Linearized kinetic description of non-equilibrium dynamics in pp and pA collisions

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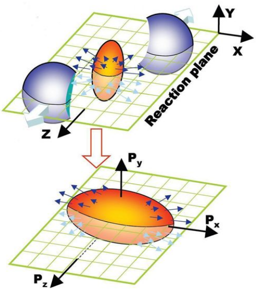
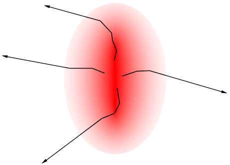
- describe QCD fireball created in a particle collision
- examine how spatial anisotropies in the initial state (ϵ_n) dynamically create momentum anisotropies in the final state (v_n) in small systems
- small densities, large gradients: hydro not applicable, need microscopic description in terms of kinetic theory
- numerical transport codes simulate these dynamics but obscure part of the mechanism $\Rightarrow$ analytical treatment for better understanding and to find parametric dependencies



Hiroshi Masui (2008)



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- microscopic description in terms of on-shell phase-space distribution of gluons:

$$f(\tau, \vec{x}_T, \eta, \vec{p}_T, y) = \frac{(2\pi)^3}{\nu_g} \frac{\mathrm{d}N}{\mathrm{d}^3x\,\mathrm{d}^3p}(\tau, \vec{x}_T, \eta, \vec{p}_T, y)$$

  - boost invariance: dependence only on $y - \eta \Rightarrow \underbrace{2}_{\vec{x}_T} + \underbrace{3}_{(\vec{p}_T, "p_{\parallel}")} + \underbrace{1}_{\tau}$ D description
  - observables expressed as constant densities $\mathrm{d}X/\mathrm{d}y$ (or ratios)
- time evolution: Boltzmann equation

$$p^\mu \partial_\mu f = C[f]$$

- will solve this both analytically and numerically!

Analytical Description

relativistic Boltzmann equation

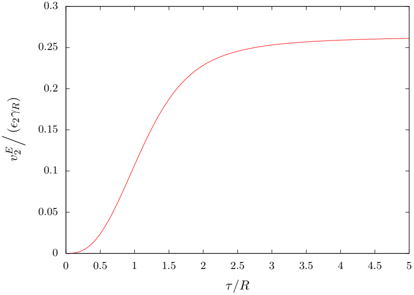
$$p^\mu \partial_\mu f = C[f]$$

- integro-differential equation, need simplifications for analytical solution
  - small system $\Rightarrow$ "eremitic expansion" in number of scatterings

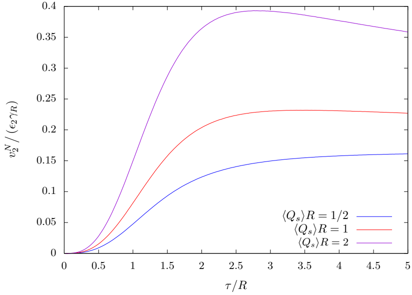
$$\begin{aligned} \text{0th order}: \quad & p^\mu \partial_\mu f^{(0)} = 0 \\ \text{1st order}: \quad & p^\mu \partial_\mu f^{(1)} = C[f^{(0)}] \\ & \text{etc...} \end{aligned}$$

  - initial condition: isotropic gaussian energy density $\epsilon(x_T)$ and linearization in small anisotropic $\delta\epsilon(\vec{x}_T)$ on top
  - for now: $C_{RTA}[f] = \frac{p_\mu u^\mu}{\tau_R}(f_{eq} - f)$, $\tau_R = 5\frac{\eta}{s}T^{-1}$
    rates will depend on opacity $\gamma_R \sim \tau_R^{-1}$

E_T-weighted



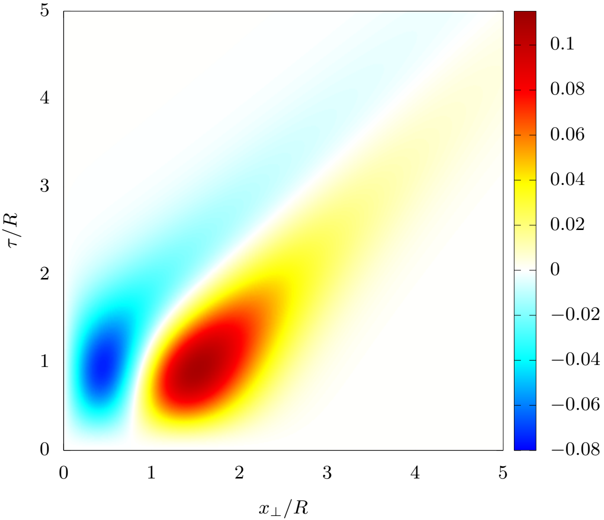
N-weighted



- energy weighted response fully determined, particle number weighted response needs parametric input
- most of the buildup happens around $\tau/R = 0.5 - 2$
- in case of large $\langle Q_s \rangle R$, loss term wins slightly at late τ

Where is v_2^E produced?

Heatmap of production rate of v_2^E



- important region ranges into outskirts of the system

Numerical nonlinear simulation

Setup of numerical nonlinear simulation

- comparing with numerical treatment:
  - find range of validity of linearizations in γ_R, ϵ_n
  - check for interesting observables
- Setup: switch to describing only moments of f

$$C_l^m = \int \frac{\mathrm{d}^2 p_T \, \mathrm{d}p_\eta}{(2\pi)^3} p^\tau Y_l^m(\theta_p, \phi_p) f$$

  - can extract only energy related observables as linear combinations of the C_l^m
- discretization: cutoff at finite l_{max}, lattice in $\vec{x}_T$

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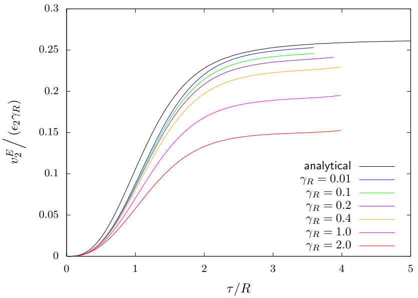
  - can extract only energy related observables as linear combinations of the C_l^m
- discretization: cutoff at finite l_{max}, lattice in $\vec{x}_T$
- taking moments of Boltzmann equation: different momentum components have different angular dependence $\Rightarrow$ non-diagonal

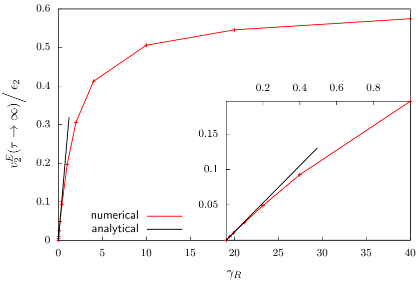
$$\partial_\tau (p^\tau f) = f \partial_\tau p^\tau - p^i \partial_i f + \frac{p^\mu u_\mu}{\tau_R} (f_{eq} - f)$$

$$\Rightarrow \partial_\tau C_l^m = \sum_{l',m'} (b_{ll'}^{mm'} + c_{ll'}^{mm'} \partial_1 + d_{ll'}^{mm'} \partial_2 + e_{ll'}^{mm'}(u^\mu)) C_{l'}^{m'} + E_l^m(u^\mu, T)$$

buildup of v_2^E



γ_R-dep. of final v_2^E



- smaller γ_R: closer to analytical result
- no significant deviation from $v_2^E \propto \epsilon_2$
- in γ_R-dependence of final values:
  - saturation for large γ_R
  - tangential to analytical result for small γ_R

So far

- successfull analytical computation of observables to linear order in opacity and eccentricity in RTA
- numerical results match for small opacities, are always linear in eccentricities
- v_2 buildup: outer regions important

Next steps

- use numerical code to examine v_2 buildup, check for interesting observables
- switch to more realistic QCD collision kernel

Backup

1. initial condition:

$$f^{(0)}(\tau_0, \vec{x}_\perp, \vec{p}_\perp, y-\eta) = \frac{(2\pi)^3}{\nu_g} \frac{\delta(y-\eta)}{\tau_0 p_\perp} F\left(\frac{Q_s(\vec{x}_\perp)}{p_\perp}\right)$$

$$\epsilon(\tau_0, \vec{x}_\perp) = \frac{\mathrm{d}E_\perp^{(0)}}{\mathrm{d}y} \frac{1}{2\pi R^2 \tau_0} \exp\left(-\frac{\vec{x}_\perp^2}{2R^2}\right) \left\{1 + \delta_n \left(\frac{x_\perp}{R}\right)^n \exp\left(-\frac{x_\perp^2}{2R^2}\right) \cos[n(\phi - \psi_n)]\right\}$$

2. eremitic expantion: compute solutions to $p^\mu \partial_\mu f^{(0)} = 0$ and $p^\mu \partial_\mu f^{(1)} = C[f^{(0)}]$

$$f^{(1)}(\tau, \vec{x}_T, \vec{p}_T, y-\eta) = \int_{\tau_0}^{\tau} \mathrm{d}\tau' \left(\frac{C[f^{(0)}]}{p^\tau}\right)\left(\tau', \vec{x}'_T, \vec{p}_T, y-\eta'\right)$$

3. collision kernel: find local rest frame and temperature using Landau matching to compute $C_{RTA}[f^{(0)}]$

$$T^{\mu\nu} = \int \frac{\mathrm{d}^3 p}{(2\pi)^3 p^\tau} p^\mu p^\nu f^{(0)} \qquad \epsilon u^\mu = u_\nu T^{\nu\mu} \qquad \epsilon = \frac{\nu_g \pi^2}{30} T^4$$

4. observables: extract momentum distribution and compute its moments
   - jacobian from milne coordinates

$$\frac{\mathrm{d}N}{\mathrm{d}^2 p_T \mathrm{d}y}(\tau) = \nu_g \int \mathrm{d}\sigma_\mu p^\mu f = \int \mathrm{d}^2 x_T \mathrm{d}\eta\, p_T \tau \cosh(y-\eta) \nu_g f(\tau, \vec{x}_T, \eta, \vec{p}_T, y)$$

   - extract relevant moments; flow harmonics $v_n(p_T)$ in terms of weighted p_T-averages:

$$V_{m,n} = \int \mathrm{d}^2 p_T\, p_T^m e^{in\phi_p} \frac{\mathrm{d}N}{\mathrm{d}^2 p_T \mathrm{d}y} \qquad v_n^{(m)} = \frac{V_{m,n}}{V_{m,0}}$$

   - in total: 6d integral over $\tau', \vec{x}_T, \eta, \vec{p}_T$. 4 computed analytically, 2 numerically

$$C_l^m = \int \frac{\mathrm{d}^2 p_T \, \mathrm{d}p_\eta}{(2\pi)^3} p^\tau Y_l^m(\theta_p, \phi_p) f$$

- $Y_l^m(\theta_p, \phi_p)$: spherical harmonics
- ϕ_p: azimuthal momentum angle
- θ_p defined by $\cos\theta_p = \frac{p_\eta}{\tau p^\tau}$; dispersion relation $p^\tau = \sqrt{p_\perp^2 + p_\eta^2/\tau^2}$

$$\partial_\tau(p^\tau f) = f\partial_\tau p^\tau - p^i \partial_i f + \frac{p^\mu u_\mu}{\tau_R}(f_{eq} - f)$$

- taking moments: $p^1 = p_T \cos\phi_p$, $p^2 = p_T \sin\phi_p$, $p_T = p^\tau \sin\theta_p$, $p_\eta = \tau p^\tau \cos\theta_p$ together with $Y_l^m(\theta_p, \phi_p)$ result in linear combination of $Y_{l'}^{m'}(\theta_p, \phi_p)$

$$\Rightarrow \partial_\tau C_l^m = \sum_{l',m'} (b_{ll'}^{mm'} + c_{ll'}^{mm'} \partial_1 + d_{ll'}^{mm'} \partial_2 + e_{ll'}^{mm'}(u^\mu)) C_{l'}^{m'} + E_l^m(u^\mu, T)$$

- E_l^m are the moments of the equilibrium term $\frac{p_\mu U^\mu}{\tau_R} f_{eq}$

Computation:

- $C[f]$ local $\Rightarrow$ change of C_l^m due to blue and purple terms can be computed for each $\vec{x}_T$-site individually
- red term needs nonlocal discretization of derivative $\Rightarrow$ treated separately in Fourier space