

Event Generation at Future Colliders: EW Corrections in Parton Showers

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Monte Carlo Challenges

Computational Challenges

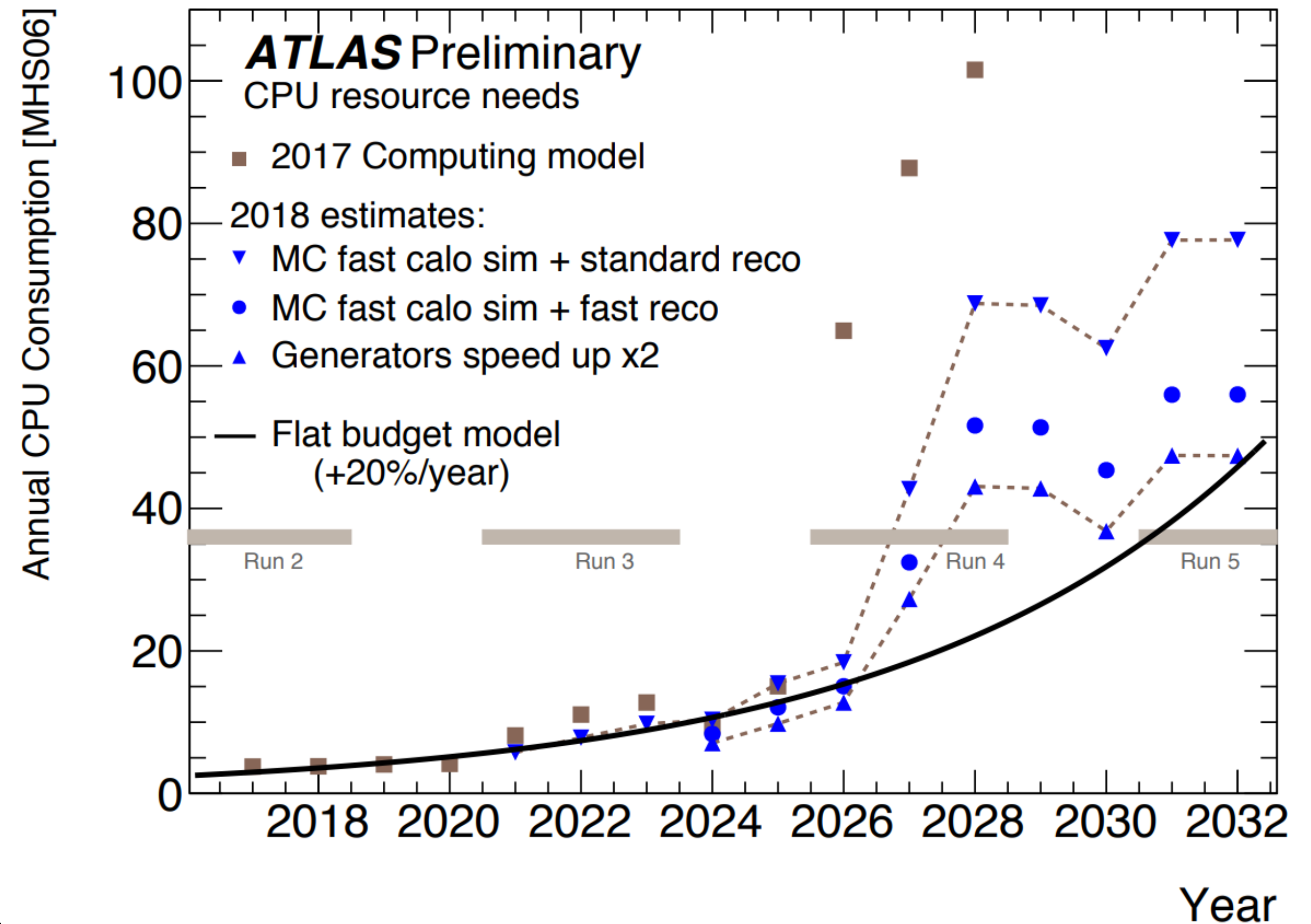
State of the art is now NLO/LO multileg merging

→ Event generation has become expensive

Improvements are required in multiple areas

Physics Challenges

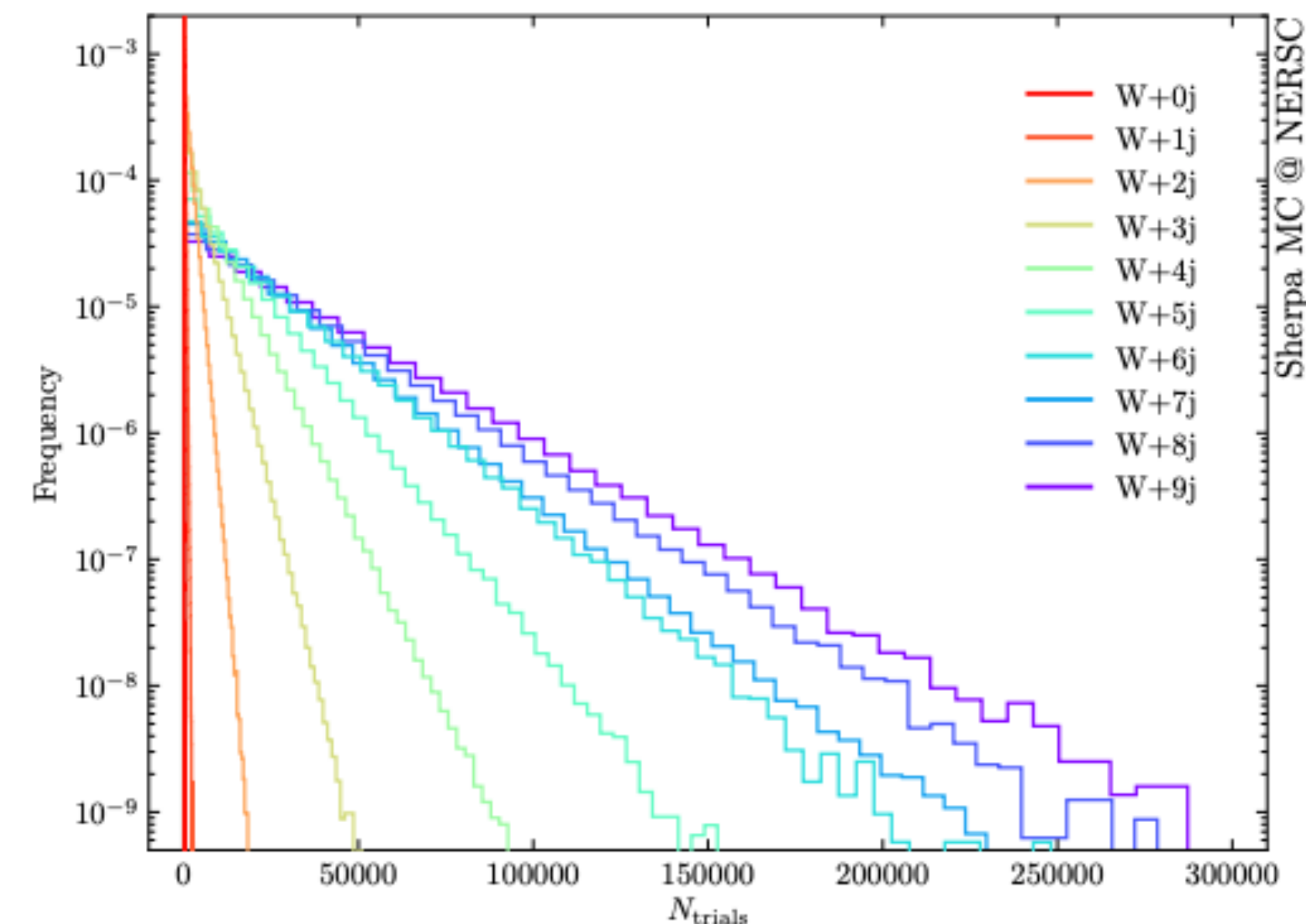
- NNLO fixed order increasingly available
 - Matching algorithms exist, but not part of standard codes yet
 - Work needed before computationally feasible
- Accuracy + subleading effects in parton showers
- Improvements & better understanding of nonperturbative effects



Matrix Element Sampling

Often still reliant on VEGAS + multi channeling

→ Many case-specific algorithms exist (FOAM, HAAG)



Gao, Isaacson, Krause 2001.05486

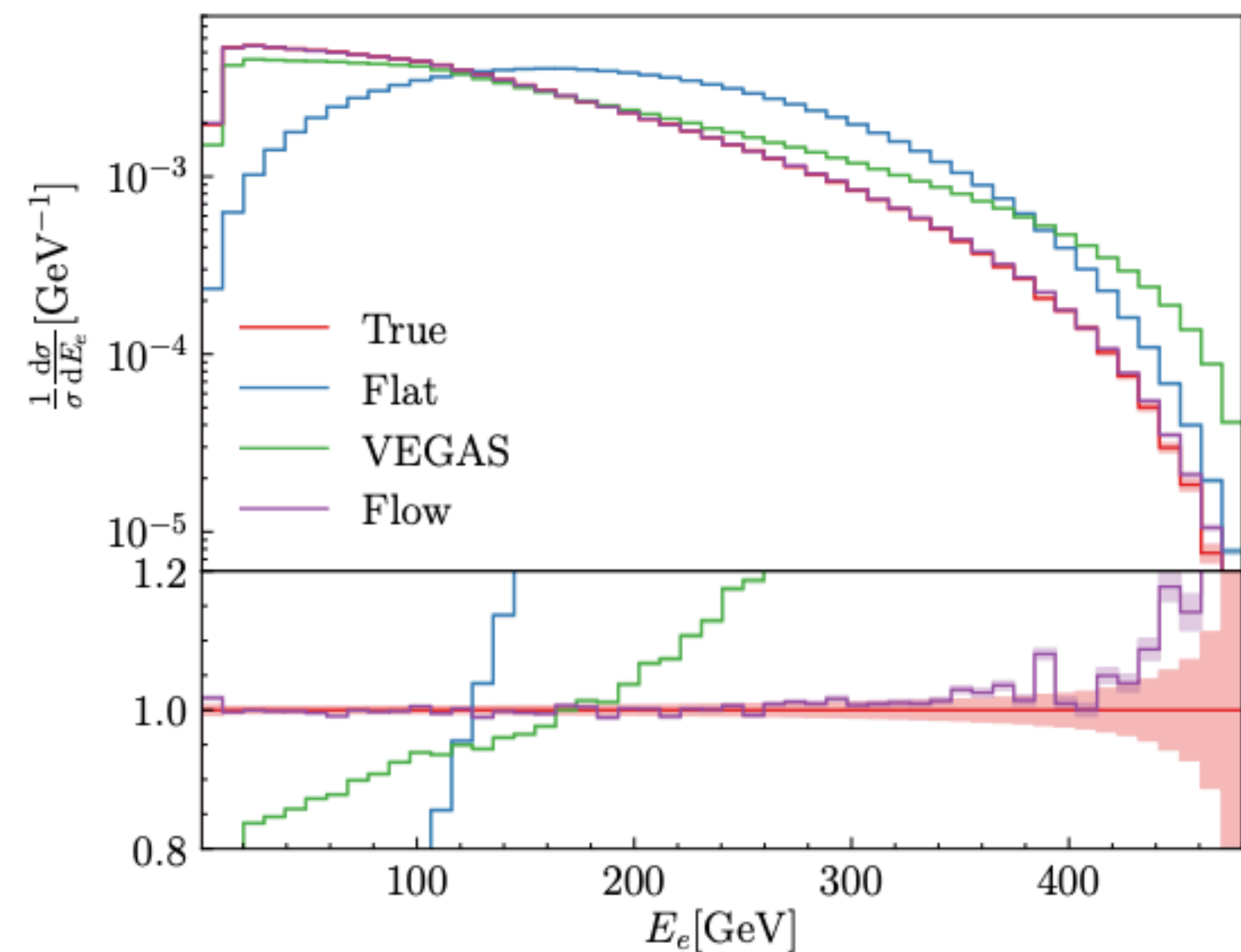
Gao, Hoche, Isaacson, Krause, Schulz 2001.10028

Bothmann, Janssen, Knobbe, Schmale, Schumann 2001.05478

Stienen, RV 2011:13445

Recent developments from generative machine learning models

unweighting efficiency $\langle w \rangle / w_{\max}$		LO QCD					NLO QCD (RS)	
		$n=0$	$n=1$	$n=2$	$n=3$	$n=4$	$n=0$	$n=1$
$W^+ + n$ jets	Sherpa	$2.8 \cdot 10^{-1}$	$3.8 \cdot 10^{-2}$	$7.5 \cdot 10^{-3}$	$1.5 \cdot 10^{-3}$	$8.3 \cdot 10^{-4}$	$9.5 \cdot 10^{-2}$	$4.5 \cdot 10^{-3}$
	NN+NF	$6.1 \cdot 10^{-1}$	$1.2 \cdot 10^{-1}$	$1.0 \cdot 10^{-2}$	$1.8 \cdot 10^{-3}$	$8.9 \cdot 10^{-4}$	$1.6 \cdot 10^{-1}$	$4.1 \cdot 10^{-3}$
	Gain	2.2	3.3	1.4	1.2	1.1	1.6	0.91
$W^- + n$ jets	Sherpa	$2.9 \cdot 10^{-1}$	$4.0 \cdot 10^{-2}$	$7.7 \cdot 10^{-3}$	$2.0 \cdot 10^{-3}$	$9.7 \cdot 10^{-4}$	$1.0 \cdot 10^{-1}$	$4.5 \cdot 10^{-3}$
	NN+NF	$7.0 \cdot 10^{-1}$	$1.5 \cdot 10^{-1}$	$1.1 \cdot 10^{-2}$	$2.2 \cdot 10^{-3}$	$7.9 \cdot 10^{-4}$	$1.5 \cdot 10^{-1}$	$4.2 \cdot 10^{-3}$
	Gain	2.4	3.3	1.4	1.1	0.82	1.5	0.91
$Z + n$ jets	Sherpa	$3.1 \cdot 10^{-1}$	$3.6 \cdot 10^{-2}$	$1.5 \cdot 10^{-2}$	$4.7 \cdot 10^{-3}$		$1.2 \cdot 10^{-1}$	$5.3 \cdot 10^{-3}$
	NN+NF	$3.8 \cdot 10^{-1}$	$1.0 \cdot 10^{-1}$	$1.4 \cdot 10^{-2}$	$2.4 \cdot 10^{-3}$		$1.8 \cdot 10^{-3}$	$5.7 \cdot 10^{-3}$
	Gain	1.2	2.9	0.91	0.51		1.5	1.1



Negative Weights

NLO corrections often lead to negative weights

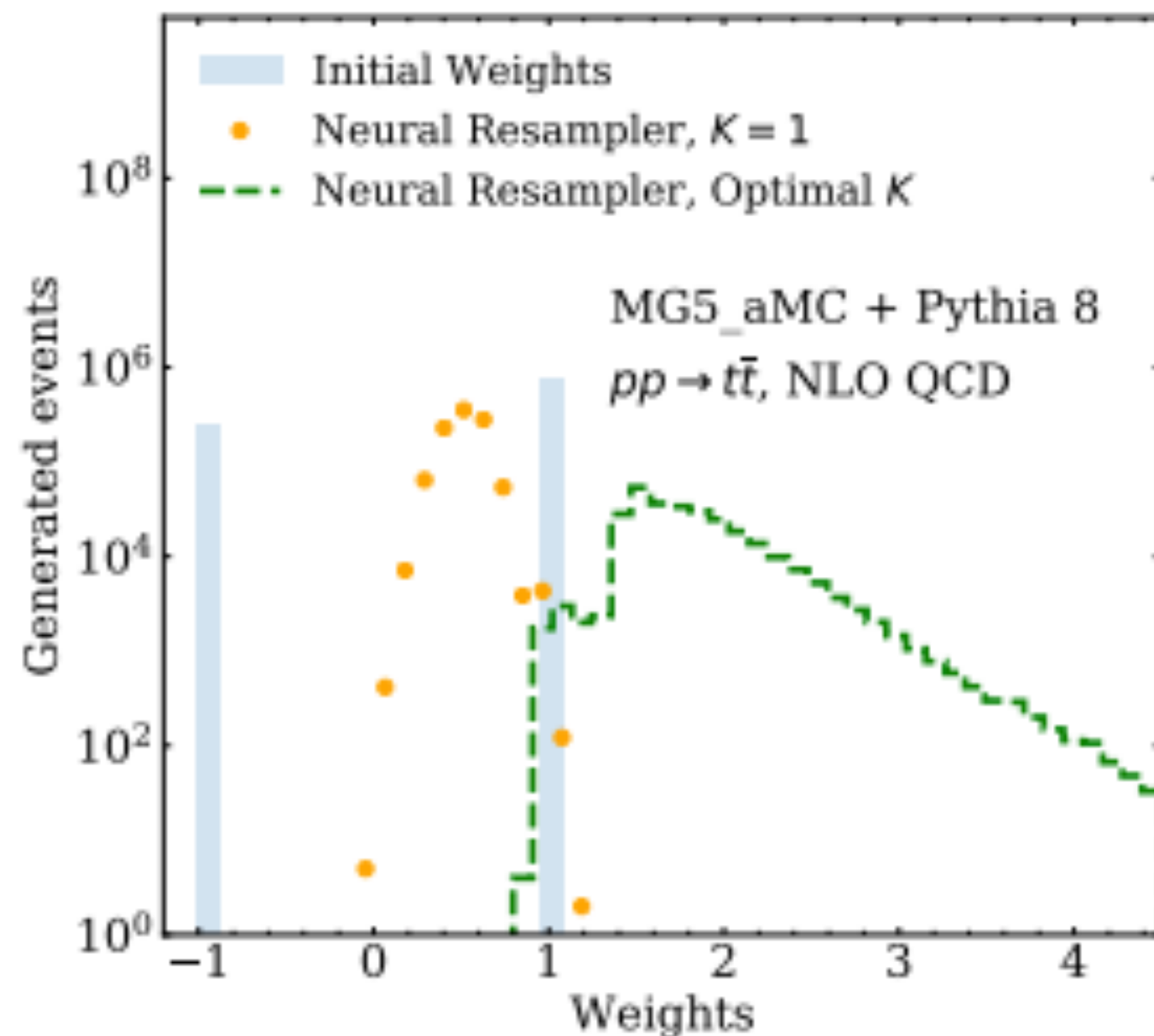
Requires a factor of $1/(1 - f_{\text{neg}})^2$ more events

Several improvements being explored

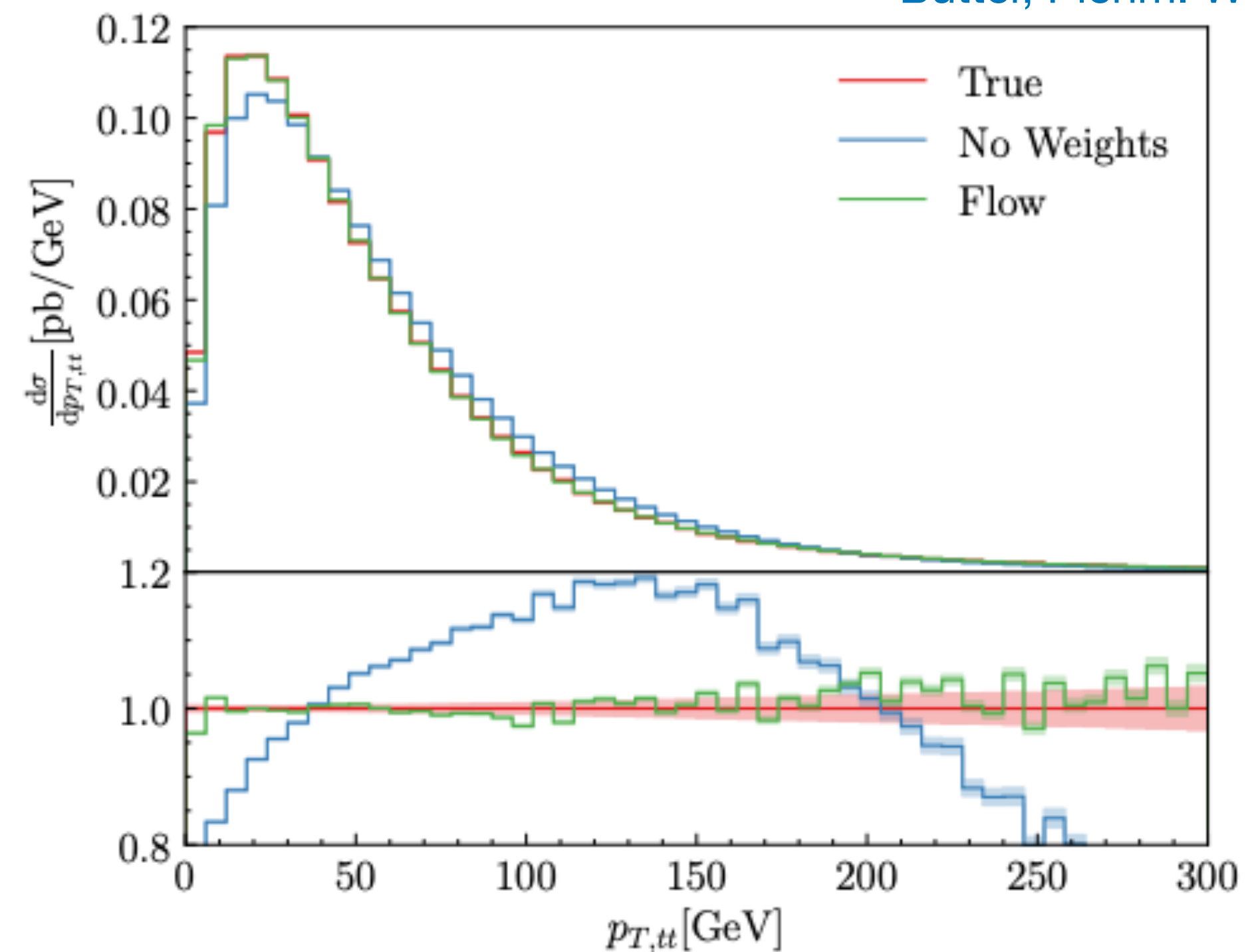
- Resampling [Andersen, Gutschow, Maier, Prestel 2005.09375](#)
[Nachman, Thaler 2007.11586](#)

- Improving MC@NLO [Frederix, Frixione, Prestel, Torrielli 2002:12716](#)

	MC@NLO			MC@NLO- Δ		
	111	221	441	Δ -111	Δ -221	Δ -441
$pp \rightarrow e^+e^-$	6.9% (1.3)	3.5% (1.2)	3.2% (1.1)	5.7% (1.3)	2.4% (1.1)	2.0% (1.1)
$pp \rightarrow e^+\nu_e$	7.2% (1.4)	3.8% (1.2)	3.4% (1.2)	5.9% (1.3)	2.5% (1.1)	2.3% (1.1)
$pp \rightarrow H$	10.4% (1.6)	4.9% (1.2)	3.4% (1.2)	7.5% (1.4)	2.0% (1.1)	0.5% (1.0)
$pp \rightarrow Hb\bar{b}$	40.3% (27)	38.4% (19)	38.0% (17)	36.6% (14)	32.6% (8.2)	31.3% (7.2)
$pp \rightarrow W^+j$	21.7% (3.1)	16.5% (2.2)	15.7% (2.1)	14.2% (2.0)	7.9% (1.4)	7.4% (1.4)
$pp \rightarrow W^+t\bar{t}$	16.2% (2.2)	15.2% (2.1)	15.1% (2.1)	13.2% (1.8)	11.9% (1.7)	11.5% (1.7)
$pp \rightarrow t\bar{t}$	23.0% (3.4)	20.2% (2.8)	19.6% (2.7)	13.6% (1.9)	9.3% (1.5)	7.7% (1.4)



- Generative Models [Stienen, RV 2011:13445](#)
[Butter, Plehm. Winterhalder 1912.08824](#)



One-Slide Parton Shower Summary

Process-independent, fully differential resummation framework $p_{\perp} \approx Q_{\text{fac}}$

Incorporates logarithms associated with soft and collinear branchings

Repeatedly sample emissions from:

Branching kernel (real corrections)



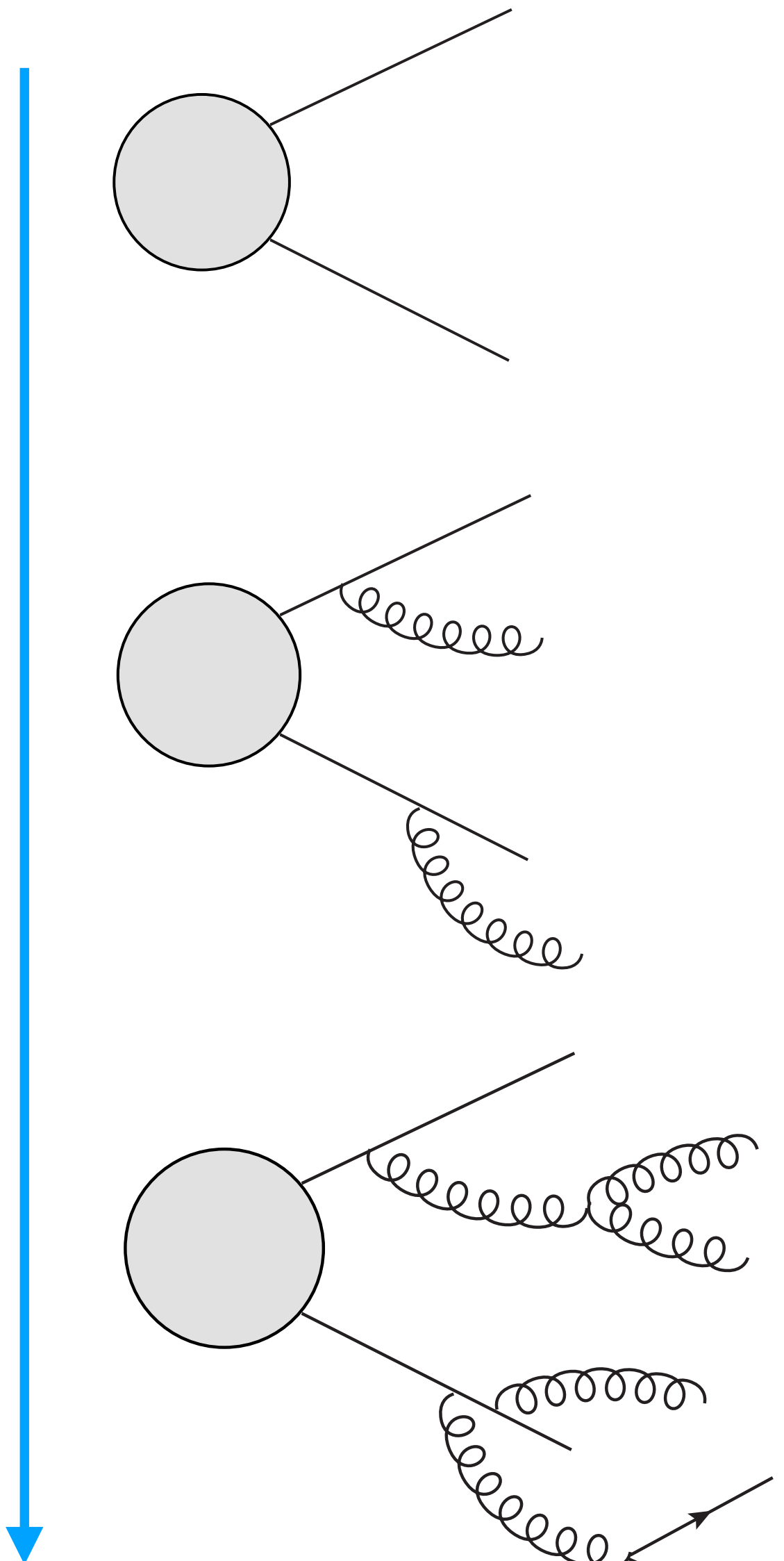
$$P_i(p_{\perp,i}^2) = B(p_{\perp,i}^2) \times \Theta(p_{\perp,i-1}^2 - p_{\perp,i}^2) \times \Delta(p_{\perp,i-1}, p_{\perp,i})$$



Sudakov factor (virtual corrections)

$$\Delta(Q_{\text{fac}}, \Lambda_{\text{QCD}}) \propto \exp\left(-\alpha_s \log^2 \frac{Q_{\text{fac}}}{\Lambda_{\text{QCD}}} + \dots\right)$$

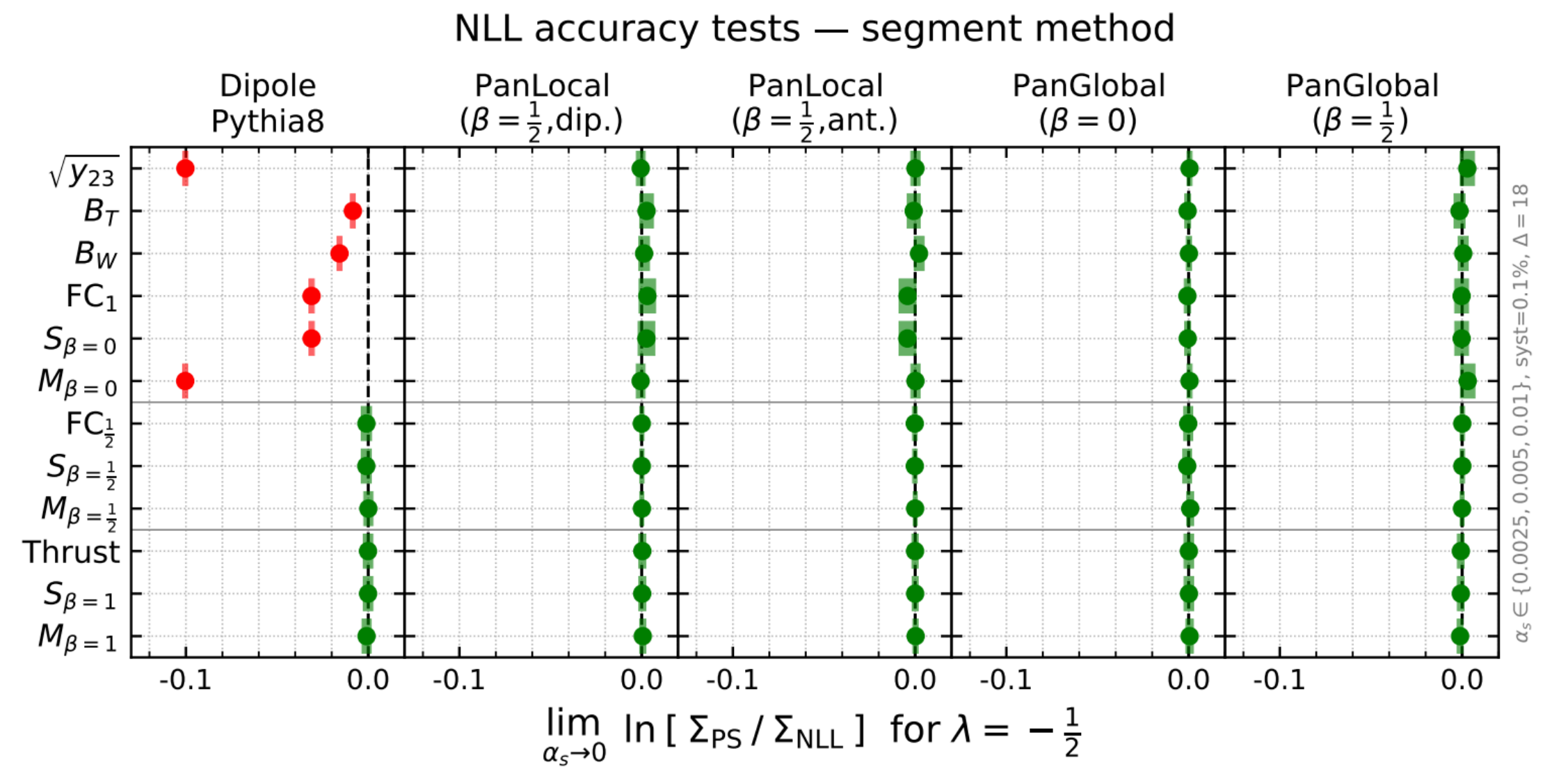
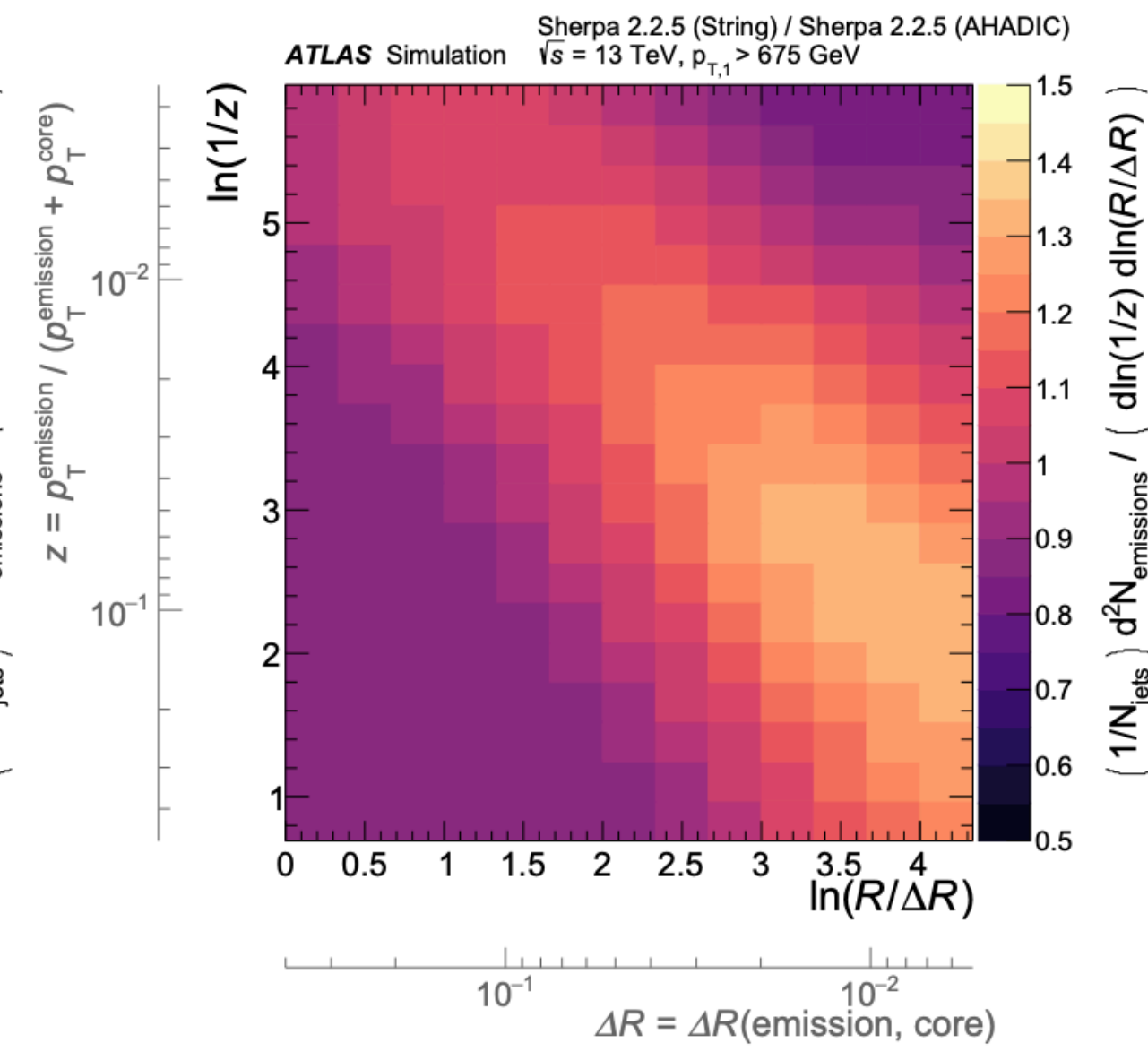
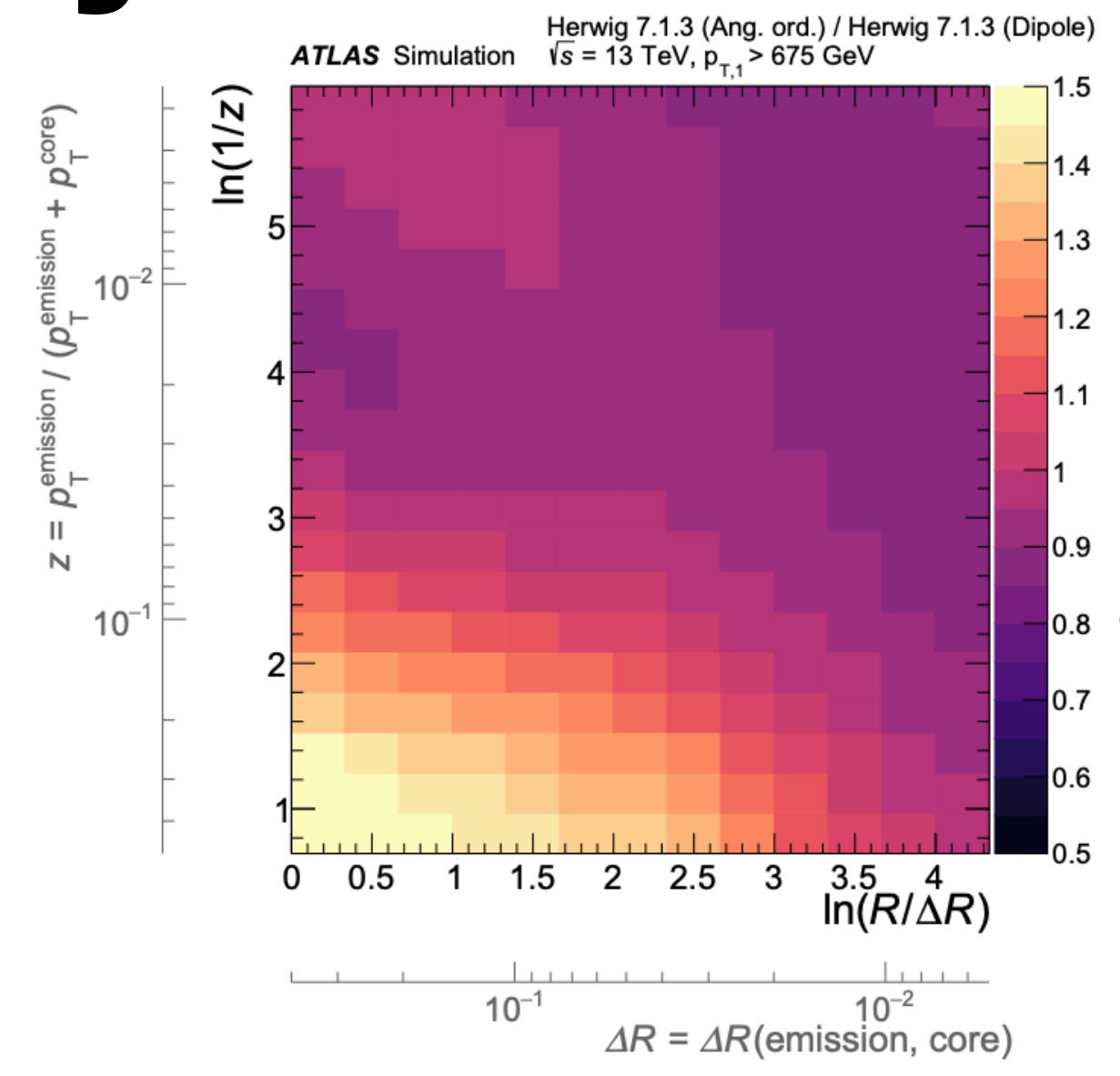
$p_{\perp} \approx \Lambda_{\text{QCD}}$



Parton Shower Accuracy ATLAS 2004.03540

Currently large differences between models
Recent significant progress:

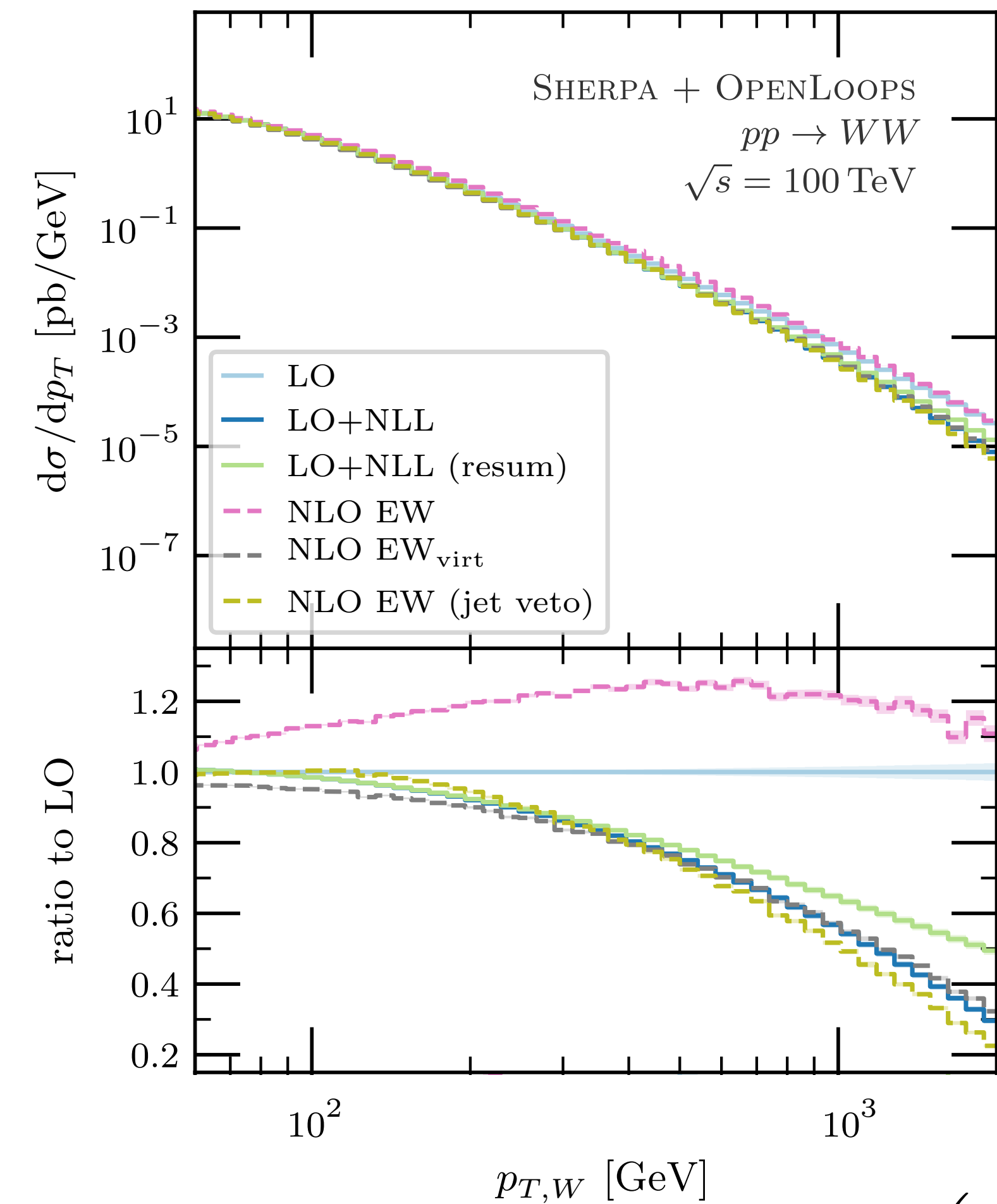
- Formal NLL accuracy
 - Dasgupta, Dreyer, Hamilton, Monni, Salam, Soyez 2002.11114
 - Nagy, Soper 2011.04773
 - Forshaw, Holguin, Platzer 2003.06400
- Inclusion of higher-order branching kernels
 - Requirement for NNLL
 - Hoche, Krauss, Prestel 1705.00982
 - Li, Skands 1611.00013
- Subleading colour effects $1/N_c^2 \sim 10\%$
 - Hamilton, Medves, Salam, Scyboz, Soyez 2011.10054
 - Nagy, Soper 1501.00778
 - Platzer, Sjodahl, Thoren 1808.00332
 - Forshaw, Holguin, Platzer 1905.08686
 - Isaacson, Prestel 1806.10102
- Electroweak corrections $\alpha/\alpha_s \sim 10\%$
 - Christiansen, Sjostrand arXiv:1401.5238
 - Krauss, Petrov, Schoenherr, Spannowsky arXiv:1403.4788
 - Chen, Han, Tweedie arXiv:1611.00788
 - Kleiss, RV 2002.09248



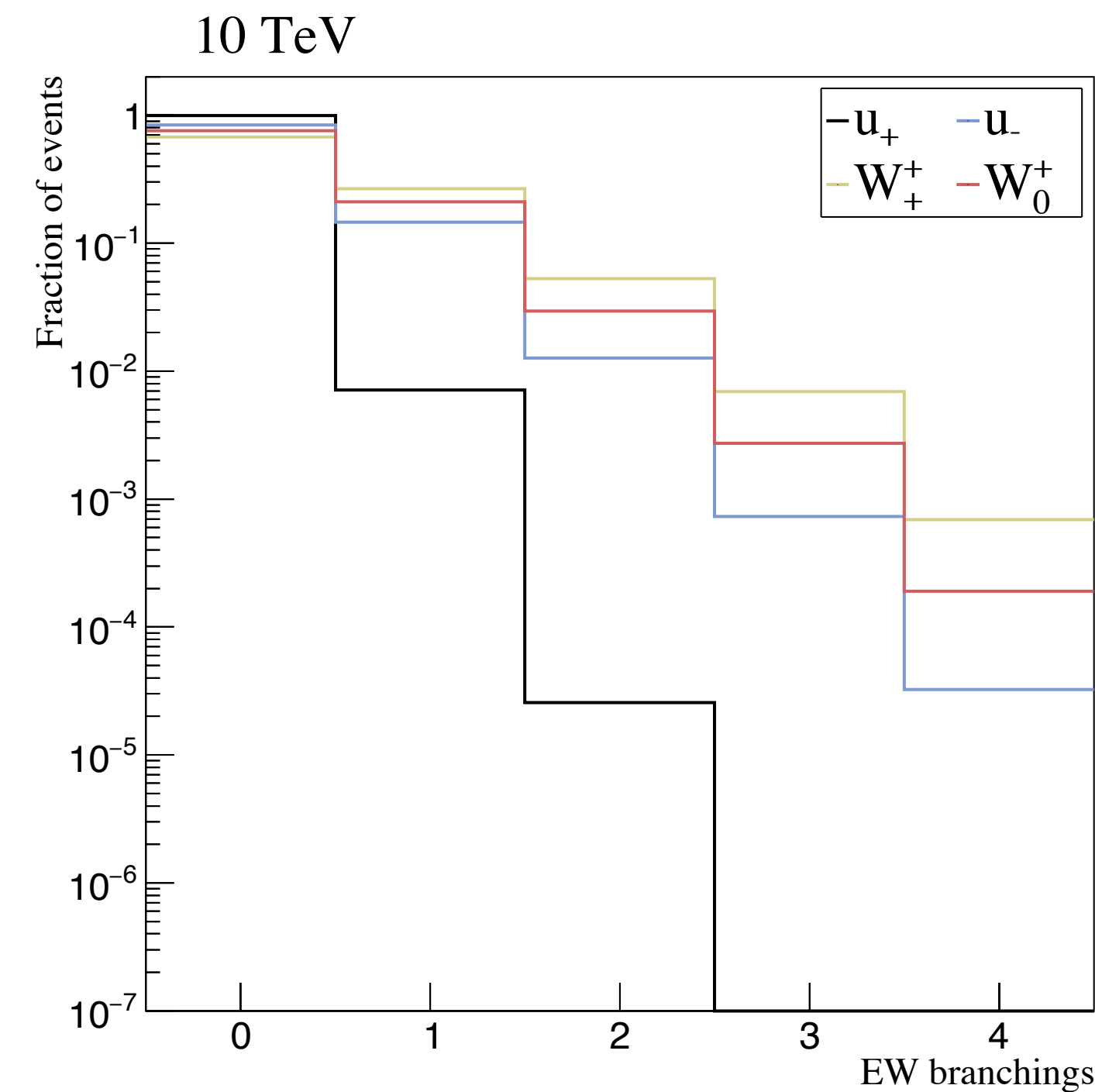
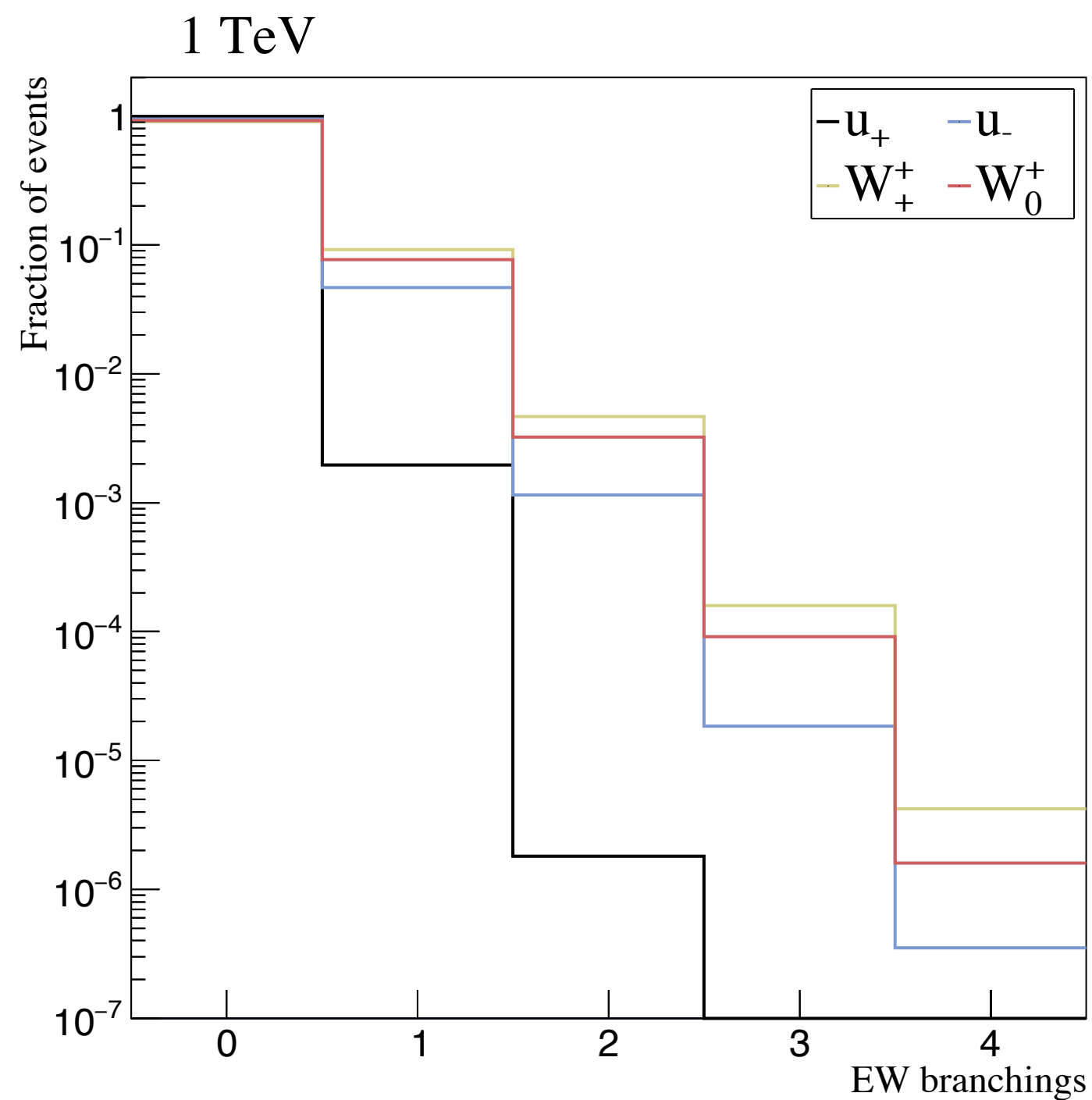
Electroweak Showers in Vincia



Why EW Showers?



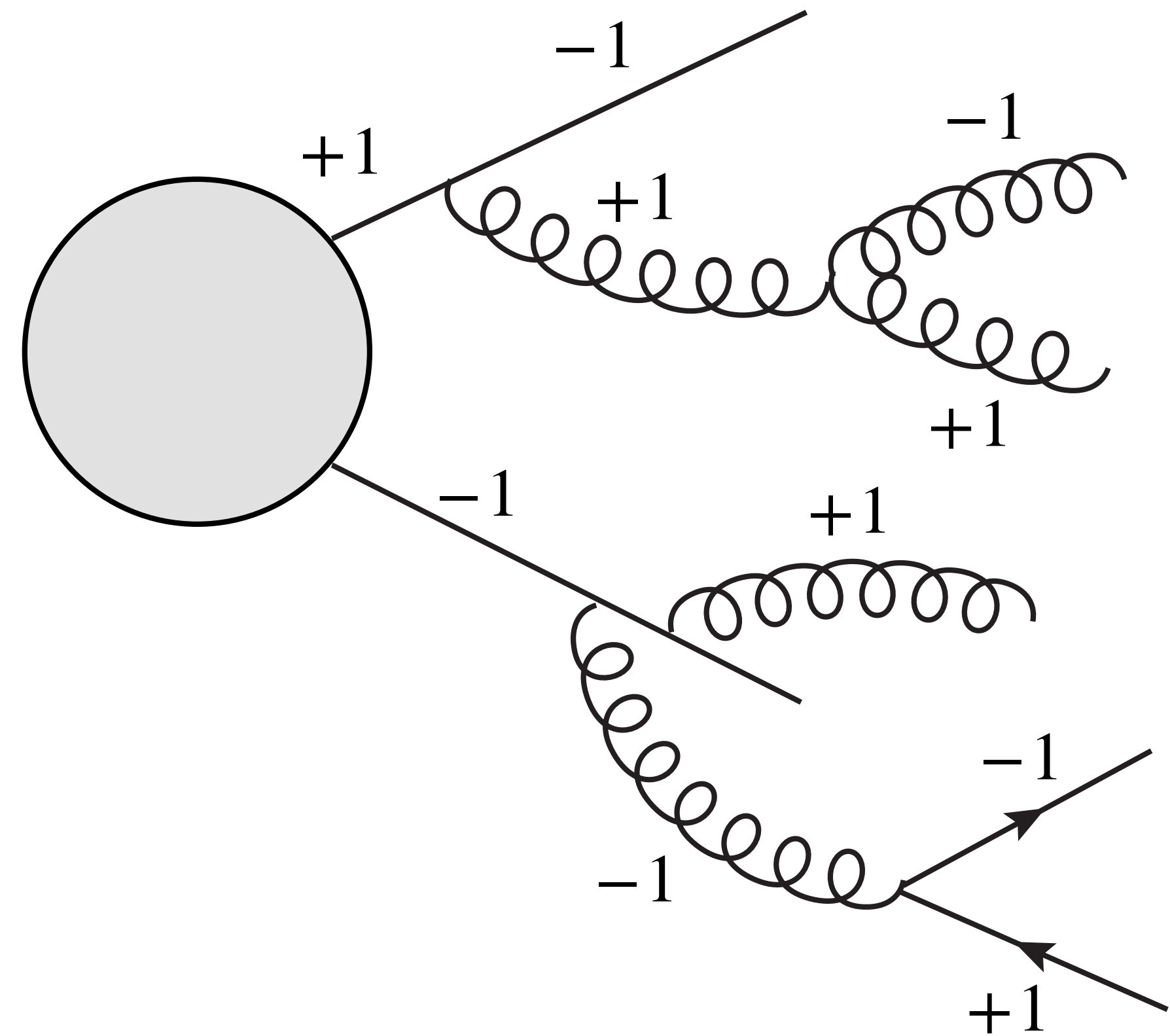
Universal incorporation $\propto \log \left(\frac{\hat{s}}{Q_{EW}^2} \right)$
of EW virtual corrections



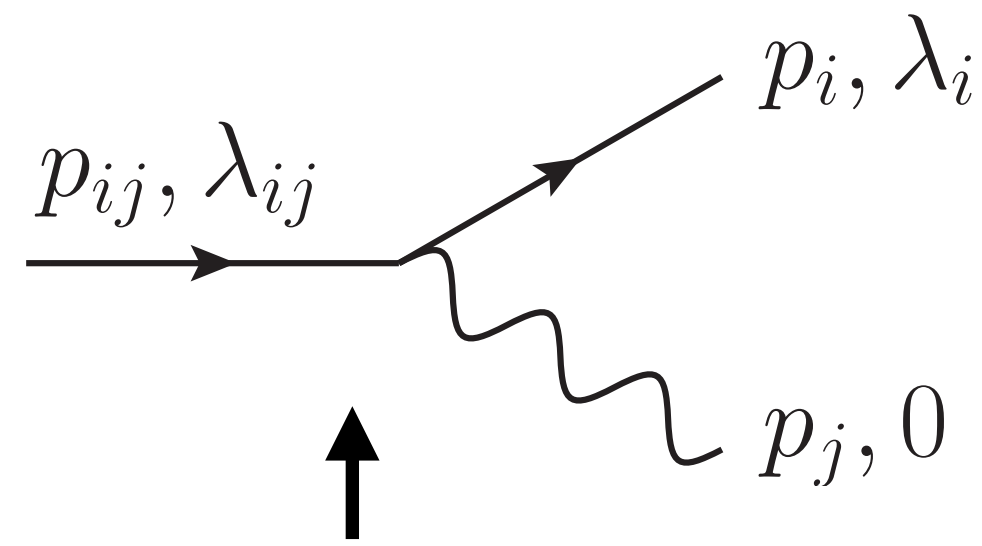
EW gauge bosons, top,
Higgs part of jets

Unique Features of the EW Sector

- Chiral nature of the EW theory \rightarrow helicity showers
- Consequences of EW symmetry breaking
- Shower vs. Resonance decays
- Neutral boson interference
- Double-counting Borns
- Bloch-Nordsieck violations



EW Symmetry Breaking & Goldstone Bosons

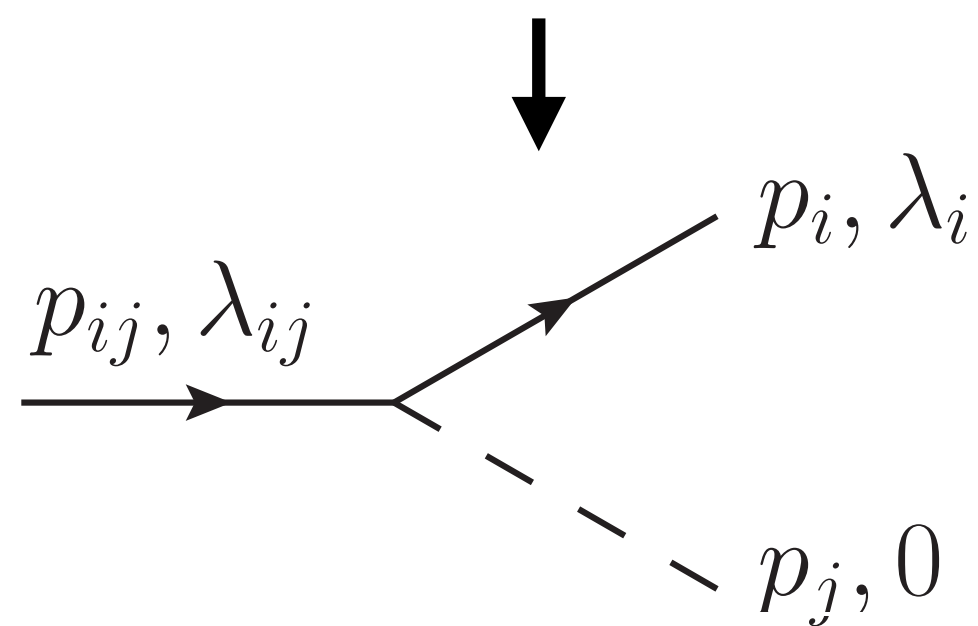


Naïve calculation of branching kernels

→ Unitarity violation

$$\epsilon_0^\mu(p) = \frac{1}{m} \left(p^\mu - \frac{m^2}{p \cdot k} k^\mu \right)$$

Consequence of gauge-dependence



Goldstone piece actually couples to Yukawa

Possible to solve with Goldstone equivalence and suitable gauge choice

Branching Kernels

λ_I	λ_i	λ_j	$V \rightarrow f\bar{f}'$
λ	λ	$-\lambda$	$\sqrt{2}\lambda(v - \lambda a)\sqrt{\tilde{Q}^2}z$
λ	$-\lambda$	λ	$\sqrt{2}\lambda(v + \lambda a)\sqrt{\tilde{Q}^2}(1 - z)$
λ	λ	λ	$\sqrt{2}\lambda\left[m_i(v + \lambda a)\sqrt{\frac{1-z}{z}} + m_j(v - \lambda a)\sqrt{\frac{z}{1-z}}\right]$
λ	$-\lambda$	$-\lambda$	0
0	λ	λ	$\sqrt{\tilde{Q}^2}\left[\frac{m_i}{m_{ij}}(v + \lambda a) + \frac{m_j}{m_{ij}}(v - \lambda a)\right]$
0	λ	$-\lambda$	$(v - \lambda a)\left[2m_{ij}\sqrt{z(1-z)} - \frac{m_i^2}{m_{ij}}\sqrt{\frac{1-z}{z}} - \frac{m_j^2}{m_{ij}}\sqrt{\frac{z}{1-z}}\right] + (v + \lambda a)\frac{m_i m_j}{m_{ij}}\frac{1}{\sqrt{z(1-z)}}$

λ_{ij}	λ_i	λ_j	$f \rightarrow f'V$ and $\bar{f} \rightarrow \bar{f}'V$
λ	λ	λ	$\sqrt{2}\lambda(v - \lambda a)\sqrt{\tilde{Q}^2}\frac{1}{\sqrt{1-z}}$
λ	λ	$-\lambda$	$\sqrt{2}\lambda(v - \lambda a)\sqrt{\tilde{Q}^2}\frac{z}{\sqrt{1-z}}$
λ	$-\lambda$	λ	$\sqrt{2}\lambda\left[m_{ij}(v - \lambda a)\sqrt{z} - m_i(v + \lambda a)\frac{1}{\sqrt{z}}\right]$
λ	$-\lambda$	$-\lambda$	0
λ	λ	0	$(v - \lambda a)\left[\frac{m_{ij}^2}{m_j}\sqrt{z} - \frac{m_i^2}{m_j}\frac{1}{\sqrt{z}} - 2m_j\frac{\sqrt{z}}{1-z}\right] + (v + \lambda a)\frac{m_i m_{ij}}{m_j}\frac{1-z}{\sqrt{z}}$
λ	$-\lambda$	0	$\sqrt{\tilde{Q}^2}\sqrt{1-z}\left[\frac{m_i}{m_j}(v - \lambda a) - \frac{m_{ij}}{m_j}(v + \lambda a)\right]$

λ_I	λ_i	$(f \rightarrow fh$ and $\bar{f} \rightarrow \bar{f}h) \times \frac{e}{2s_w} \frac{m_f}{m_w}$
λ	λ	$m_f\left[\sqrt{z} + \frac{1}{\sqrt{z}}\right]$
λ	$-\lambda$	$\sqrt{1-z}\sqrt{\tilde{Q}^2}$

λ_I	λ_i	$V \rightarrow Vh \times g_h$
λ	λ	-1
λ	$-\lambda$	0
0	λ	$\frac{1}{m_{ij}}\frac{\lambda}{\sqrt{2}}\sqrt{\tilde{Q}^2}\sqrt{z(1-z)}$
λ	0	$\frac{1}{m_i}\frac{\lambda}{\sqrt{2}}\sqrt{\tilde{Q}^2}\sqrt{\frac{1-z}{z}}$
0	0	$\frac{1}{2}\frac{m_j^2}{m_i^2} + \frac{1-z}{z} + z$

λ_i	λ_i	$h \rightarrow VV \times g_V$
λ	λ	0
λ	$-\lambda$	-1
0	λ	$\frac{1}{m_i}\frac{\lambda}{\sqrt{2}}\sqrt{\tilde{Q}^2}\sqrt{\frac{1-z}{z}}$
λ	0	$\frac{1}{m_j}\frac{\lambda}{\sqrt{2}}\sqrt{\tilde{Q}^2}\sqrt{\frac{z}{1-z}}$
0	0	$\frac{1}{2}\frac{m_{ij}^2}{m_i^2} - 1 - \frac{1-z}{z} - \frac{z}{1-z}$

λ_i	λ_j	$h \rightarrow f\bar{f} \times \frac{e}{2s_w} \frac{m_f}{m_w}$
λ	λ	$\sqrt{\tilde{Q}^2}$
λ	$-\lambda$	$m_f\left[\sqrt{\frac{1-z}{z}} - \sqrt{\frac{z}{1-z}}\right]$

λ_I	λ_i	λ_j	$V \rightarrow V'V'' \times g_V$
λ	λ	λ	$\sqrt{2}\lambda\sqrt{\tilde{Q}^2}\sqrt{\frac{1}{z(1-z)}}$
λ	λ	$-\lambda$	$\sqrt{2}\lambda\sqrt{\tilde{Q}^2}z\sqrt{\frac{z}{1-z}}$
λ	$-\lambda$	λ	$\sqrt{2}\lambda\sqrt{\tilde{Q}^2}(1-z)\sqrt{\frac{1-z}{z}}$
λ	$-\lambda$	$-\lambda$	0
0	λ	λ	0
0	λ	$-\lambda$	$m_{ij}(2z - 1) + \frac{m_j^2}{m_{ij}} - \frac{m_i^2}{m_{ij}}$
λ	0	λ	$m_i\left(1 + 2\frac{1-z}{z}\right) + \frac{m_j^2}{m_i} - \frac{m_{ij}^2}{m_i}$
λ	0	$-\lambda$	0
λ	λ	0	$m_j\left(1 + 2\frac{z}{1-z}\right) + \frac{m_i^2}{m_j} - \frac{m_{ij}^2}{m_j}$
λ	$-\lambda$	0	0
λ	0	0	$\frac{\lambda}{\sqrt{2}}\frac{m_i^2 + m_j^2 - m_{ij}^2}{m_i m_j}\sqrt{\tilde{Q}^2}\sqrt{z(1-z)}$
0	λ	0	$\frac{\lambda}{\sqrt{2}}\frac{m_{ij}^2 + m_j^2 - m_i^2}{m_{ij} m_j}\sqrt{\tilde{Q}^2}\sqrt{\frac{1-z}{z}}$
0	0	λ	$\frac{\lambda}{\sqrt{2}}\frac{m_{ij}^2 + m_i^2 - m_j^2}{m_{ij} m_i}\sqrt{\tilde{Q}^2}\sqrt{\frac{z}{1-z}}$
0	0	0	$\frac{1}{2}\frac{m_{ij}^3}{m_i m_j}(2z - 1) - \frac{m_i^3}{m_{ij} m_j}\left(\frac{1}{2} + \frac{1-z}{z}\right) + \frac{m_j^3}{m_{ij} m_i}\left(\frac{1}{2} + \frac{z}{1-z}\right) + \frac{m_i m_j}{m_{ij}}\left(\frac{1-z}{z} - \frac{z}{1-z}\right) + \frac{m_{ij} m_i}{m_j}(1-z)\left(2 + \frac{1-z}{z}\right) - \frac{m_{ij} m_j}{m_i}z\left(2 + \frac{z}{1-z}\right)$

Resonance Matching

Branchings like $t \rightarrow bW$, $Z \rightarrow q\bar{q}$ etc.

- Large scales:
EW shower offers best description
- Small scales:
Breit-Wigner distribution

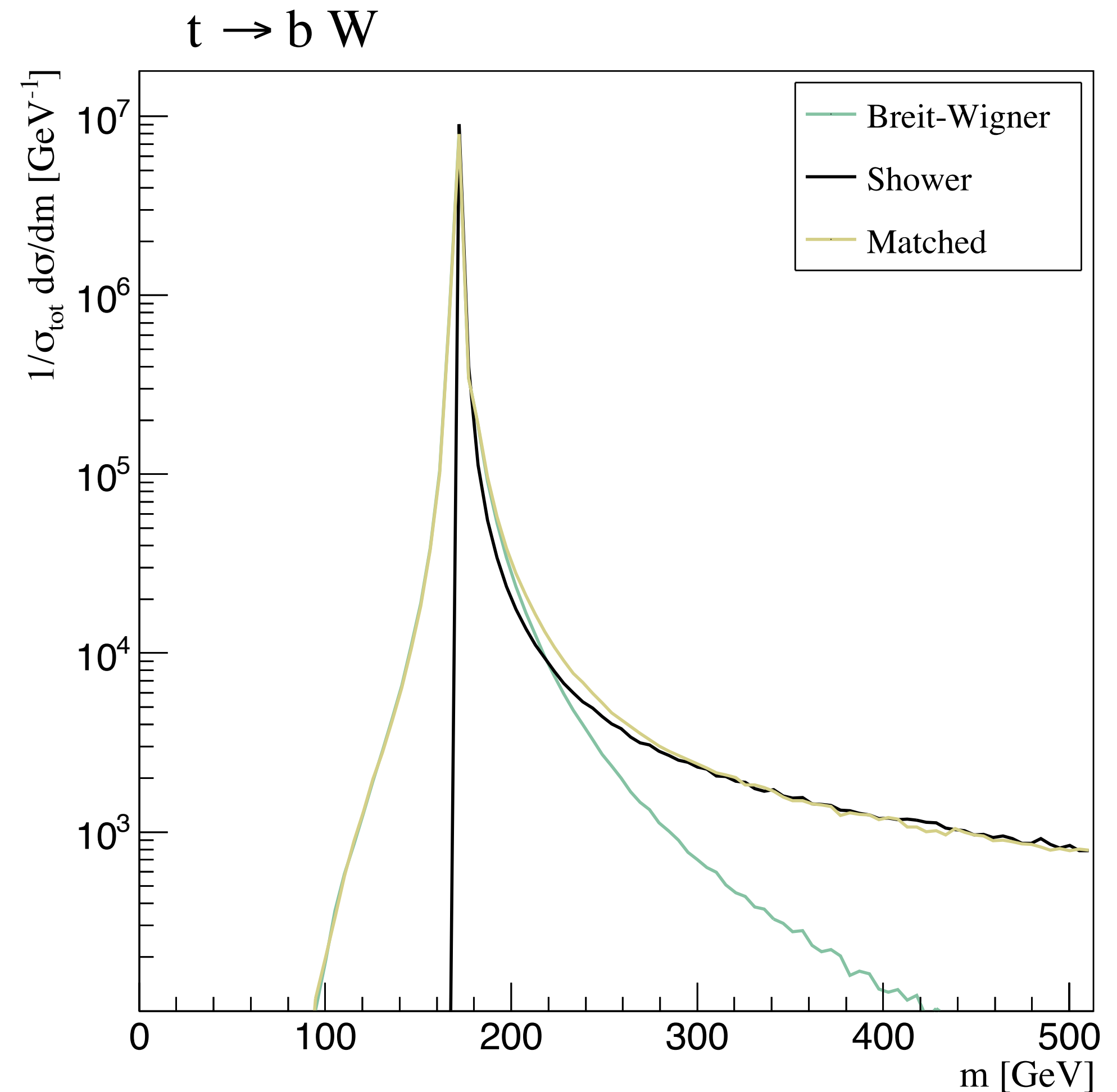
$$\text{BW}(Q^2) \propto \frac{m_0 \Gamma(m)}{Q^4 + m_0^2 \Gamma(m)^2}$$

Matching:

- Sample mass from Breit-Wigner upon production
- Suppress shower by factor

$$\frac{Q^4}{(Q^2 + Q_{\text{EW}}^2)^2}$$

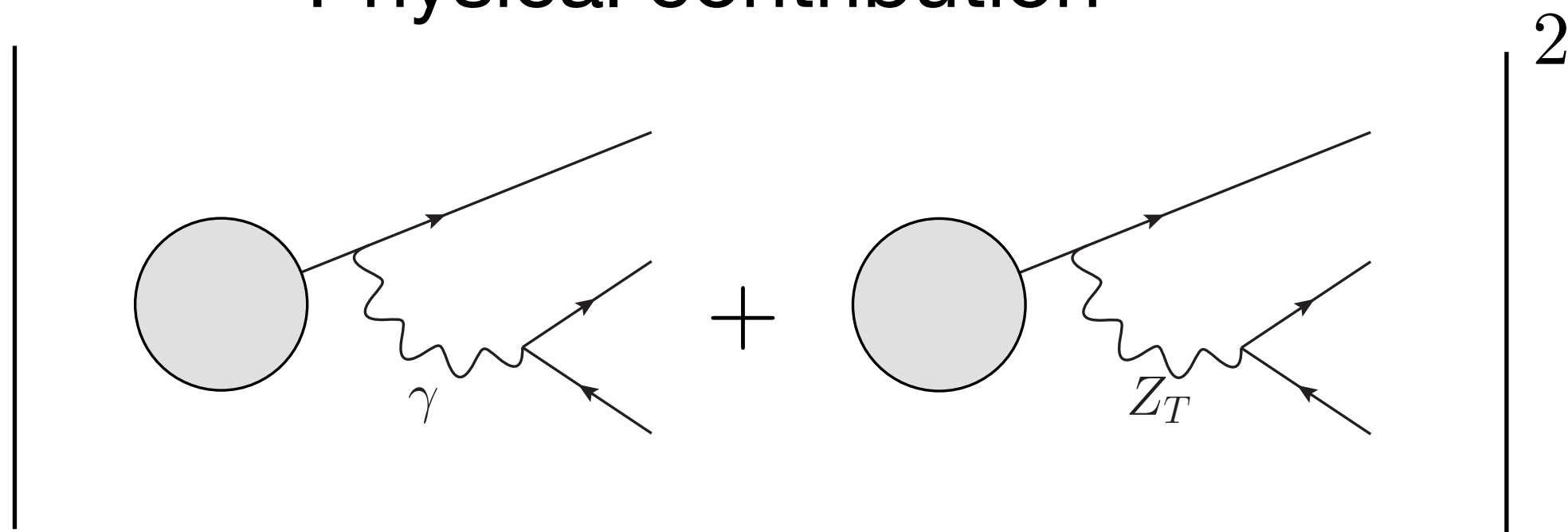
- Decay when shower hits off-shellness scale



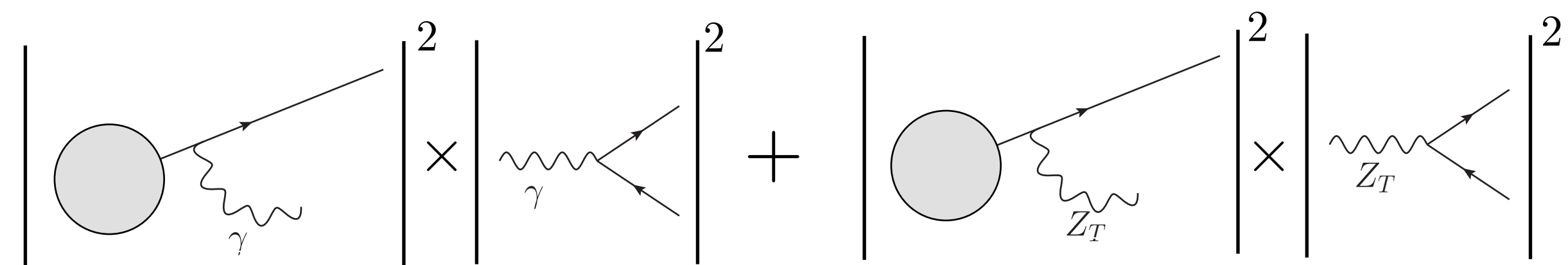
Neutral Boson Interference

Interference between γ, Z_T and h, Z_L

Physical contribution



Shower approximation

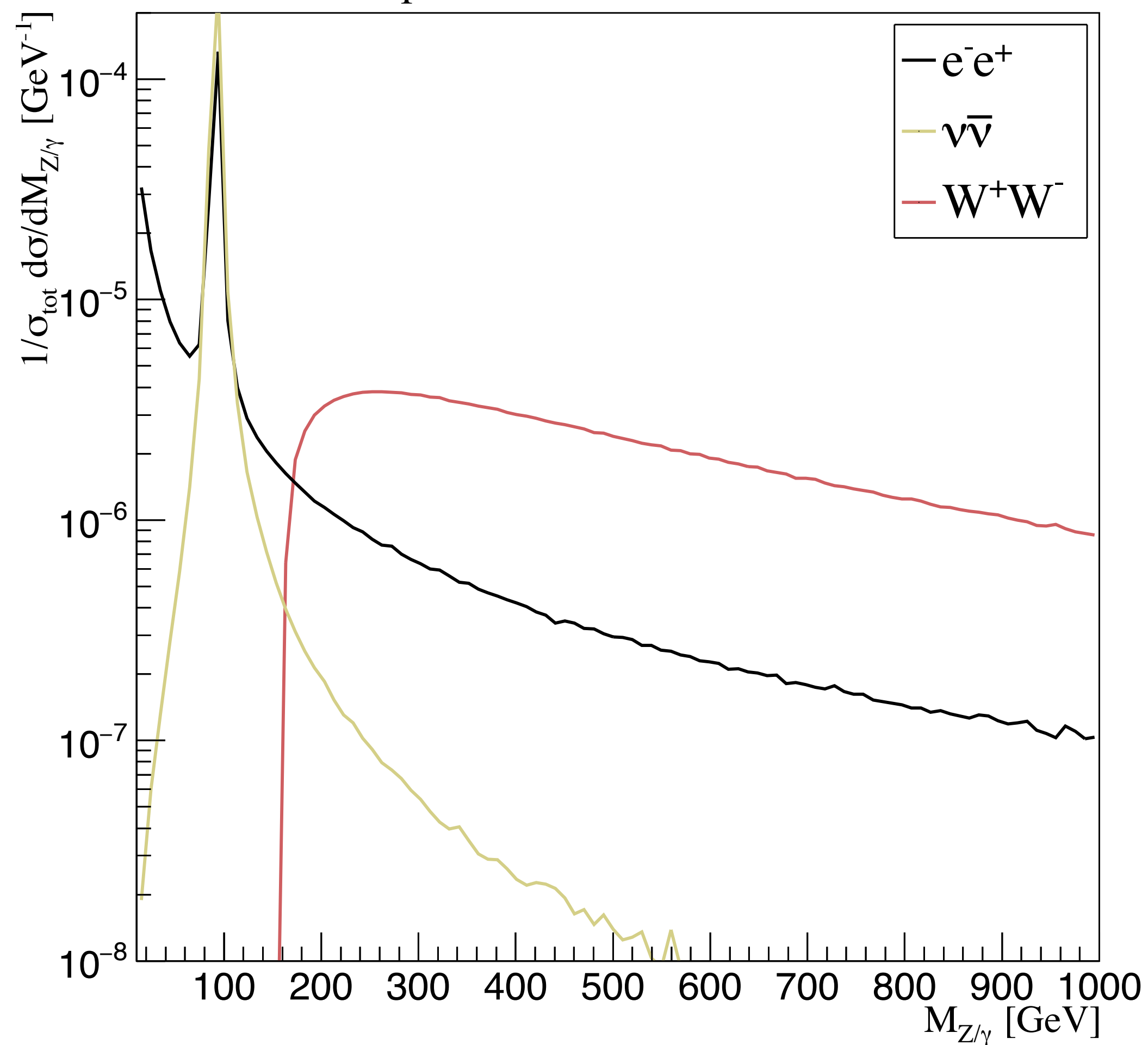


- Complicated solution: Evolve density matrices
 → Very computationally expensive
- Simple solution: Apply event weight
 → Does not get Sudakov right

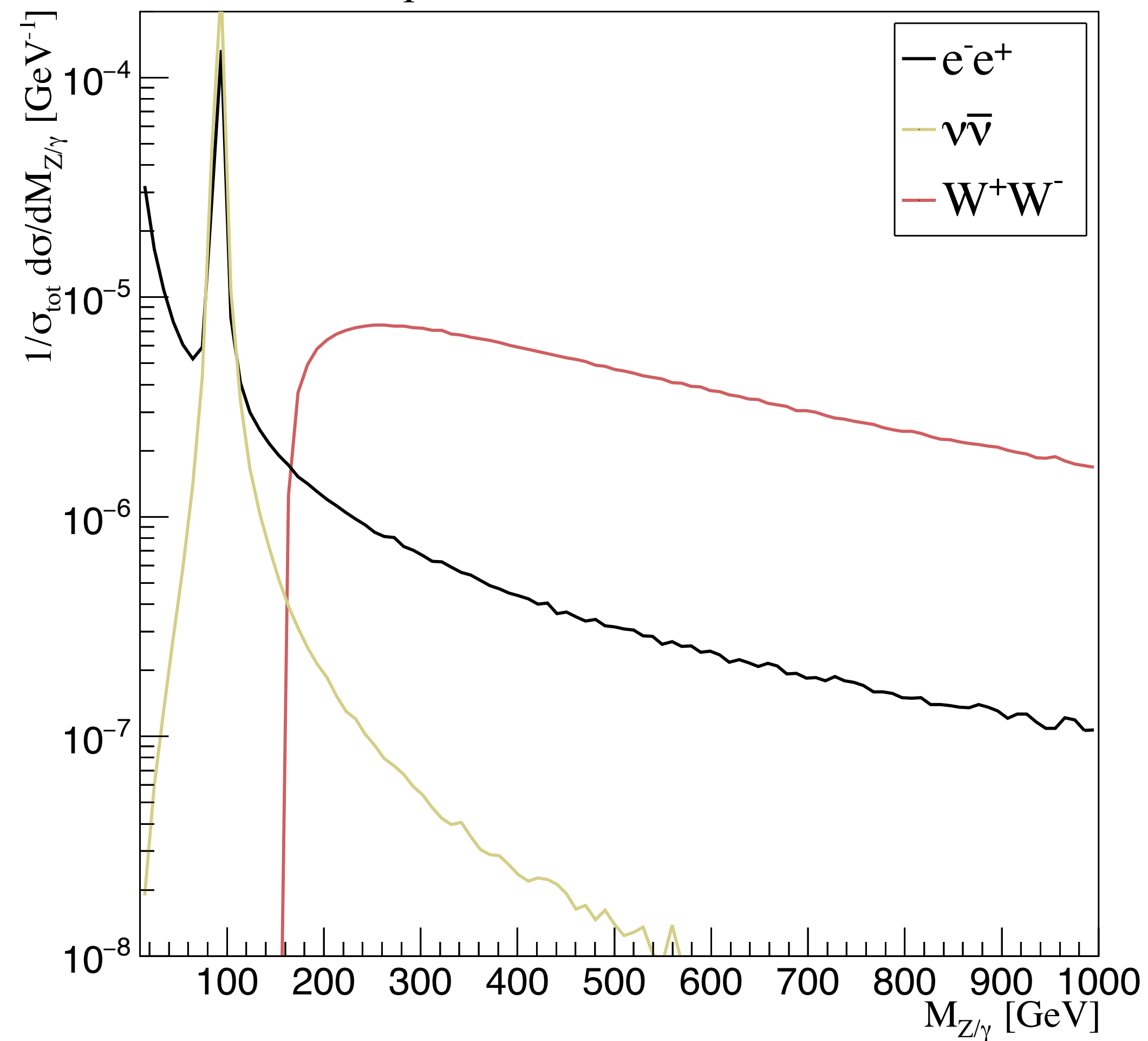
$$w = \frac{\left| \begin{array}{c} \text{grey circle} \\ \times \text{ wavy line } \gamma \\ \times \text{ fermion lines} \end{array} \right|^2 + \left| \begin{array}{c} \text{grey circle} \\ \times \text{ wavy line } Z_T \\ \times \text{ fermion lines} \end{array} \right|^2}{\left| \begin{array}{c} \text{grey circle} \\ \times \text{ wavy line } \gamma \\ \times \text{ fermion lines} \end{array} \right|^2 + \left| \begin{array}{c} \text{grey circle} \\ \times \text{ wavy line } Z_T \\ \times \text{ fermion lines} \end{array} \right|^2}$$

Bosonic Interference

$e^- \rightarrow e^- \gamma / Z_T \rightarrow e^- X$ (No interference)

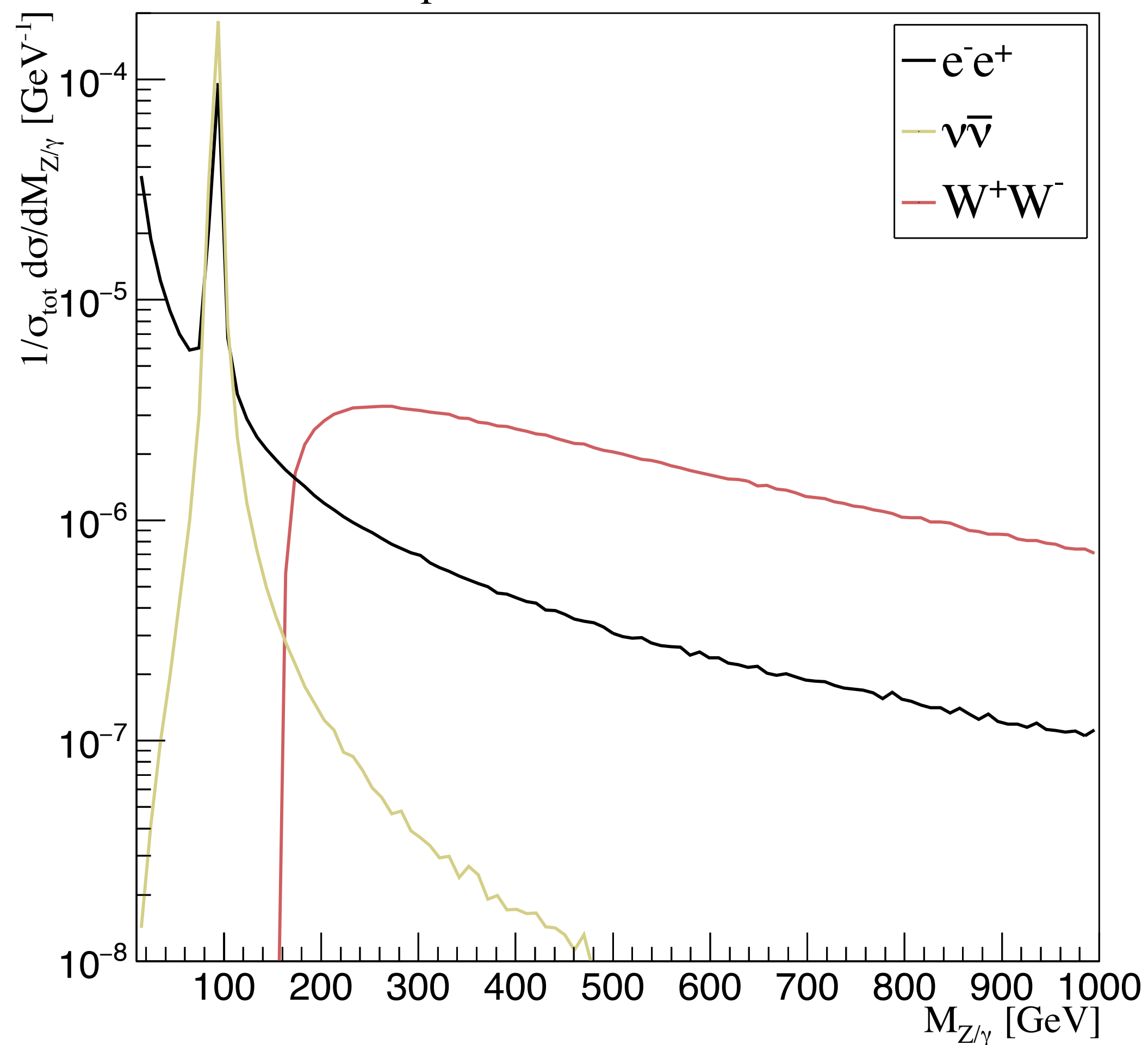


$e^- \rightarrow e^- \gamma / Z_T \rightarrow e^- X$ (Interference)

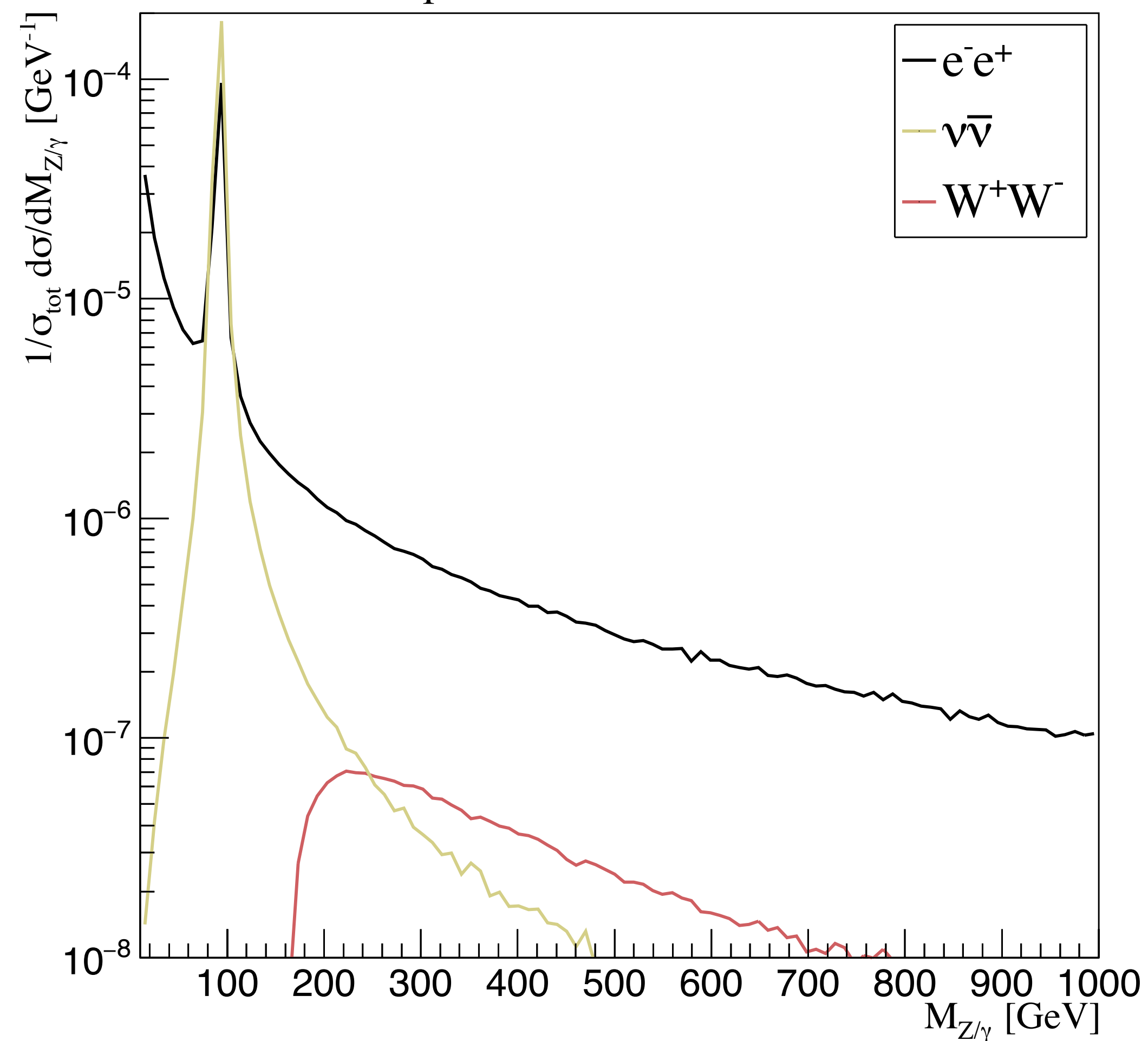


Bosonic Interference

$e_+ \rightarrow e_+ \gamma / Z_T \rightarrow e_+ X$ (No interference)

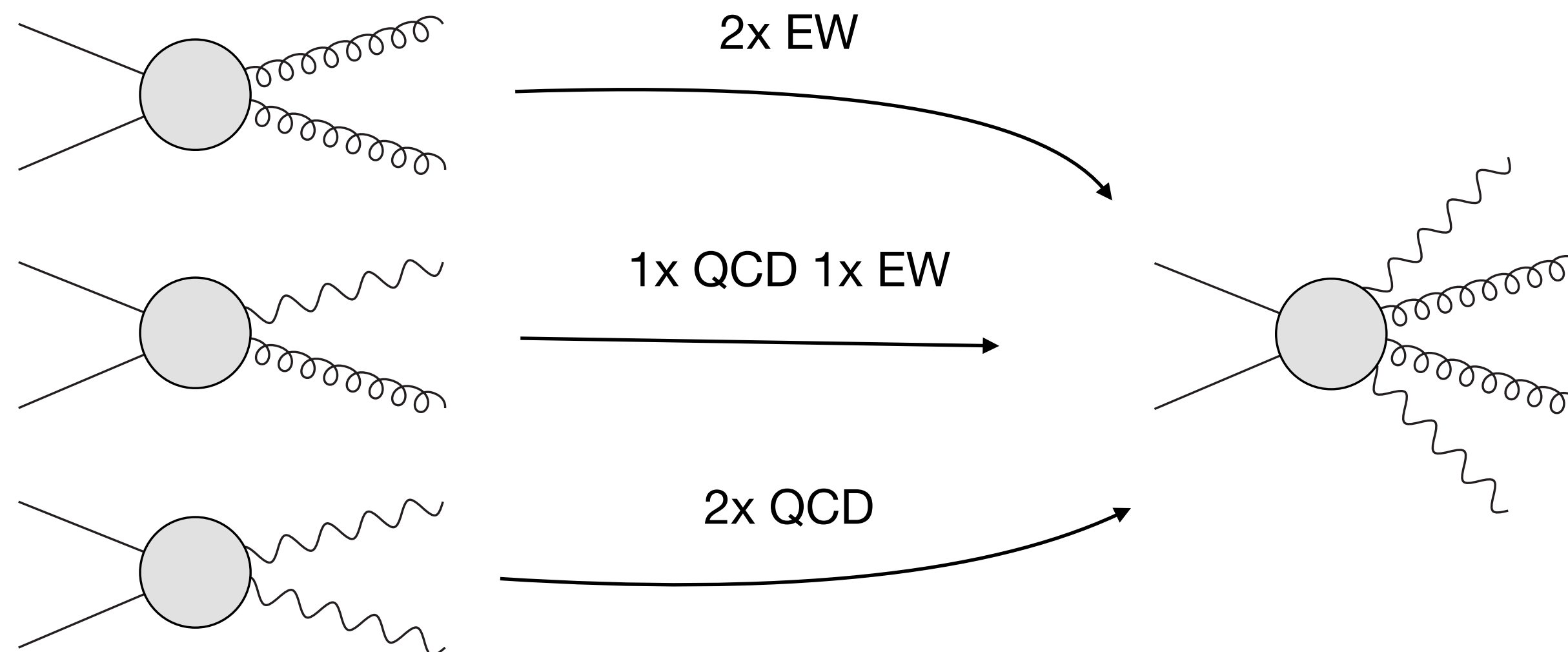


$e_+ \rightarrow e_+ \gamma / Z_T \rightarrow e_+ X$ (Interference)

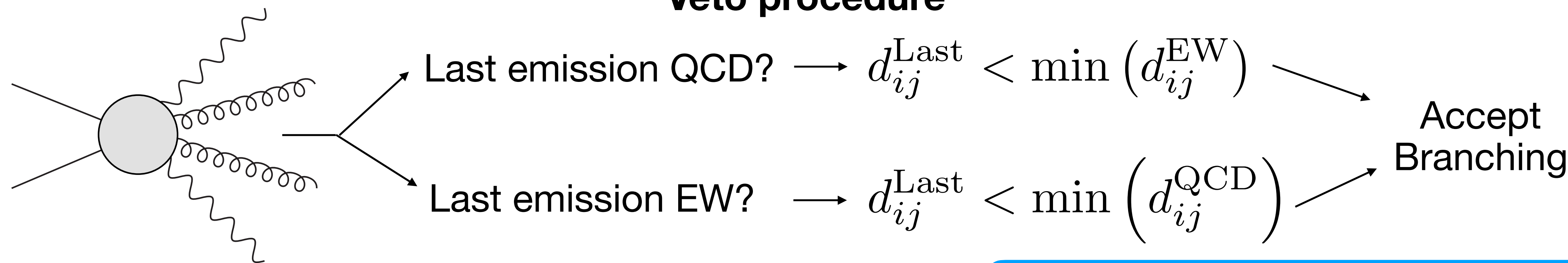


Overlap Veto

Double counting problem

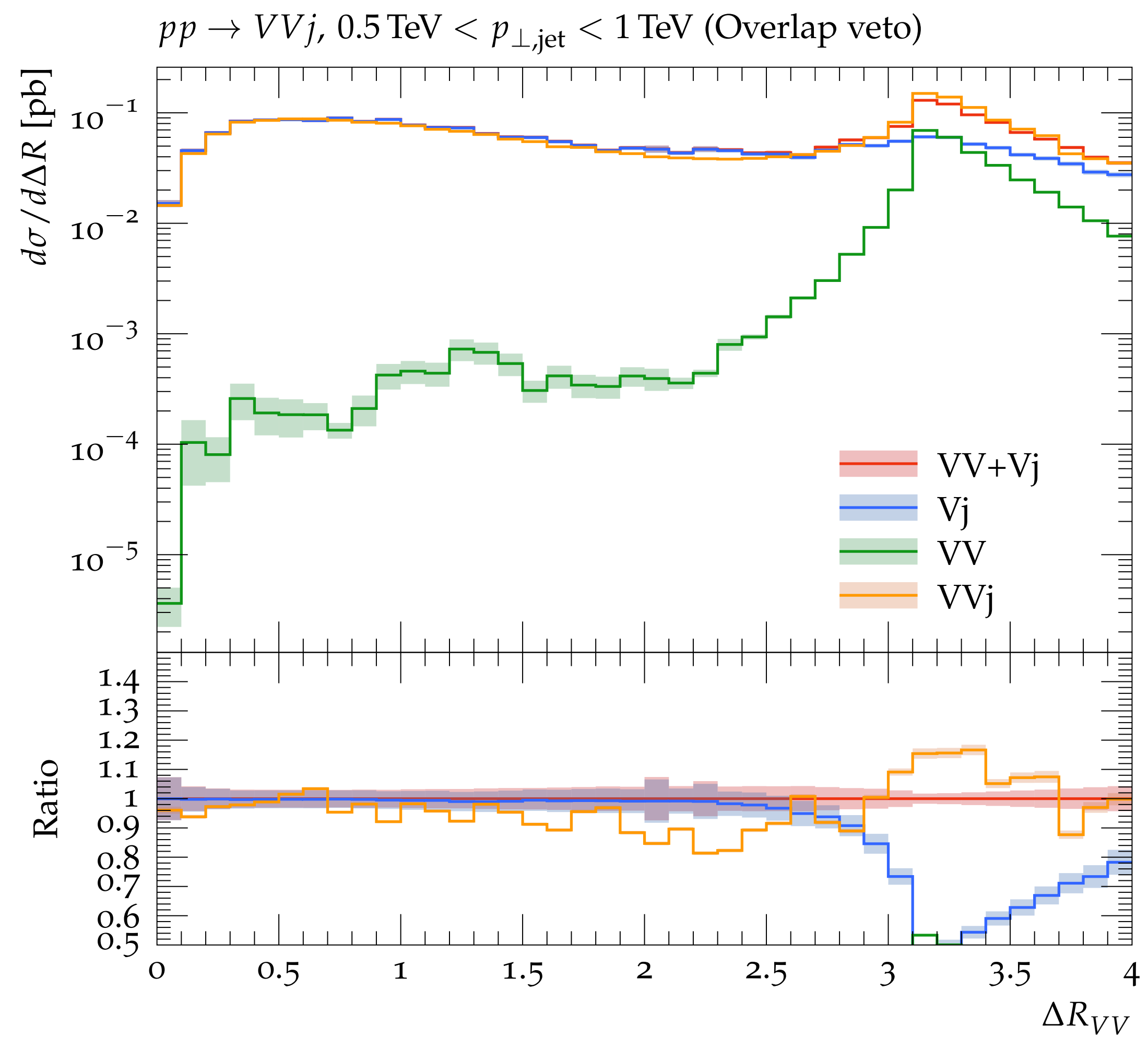
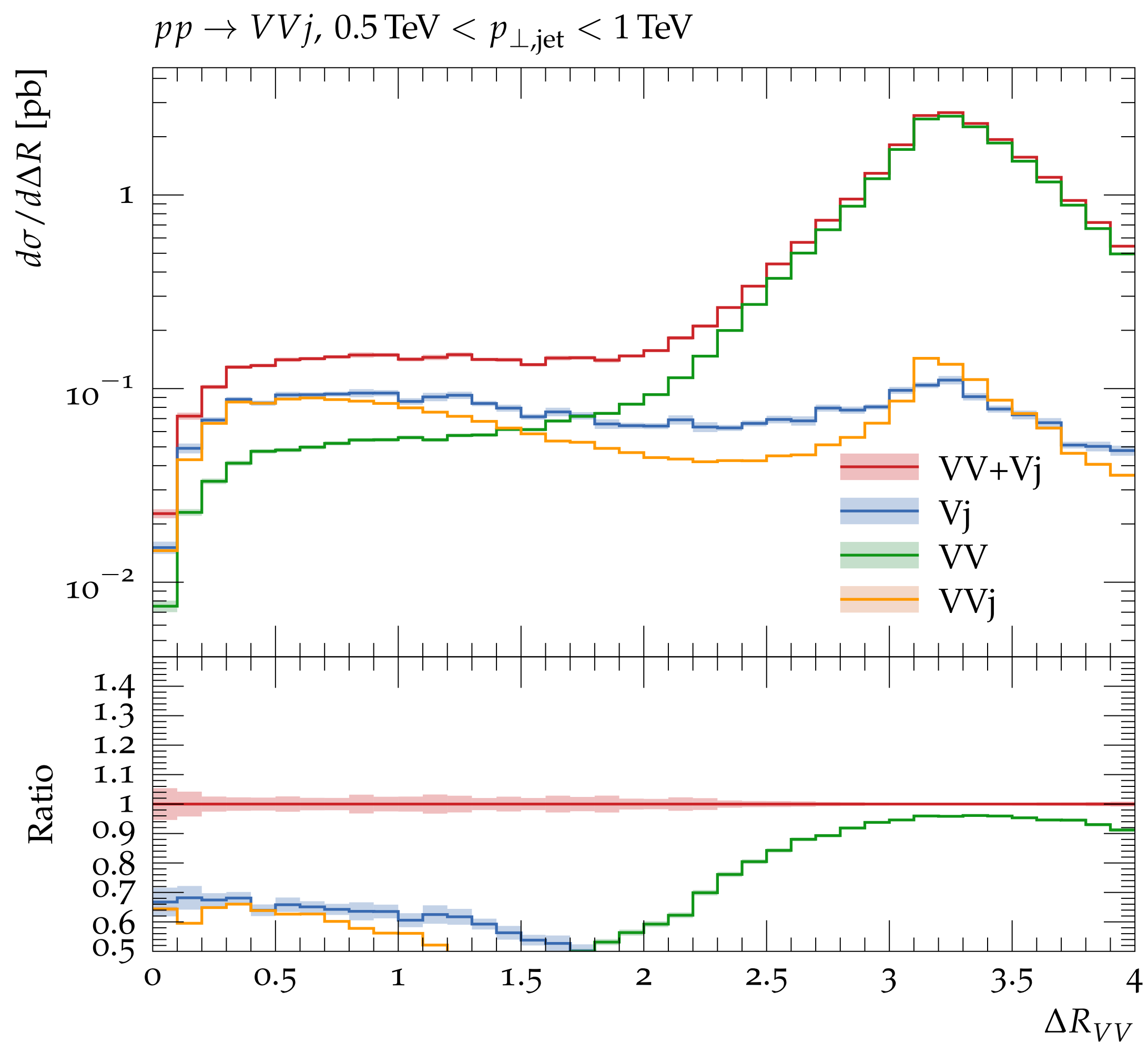


Veto procedure



$$d_{ij} = \min(k_{T,i}^2, k_{T,j}^2) \frac{\Delta_{ij}}{R} + m_i^2 + m_j^2 - m^2$$

Overlap Veto

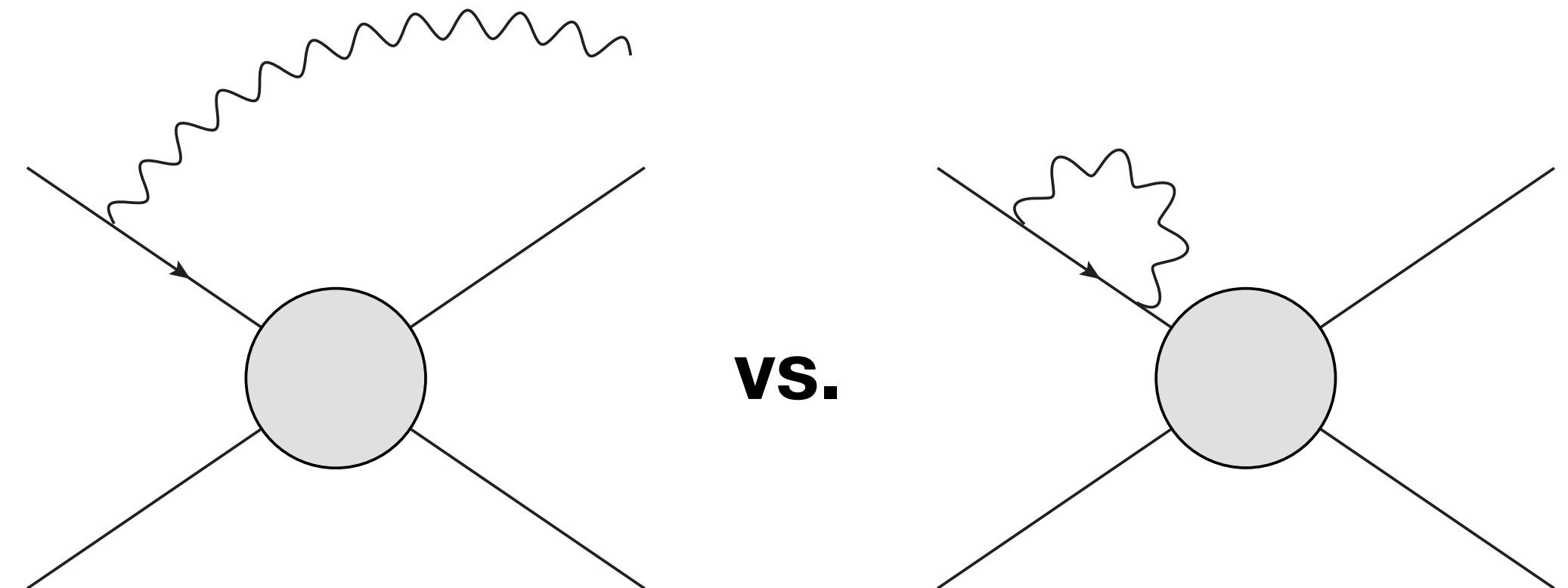


Bloch-Nordsieck Violations

BN / KLN Theorems: Real and virtual singularities cancel

Requirement: Summing over gauge indices

W radiation from the initial state:
PDFs are not isospin symmetric
→ Incomplete cancellation



Effects not large at LHC, but will be significant at higher energies

No straightforward solution in shower language

Conclusions

Many interesting challenges for the development of event generators

Electroweak corrections in parton showers come with unique challenges

- Chiral nature of the EW theory ✓
- Consequences of EW symmetry breaking ✓
- Shower vs. Resonance decays !
- Neutral boson interference !
- Double-counting Borns ✓
- Bloch-Nordsieck violations ✗

EW shower will be publicly available as part of Vincia in the upcoming Pythia 8.304 release

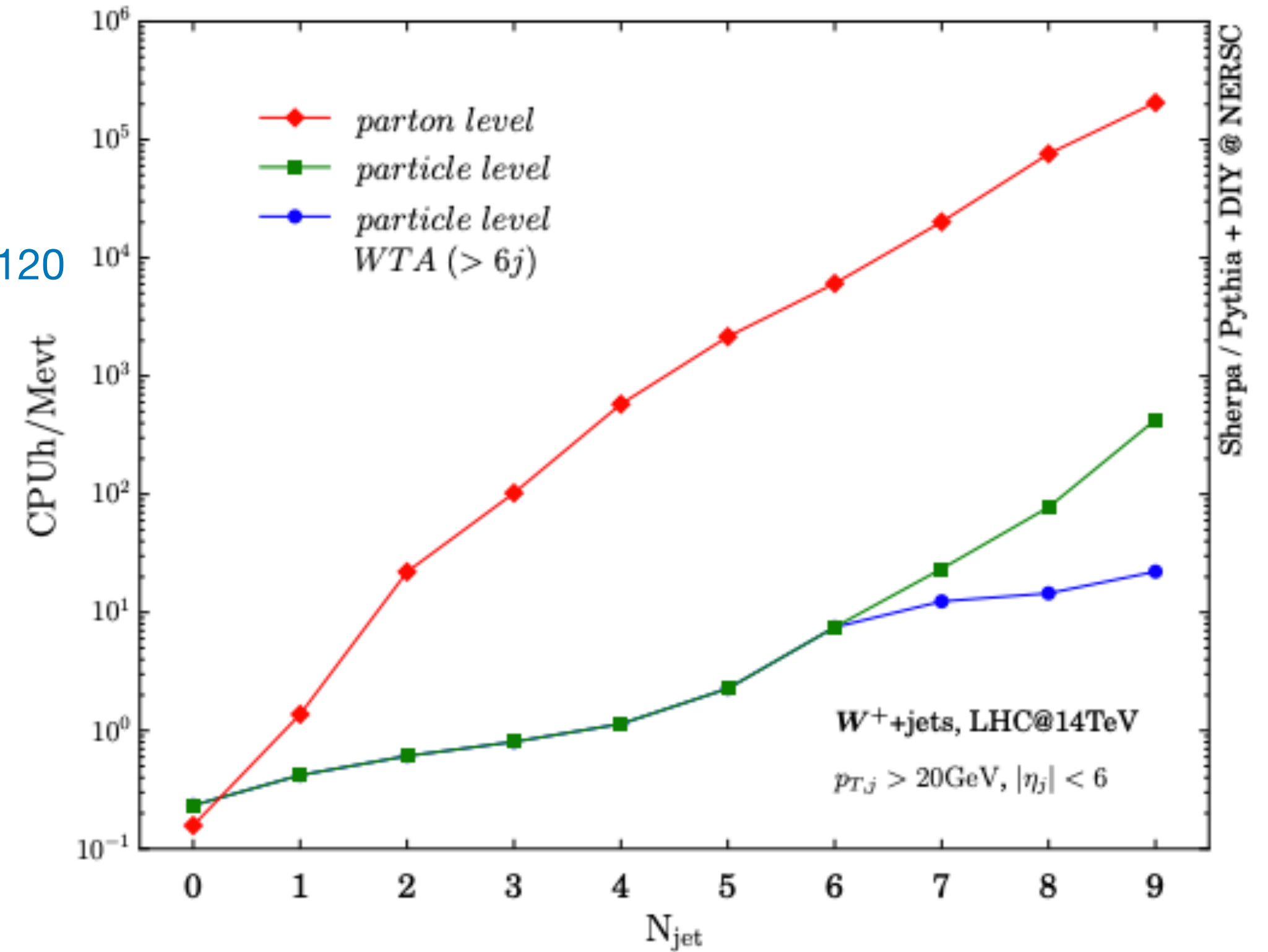
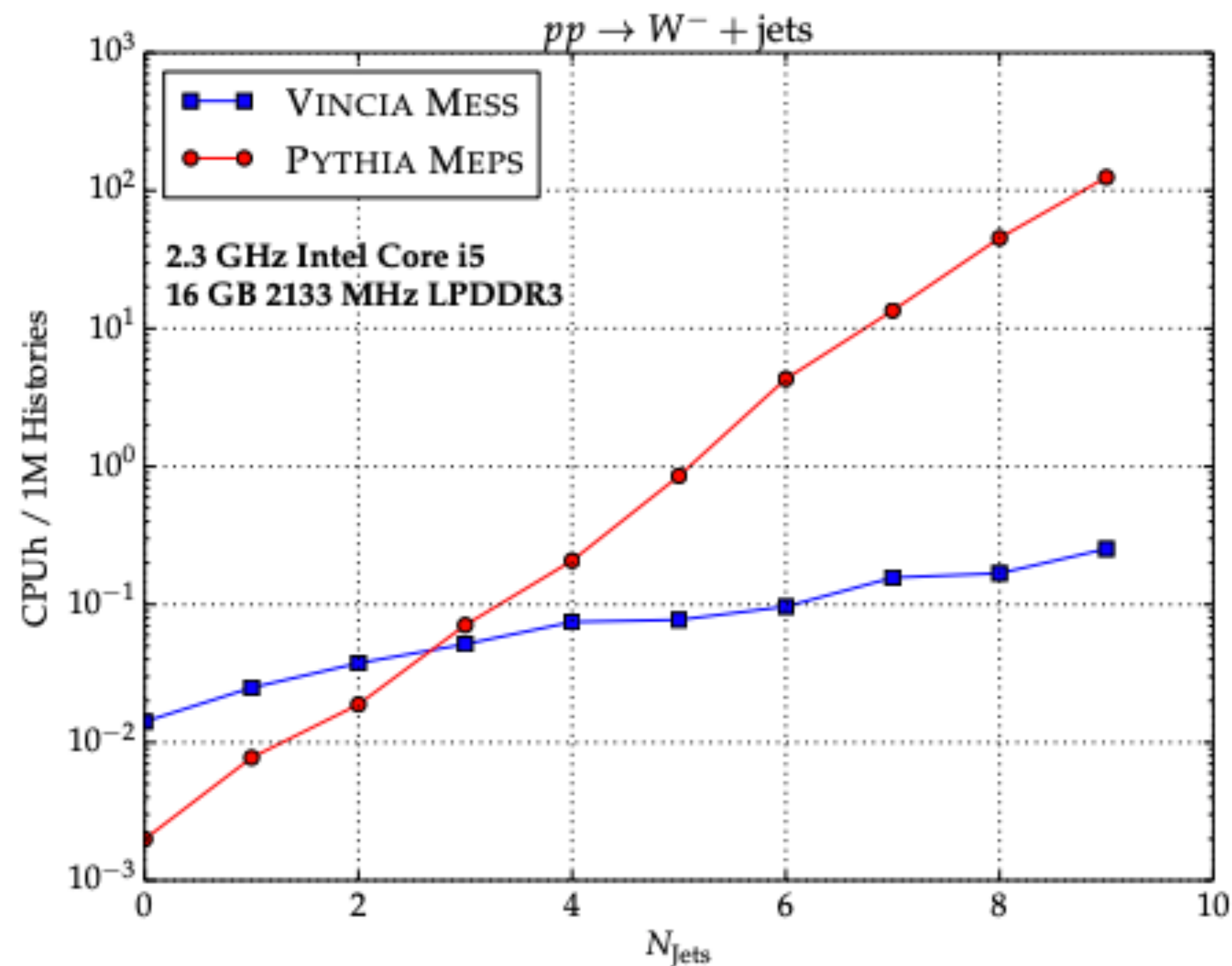
Backup

Efficient Multileg Merging

Hoche, Prestel, Schulz 1905.05120

Multileg merging is expensive for two reasons

- Sampling high-multiplicity MEs
- Reconstructing shower histories



The Vincia parton shower: sector showering
Unique shower history → fast clustering

Brooks, Preuss 2008.09468

Spinor-Helicity formalism

Fermion

$$u_{\pm}(p) = \frac{1}{\sqrt{2p \cdot k}} (\not{p} + m) u_{\mp}(k)$$

$$v_{\pm}(p) = \frac{1}{\sqrt{2p \cdot k}} (\not{p} - m) u_{\mp}(k)$$

$k \rightarrow$ helicity for massive fermions

Spin points in direction of motion

Gauge boson

$$\epsilon_{\pm}^{\mu}(p) = \pm \frac{1}{\sqrt{2}} \frac{1}{2p \cdot k} \bar{u}_{\mp}(k) \not{p} \gamma^{\mu} u_{\pm}(k)$$

$$\epsilon_0^{\mu}(p) = \frac{1}{m} \left(p^{\mu} - \frac{m^2}{p \cdot k} k^{\mu} \right)$$

$k \rightarrow$ gauge choice

Purely transverse & longitudinal

$$k = (1, -\vec{e}_p)$$

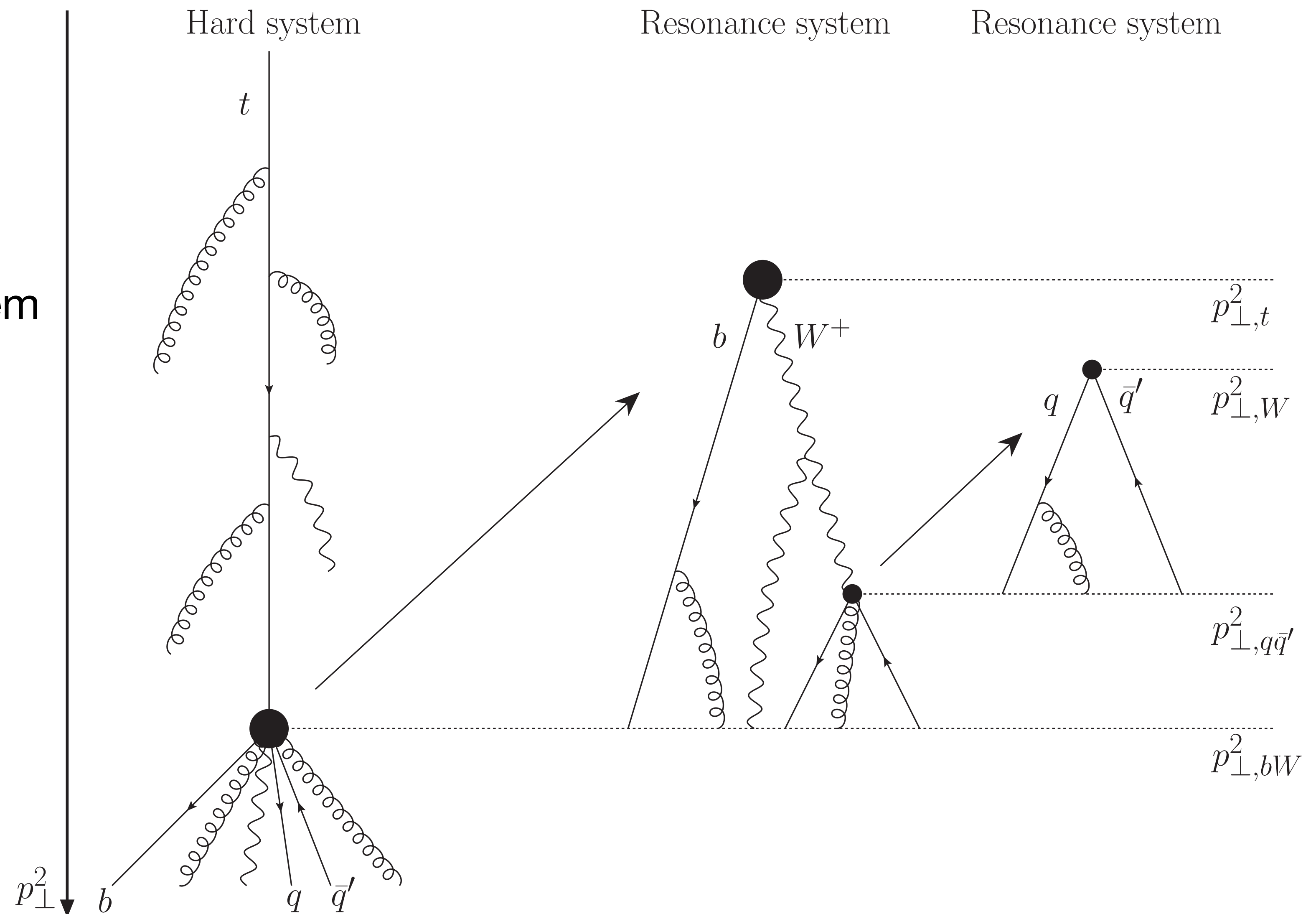
Resonance Matching

Pythia

- Narrow width approximation
- Decay showers after hard system

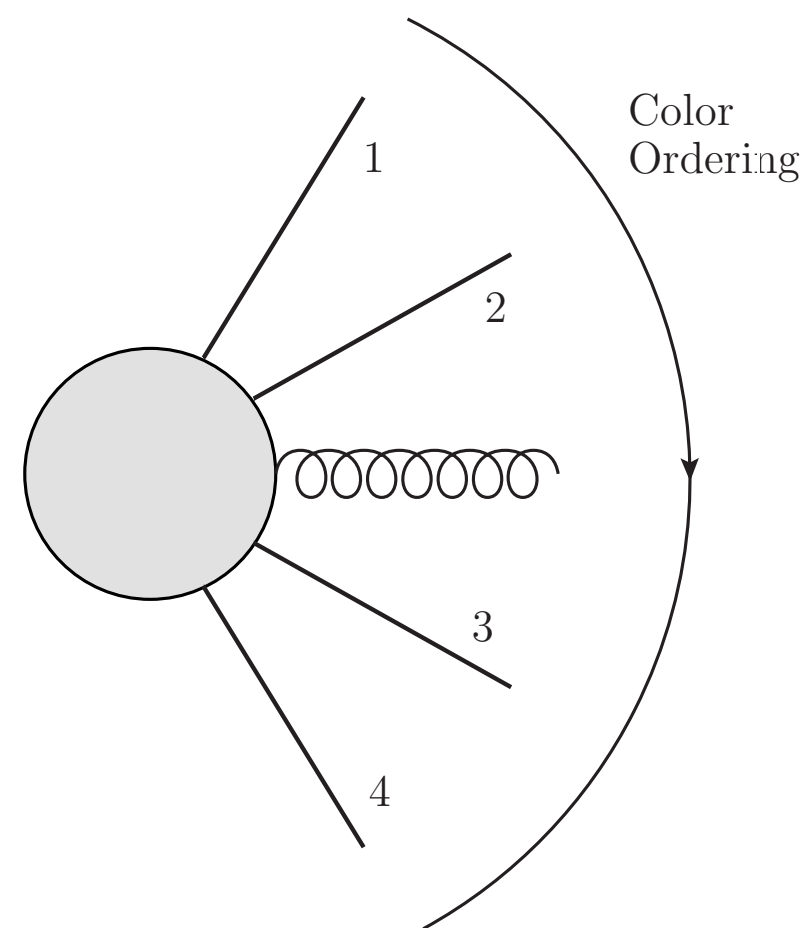
Vincia

- Decays part of hard system
- Natural treatment of finite width effects



Recoiler Selection

In QCD recoiler determined by colour structure



Gluon splitting: recoiler ambiguous

In EW no such guidance exists

$$\begin{aligned}
 \left| \text{Vertex} \right|^2 &= \frac{\left| \text{Diagram 1} \right|^2}{\left| \text{Diagram 2} \right|^2 + \left| \text{Diagram 3} \right|^2} \left| \text{Vertex} \right|^2 \\
 &+ \frac{\left| \text{Diagram 4} \right|^2}{\left| \text{Diagram 5} \right|^2 + \left| \text{Diagram 6} \right|^2} \left| \text{Vertex} \right|^2
 \end{aligned}$$

Probabilistic choice to avoid back reaction effects