverse bosons  $\Rightarrow \sim 6\%$

## Distribution in transverse momentum of the leading jet

Denner et al. '19



- $\mathcal{O}(\alpha^7) \sim -30\%$   
 at  $p_{T,j1} = 800$  GeV  
 (Sudakov logarithms)  
 dominant correction
- $\mathcal{O}(\alpha_s \alpha^6) \lesssim 10\%$   
 for  $p_{T,j1} > 100$  GeV  
 small QCD scale uncertainty  
 owing to dynamical scale  
 $\mu = \sqrt{p_{T,j1} p_{T,j2}}$
- large correction for small  
 $p_{T,j1}$  due to phase-space  
 suppression at LO  
 (all jets have small  $p_T$ )  
 redistribution of events at  
 NLO

## Experimentally

- Semileptonic final state offer more statistics
- much stronger QCD background
- hadronically decaying vector boson can be reconstructed using jet-substructure techniques  $\Rightarrow$  6.5% at  $3\text{ab}^{-1}$  and 27 TeV [Cavaliere et al. '18](#)
- first results from ATLAS [1905.07714](#) ( $2\sigma$  significance) and CMS [1905.07445](#)

## theoretically

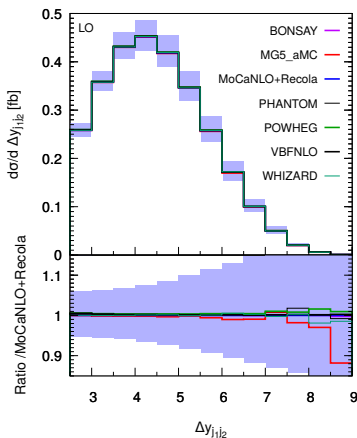
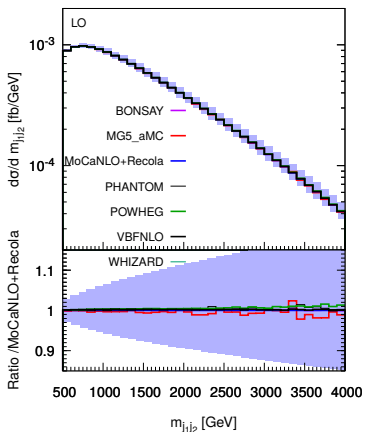
- **Proliferation of partonic channels in full calculation**  
60 quark-induced partonic channels for  $pp \rightarrow \mu^+ \mu^- e^+ e^- jj$ ,  
+ 40 gluon-induced channels (+ b-induced channels)  
even more channels for semi-leptonic final states (4-quark final states)
- LO diagrams of orders  $\mathcal{O}(g^6)$ ,  $\mathcal{O}(g^4 g_s^2)$ , +  $\mathcal{O}(g^2 g_s^4)$   
 $\Rightarrow$  need strategy to simplify calculation
- consider only contributions involving a virtual  $VV'$  pair in theoretical calculation to reduce number of contributions  
**use double-pole approximation** to calculate NLO corrections  
(gauge invariant, accuracy of DPA 1% for  $pp \rightarrow \mu^+ \nu_\mu e^+ \nu_e jj$ )  
 $\Rightarrow$  **calculation of NLO corrections should be feasible**

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Comparison of codes with VBS approximation (BONSAY, POWHEG, VBFNLO) and without (MoCaNLO+RECOLA, MG5\_AMC, PHANTOM, WHIZARD)

$$pp \rightarrow \mu^+ \nu_\mu e^+ \nu_e jj$$

Ballestrero et al. '18 (VBSCAN)

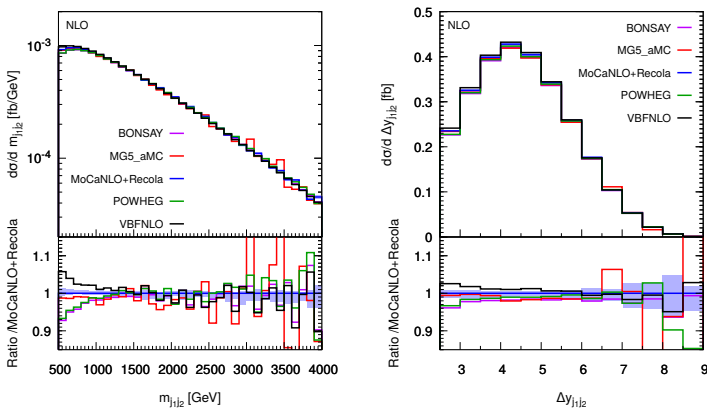


Differences between codes below  $\sim 1\%$  in fiducial region  
 $\Rightarrow$  accuracy of VBS approximation below  $\sim 1\%$  at LO

Comparison of codes with VBS approximation (BONSAY, POWHEG VBFNLO) and without VBS approximation (MoCANLO+RECOLA, MG5\_AMC)

$$pp \rightarrow \mu^+ \nu_\mu e^+ \nu_e jj$$

Ballestrero et al. '18 (VBSCAN)



differences up to 10% outside the QCD scale uncertainty band

POWHEG, Bonsay: no  $s$  channel  $\Rightarrow$  reduction at small  $M_{jj}$

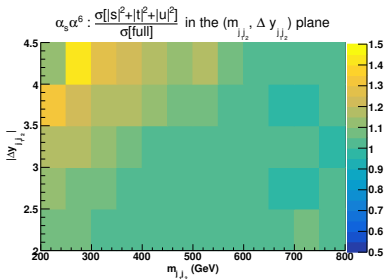
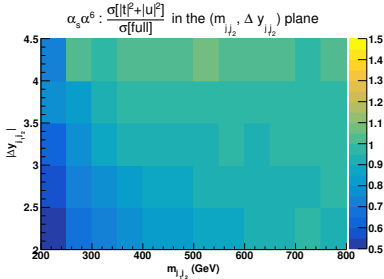
VBFNLO: no interference  $\Rightarrow$  enhancement at small  $M_{jj}$



Comparison of codes with VBS approximation (VBFNLO)  
and without VBS approximation (MOCANLO+RECOLA)

$$pp \rightarrow \mu^+ \nu_\mu e^+ \nu_e jj$$

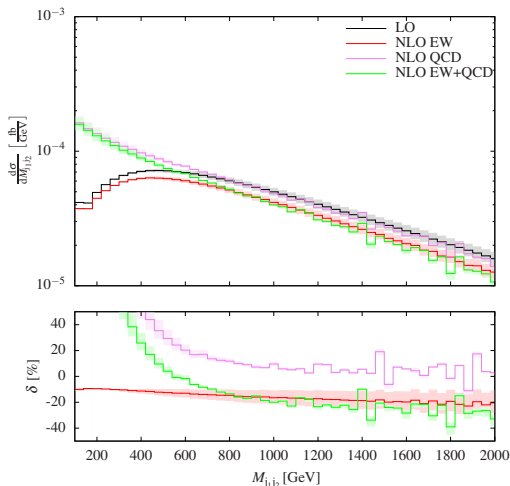
Ballestrero et al. '18 (VBSCAN)



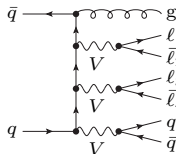
- approximations worse at NLO than at LO:  
difference of up to 20% in fiducial region  $M_{jj} > 500 \text{ GeV}$ ,  $\Delta y_{jj} > 2.5$   
(gluon bremsstrahlung fakes tagging jet in  $s$  channel)
- difference for fiducial cross section: ( $M_{jj} > 500 \text{ GeV}$ ,  $\Delta y_{jj} > 2.5$ )  
 $|t| + |u|$  approximation:  $\sim -2\%$        $|s| + |t| + |u|$  approximation:  $\sim +1\%$
- difference for inclusive cross section: ( $M_{jj} > 200 \text{ GeV}$ ,  $\Delta y_{jj} > 2$ )  
 $|t| + |u|$  approximation:  $-6\%$        $|s| + |t| + |u|$  approximation:  $+2.6\%$

$$pp \rightarrow \mu^+ \mu^- e^+ e^- jj$$

Denner et al. '20



- Loose VBS cut:  $M_{jj} > 100$  GeV based on 1708.02812 (CMS)
- $s$ -channel NLO contribution involving tri-boson prod.



Less suppression at NLO owing to extra gluon jet

- 24% NLO QCD corrections to fiducial cross section
- ⇒ include tri-boson contrib. for loose VBS cuts

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## Preliminaries

- All information about polarised cross-sections is within angular distributions of final-state particles.
- Extracting polarised observables simplifies interpretation and theoretical analysis.

## Polarized observables

- are important probes of Standard Model gauge and Higgs sectors,
- may provide discrimination power between SM and beyond-SM physics.

## Longitudinal polarisation mode of vector bosons is

- a consequence of the Electroweak Symmetry Breaking,
- very sensitive to deviations from SM:  
 unitarity of cross sections with longitudinally polarised vector bosons realized in SM via cancellation of different contributions

⇒ Extract experimental results for cross-sections with longitudinally polarised vector bosons.



- Massive vector bosons appear only as virtual particles  $\Rightarrow$ 
  - no unique definition of vector-boson polarisations
  - diagrams without resonant vector bosons contribute to physical final state
- vector bosons are massive  $\Rightarrow$   
 definition of polarisation depends on frame and on mass

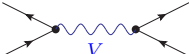
Different definitions of polarised cross sections in the literature:

- Definition via projections on LO decay-angle distributions  
 Baglio, Le Duc '18, '19
  - tailored to inclusive LO predictions
  - assumes small non-resonant background
  - only applicable for one polarised vector boson
  - results depend on cuts, background and NLO corrections
- Definition based on on-shell production and decay with spin correlations Franzosi et al. [Madgraph] '19
  - neglects non-resonant contributions
  - only available for LO

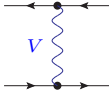
Idea: use **pole approximation** to extract resonant contributions in gauge-invariant way **Ballestrero, Maina, Pelliccioli '17, '19**

Formulation developed by **Denner, Pelliccioli '20**

- Not all diagrams involve required resonances  
resonant diagrams

$$\frac{R(k^2)}{k^2 - M^2 + iM\Gamma} =$$


non-resonant diagrams

$$N(k^2) =$$


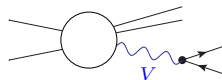
- split full matrix element into resonant part and non-resonant part using **pole expansion** (gauge-invariant)

$$\begin{aligned} \mathcal{A} &= \frac{R(k^2)}{k^2 - M^2 + iM\Gamma} + N(k^2) \\ &= \frac{R(M^2)}{k^2 - M^2 + iM\Gamma} + \frac{R(k^2) - R(M^2)}{k^2 - M^2} + N(k^2) = \mathcal{A}_{\text{res}} + \mathcal{A}_{\text{nonres}} \end{aligned}$$

- **consider non-resonant part as irreducible background**: no resonance

## Separate polarisation modes of resonant amplitude

split propagator numerator of resonant particle



$$\mathcal{A}_{\text{res}} = \mathcal{P}_\mu \frac{-g^{\mu\nu}}{k^2 - M_W^2 + i\Gamma_W M_W} \mathcal{D}_\nu = \mathcal{P}_\mu \frac{\sum_\lambda \varepsilon_\lambda^{\mu*}(k) \varepsilon_\lambda^\nu(k)}{k^2 - M_W^2 + i\Gamma_W M_W} \mathcal{D}_\nu$$

$$= \sum_{\lambda=L,\pm} \frac{\mathcal{M}_\lambda^{\text{prod}} \mathcal{M}_\lambda^{\text{dec}}}{k^2 - M_W^2 + i\Gamma_W M_W} =: \sum_{\lambda=L,\pm} \mathcal{A}_\lambda,$$

$$|\mathcal{A}_{\text{res}}|^2 = \sum_\lambda |\mathcal{A}_\lambda|^2 + \sum_{\lambda \neq \lambda'} \mathcal{A}_\lambda^* \mathcal{A}_{\lambda'}$$

- incoherent sum  $\sum_\lambda |\mathcal{A}_\lambda|^2$ :  $|\mathcal{A}_\lambda|^2 \propto$  “**polarised cross sections**”
- **interferences**  $\sum_{\lambda \neq \lambda'} \mathcal{A}_\lambda^* \mathcal{A}_{\lambda'}$   
 vanish for quantities fully inclusive in decay products but not in general

Polarisation vectors are defined in specific frames. Natural choices are the **(di-boson-)centre-of-mass frame** and the **laboratory frame**.

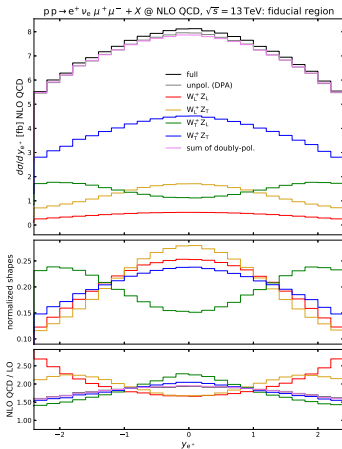
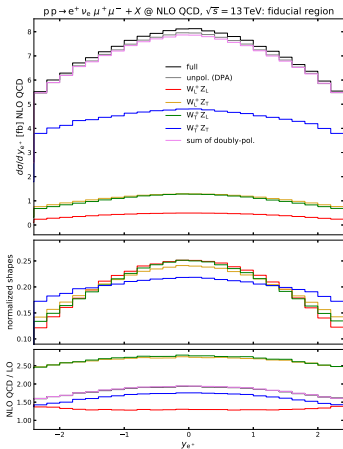
- Method is **applicable to arbitrary processes and multiple resonances** at LO, NLO and beyond.
- needs **pole approximation** (or double-pole approximation) **for** all NLO contributions including **subtraction terms!**
- results at NLO QCD exist for
  - $pp \rightarrow \mu^+ \nu_\mu e^+ \nu_e$  ( $W^+W^-$  production) **Denner, Pelliccioli '20** and
  - $pp \rightarrow \mu^+ \mu^- e^+ \nu_e$  ( $W^+Z$  production) **Denner, Pelliccioli '20**
- results at LO exist for VBS for  $ss$ - $WW$ ,  $WZ$ ,  $ZZ$ ,  $os$ - $WW$   
**Ballestrero, Maina, Pelliccioli '17, '19, '20 [PHANTOM]**
- generalisation in progress towards VBS at NLO QCD and NLO EW

### Method allows to separate

- polarised cross sections in arbitrary frames
- interference contributions between polarisations
- irreducible background.



$pp \rightarrow e^+ \nu_e \mu^+ \mu^- + X$ : Distributions in the positron rapidity in the fiducial region for polarisations defined in the CM (left) and in the LAB (right) frame.



Distributions for pol. cross sections defined in different frames differ considerably!

- 1 Introduction
- 2 Full NLO corrections to vector-boson pair plus jet-pair production
- 3 Electroweak corrections to vector-boson scattering (VBS)
- 4 Quality of VBS approximation
- 5 Polarised VBS
- 6 Conclusion**

## Status of NLO calculations for VBS

- NLO QCD corrections matched to PS available for all VBS processes  
NLO QCD corrections at level of few percent if  $p_{T,j}$  or  $M_{jj}$  not small
- VBS approximation might not be sufficient at NLO Ballestrero et al. '18  
NLO-QCD tri-boson contributions of  $\mathcal{O}(20\%)$  for loose VBS cuts
- electroweak corrections for VBS
  - full NLO EW corrections known for
    - $pp \rightarrow \mu^+ \nu_\mu e^+ \nu_e jj (W^+ W^+)$  Biedermann et al. '16, '17
    - $pp \rightarrow \mu^+ \mu^- e^+ \nu_e jj (WZ)$  Denner et al. '19
    - $pp \rightarrow \mu^+ \mu^- e^+ e^- jj (ZZ)$  Denner et al. '20
  - **-16% EW corrections for fiducial cross section**  
intrinsic feature of VBS, reproducible by simple approximations
  - EW corrections in distributions even larger  
-40% for  $p_{T,j_1} = 800 \text{ GeV}$
- NLO EW corr. for  $W^+ W^+$  scattering matched to QED PS Denner et al. '19
- full NLO corrections for  $W^+ W^+$  scattering Denner et al. '17  
only measurement of full process is well-defined!

Significant theoretical progress in VBS in recent years!

## Expected progress in theoretical predictions to VBS

- NLO EW corrections for  $pp \rightarrow \mu^+ \nu_\mu \bar{\nu}_e e^- jj$  ( $W^+ W^-$ ) (in progress)
- predictions for VBS with **semileptonic final states** (needed?)
- NLO corrections for **polarised VBS** within reach
- matching to **EW parton showers** (long term project)
- predictions for VBS within **extended models** feasible once LO and NLO matrix elements available
- predictions for VBS within **SMEFT** including (approximative) NLO corrections  $\Rightarrow$  need to extend/combine tools

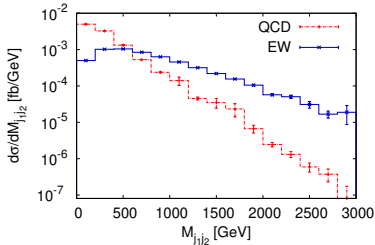
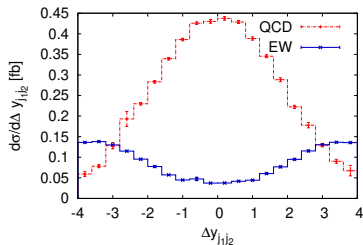
Input on priorities would be useful!



## 7 Backup

Jäger, Zanderighi '11

$\sqrt{s} = 7$  TeV, NLO QCD, basic cuts:  $p_{T,j} > 20$  GeV



EW production:



- large rapidity separation  $\Delta y_{jj}$
- dominant for large  $M_{jj}$
- $\sigma_{EW}^{\text{inclusive}} = 1.10$  fb
- $\sigma_{EW}^{\text{VBFcuts}} = 0.201$  fb

QCD production:



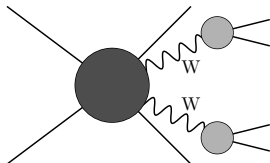
- small rapidity separation  $\Delta y_{jj}$
- prefers small  $M_{jj}$
- $\sigma_{QCD}^{\text{inclusive}} = 2.12$  fb 192%
- $\sigma_{QCD}^{\text{VBFcuts}} = 0.0074$  fb 3.7%

VBF cuts:  $M_{jj} > 600$  GeV,  $|\Delta y_{jj}| > 4$ ,  $y_{j1} \times y_{j2} < 0$

Leading order:

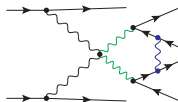
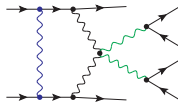
$$\mathcal{M}_{\text{LO,DPA}}^{qq \rightarrow WW qq \rightarrow 4f qq} = \sum_{\lambda_{W_1}, \lambda_{W_2}} \frac{[\mathcal{M}_{\text{LO}}^{qq \rightarrow WW qq}(\lambda_{W_1}, \lambda_{W_2}) \mathcal{M}_{\text{LO}}^{W \rightarrow 2f}(\lambda_{W_1}) \mathcal{M}_{\text{LO}}^{W \rightarrow 2f}(\lambda_{W_2})]_{\text{on-shell}}}{(p_{W_1}^2 - M_W^2 + iM_W \Gamma_W)(p_{W_2}^2 - M_W^2 + iM_W \Gamma_W)}$$

- only contributions with two resonant W bosons  $\Rightarrow$  dominant contribution
- momenta in numerator projected on shell  $\Rightarrow$  gauge invariance



## NLO:

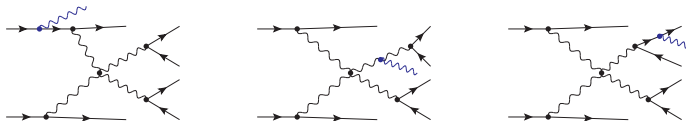
- **factorizable corrections:**  
 corrections to production  
 or decay matrix elements
- **non-factorizable corrections:**  
 IR-singular corrections connecting  
 production and decay  
 $\Rightarrow$  universal correction factors  
 Denner et al. '00; Accomando et al. '04;  
 Dittmaier, Schwan '15



## Implementation

- DPA applied only to squared matrix element for (subtracted) virtual corrections
- leading order and real corrections treated exactly
- phase-space integration treated exactly
- naive error estimate:  $\mathcal{O}(\Gamma_W/M_W) \times \delta_{EW} \sim \mathcal{O}(0.2\%)$
- DPA worse, where non-doubly-resonant contributions sizeable



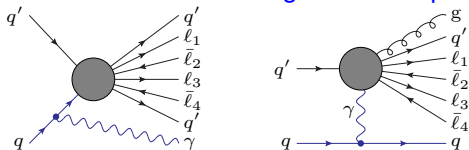


## soft and collinear singularities

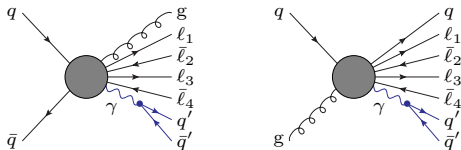
- **Catani–Seymour dipole subtraction**    Catani, Seymour '96; Dittmaier '99
- **recombination of collinear parton–photon, lepton–photon, and parton–parton pairs** (jet clustering)  
 ⇒ cancellation of soft and final-state collinear singularities
- initial-state collinear singularities cancelled by  $\overline{\text{MS}}$  redefinition of PDFs
- final-state photon splitting into quark pairs at  $\mathcal{O}(\alpha_s \alpha^6)$  requires **photon-to-quark conversion function**    Denner, Dittmaier, Pellen, Schwan '19
- photon–jet separation at  $\mathcal{O}(\alpha_s^2 \alpha^5)$  requires **quark-to-photon fragmentation function**    Glover, Morgan '93

## Phase-space integration with multi-channel Monte Carlo codes

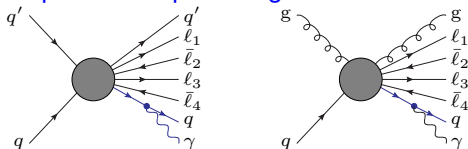
- initial-state collinear singularities  $\Rightarrow$  parton distribution functions



- final-state  $\gamma \rightarrow q\bar{q}$  splitting  $\Rightarrow$  photon-to-quark conversion function



- final-state collinear singularities, photon-jet separation  $\Rightarrow$  photon-to-quark fragmentation function



Energy: 13 TeV

## PDFs

NNPDF3.0QED [Ball et al. '13, '14](#)

factorization and renormalization scales:  $\mu_F = \mu_R = \sqrt{p_{T,j1} p_{T,j2}}$

## Recombination / jet clustering

Anti- $k_T$  algorithm with  $R = 0.4$  [Cacciari, Salam, Soyez '08](#)

recombination of photons with charged partons with  $R = 0.1$

Cuts: based on [ATLAS 1405.6241](#), [1611.02428](#) and [CMS 1410.6315](#)

$$p_{T,j} > 30 \text{ GeV}, \quad |y_j| < 4.5, \quad \Delta R_{j\ell} > 0.3$$

$$p_{T,\ell} > 20 \text{ GeV}, \quad |y_\ell| < 2.5, \quad \Delta R_{\ell\ell} > 0.3, \quad \Delta R_{ij} = \sqrt{(\Delta y_{ij})^2 + (\Delta \phi_{ij})^2}$$

$$E_{T,\text{miss}} > 40 \text{ GeV}$$

$$M_{jj} > 500 \text{ GeV}, \quad |\Delta y_{jj}| > 2.5 \quad (\text{VBF cuts})$$

require  $\geq 2$  jets, 2 same-sign leptons and missing energy

Energy: 13 TeV (14 TeV)

## PDFs

NNPDF3.1QED [Ball et al. '14](#), [Bertone et al. '17](#)

factorization and renormalization scales:  $\mu_F = \mu_R = \sqrt{p_{T,j_1} p_{T,j_2}}$

## Recombination / jet clustering

Anti- $k_T$  algorithm with  $R = 0.4$  [Cacciari, Salam, Soyez '08](#)

recombination of photons with charged partons with  $R = 0.4$

Cuts: loose fiducial region of [CMS 1901.04060](#)

$$p_{T,j} > 30 \text{ GeV}, \quad |y_j| < 4.7, \quad \Delta R_{j\ell} > 0.4 \quad \Delta R_{ij} = \sqrt{(\Delta y_{ij})^2 + (\Delta \phi_{ij})^2}$$

$$p_{T,\ell} > 20 \text{ GeV}, \quad |y_\ell| < 2.5, \quad M_{3\ell} > 100 \text{ GeV}, \quad M_{\ell\ell} > 4 \text{ GeV}$$

$$|M_{\mu^+\mu^-} - M_Z| < 15 \text{ GeV}$$

$$M_{jj} > 500 \text{ GeV}, \quad |\Delta y_{jj}| > 2.5 \quad (\text{VBF cuts})$$

require  $\geq 2$  jets, 3 leptons

Energy: 13 TeV

## PDFs

NNPDF3.1QED [Ball et al. '14](#), [Bertone et al. '17](#)

factorization and renormalization scales:  $\mu_F = \mu_R = \sqrt{p_{T,j_1} p_{T,j_2}}$

## Recombination / jet clustering

Anti- $k_T$  algorithm with  $R = 0.4$  [Cacciari, Salam, Soyez '08](#)

recombination of photons with charged partons with  $R = 0.4$

Cuts: inspired by [CMS 1708.02812](#)

$$p_{T,j} > 30 \text{ GeV}, \quad |y_j| < 4.7, \quad \Delta R_{j\ell} > 0.4, \quad \Delta R_{ij} = \sqrt{(\Delta y_{ij})^2 + (\Delta \phi_{ij})^2}$$

$$p_{T,\ell} > 20 \text{ GeV}, \quad |y_\ell| < 2.5, \quad \Delta R_{\ell\ell'} > 0.05, \quad M_{\ell+\ell'} > 4 \text{ GeV}$$

$$60 \text{ GeV} < M_{\ell+\ell'} < 120 \text{ GeV}$$

inclusive setup:  $M_{jj} > 100 \text{ GeV}$ , **VBS setup**  $M_{jj} > 500 \text{ GeV}$

require  $\geq 2$  jets, 4 leptons

Process:  $pp \rightarrow \mu^+ \mu^- e^+ \nu_e jj$       $\sqrt{s} = 13 \text{ TeV}$ 

Denner et al. '19

Order	$\mathcal{O}(\alpha^6)$	$\mathcal{O}(\alpha_s \alpha^5)$	$\mathcal{O}(\alpha_s^2 \alpha^4)$	Sum
$\sigma_{\text{LO}} [\text{fb}]$	$0.2551^{+9.0\%}_{-7.8\%}$	$0.0068^{+18\%}_{-14\%}$	$1.097^{+37\%}_{-25\%}$	1.359
$\Delta[\%]$	18.8	0.5	80.7	100

Contribution	$\gamma$ -induced	bottom
$\Delta\sigma_{\text{LO}} [\text{fb}]$	$0.00099^{+11.0\%}_{-9\%}$	$0.195^{+3.6\%}_{-7.2\%}$
$\Delta\sigma_{\text{LO}} / \sigma_{\text{LO}}^{\mathcal{O}(\alpha^6)} [\%]$	0.4	76.2

- very large **QCD** contribution mainly due to gluon PDF
- **EW** contributions smaller than for  $W^+W^+$  (Z boson)
- small **interference** (colour and kinematic suppression)
- **photon-induced** ( $\gamma\gamma$ ) contribution completely irrelevant
- **bottom** contribution important, dominated by  $tZ + j$  production  
 $\Rightarrow$  different process, eliminate via b tagging

Process:  $pp \rightarrow \mu^+ \mu^- e^+ \nu_e jj$ ,  $\sqrt{s} = 13 \text{ TeV}$

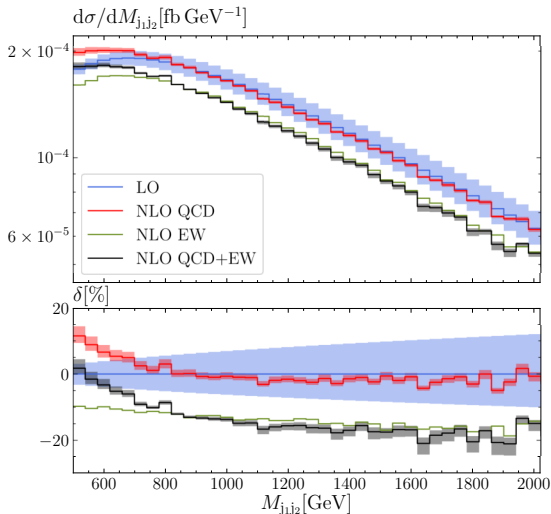
Denner et al. '19

Order	$\mathcal{O}(\alpha^6)$ LO	$\mathcal{O}(\alpha^7)$ NLO EW	$\mathcal{O}(\alpha_s \alpha^6)$ NLO QCD	$\mathcal{O}(\alpha^7) + \mathcal{O}(\alpha_s \alpha^6)$ NLO EW+QCD
$\sigma [\text{fb}]$	$0.2551^{+9.0\%}_{-7.8\%}$	$0.2142^{+8.5\%}_{-7.4\%}$	$0.2506^{+1.0\%}_{-1.0\%}$	$0.2097^{+1.3\%}_{-2.2\%}$
$\delta [\%]$	100%	-16.0%	-1.8%	-17.8%

- large EW corrections similar to  $W^+W^+$  scattering
- rather small QCD corrections
- corrections are larger in distributions
- bottom-quark contributions omitted (b-tagging)
- photon-induced contributions at NLO omitted (small)

Process:  $pp \rightarrow \mu^+ \mu^- e^+ \nu_e jj$        $\sqrt{s} = 13 \text{ TeV}$

Denner et al. '19



- $\mathcal{O}(\alpha^7) \sim -18\%$  at 2 TeV (Sudakov logarithms) dominant correction
- $\mathcal{O}(\alpha_s \alpha^6) \lesssim 10\%$  for  $M_{j_1 j_2} > 600 \text{ GeV}$  small QCD scale uncertainty owing to dynamical scale  $\mu = \sqrt{p_{T,j_1} p_{T,j_2}}$
- small  $M_{j_1 j_2} \Rightarrow$  small  $p_{T,j_1} \Rightarrow$  large positive QCD corrections accidental cancellation of QCD and EW corrections for small  $M_{j_1 j_2}$



Process:  $pp \rightarrow \mu^+ \mu^- e^+ e^- jj$       $\sqrt{s} = 13 \text{ TeV}$ 

Denner et al. '20

Order	$\mathcal{O}(\alpha^6)$	$\mathcal{O}(\alpha_s \alpha^5)$	$\mathcal{O}(\alpha_s^2 \alpha^4)$	$\mathcal{O}(\alpha_s^4 \alpha^4)$	Sum
$M_{j_1 j_2} > 100 \text{ GeV}$					
$\sigma_{\text{LO}}[\text{fb}]$	$0.097683^{+6.8\%}_{-6.0\%}$	0.008628	1.062478	0.12101	1.28980
fraction[%]	7.57	0.67	82.38	9.38	100
$M_{j_1 j_2} > 500 \text{ GeV}$					
$\sigma_{\text{LO}}[\text{fb}]$	$0.073676^{+8.6\%}_{-7.5\%}$	0.005567	0.136143	0.01345	0.22883
fraction[%]	32.20	2.43	59.49	5.88	100

- very large **QCD** contribution reduced by  $M_{j_1 j_2}$  cut
- **EW** contributions even smaller than for  $W^+Z$  (Z boson)
- small **interference** (colour and kinematic suppression)
- **loop-induced gg** contribution reduced by  $M_{j_1 j_2}$  cut
- photon-induced and bottom contributions ( $< 3\%$ ) omitted

Process:  $pp \rightarrow \mu^+ \mu^- e^+ e^- jj$        $\sqrt{s} = 13 \text{ TeV}$ 

Denner et al. '20

Order	$\mathcal{O}(\alpha^6)$	$\mathcal{O}(\alpha^6 + \alpha^7)$	$\mathcal{O}(\alpha^6 + \alpha_s \alpha^6)$	$\mathcal{O}(\alpha^6 + \alpha^7 + \alpha_s \alpha^6)$
$M_{j_1 j_2} > 100 \text{ GeV}$				
$\sigma [\text{fb}]$	$0.09768^{+6.8\%}_{-6.0\%}$	$0.08211^{+6.3\%}_{-5.6\%}$	$0.12078^{+3.8\%}_{-3.5\%}$	$0.10521^{+3.0\%}_{-2.8\%}$
$\delta [\%]$	100	-15.9	+23.6	+7.7
$M_{j_1 j_2} > 500 \text{ GeV}$				
$\sigma [\text{fb}]$	$0.07368^{+8.6\%}_{-7.5\%}$	$0.06069^{+8.2\%}_{-7.1\%}$	$0.07375^{+1.2\%}_{-1.3\%}$	$0.06077^{+1.2\%}_{-1.6\%}$
$\delta [\%]$	100	-17.6	0.1	-17.5

- large EW corrections similar to  $W^+ W^+$  scattering
- QCD corrections large for  $M_{j_1 j_2} > 100 \text{ GeV}$ , small for  $M_{j_1 j_2} > 500 \text{ GeV}$
- corrections are larger in distributions
- bottom-quark contributions omitted (b-tagging)
- photon-induced contributions at NLO omitted (small)