# Momentum resolution correction in femtoscopic measurements

Paweł Szymański (for the STAR Collaboration)

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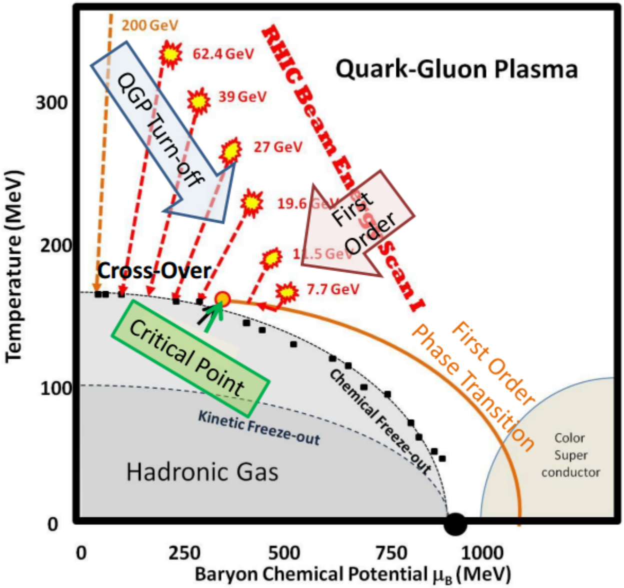
Supported by the Polish National Science Centre grant no. 2017/27/B/ST2/01947
Supported by IDUB-POB-FWEiTE-1 project by WUT

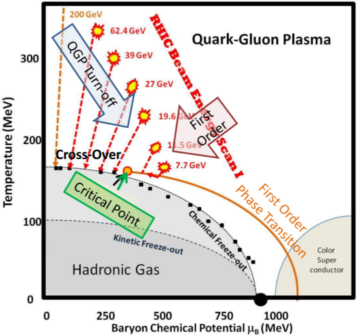
# Beam Energy Scan at STAR



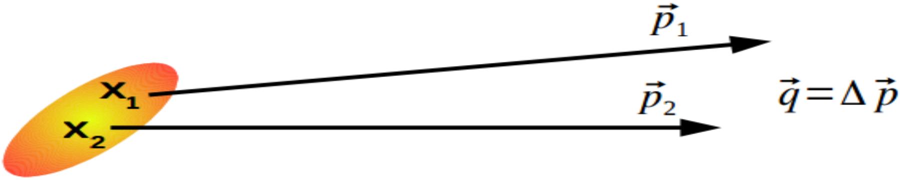
# BES goals

**Analyze all BES energies and find answers:**

- Search for turn-off of QGP signatures
- Search for the QCD critical point
- Search for the signals of phase transition/phase boundary



# Correlation Function




- analyze many pairs of particles $(\vec{p_1}, \vec{x_1})$ and $(\vec{p_2}, \vec{x_2})$ with relative momentum $\vec{q} = \vec{p_1} - \vec{p_2}$

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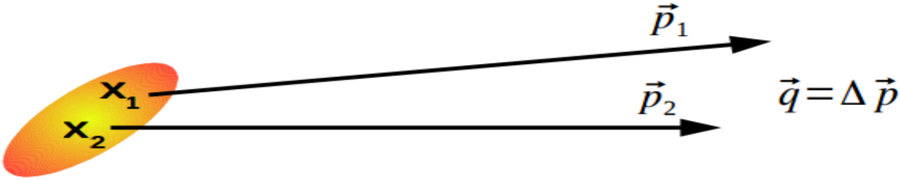


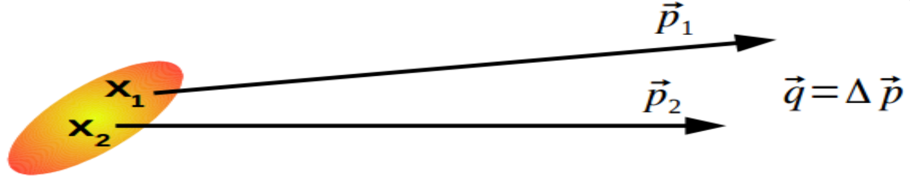

- analyze many pairs of particles $(\vec{p_1}, \vec{x_1})$ and $(\vec{p_2}, \vec{x_2})$ with relative momentum $\vec{q} = \vec{p_1} - \vec{p_2}$
- calculate correlation function (CF) of pairs:

$$CF(\vec{p_1}, \vec{p_2}) = \frac{P_2(\vec{p_1}, \vec{p_2})}{P_1(\vec{p_1})P_1'(\vec{p_2})}$$

$P_2(\vec{p_1}, \vec{p_2})$ — probability of observing two particles with momentum $\vec{p_1}$ and $\vec{p_2}$ at the same time and the same place

$P_1(\vec{p_1})$, $P_1'(\vec{p_2})$ — probability of observing two particles with momentum $\vec{p_1}$ and $\vec{p_2}$ separately

# Correlation Function

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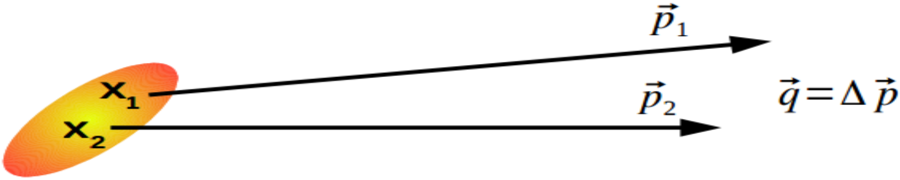
experimental correlation function:

$$CF(\vec{q}) = \frac{A(\vec{q})}{B(\vec{q})}$$

# Correlation Function

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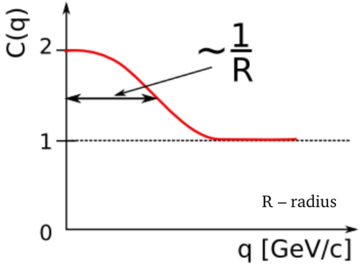


- calculate size of the source

$P_2(\vec{p_1}, \vec{p_2})$ — probability of observing two particles with momentum $\vec{p_1}$ and $\vec{p_2}$ at the same time and the same place

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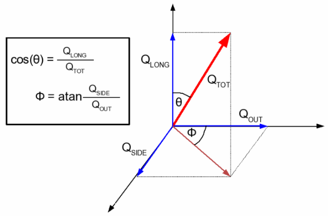
SH representation of 3D correlation function as a set of 1D plots

$$C(\mathbf{q}) = \sum_{l,m} C_l^m(q) Y_l^m(\theta, \phi) \qquad C_l^m(q) = \int_\Omega C(q, \theta, \phi) Y_l^m(\theta, \phi) d\Omega$$

$\Omega$ - full solid angle
$Y_l^m(\theta, \phi)$ - spherical harmonic function
$q = |\mathbf{q}|$ - pair relative momentum
$\theta$ and $\phi$ - polar and azimuthal angle




P. Danielewicz and S. Pratt.
Phys. Lett B618, 60 (2005)
Phys. Rev. C75, 034907 (2007)

Z. Chajecki and M. Lisa
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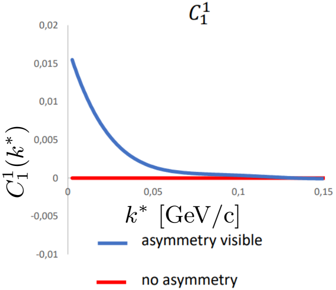
$C_0^0 \rightarrow$ sensitive to the size of the emitting source (shapes same as correlation function)

$C_1^1 \rightarrow$ sensitive to the spacetime emission asymmetry

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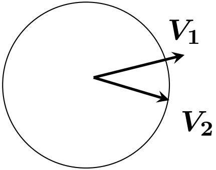
# Non-identical particle combinations

## Time asymmetry

$t_1 \neq t_2$
$\Delta r = 0$

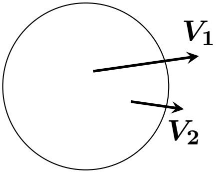



$t_1 > t_2$ - Catching up

$t_1 < t_2$ - Run away

## Space asymmetry

$t_1 = t_2$
$\Delta r \neq 0$

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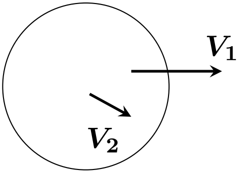


Run away

t — emission time
r — emission point distance from the center

R. Lednicky, et al.,
Phys. Lett. B373,
30-34 (1996)

**Catching up**
longer interaction,
strong correlation
**Running away**
shorter interaction,
weaker correlation

# Non-identical particle combinations

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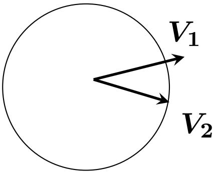


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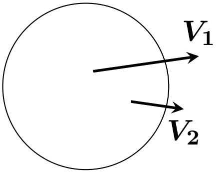


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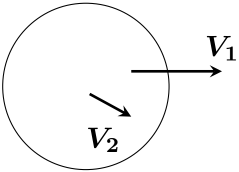


Run away

$C_+(k^*)$ — pions catch up with kaons
$C_-(k^*)$ — pions move away from kaons

Double Ratio:

$$DR(k^*) = \frac{C_+(k^*)}{C_-(k^*)}$$

# THERMINATOR model

**THERMal** heavy-**I**o**N** Gener**ATOR**

- Generates collisions of relativistic ions
- Uses Monte Carlo methods
- Implements thermal models of particle production with single freeze-out

THERMINATOR: THERMal heavy-IoN generATOR
A. Kisiel,T. Tałuć, W. Broniowski, W. Florkowski.
Comput.Phys.Commun. 174 (2006) 669-687

THERMINATOR is a Monte Carlo event generator designed for studying of particle production in relativistic heavy-ion collisions performed at such experimental facilities as the SPS, RHIC, or LHC. The program implements thermal models of particle production with single freeze-out.

# Thermiantor for BES program

| $\sqrt{s_{NN}}[GeV]$ | T [MeV] | $\mu_B$ [MeV] | $\mu_S$ [MeV] | $\mu_{I_3}$ [MeV] |
|---|---|---|---|---|
| 7.7 | 138.95 | 406.36 | 93.026 | -10.378 |
| 11.5 | 150.12 | 303.22 | 70.369 | -7.825 |
| 19.6 | 156.17 | 196.77 | 45.951 | -5.189 |
| 27 | 157.60 | 148.99 | 34.991 | -4.006 |
| 39 | 158.38 | 106.89 | 25.335 | -3.064 |
| 62.4 | 158.78 | 68.92 | 16.626 | -2.024 |

T - temperature
$\mu_B$ - barion potential
$\mu_S$ - strangeness potential
$\mu_{I_3}$ - third component of isospin

"Adaptation of the therminator model for BES program",
H. Zbroszczyk
Proc.SPIE Int.Soc.Opt.Eng. 11581 (2020) 1158104

# Thermintor for BES program

| $\sqrt{s_{NN}}[GeV]$ | $\tau$ [fm] | $\rho_{max}$ [fm] | $V_T$ |
|---|---|---|---|
| 7.7 | 8.3 | 8 | 0.65 |
| 11.5 | 8.35 | 8 | 0.8 |
| 19.6 | 8.75 | 8.2 | 0.85 |
| 27 | 8.75 | 8.85 | 0.8 |
| 39 | 8.6 | 8.7 | 0.75 |
| 62.4 | 9.4 | 9 | 0.75 |

$V_T$ - parameter specific to the Blast-Wave model, denoting velocity

$\tau$, $\rho_{max}$ - geometric parameters

$\rho_{max}^2 \cdot \tau \simeq V$

$V$ is the volume of source

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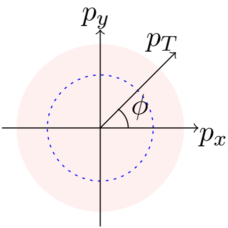
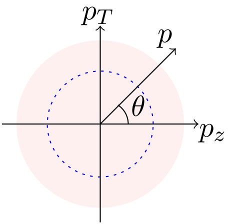
# Momentum resolution

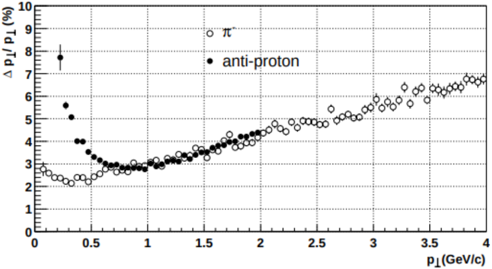



$$p_T = R|q||\vec{B}| = p\sin\theta \qquad p_z = p\cos\theta$$

$$p_x = p_T\cos\phi \qquad p_y = p_T\sin\phi$$

R - radius of fitted helise to points of particle track in TPC




Transverse momentum ($p_T$) resolution of the STAR TPC for $\pi^-$ and anti-protons in the 0.25 T magnetic field ($\vec{B}$) (embedding)

Nucl.Instrum.Meth.A499:659-678,2003

# Momentum resolution

**How to add momentum resolution effect?**

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**How to add momentum resolution effect?**

Resolution of momentum, azimuthal and polar angles:

$$\Delta p = a_p + b_p p^{\alpha_p} + c_p p$$

$$\Delta \phi = a_\phi + b_\phi \phi^{\alpha_\phi}$$

$$\Delta \theta = a_\theta + b_\theta \theta^{\alpha_\theta}$$

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Standard deviations:

$$\sigma_{p_x}^2 = (|p_x|\frac{\Delta p}{p})^2 + (|p_y|\Delta\phi)^2 + (|\frac{p_x}{\operatorname{tg}\theta}|\Delta\theta)^2$$

$$\sigma_{p_y}^2 = (|p_y|\frac{\Delta p}{p})^2 + (|p_x|\Delta\phi)^2 + (|\frac{p_y}{\operatorname{tg}\theta}|\Delta\theta)^2$$

$$\sigma_{p_z}^2 = (|p_z|\frac{\Delta p}{p})^2 + (|p_z \operatorname{tg}\theta|\Delta\theta)^2$$

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$$\sigma_{p_z}^2 = (|p_z|\frac{\Delta p}{p})^2 + (|p_z \operatorname{tg}\theta|\Delta\theta)^2$$

Smeared momentum:

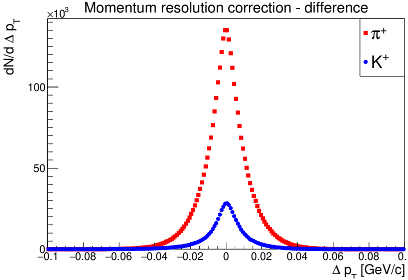
$$p_x^{smeared} = p_x^{real} + \partial p_x$$

$$p_y^{smeared} = p_y^{real} + \partial p_y$$

$$p_z^{smeared} = p_z^{real} + \partial p_z$$

where $\partial p_x$, $\partial p_y$ and $\partial p_z$ calulated from Gaussian distribution with the above standard deviations and mean 0

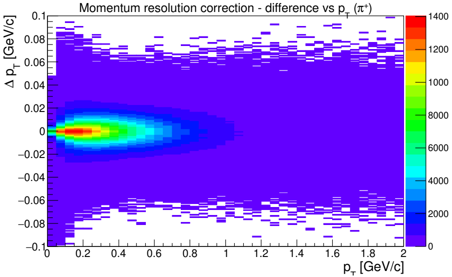
# Momentum resolution



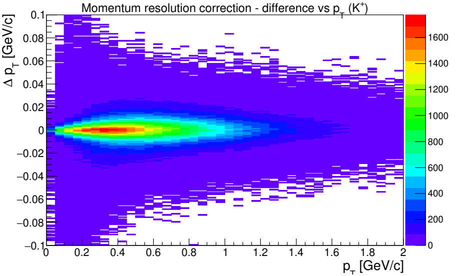

From Therminator 2

$$p_T = \sqrt{p_x^2 + p_y^2}$$

$$\Delta p_T = p_T^{raw} - p_T^{smeared}$$



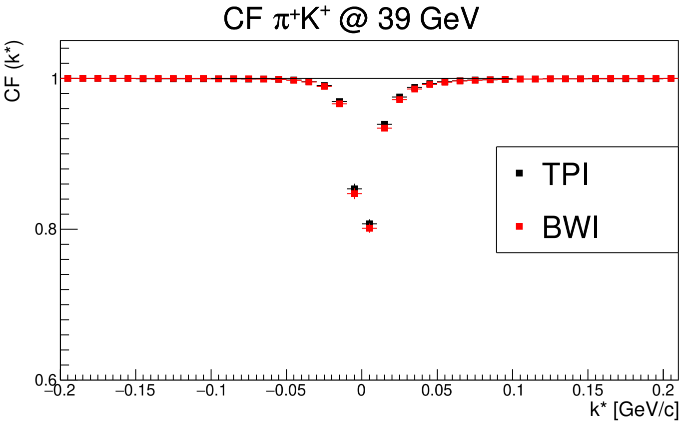

positive pion




positive kaon

# Software for calculating correlation functions

- **BWIntegrate** — software created by Richard Lednicky (using Lednicky weights)
- **TPI** — software written by Adam Kisiel (using wave function)




Obtained functions are similar

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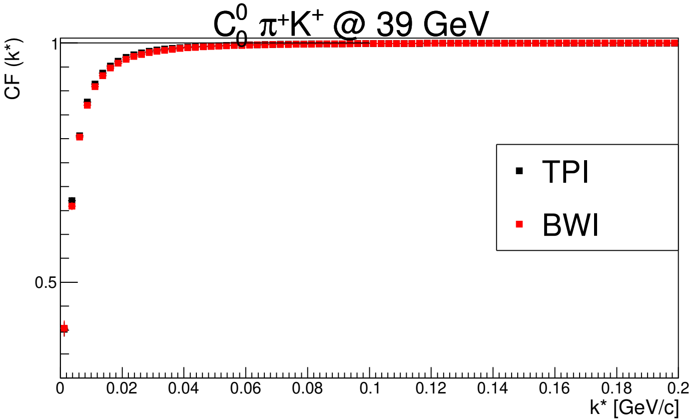


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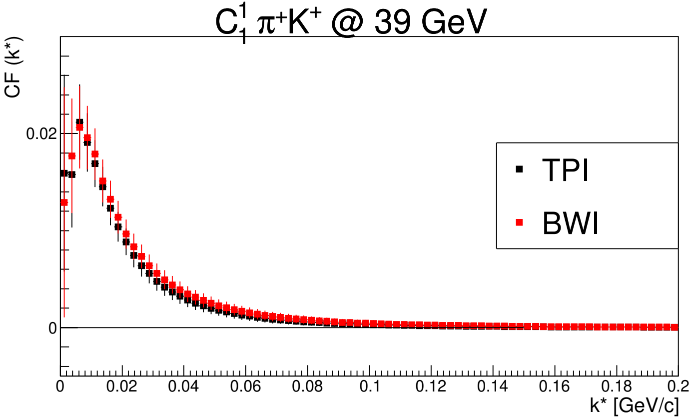


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Therminator2

$\mathrm{CF}^{raw}$

interaction from $\mathrm{p}^{raw}$

$k^*$ from $\mathrm{p}^{raw}$

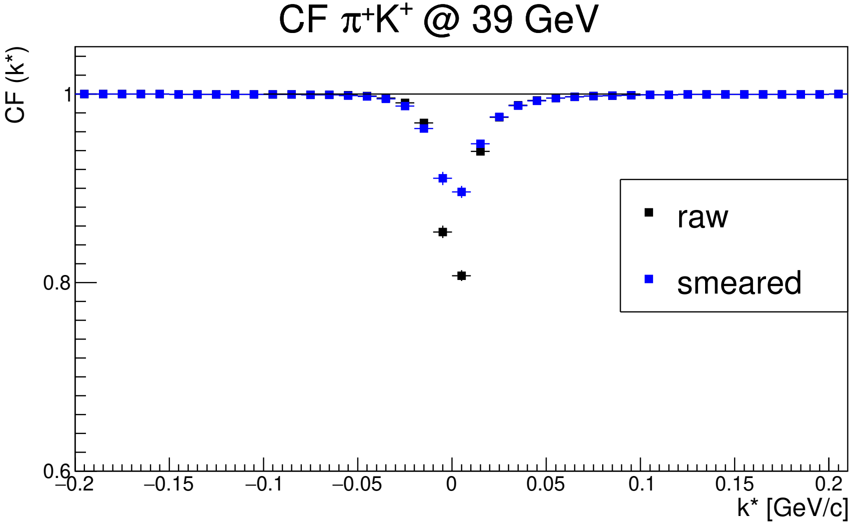
$\mathrm{CF}^{smear}$

interaction from $\mathrm{p}^{raw}$

$k^*$ from $\mathrm{p}^{smear}$

- Particles know their momenta → **weights from true (raw) momenta**
- We measure momentum of the particles → $\mathbf{k}^*$ **from smeared momenta**

# $\pi^+ K^+$ CF @ 39 GeV




Visible effect at low $k^*$

Only TPI

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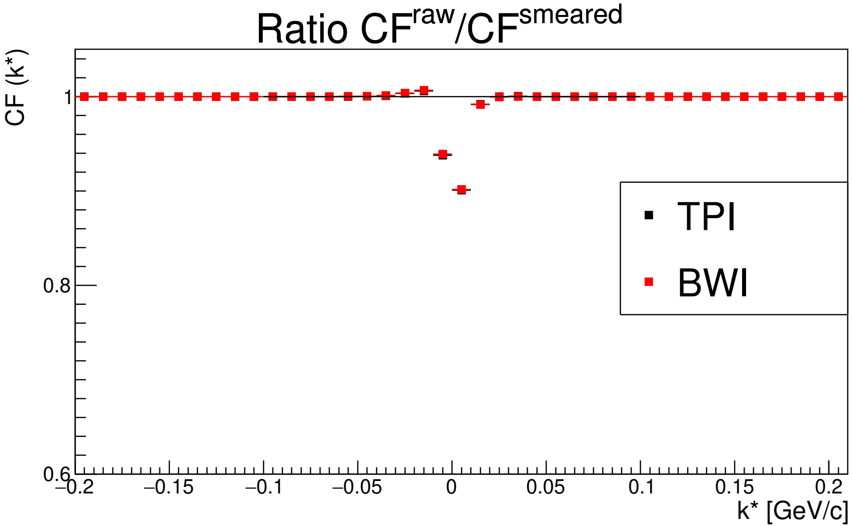
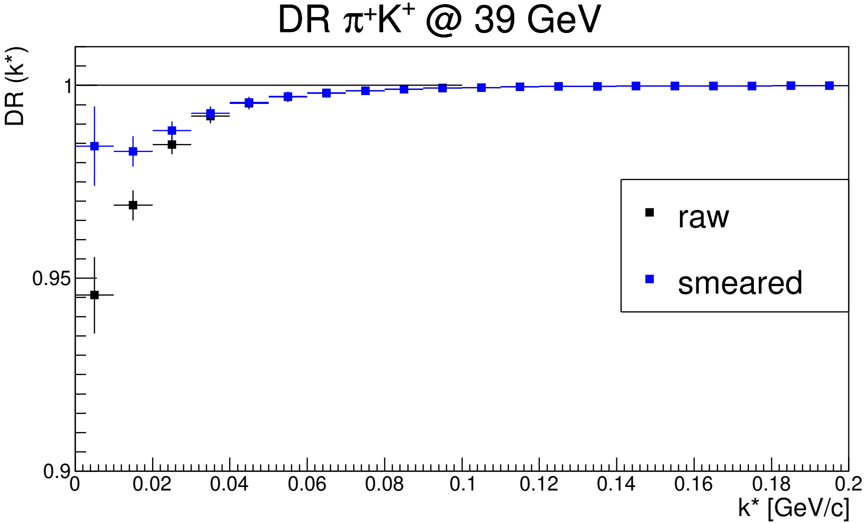



At $|k^*| < 0.02$ visible effect

TPI/BWI comparison

# $\pi^+K^+$ Double Ratio (DR) @ 39 GeV

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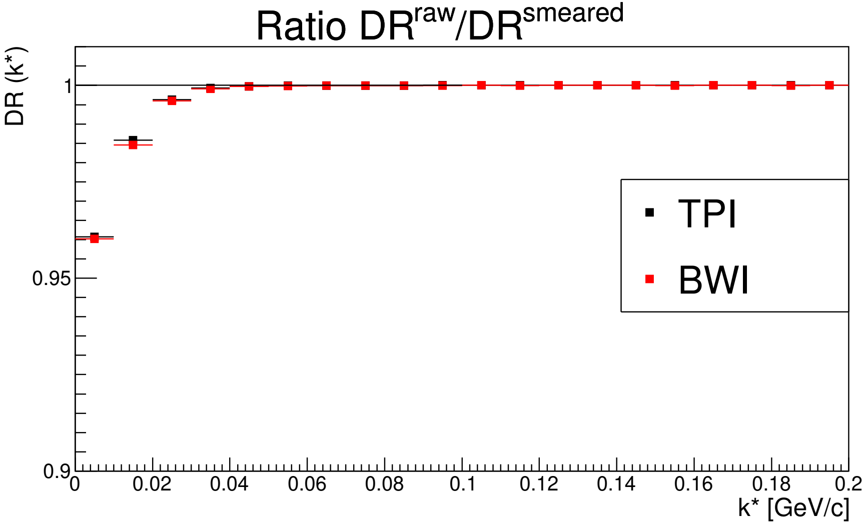
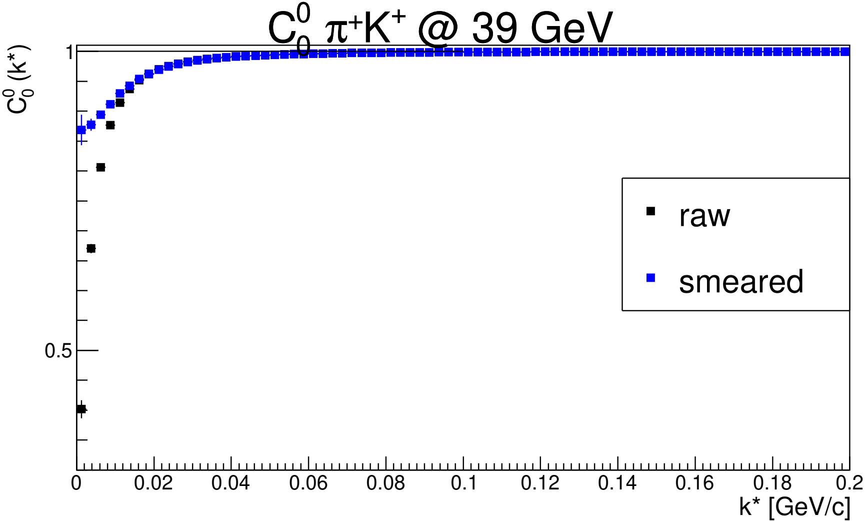



At $|k^*| < 0.04$ visible effect

TPI/BWI comparison

# $\pi^+ K^+ \; C_0^0$ @ 39 GeV

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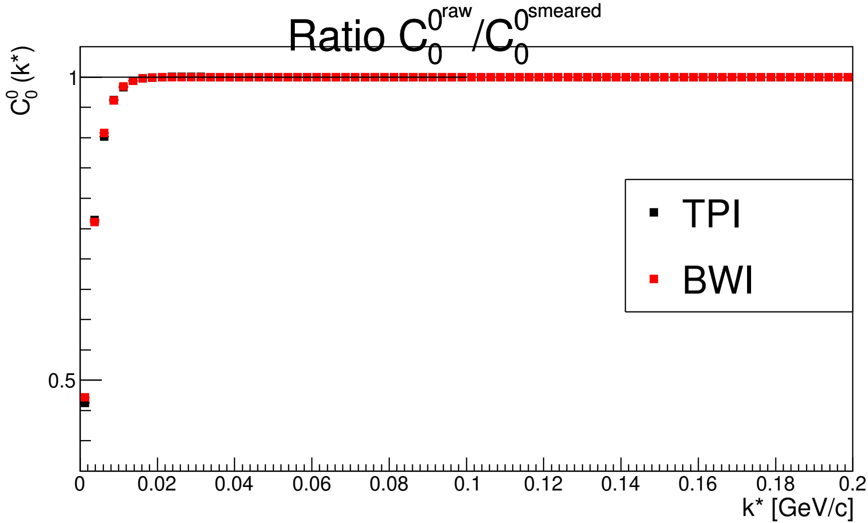


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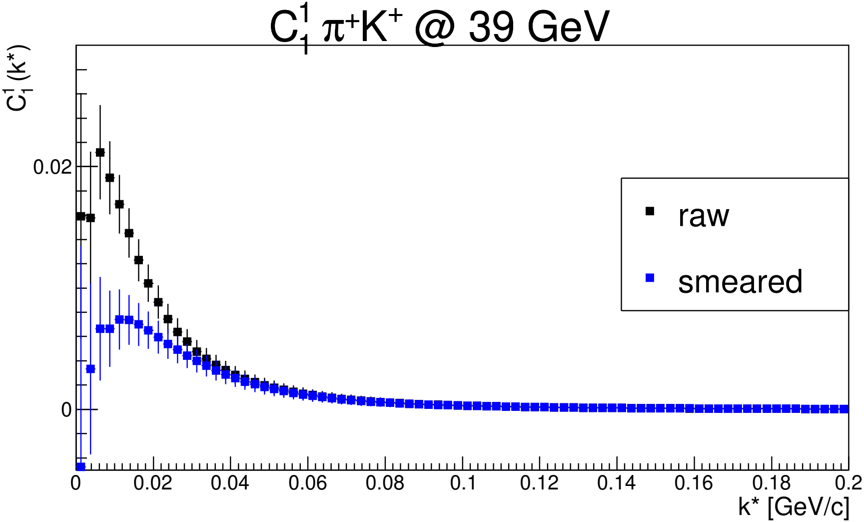
TPI/BWI comparison

# $\pi^+ K^+ \; C_1^1$ @ 39 GeV

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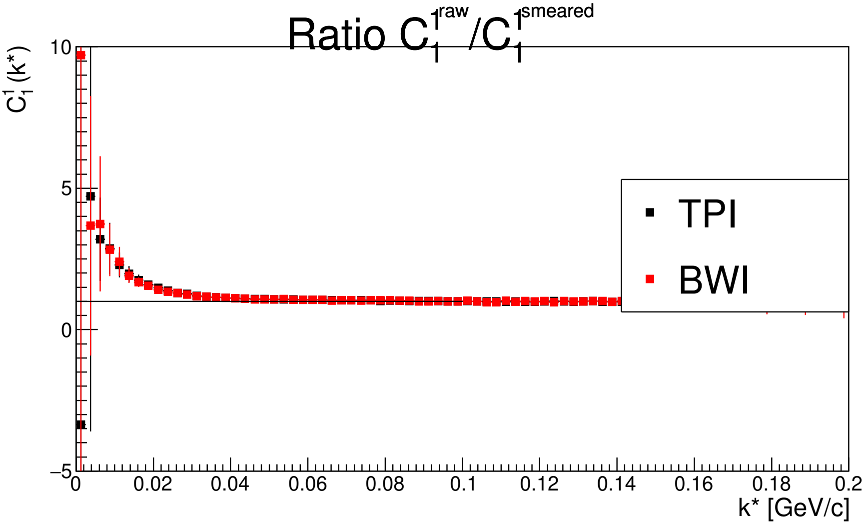


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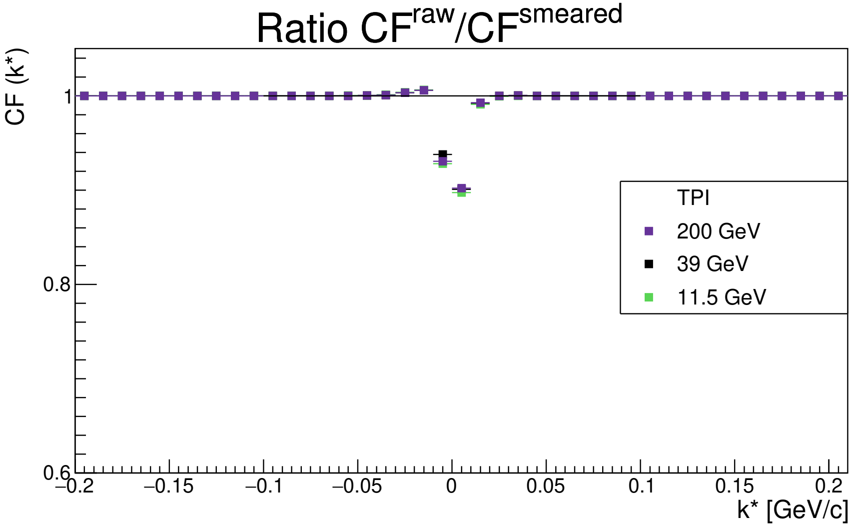


similar effect

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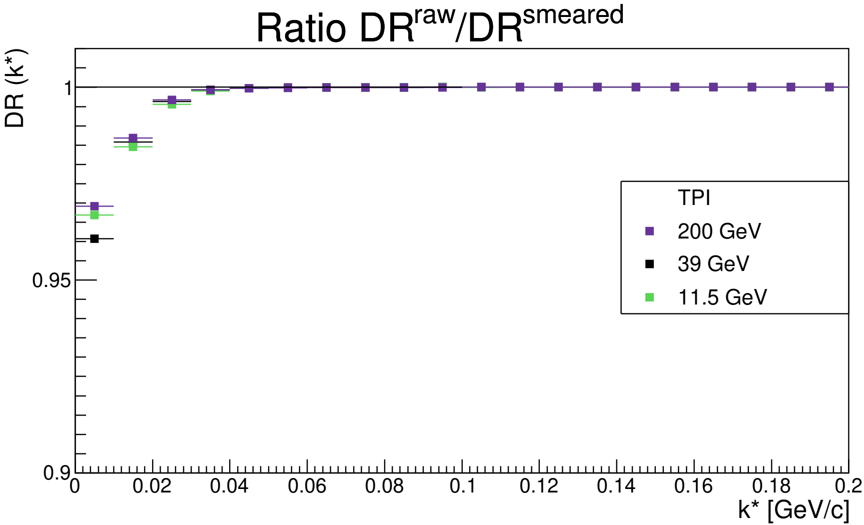
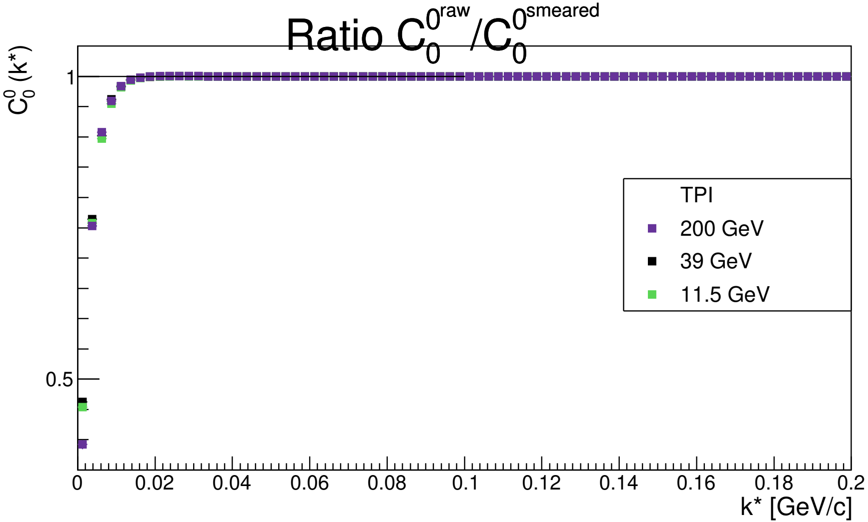


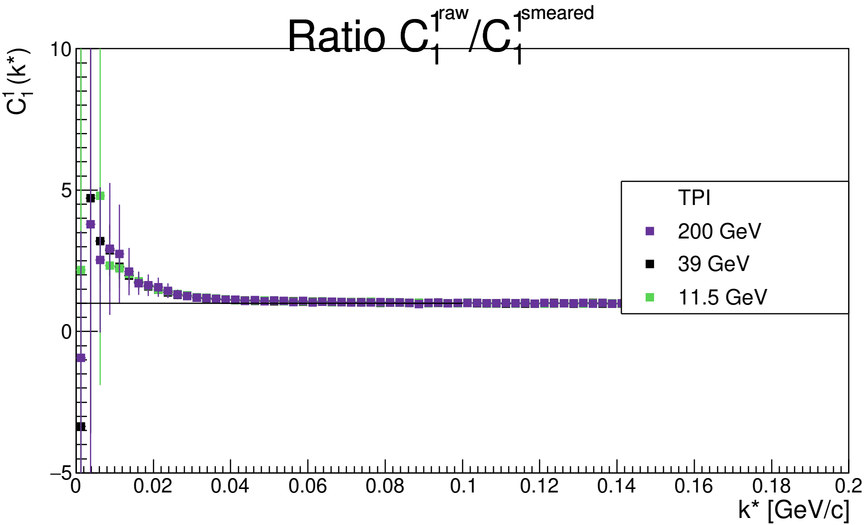

similar effect

# Comparison for other energies — $C_0^0$




similar effect

# Comparison for other energies — $C_1^1$




similar effect

# Summary

Studies of momentum resolution effect using Therminator 2 model

- Visible effect of momentum resolution (MR) on CF
- MR should be taken into account in femtoscopic analysis
- Similar effect for energies 11.5, 39 and 200 GeV
  - with the same parameters of MR!
- Wider impact on DR and $C_1^1$ function (asymmetry)

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**Thank you for your attention!**