Pre-Equilibrium Quark Gluon Plasma and its Connection to Hydrodynamics

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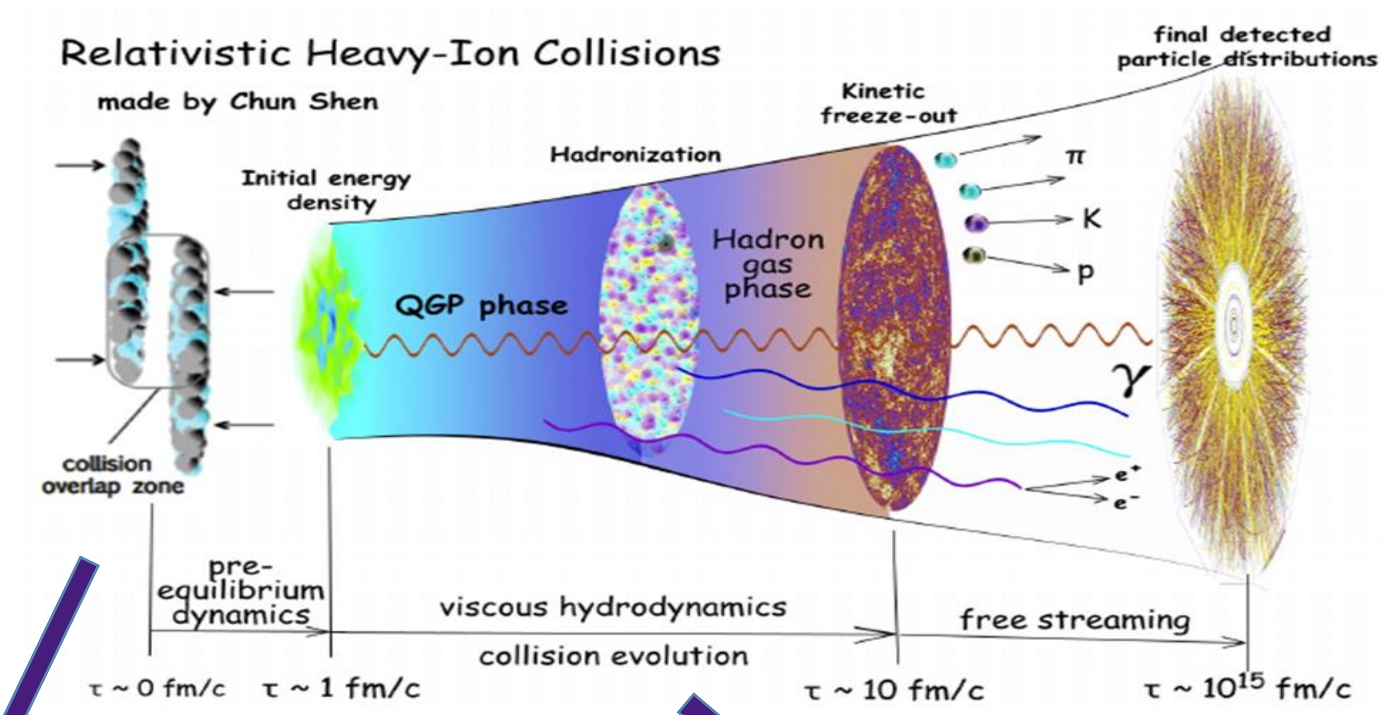
Pre-Equilibration Quark-Gluon Plasma



- Initial Collision
- Off-Thermal
- Gluon Saturation

- Pre-Equilibrium QGP
- Thermalization
- Chemical Equilibration

- Hydrodynamic
- Thermal
- Gluon/Quarks

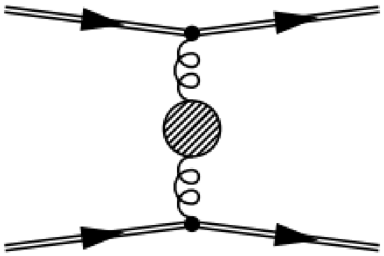
Effective Kinetic QCD

Effective Kinetic Theory (Arnold, Moore, Yaffe) at LO

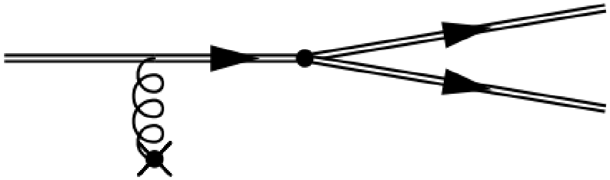
AMY,JHEP01 (2003) 030
AMY,JHEP0206(2002)030
Kurkela, Mazeliauskas,PRD99 (2019) 054018

$$\frac{\partial}{\partial t} f_a(\vec{p},t) = -C_a^{2\leftrightarrow 2}[f](\vec{p},t) - C_a^{1\leftrightarrow 2}[f](\vec{p},t) - C_a^{\mathrm{z-exp}}[f](\vec{p},t) \qquad a = g, u, \bar{u}, d, \bar{d}, s, \bar{s}$$

Explicitly solve Boltzmann equation for massless gluon and 3 light quarks/anti-quarks as an integro-differential equation including 2 ↔ 2 elastic processes and 1 ↔ 2 inelastic processes



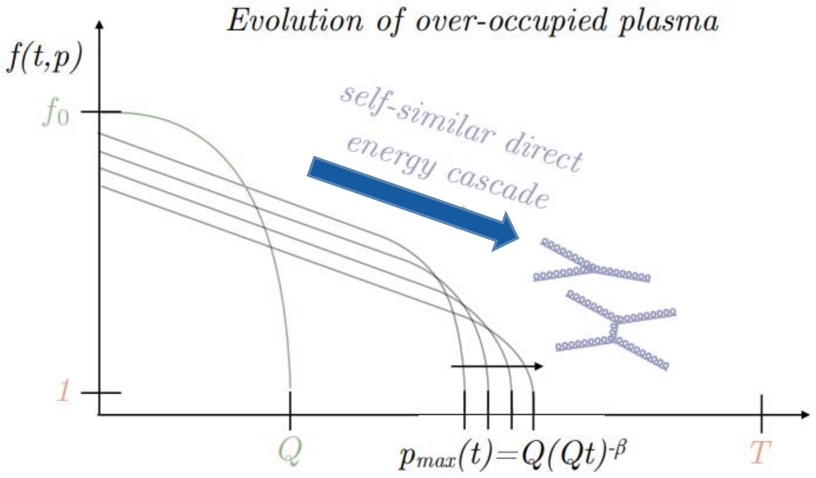
2 ↔ 2: Color screening by Debye mass fit to HTL calculation



1 ↔ 2: Collinear radiation including LPM effect via effective vertex resummation

Turbulence in QGP

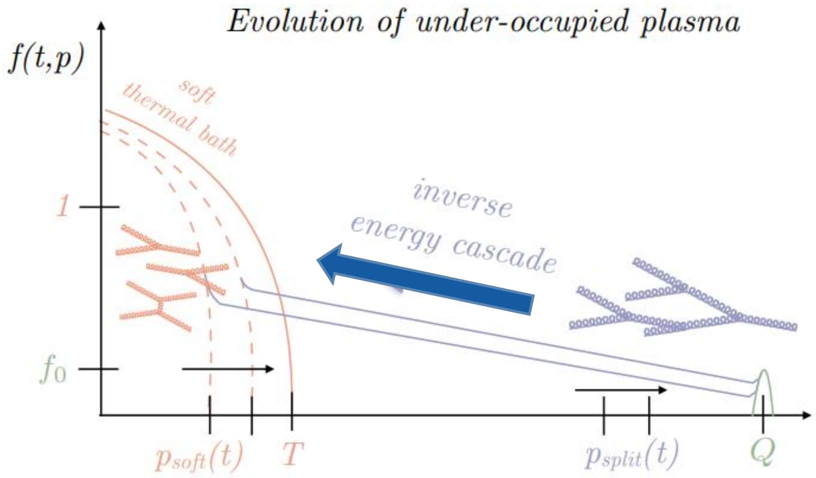
Weak-Coupling Thermalization



Over-occupied systems
<p> << T

Direct energy cascade

Far from equilibrium
Large separation of scales

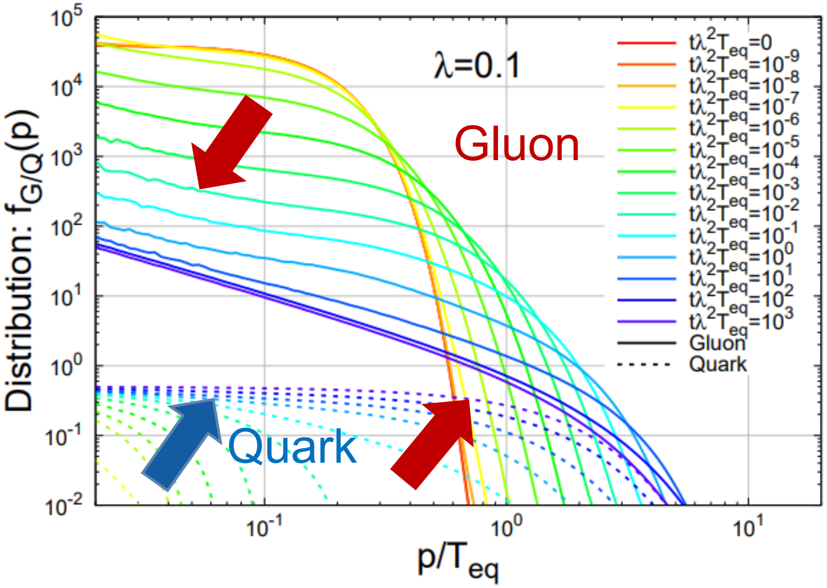


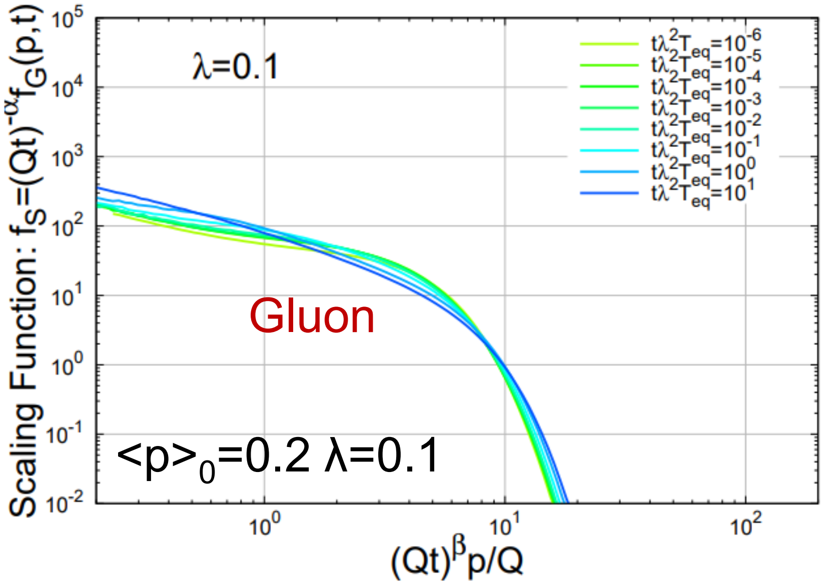
Under-occupied systems
<p> >> T

Inverse energy cascade

Schlichting, Teaney, Ann. Rev. of Nuc & Part. Sci. 69:447 (2019)

Over-Occupied Plasma





Self-similar Turbulence

$$f_G(p,t) = (Qt)^{\alpha} f_S\left((Qt)^{\beta}\frac{p}{Q}\right)$$

Universal Scaling Function

$$f_S\left((Qt)^{\beta}\frac{p}{Q}\right)$$

Scaling Exponents from pure Yang-Mills plasma

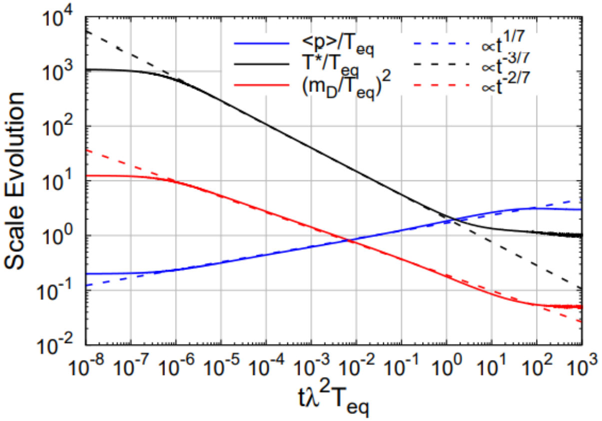
α=-4/7, β=-1/7

Also work for QCD plasma: Gluon domination

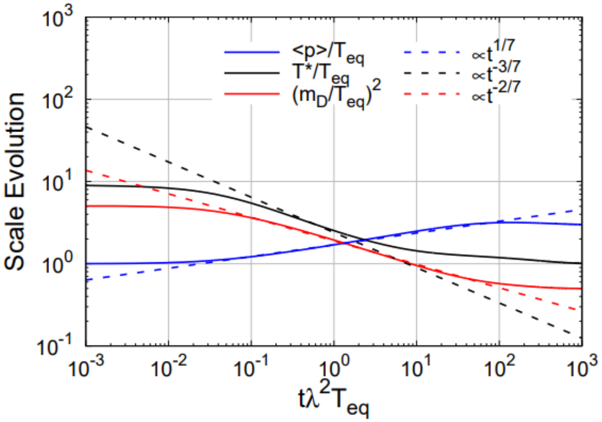
Quark spectra are following gluon spectrum

Berges, Boguslavski, Schlichting, Venugopalan, PRD89(2014)114007
Abraao York, Kurkela, Lu, Moore, PED89(2014)074036

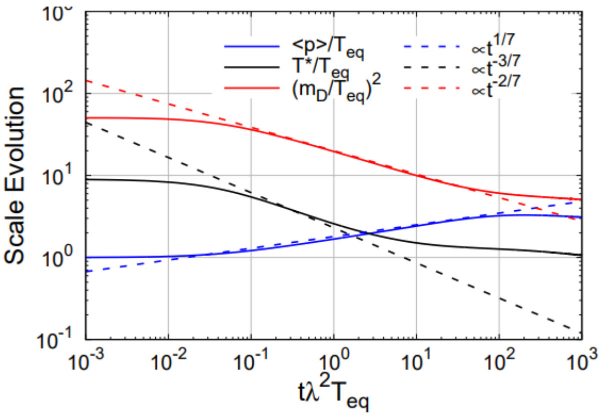
Over-Occupied Plasma



$\langle p\rangle_0=0.2$ $\lambda=0.1$



$\langle p\rangle_0=1.0$ $\lambda=1.0$



$\langle p\rangle_0=1.0$ $\lambda=10.0$

Self-similar Scaling: pow-law evolution

Not limited to pure Yang-Mills

Even work for stronger coupling

Chemical Equilibration

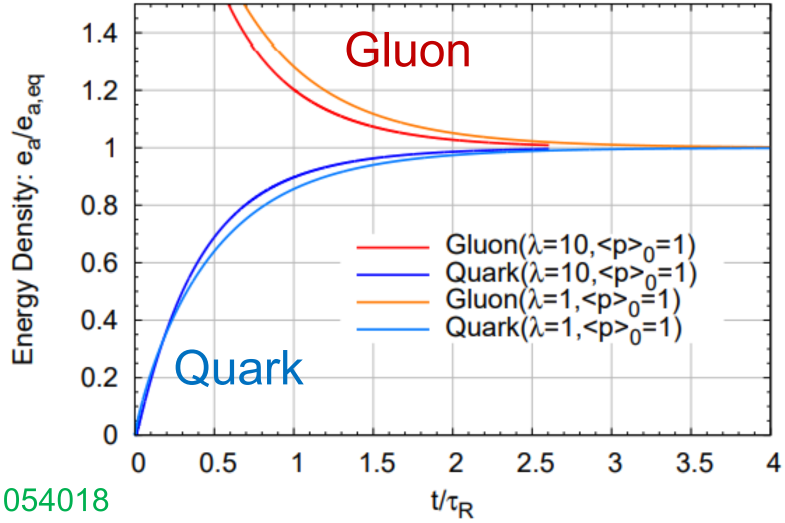
Later than kinetic equilibration

Equilibration relaxation time

$$\tau_R = \frac{4\pi\eta/s}{T_{eq}}$$

t'Hooft coupling

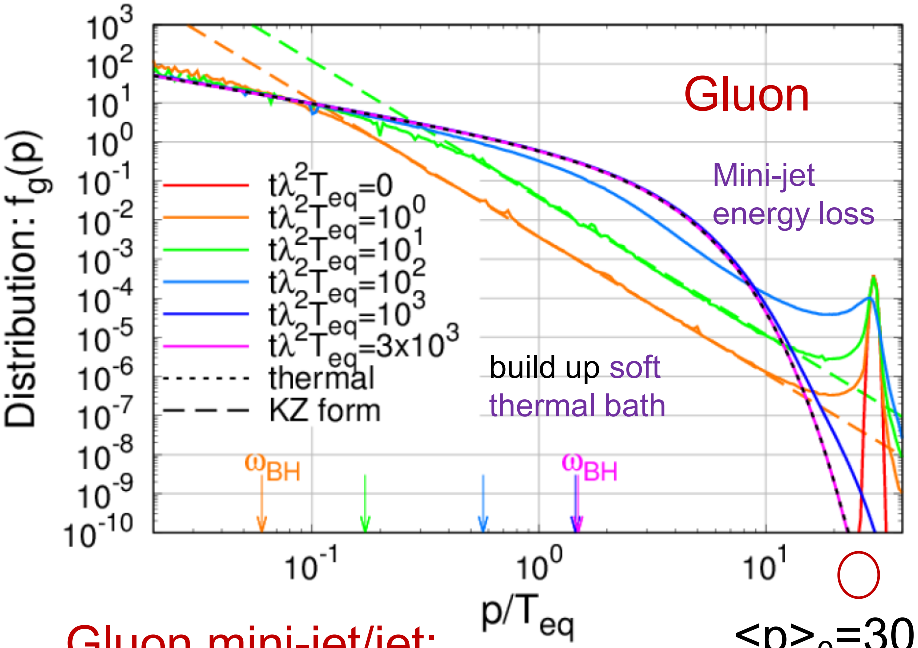
$$\lambda = 4\pi\alpha_s N_c$$

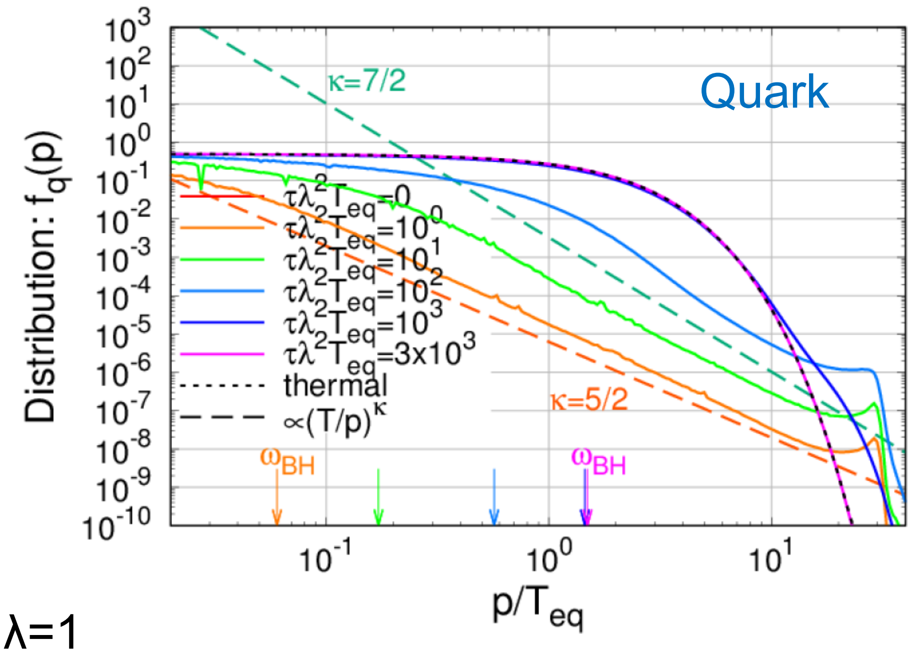


Kurkela, Mazeliauskas,PRD99 (2019) 054018

KOMPOST, PRC99 (2019) 034910

Under-Occupied Plasma



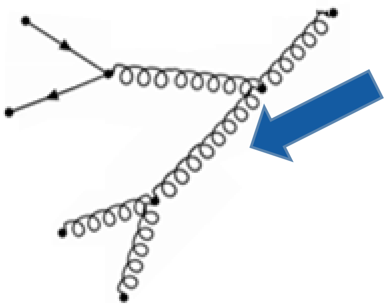


$\langle p\rangle_0=30$, $\lambda=1$

Gluon mini-jet/jet:

- Kolmogorov-Zakharov spectrum (exponent $\kappa=7/2$ for gluon)

$$f_{KZ}(\vec{p},t) = \eta(t)\left(\frac{\langle p\rangle_0}{p}\right)^{\kappa}$$

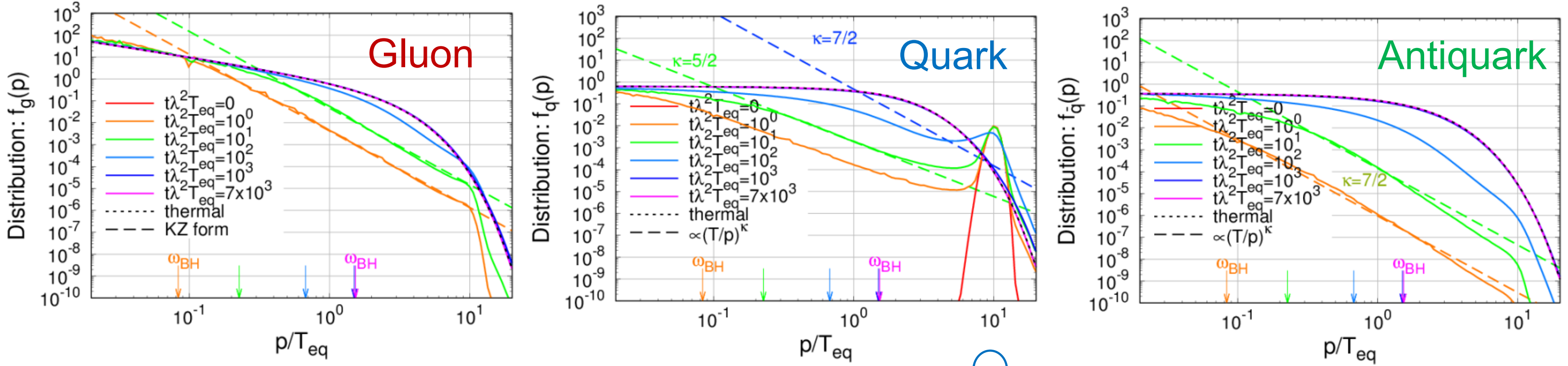


- Bottom-up thermalization

R. Baier, et al. PLB502(2001)51

1. Emission of (soft) quarks and gluons
2. Radiative breakup by multiple branchings -> build up soft thermal bath
3. Mini-jet energy loss -> heating up thermal bath

Under-Occupied Plasma



$\langle p\rangle_0=10,\ \lambda=1$

Quark mini-jet/jet:

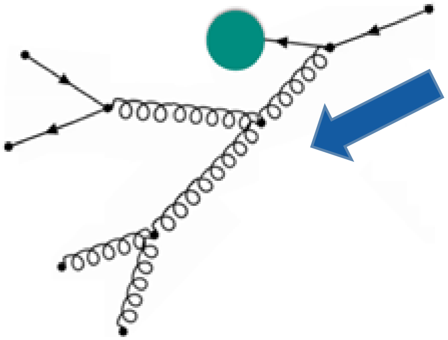
- Bottom up thermalization

- Kolmogorov-Zakharov spectrum

1. Quark follows κ=5/2 to κ=7/2
2. Gluon follows κ=7/2
3. Antiquark follows gluon (secondary production)

- Same pattern as for in-medium mini-jet/jet evolution with unified description of soft and hard sectors
- Equilibration of Jets

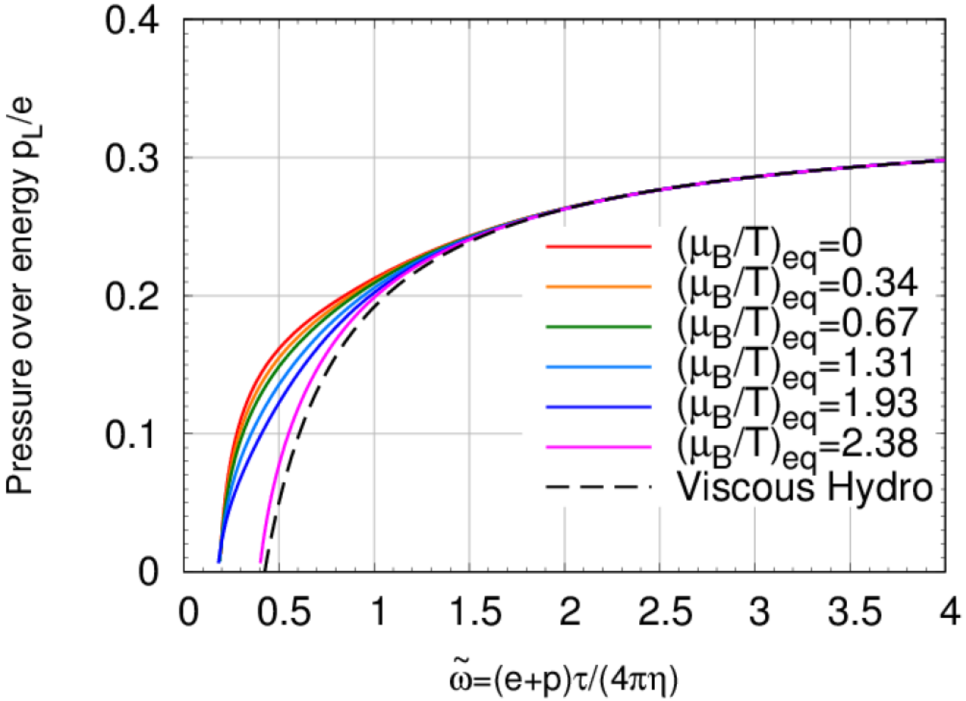
Soudi, Schlichting, 2008.04928



Hydrodynamization of QGP

Hydrodynamization

System initially highly anisotropic with CGC inspired gluon dist. & finite baryon/charge density



1st-order hydrodynamics near equilibrium

$$\frac{p_L}{e} = \frac{1}{3} - \frac{4}{9\pi}\left(\frac{\eta T_{\mathrm{eff}}}{e+p}\right)\frac{4\pi}{\tau T_{\mathrm{eff}}}$$

const.

Isotropization:

Larger chemical potential

↓

Larger fraction of quarks

↓

Slower isotropization

Ineffectiveness of quark interaction:

Spin degeneracy

Quantum statistics

Insensitive to initial conditions:

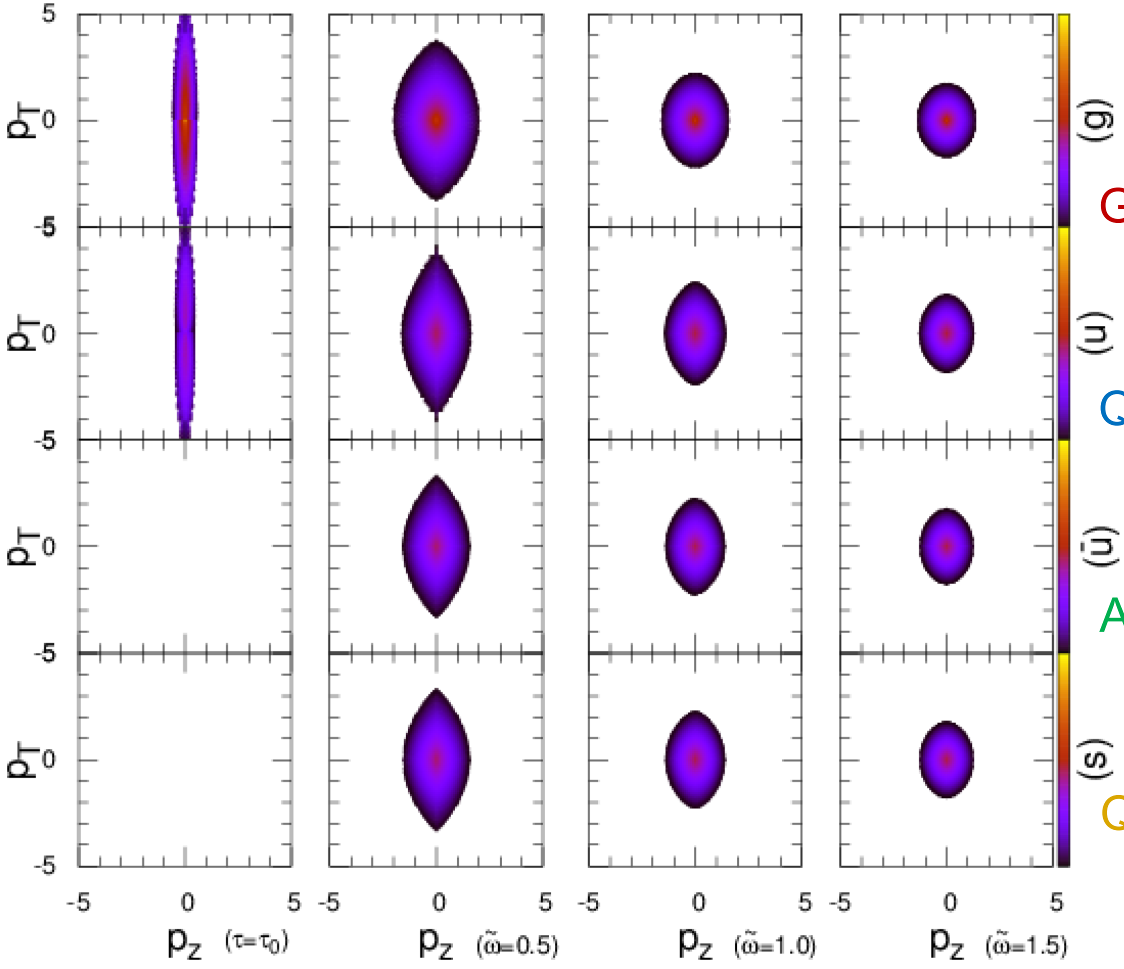
Non-equilibrium attractors from kinetic theory

↓

Effective constitutive relations

far-from equilibrium $\frac{p_L}{e} = f(\tilde{\omega})$

Isotropization



Gluon

Quark

Antiquark

Quark

$\mu_B/T=0.34$

Gluon isotropy faster

Quarks isotropy slower

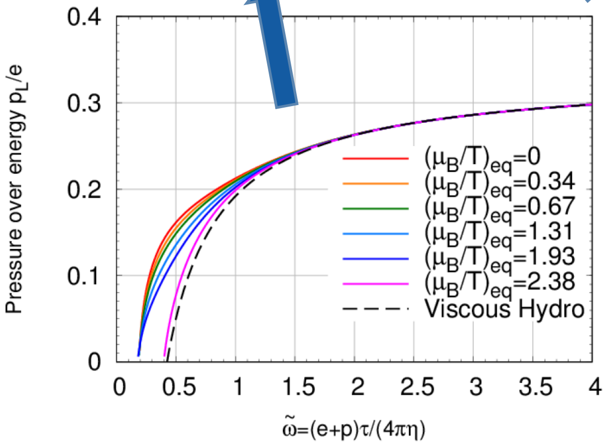
Systems Isotropization:

$p_L/e \rightarrow 1/3$

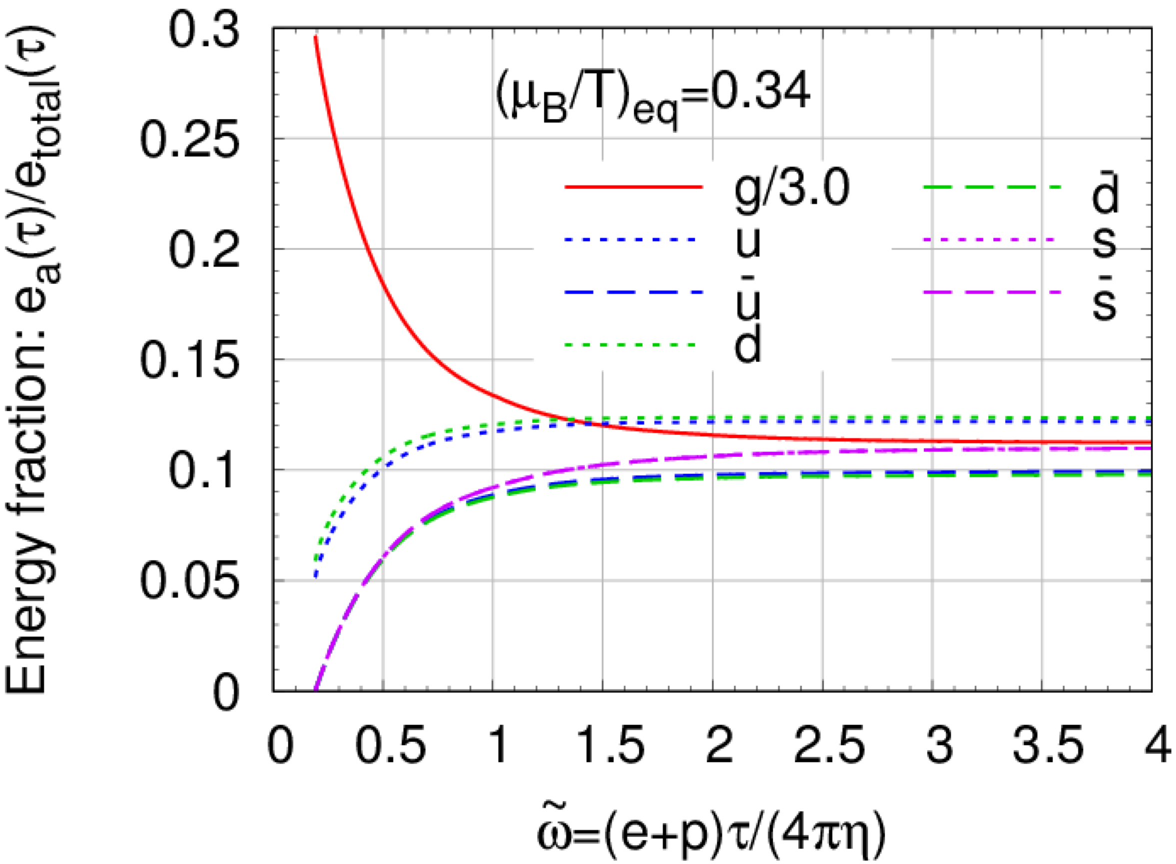
with long time

Hydrodynamization:

$$\sim 1.5 \left(\frac{\eta T_{\rm eff}}{e+p}\right)\frac{4\pi}{T_{\rm eff}}$$



Kinetic and Chemical Equilibration



Chemical Reaction:
Energy transfer
Quark/antiquark produced in pairs

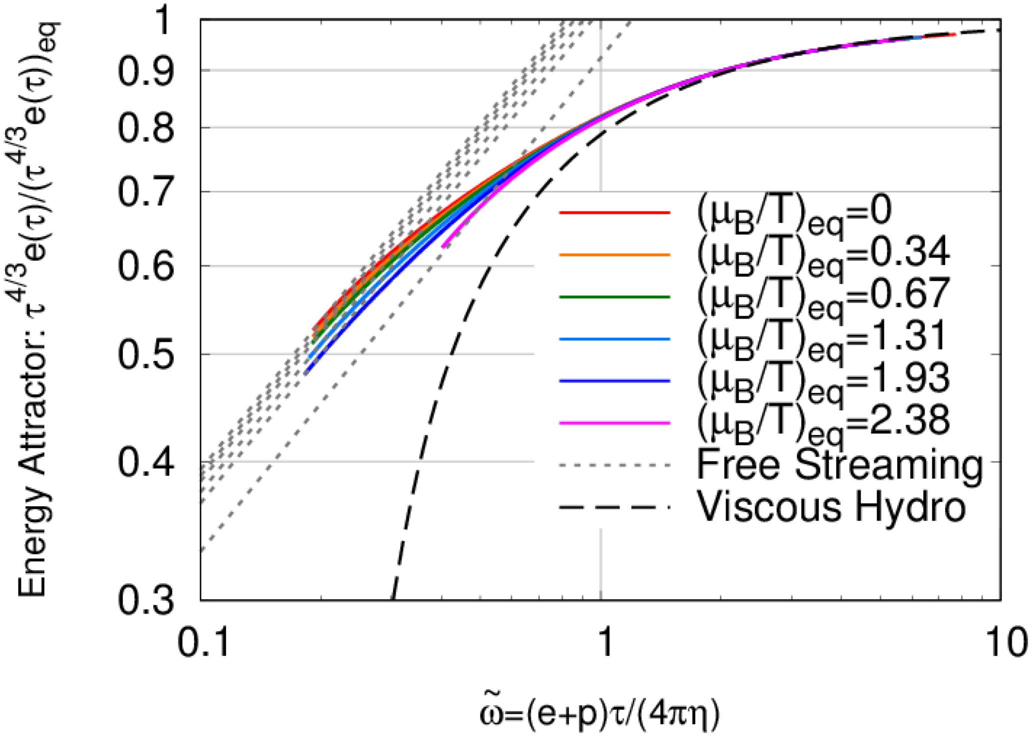
Chemical equilibration:

$$\sim 2\left(\frac{\eta T_{\rm eff}}{e+p}\right)\frac{4\pi}{T_{\rm eff}}$$

Quark/antiquark asymmetry:
More quarks than antiquarks at finite density
Net baryon density conserved

With all light parton degrees of freedom:
Realistic matching to hydrodynamics at finite density
(heavy-ion collisions at RHIC, forward rapidity at LHC, etc…)

Energy Attractor



Energy attractor:

$$\mathcal{E}\left(\tilde{\omega} = \left(\frac{e+p}{\eta T_{\rm eff}}\right)\frac{\tau T_{\rm eff}}{4\pi}\right) = \frac{\tau^{\frac{4}{3}}e}{\left(\tau^{\frac{4}{3}}e\right)_{\rm eq}}$$

Asymptotes:

$$\mathcal{E}(\tilde{\omega} \gg 1) \simeq 1 - \frac{2}{3\pi\tilde{\omega}}$$ Hydrodynamics

$$\mathcal{E}(\tilde{\omega} \ll 1) \simeq C_\infty^{-1}\tilde{\omega}^{\frac{4}{9}}$$ Free streaming

Pre-equilibrium description connects initial state to hydrodynamics

$$\left(\tau^{\frac{4}{3}}e\right)_{\tilde{\omega}} = \left(4\pi\frac{\eta T_{\rm eff}}{e+p}\right)^{\frac{4}{9}}\left(\frac{\pi^2\nu_{\rm eff}}{30}\right)^{\frac{1}{9}}\boxed{(e\tau)_0^{\frac{8}{9}}}C_\infty\mathcal{E}(\tilde{\omega})$$

$$(\tau\Delta n_f)_{\tilde{\omega}} = \boxed{(\tau\Delta n_f)_0}$$

Input to hydrodynamics through pre-equilibrium evolution

Giacalone, Mazeliauskas, Schlichting PRL123(2019)26

Entropy Production and Scale Fixing

In equilibrium:

Entropy:

$$\boxed{(s\tau)_{\mathrm{eq}}} = \frac{\left(e + p - \sum_f \mu_f \Delta n_f\right)\tau}{T}$$

Net Baryon Number:

$$\boxed{\Delta n_B} = \frac{1}{3}\Delta n_u + \frac{1}{3}\Delta n_d$$

Fixed from experimental/lattice data

Charged particle multiplicity:

$$\frac{dN_{ch}}{d\eta} \simeq \frac{N_{ch}}{JS}\boxed{(\tau s)_{\mathrm{eq}}}S_T \simeq 0.12(\tau s)_{\mathrm{eq}}S_T$$

Entropy per baryon:

$$\frac{S}{N_B} = \boxed{\left(\frac{\tau s}{\tau \Delta n_B}\right)_{\mathrm{eq}}}$$

Non-equilibrium from attractor:

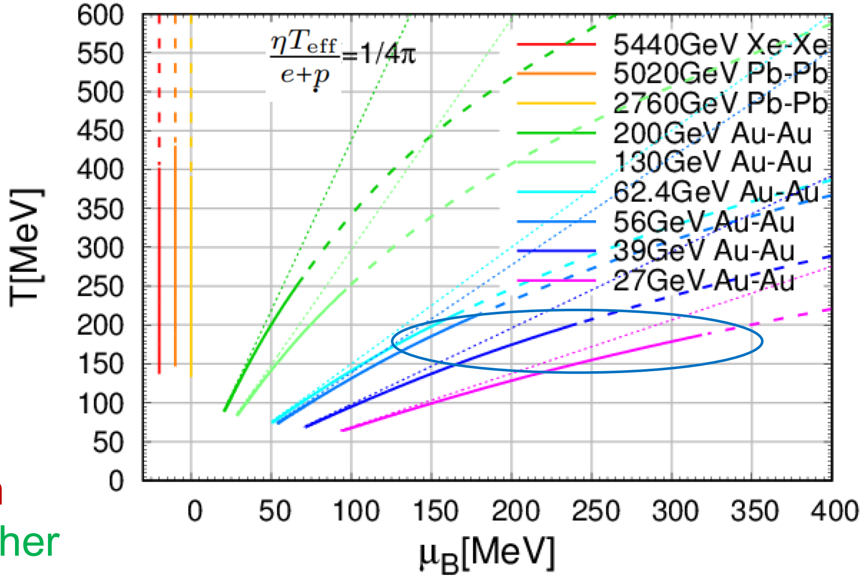
$$\left(\tau^{\frac{4}{3}}e\right)_{\tilde{\omega}} = \left(4\pi\frac{\eta T_{\mathrm{eff}}}{e+p}\right)^{\frac{4}{9}}\left(\frac{\pi^2\nu_{\mathrm{eff}}}{30}\right)^{\frac{1}{9}}(e\tau)_0^{\frac{8}{9}}C_\infty\mathcal{E}(\tilde{\omega})$$

$$(\tau\Delta n_f)_{\tilde{\omega}} = (\tau\Delta n_f)_0$$

Landau matching:

Time-dependent T and μ

Non-equilibrium trajectory to higher baryon density



To what extent does hydro occur at lower-energy ?

Summary

- Turbulence in QGP
  - Over-occupied system follows a self-similar universal scaling, not limited to pure Yang-Mills theory but also for QCD, even for moderately strongly coupled system
  - Under-occupied system follows a bottom-up thermalization
- Hydrodynamization of QGP
  - Ineffectiveness of quarks interaction in isotropization / equilibration
  - Kinetic theory provides effective constitutive relation far from equilibrium
  - Hydrodynamization ~ 1.5 Kinetic equilibration time << Isotropization time
  - Realistic matching to hydrodynamics at finite density with universal attractor and fixed certain scales from experiments (charged particle multiplicity, baryon density, etc…)