Early time anisotropies in kinetic theory

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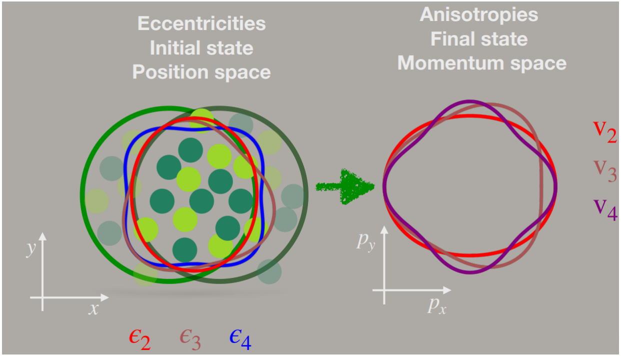
Motivation and Procedure

- How does anisotropic flow develop on a system ?
- Do all v_n behave equally ?
- We solve the Boltzmann equation analytically

Early time

- Eccentricities generate momentum anisotropies
- We focus on the early time behaviour of the system.



General particle density

The distribution is decomposed in a series as

$$f \equiv f(t, \vec{r}, \vec{p}) = f_0 + t(\partial_t f)_{t=0} + \frac{t^2}{2}(\partial_t^2 f)_{t=0} + \mathcal{O}(t^3)$$

The Boltzmann equation: $\partial_t f = -\frac{\vec{p}}{E}\vec{\nabla}_r f + C$

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$$f = f_0 + t(-\frac{\vec{p}}{E}\vec{\nabla}_r f + C)_{t=0} + \frac{t^2}{2}(\frac{(\vec{p}\vec{\nabla}_r)^2}{E^2} f + \partial_t C - \frac{\vec{p}}{E}\vec{\nabla}_r C)_{t=0} + \mathcal{O}(t^3)$$

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Or

$$f_{f.s.} = f_0 - t\frac{\vec{p}}{E}\vec{\nabla}_r f_0 + t^2\frac{(\vec{p}\cdot\vec{\nabla}_r)^2}{2E^2} f_0 + \mathcal{O}(t^3)$$

$$f = f_{f.s.} + tC_0 + \frac{t^2}{2}(\partial_t C - \frac{\vec{p}}{E}\vec{\nabla}_r C)_{t=0} + \mathcal{O}(t^3)$$

Anisotropic flow coefficients

$$v_n = \frac{\int f \cos(n\phi) d^3x d^3p}{\int f d^3x d^3p}$$

For $2 \leftrightarrow 2$ interactions we have

$$C \propto \int (f_3 f_4 - f_1 f_2) v_r \sigma d^3 p_{2,3,4}$$

The gain term has an small impact overall (Anisotropic flow, N. Kersting)

$$C \propto - \int f_1 f_2 v_r \sigma d^3 p_2$$

Flow for a large Knudsen number

We only consider terms at order σ.

$$Kn \propto \lambda_{m.f.p} \rightarrow \lambda_{m.f.p} \approx \frac{1}{n\sigma} \rightarrow Kn \propto \sigma^{-1}$$

The initial distribution is required to

- Vanish at $r \rightarrow \infty$
- Be initially isotropic in momentum (i.e. $v_n(t = 0) = 0$)
- Contain massless particles

We will work in 2d from now.

Elliptic flow for a large Knudsen number

$$\frac{dv_2}{dt} \propto -\int v_r \sigma f_1 f_2 \cos(2\phi_1)$$

$\int v_r f_{1,0} f_{2,0} \cos(2\phi_1) = 0 \rightarrow$ No anisotropies initially

$t \int v_r f_{1,0} \frac{\vec{p_2}}{E_2} \vec{\nabla}_r f_{2,0} \cos(2\phi_1) = 0 \rightarrow$ Odd in momentum

$t^2 \int v_r \cos(2\phi_1)(f_{1,0} \frac{(\vec{p_2}\vec{\nabla}_r)^2 f_{2,0}}{2E_2} + ...)\sigma \propto t^2 \sigma \cos(2\theta) f_0 \propto \epsilon_2$

v_n

For the elliptic flow we found

$$v_2 \propto t^3 \epsilon_2$$

For the triangular and quadrangular flow coefficients

$$v_3 \propto t^4 \epsilon_3 \qquad v_4 \propto t^5 (\alpha \epsilon_2^2 + \beta \epsilon_4)$$

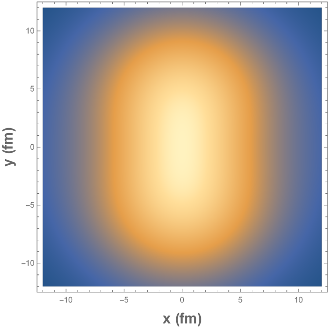
Therefore

$$v_n \propto t^{n+1}$$

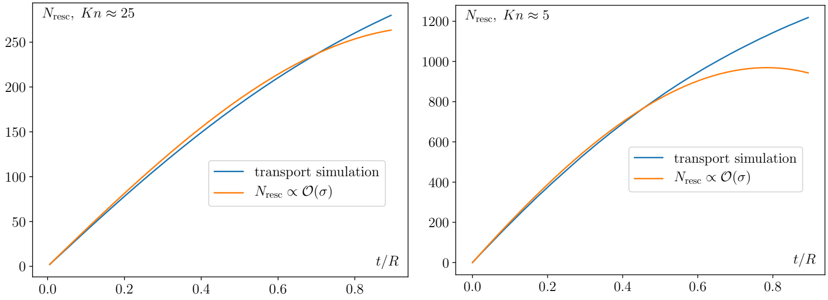
Suggested behaviour B. H. Alver, C. Gombeaud, M. Luzum, and J.-Y. Ollitrault, Phys.Rev.C 82 (2010), 034913

Special case

$$f_0 = \frac{N}{4\pi^2 R^2 T^2} e^{-\frac{r^2}{2R^2}} [1 - 4\epsilon_2 \frac{r^2}{R^2} \cos(2(\theta - \psi_2)) e^{-\frac{r^2}{2R^2}}] e^{-p/T}$$



Number of rescatterings



For the transport simulation see Fluctuations of anisotropic flow in transport, H. Roch, Thursday 8:30.

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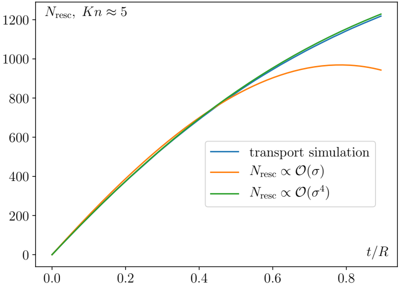
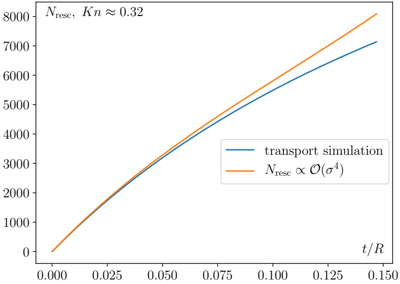
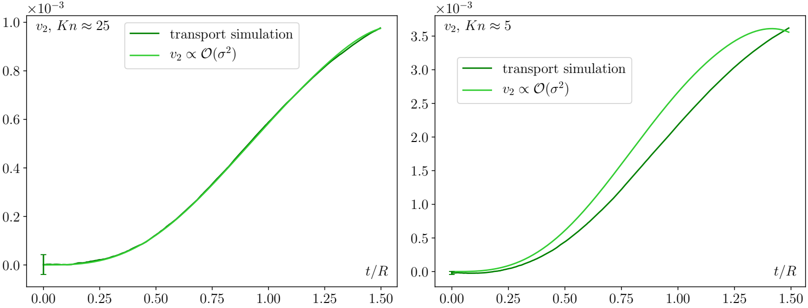




Even small Kn can be described.

Elliptic flow

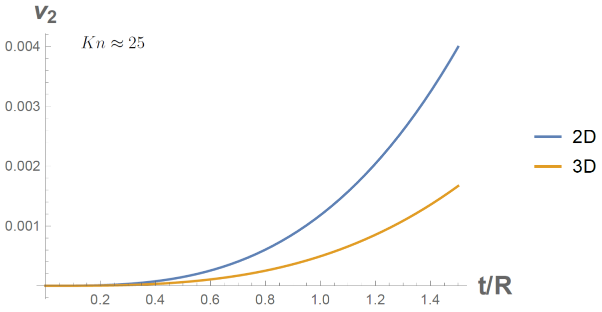
The behaviour of v_2 is captured specially at large Kn.



t/R $\approx$ 1.5 but the expansion is still well behaved, see more Anisotropic flow in non-equilibrated systems , N. Kersting.

Going to 3 dimensions

In 3d the system develops flow slower for the same Knudsen number (Kn = 25).



Summary and Outlook

- The anisotropic flow coefficients scale as $v_n \propto t^{n+1}$ for kinetic theory.
- Good comparison for N_{resc} and v_2 with a transport simulation for different Kn.
- Move towards smaller K_n.
- Exploring further the effects of 3d and study massive systems.

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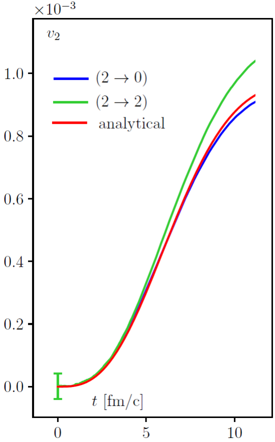
Thank You

Gain term



The inclusion of $2 \to 2$ interactions has an small impact on v_2.

Quantum effects

- Quantum effects (bosons,fermions) change the collision kernel
- Assuming that we have a gas of mesons, the elliptic flow becomes

$$\frac{dv_2^c}{dt} \propto -\int v_r \sigma f_1 f_2 (1+f_3)(1+f_4) cos(2\phi_1) d^2x d^2p_1 d^2p_2 d^2p_3 d^2p_4$$

- All "quantum" terms are zero, for example

$$\int t^2 \frac{(\vec{p_3} \cdot \vec{\nabla}_r)^2}{2E^2} f_{3,0} v_r \sigma cos(2\varphi_1) f_{1,0} f_{2,0} f_{4,0} d^2x d^2p_{1,2,3,4}$$

- What we learn: Only quantitative but no qualitative differences when including quantum effects

Massive

- Including mass implies a more complicated form of the relative velocity

$$v_r = \sqrt{(1 - cos(\phi_1 - \phi_2)\beta_1\beta_2)^2 - (1 - \beta_1^2)(1 - \beta_2^2)}$$

Assumption $\rightarrow$ Particles 1 and 2 have the same mass and the same fixed energy i.e. $m_1 = m_2 = m \quad E_1 = E_2 = E = ct$

$$v_r = \sqrt{(1 - cos(\phi_1 - \phi_2)(1 - m^2/E^2))^2 - (1 - (1 - m^2/E^2))^2}$$

- Expanding up tor order m^2

$$v_r = 1 - (1 - \frac{m^2}{E^2})cos(\phi_1 - \phi_2)$$

- What we learn: No qualitative differences