

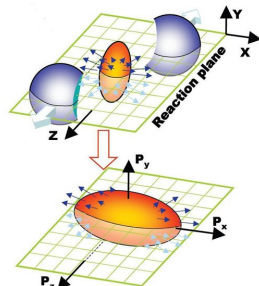
# Linearized kinetic description of non-equilibrium dynamics in pp and pA collisions

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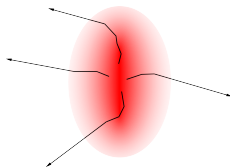
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- | describe QCD fireball created in a particle collision
- | examine how spatial anisotropies in the initial state ( $n$ ) dynamically create momentum anisotropies in the final state ( $v_n$ ) **in small systems**
- | small densities, large gradients: hydro not applicable, need microscopic description in terms of kinetic theory
- | numerical transport codes simulate these dynamics but obscure part of the mechanism
  - ) analytical treatment for better understanding and to find parametric dependencies



Hiroshi Masui (2008)



- microscopic description in terms of on-shell phase-space distribution of gluons:

$$f(\vec{x}_T; \vec{p}_T; y) = \frac{(2\pi)^3}{g} \frac{dN}{d^3x d^3p}(\vec{x}_T; \vec{p}_T; y)$$

- boost invariance: dependence only on  $y$  )  $\underbrace{2}_{\vec{x}_T} + \underbrace{3}_{(\vec{p}_T, "p_k")}$  +  $\underbrace{1}_{\tau}$  D description
  - observables expressed as constant densities  $dX=dy$  (or ratios)

- time evolution: Boltzmann equation

$$p^\mu @_\mu f = C[f]$$

- will solve this both analytically and numerically!

## Analytical Description

relativistic Boltzmann equation

$$p^\mu @_\mu f = C[f]$$

| integro-differential equation, need simplifications for analytical solution

- small system ) "eremitic expansion" in number of scatterings

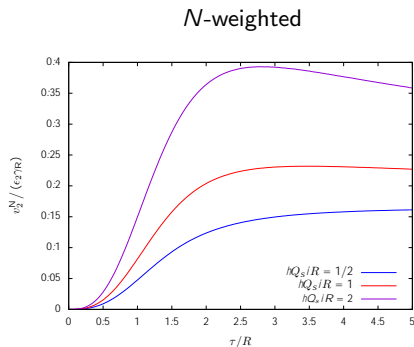
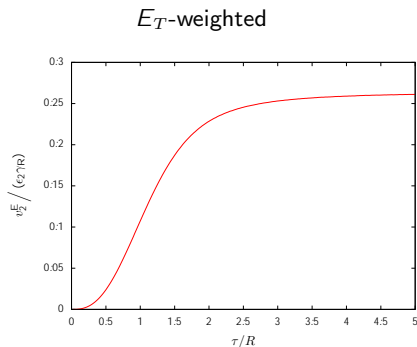
$$0\text{th order : } p^\mu @_\mu f^{(0)} = 0$$

$$1\text{st order : } p^\mu @_\mu f^{(1)} = C[f^{(0)}]$$

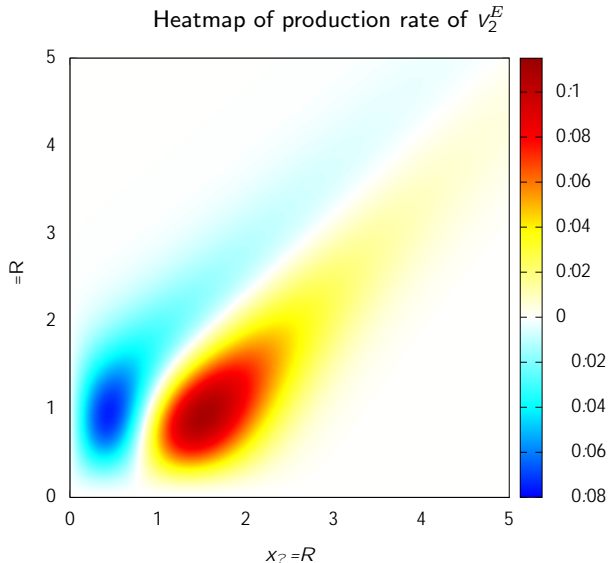
etc:::

- initial condition: isotropic gaussian energy density ( $\chi_T$ ) and linearization in small anisotropic ( $\chi_T$ ) on top

- for now:  $C_{RTA}[f] = \frac{p_\mu u^\mu}{\tau_R} (f_{eq} - f)$ ,  $R = 5 \frac{\eta}{s} T^{-1}$   
rates will depend on opacity  $R^{-1}$



- | energy weighted response fully determined, particle number weighted response needs parametric input
- | most of the buildup happens around  $\tau/R = 0.5 \quad 2$
- | in case of large  $hQ_s i R$ , loss term wins slightly at late



- | important region ranges into outskirts of the system

## Numerical nonlinear simulation



- | comparing with numerical treatment:
  - find range of validity of linearizations in  $R, n$
  - check for interesting observables

- | Setup: switch to describing only moments of  $f$

$$C_l^m = \int \frac{d^2 p_T d p_\eta}{(2\pi)^3} p^\tau Y_l^m(p; p) f$$

- can extract only energy related observables as linear combinations of the  $C_l^m$
- | discretization: cutoff at finite  $l_{max}$ , lattice in  $X_T$

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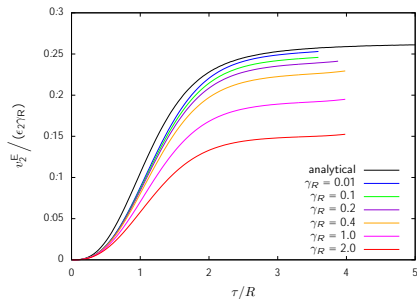
$$C_l^m = \int \frac{d^2 p_T d p_\eta}{(2\pi)^3} p^\tau Y_l^m(p; p) f$$

- can extract only energy related observables as linear combinations of the  $C_l^m$
- | discretization: cutoff at finite  $l_{max}$ , lattice in  $X_T$
- | taking moments of Boltzmann equation: different momentum components have different angular dependence ) non-diagonal

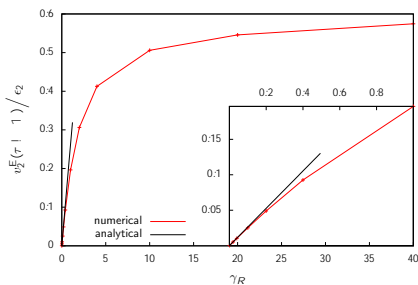
$$@_\tau(p^\tau f) = f @_\tau p^\tau + p^i @_i f + \frac{p^\mu u_\mu}{R} (f_{eq} - f)$$

$$) @_\tau C_l^m = \sum_{l^0, m^0} (b_{ll^0}^{mm^0} + c_{ll^0}^{mm^0} @_1 + d_{ll^0}^{mm^0} @_2 + e_{ll^0}^{mm^0}(u^\mu)) C_{l^0}^{m^0} + E_l^m(u^\mu; T)$$

buildup of  $v_2^E$



$R$ -dep. of final  $v_2^E$



- | smaller  $\gamma_R$ : closer to analytical result
- | no significant deviation from  $v_2^E / \epsilon_2 \approx 1/2$
- | in  $\gamma_R$ -dependence of final values:
  - saturation for large  $\gamma_R$
  - tangential to analytical result for small  $\gamma_R$

## So far

- | successfull analytical computation of observables to linear order in opacity and eccentricity in RTA
- | numerical results match for small opacities, are always linear in eccentricities
- |  $v_2$  buildup: outer regions important

## Next steps

- | use numerical code to examine  $v_2$  buildup, check for interesting observables
- | switch to more realistic QCD collision kernel

Backup

1. initial condition:

$$f^{(0)}(\tau_0, \vec{x}_\tau, \vec{p}_\tau, y, \eta) = \frac{(2\pi)^3 \delta(y - \eta)}{\nu_g \tau_0 p_\tau} F\left(\frac{Q_s(\vec{x}_\tau)}{p_\tau}\right)$$

$$\epsilon(\tau_0, \vec{x}_\tau) = \frac{dE_\tau^{(0)}}{dy} \frac{1}{2\pi R^2 \tau_0} \exp\left(-\frac{\vec{x}_\tau^2}{2R^2}\right) \left\{ 1 + \delta_n \left(\frac{x_\tau}{R}\right)^n \exp\left(-\frac{x_\tau^2}{2R^2}\right) \cos[n(\phi - \psi_n)] \right\}$$

2. eremitic expansion: compute solutions to  $p^\mu @_\mu f^{(0)} = 0$  and  $p^\mu @_\mu f^{(1)} = C[f^{(0)}]$

$$f^{(1)}(\tau, \vec{x}_T, \vec{p}_T, y, \eta) = \int_{\tau_0}^{\tau} d\tau^0 \left( \frac{C[f^{(0)}]}{p^\tau} \right) (\tau^0, \vec{x}_T^0, \vec{p}_T, y, \eta^0)$$

3. collision kernel: find local rest frame and temperature using Landau matching to compute  $C_{RTA}[f^{(0)}]$

$$T^{\mu\nu} = \int \frac{d^3p}{(2\pi)^3 p^\tau} p^\mu p^\nu f^{(0)} \quad \epsilon u^\mu = u_\nu T^{\nu\mu} \quad \epsilon = \frac{\nu_g \pi^2}{30} T^4$$

4. observables: extract momentum distribution and compute its moments

- jacobian from milne coordinates

$$\frac{dN}{d^2p_T dy}(\tau) = \nu_g \int d\sigma_\mu p^\mu f = \int d^2x_T d\eta p_T \tau \cosh(y - \eta) \nu_g f(\tau, \vec{x}_T, \eta, \vec{p}_T, y)$$

- extract relevant moments; flow harmonics  $v_n(\rho_T)$  in terms of weighted  $\rho_T$ -averages:

$$V_{m,n} = \int d^2p_T p_T^m e^{in\phi_p} \frac{dN}{d^2p_T dy} \quad v_n^{(m)} = \frac{V_{m,n}}{V_{m,0}}$$

- in total: 6d integral over  $\tau^0; \vec{x}_T^0; \vec{p}_T$ . 4 computed analytically, 2 numerically

$$C_l^m = \int \frac{d^2 p_T d p_\eta}{(2\pi)^3} p^\tau Y_l^m(p; p) f$$

- $Y_l^m(p; p)$ : spherical harmonics
- $p$ : azimuthal momentum angle
- $p$  defined by  $\cos p = \frac{p_\eta}{\tau p^\tau}$ ; dispersion relation  $p^\tau = \sqrt{p_\gamma^2 + p_\eta^2} = 2$

$$\partial_\tau(p^\tau f) = f \partial_\tau p^\tau + p^i \partial_i f + \frac{p^\mu u_\mu}{R} (f_{eq} - f)$$

taking moments:  $p^1 = p_T \cos p$ ,  $p^2 = p_T \sin p$ ,  $p_T = p^\tau \sin p$ ,  $p_\eta = p^\tau \cos p$   
 together with  $Y_l^m(p; p)$  result in linear combination of  $Y_{l0}^{m0}(p; p)$

$$\partial_\tau C_l^m = \sum_{l^0, m^0} (b_{ll^0}^{mm^0} + c_{ll^0}^{mm^0} \partial_1 + d_{ll^0}^{mm^0} \partial_2 + e_{ll^0}^{mm^0}(u^\mu)) C_{l^0}^{m^0} + E_l^m(u^\mu; T)$$

$E_l^m$  are the moments of the equilibrium term  $\frac{p_\mu u^\mu}{\tau R} f_{eq}$

Computation:

- change of  $C_l^m$  due to blue and purple terms can be computed for each  $x_T$ -site individually
- red term needs nonlocal discretization of derivative ) treated separately in Fourier space