The electroweak parton distribution functions at high-energy lepton colliders

Keping Xie

University of Pittsburgh

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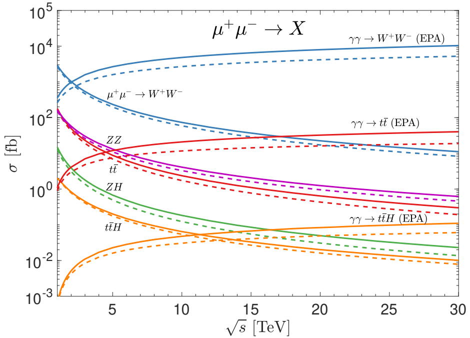
In collaboration with Tao Han and Yang Ma
2007.14300 and 2102.xxxxx

The SM processes at high-energy muon colliders

[T. Han, Y. Ma, KX 2007.14300]



- The annihilation cross sections decrease as $1/s$. Solid falling lines include ISR, which enhances about 50%.
- The fusion ones increase as $\log^p(s)$, which take over at high energies. Dashed lines correspond to scale choice $Q = \sqrt{\hat{s}}/2$.
- Question: How to treat photon (and electroweak bosons) properly at high energies when W/Z become active?

EW physics at high energies

- At high energies, every particle become massless

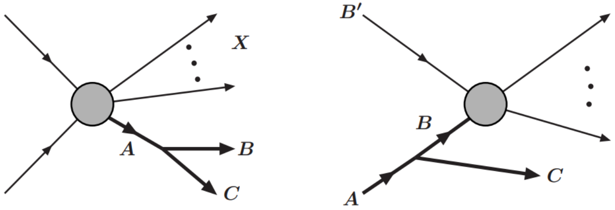
$$\frac{v}{E} : \frac{v}{100\ \mathrm{TeV}} \sim \frac{\Lambda_{\mathrm{QCD}}}{100\ \mathrm{GeV}},\ \frac{v}{E}, \frac{m_t}{E}, \frac{M_W}{E} \to 0!$$

- The splitting phenomena dominate due to large log enhancement
- The EW symmetry is restored: $SU(2)_L \times U(1)_Y$ unbroken
- Goldstone Boson Equivalence:

$$\varepsilon_L^{\mu}(k) = \frac{E}{M_W}(\beta_W, \hat{k}) \simeq \frac{k^{\mu}}{M_W} + \mathscr{O}(\frac{M_W}{E})$$

  The violation terms is power counted as $v/E \to$ QCD higher twist effects Λ_{QCD}/Q [G. Cuomo, A. Wulzer, arXiv:1703.08562; 1911.12366].
- We mainly focus on the splitting phenomena, which can be factorized and resummed as the EW PDFs in the ISR, and the Fragementaions/Parton Shower in the FRS.
- Other interesting aspects: the polarized EW boson scattering, top-Yukawa coupling effect

Factorization of the EW splittings



$$d\sigma \simeq d\sigma_X \times d\mathscr{P}_{A\to B+C}\,, \quad E_B \approx zE_A, \quad E_C \approx \bar{z}E_A, \quad k_T \approx z\bar{z}E_A\theta_{BC}$$

$$\frac{d\mathscr{P}_{A\to B+C}}{dz\,dk_T^2} \simeq \frac{1}{16\pi^2}\frac{z\bar{z}|\mathscr{M}^{(\mathrm{split})}|^2}{(k_T^2 + \bar{z}m_B^2 + zm_C^2 - z\bar{z}m_A^2)^2}, \quad \bar{z} = 1 - z$$

- The dimensional counting: $|\mathscr{M}^{(\mathrm{split})}|^2 \sim k_T^2$ or m^2
- To validate the fractorization formalism
  - The observable σ should be infra-red safe
  - Leading behavior comes from the collinear splitting

[Ciafaloni et al., hep-ph/0004071; 0007096; C. Bauer, Ferland, B. Webber et al., arXiv:1703.08562;1808.08831]

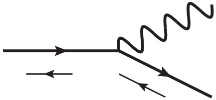
[A. Manohar et al., 1803.06347; T. Han, J. Chen & B. Tweedie, arXiv:1611.00788]

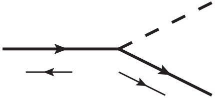
Splitting functions: EW

- Starting from the unbroken phase: all massless

$$\mathscr{L}_{SU(2)\times U(1)} = \mathscr{L}_{gauge} + \mathscr{L}_{\phi} + \mathscr{L}_{f} + \mathscr{L}_{Yukawa}$$

- Particle contents:
  - Chiral fermions $f_{L,R}$
  - Gauge bosons: $B, W^{0,\pm}$
  - Higgs $H = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix} = \begin{pmatrix} \phi^+ \\ \frac{1}{\sqrt{2}}(h - i\phi^0) \end{pmatrix}$
- Splitting functions [See Ciafaloni et al. hep-ph/0505047, Chen et al. 1611.00788 for complete lists.]

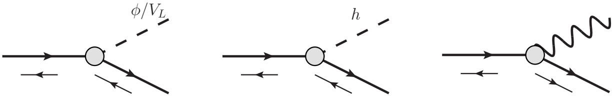




	$\frac{1}{8\pi^2}\frac{1}{k_T^2}\frac{1+\bar{z}^2}{z}$		$\frac{1}{8\pi^2}\frac{1}{k_T^2}\frac{z}{2}$	
$f_{s=L,R} \to$	$V_T f_s^{(\prime)}$	$[BW]_T^0 f_s$	$H^{0*} f_{-s}$	$\phi^{\pm} f'_{-s}$
	$g_V^2 (Q_{f_s}^V)^2$	$g_1 g_2 Y_{f_s} T_{f_s}^3$	$y^2_{f_R^{(\prime)}}$	
	Infrared, collinear singularites (P_{gq})		Collinear singularity chirality-flip, Yukawa	

Corrections to the GET in the EWSB

- New fermion splitting: $P \sim \frac{v^2}{k_T^2}\frac{\mathrm{d}k_T^2}{k_T^2}$
- V_L is of IR, h has no IR



	$\frac{1}{16\pi^2}\frac{v^2}{k_T^4}\frac{1}{z}$	$\frac{1}{16\pi^2}\frac{v^2}{k_T^4}$	$\frac{1}{16\pi^2}\frac{v^2}{k_T^4}$
$f_s \to$	$V_L f_s^{(')}(V \neq \gamma)$	$h f_s$	$V_T f_{-s}^{(')}$
	Chirality conserving non-zero for massless f		Chirality flipping $\sim m_f$

- The PDFs for W_L/Z_L behaves as constants, which does not run at the leading log: "Bjorken scaling" restoration (higher-twist effects)

$$f_{V_L/f}(x, Q^2) \sim \alpha \frac{1-x}{x}$$

PDFs and Fragmentations (parton showers)

- Initial state radiation (ISR), PDFs (DGLAP):

$$f_B(z,\mu^2) = \sum_A \int_z^1 \frac{d\xi}{\xi} f_A(\xi) \int_{m^2}^{\mu^2} d\mathscr{P}_{A\to B+C}(z/\xi, k_T^2)$$

$$\frac{\partial f_B(z,\mu^2)}{\partial \mu^2} = \sum_A \int_z^1 \frac{d\xi}{\xi} \frac{d\mathscr{P}_{A\to B+C}(z/\xi,\mu^2)}{dz dk_T^2} f_A(\xi,\mu^2)$$

- Final state radiation (FSR): Fragmentations (parton showers):

$$\Delta_A(t) = \exp\left[-\sum_B \int_{t_0}^t \int dz \mathscr{P}_{A\to B+C}(z) \right],$$

$$f_A(x,t) = \Delta_A(t) f_A(x,t_0) + \int_{t_0}^t \frac{dt'}{t'} \frac{\Delta(t)}{\Delta(t')} \int \frac{dz}{z} \mathscr{P}_{A\to B+C}(z) f_A(x/z,t')$$

- Very important formulation for the LHC physics, and future colliders.

The novel features of the EW PDFs

- The EW PDFs must be polarized due to the chiral nature of the EW theory

$$f_{V_+/A_+} \neq f_{V_-/A_-}, \qquad f_{V_+/A_-} \neq f_{V_-/A_+},$$
$$\hat{\sigma}(V_+B_+) \neq \hat{\sigma}(V_-B_-), \quad \hat{\sigma}(V_+B_-) \neq \hat{\sigma}(V_-B_+)$$

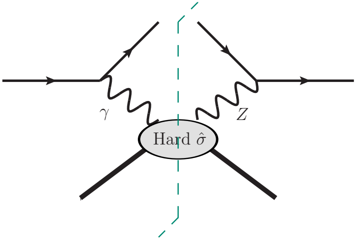
We are not able to factorize the cross sections in an unporlarized form.

$$\sigma \neq f_{V/A}\hat{\sigma}(VB), \; f_{V/A} = \frac{1}{2}\sum_{\lambda,s_1} f_{V_\lambda/A_{s_1}}, \; \hat{\sigma}(VB) = \frac{1}{4}\sum_{\lambda,s_2}\hat{\sigma}(V_\lambda B_{s_2})$$

- The interference gives the mixed PDFs [Bauer '17, '18, Manohar '18 , Tao '16.]

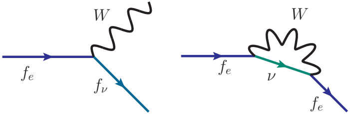
$$f_{\gamma Z} \sim \langle\Omega|A^{\mu\nu}Z_{\mu\nu}|\Omega\rangle + \text{h.c.},$$

similarly for f_{hZ_L}.

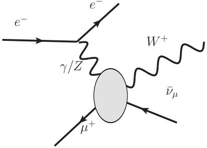


- Bloch-Nordsieck theorem violation due to the non-cancelled divergence in $f \to f'V$: cutoff M_V/Q or redefinition

$$f_1 \sim f_e + f_\nu, \; f_3 \sim f_e - f_\nu$$



A toy example: $e^-\mu^+ \to e^- W^+ \bar{\nu}_\mu$



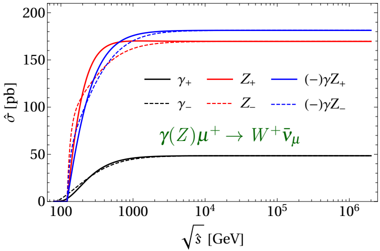
- EWA: $f_{V_\lambda/e_s^\pm}(x,Q) = \frac{1}{8\pi^2} g_1 g_2 P_{V_\lambda/e_s^\pm}(x) \log\left(Q^2/m_Z^2\right)$

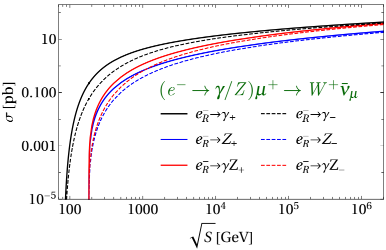
$g_L = \frac{g_2}{c_W}\left(-\frac{1}{2} + s_W^2\right) < 0,\ g_R = \frac{g_2}{c_W} s_W^2 > 0,\ g_e = -e$

	e_L^-	e_R^-	e_L^+	e_R^+
Z_-	$g_L^2 \frac{1}{x}$	$g_R^2 \frac{(1-x)^2}{x}$	$g_L^2 \frac{(1-x)^2}{x}$	$g_R^2 \frac{1}{x}$
Z_+	$g_L^2 \frac{(1-x)^2}{x}$	$g_R^2 \frac{1}{x}$	$g_L^2 \frac{1}{x}$	$g_R^2 \frac{(1-x)^2}{x}$
γZ_-	$g_e g_L \frac{1}{x}$	$g_e g_R \frac{(1-x)^2}{x}$	$g_e g_L \frac{(1-x)^2}{x}$	$g_e g_R \frac{1}{x}$
γZ_+	$g_e g_L \frac{(1-x)^2}{x}$	$g_e g_R \frac{1}{x}$	$g_e g_L \frac{1}{x}$	$g_e g_R \frac{(1-x)^2}{x}$

- The contribution of the mixed PDF $f_{\gamma Z}$ can be either constructive or destructive

$$\sigma = \sum_{\lambda, s_1, s_2} f_{V_\lambda/e_{s_1}^-} \hat{\sigma}(V_\lambda \mu_{s_2}^+ \to W^+ \bar{\nu}_\mu)$$





The PDF evolution

- The DGLAP equations

$$\frac{\mathrm{d}f_i}{\mathrm{d}\log\mu^2} = \sum_I \frac{\alpha_I}{2\pi} \sum_j P_{ij}^I \otimes f_j$$

- The initial conditions

$$f_{\ell/\ell}(x, m_\ell^2) = \delta(1-x)$$

- Three regions and two matchings
  - $m_\ell < \mu < \mu_{\mathrm{QCD}}$: QED
  - $\mu = \mu_{\mathrm{QCD}} \lesssim 1$ GeV: $f_q \propto P_{q\gamma} \otimes f_\gamma, f_g = 0$
  - $\mu_{\mathrm{QCD}} < \mu < \mu_{\mathrm{EW}}$: QED$\otimes$QCD
  - $\mu = \mu_{\mathrm{EW}} = M_Z$: $f_\nu = f_t = f_W = f_Z = f_{\gamma Z} = 0$
  - $\mu_{\mathrm{EW}} < \mu$: EW$\otimes$QCD.

$$\begin{pmatrix} f_B \\ f_{W^3} \\ f_{BW^3} \end{pmatrix} = \begin{pmatrix} c_W^2 & s_W^2 & -2c_W s_W \\ s_W^2 & c_W^2 & 2c_W s_W \\ c_W s_W & -c_W s_W & c_W^2 - s_W^2 \end{pmatrix} \begin{pmatrix} f_\gamma \\ f_Z \\ f_{\gamma Z} \end{pmatrix}$$

- We work in the (B, W) basis. The technical details can be referred to the backup slides.

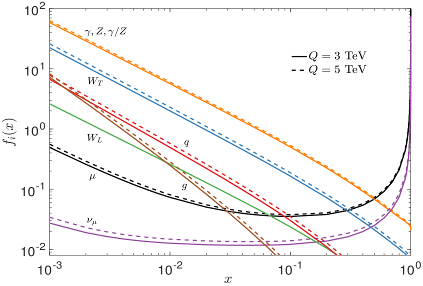
EWPDFs at a muon collider

- The sea leptonic and quark PDFs

$$\nu = \sum_i (\nu_i + \bar{\nu}_i), \ \ \ell\text{sea} = \bar{\mu} + \sum_{i\neq\mu} (\ell_i + \bar{\ell}_i), \ \ q = \sum_{i=d}^{t} (q_i + \bar{q}_i)$$

- The averaged momentum fractions [percentage]: $\langle x f_i \rangle = \int x f_i(x) \mathrm{d}x$

Q	μ	$\gamma, Z, \gamma Z$	$W^{\pm}$	ν	ℓsea	q	g
M_Z	97.9	2.06	0	0	0.028	0.035	0.0062
3 TeV	91.5	3.61	1.10	3.59	0.069	0.13	0.019
5 TeV	89.9	3.82	1.24	4.82	0.077	0.16	0.022



- W_L does not evolve, reflecting the residue of the EW broken (high-twist) effects
- We have neutrinos, quarks, and gluon, and everything as partons [Tao, Yang, Xie, 2007.14300].

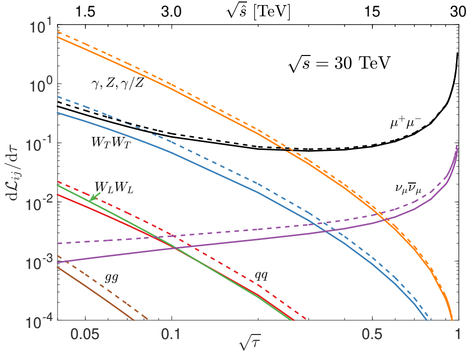
The parton luminosities

- Production cross sections

$$\sigma(\ell^+\ell^- \to F + X) = \int_{\tau_0}^{1} d\tau \sum_{ij} \frac{d\mathscr{L}_{ij}}{d\tau}\ \hat{\sigma}(ij \to F),\ \tau = \hat{s}/s$$

- Partonic luminosities

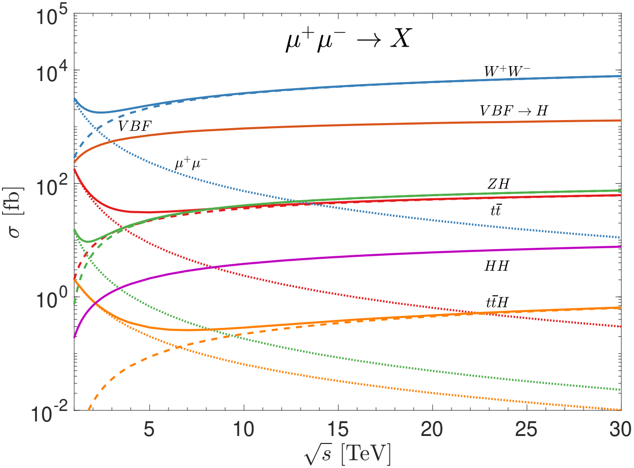
$$\frac{d\mathscr{L}_{ij}}{d\tau} = \frac{1}{1+\delta_{ij}} \int_{\tau}^{1} \frac{d\xi}{\xi} \left[f_i(\xi, Q^2) f_j\left(\frac{\tau}{\xi}, Q^2\right) + (i \leftrightarrow j) \right]$$



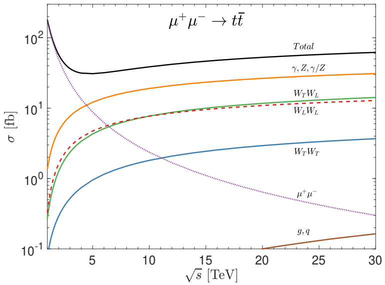
Semi-inclusive processes

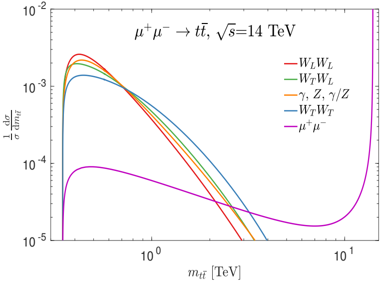
Just like in hadronic collisions:

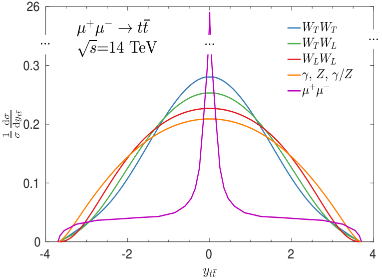
$$\mu^+\mu^- \to \text{exclusive particles} + \text{remnants}$$



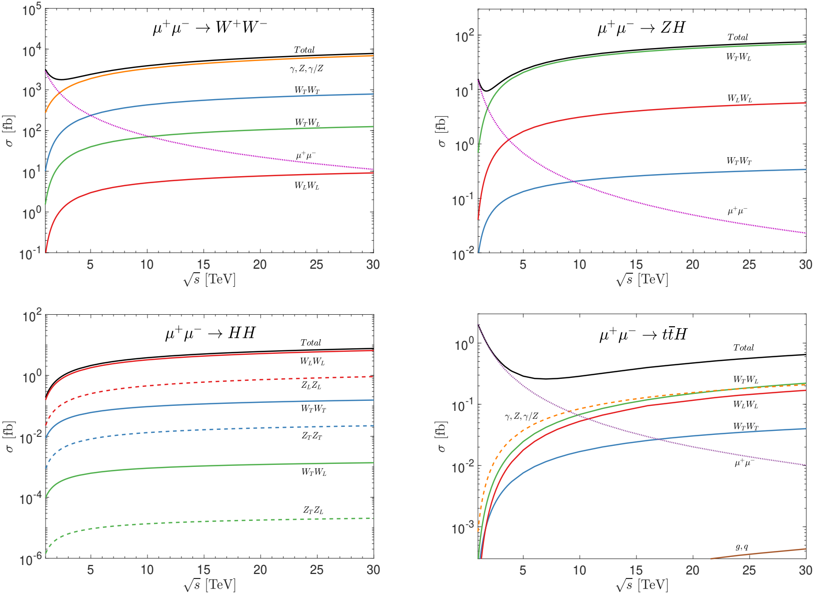
The decomposition and distributions







Other processes: $W^+W^-, ZH, HH, t\bar{t}H$



Summary and prospects

- At high energies, all particles become massless. The EW symmetry is asymptotically restored.
- The splitting phenomena dominate at high energies. The ISR can be factorized as the PDFs, the FSR as Fragmentations (parton shower).
- The EW PDFs are polarized, as well as the hard partonic cross sections, because of the chiral nature of the EW theory.
- The interference gives mixed PDFs, which can be either positive or negative. The contribution can be either constructive or destructive.
- Near the threshold (at low energies), the factorization breaks down. We need to match to the fixed-order calculation.
- The longitudinal PDFs (f_{V_L}) do not run at the leading log. But the contribution is very important due to the large Yukawa coupling.
- Bloch-Nordsieck theorem violation: Factorization breaks down for the insufficiently inclusive processes.
  - Cutoff (M_V/Q) to regulate the divergence;
  - Formulate fully inclusive observables to cancel all the divergence.

The contributions from $\nu\bar{\nu}$ and q, g