A proposal of a new conventional formula for Bose-Einstein correlations

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Outline

1. Introduction
2. Analysis of L3 BEC at Z^0-pole
3. Analysis of CMS BEC at 0.9 and 7 TeV
4. Concluding remarks

1. Introduction : Our framework

- To explain the KNO scaling violation, double-Negative Binomial Distribution (double-NBD) and triple-NBD were proposed by Giovannini et al and Zborovsky.

$$P(n,\,\langle n\rangle) = \sum_{i=1}^{2\ \text{or}\ 3} \alpha_i P_{\mathrm{NBD}_i}(n,\,\langle n_i\rangle,\,k_i)$$

$$= \sum_{i=1}^{2\ \text{or}\ 3} \alpha_i \frac{\Gamma(n+k_i)}{\Gamma(n+1)\Gamma(k_i)} \frac{(\langle n_i\rangle/k_i)^n}{(1+\langle n_i\rangle/k_i)^{n+k_i}}$$

- The triple-NBD is probably related to the following processes in proton + proton collisions:

$$p+p\begin{cases}\text{ND (Non-diffractive dissociation)}\\ \text{SD (Single diffractive dissociation)}\\ \text{DD (Double diffractive dissociation)}\end{cases}$$

Second conventional formula for the B-E correlation

- The conventional formula (CF_{I}) has been used in B-E correlations (BEC) analysis for half a century.

$$\mathrm{CF_I} = [1.0 + \lambda E_{\mathrm{BEC}}(R,\, Q)]$$

where λ is degree of coherence, E_{BEC} is B-E exchange function, $Q = \sqrt{-(p_1 - p_2)^2}$ is momentum transfer, R is radius of interaction.

- Based on the correspondence of triple-NBD and three processes in p+p collision, we derived a second conventional formula (*i.e.*, two-component interference formula) named CF_{II} :

$$\mathrm{CF_{II}} = [1.0 + \lambda_1 E_{\mathrm{BEC_1}}(R_1,\, Q) + \lambda_2 E_{\mathrm{BEC_2}}(R_2,\, Q)]$$

  - The contribution from DD is small, because of coherence property ($i.e.$, large k_3 (parameter of NBD)).
- Hereafter, we use $CF_{II} \times$ [long-range correlation(LRC)] for analysis of the data.
- Many experimental groups use $\mathrm{LRC}_{(\delta)} = c(1.0 + \delta Q)$. (*c* is normalization.)

My talk is based on the following papers with preliminary analysis.
TM and MB, IJMP A35 (2020) 2050052, MB and TM , IJMP A34 (2019) 1950203
TM and MB, JPS Conf. Proc. 26 (2019) 031032, MB and TM, EPJ A54 (2018) 105

2. Analysis of L3 BEC at Z^0-pole

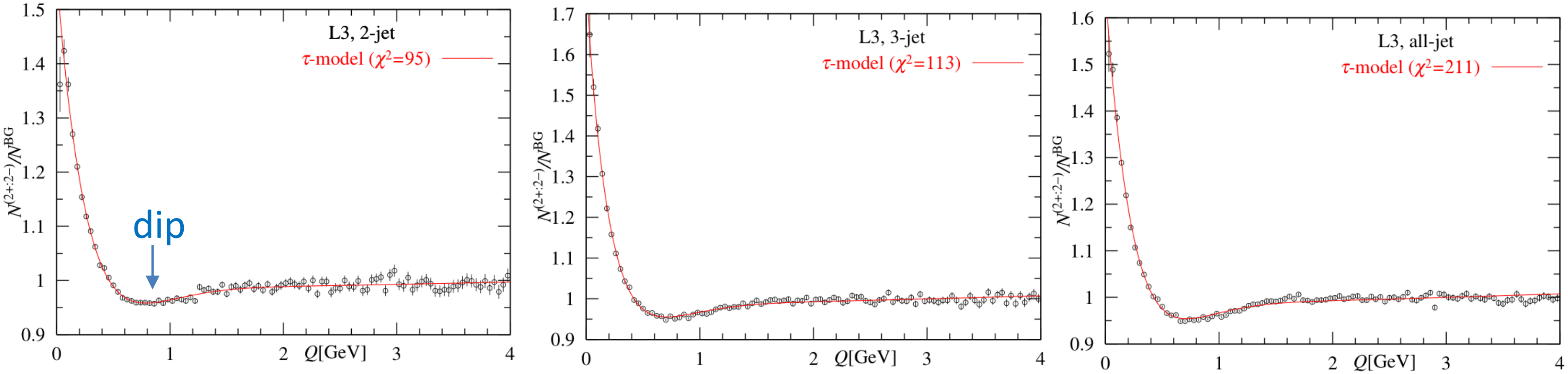
T.Csorgo, J.Zimanyi, Nucl.Phys.A517(1990)588
T.Csorgo, W.Kittel, W.J.Metzger, T.Novak, Phys.Lett.B663(2008)214

- L3 collaboration used a τ-model formula based on the Levy process for analysis of BEC.

$$F_\tau(e^+e^-) = \left[1 + \lambda \cos(R_a Q)^{2\alpha_\tau} \exp(-(RQ)^{2\alpha_\tau})\right] \times \mathrm{LRC}_{(\delta)}$$

$$\text{with } R_a^{2\alpha_\tau} = \tan(\alpha_\tau \pi/2) R^{2\alpha_\tau}$$

$$\mathrm{LRC}_{(\delta)} = c(1.0 + \delta Q)$$



Analysis of L3 BEC at Z^0-pole by τ-model

event	R (fm)	λ	α_τ	δ (GeV^{-1})	c	χ^2/dof
2-jet	0.78±0.04	0.61±0.03	0.44±0.01	0.005±0.001	0.979±0.002	95/95
3-jet	0.99±0.04	0.85±0.04	0.41±0.01	0.008±0.001	0.977±0.001	112/95
all-jet	0.86±0.03	0.71±0.02	0.44±0.01	0.008±0.001	0.977±0.001	211/95

Analysis of L3 BEC at Z−pole by our $\mathrm{CF_{II}}$

- $\mathrm{CF_{II}} \times \mathrm{LRC}_{(\delta)}$

$$\mathrm{CF_{II}} = [1.0 + \lambda_1 E_{\mathrm{BEC}_1}(R_1,\, Q) + \lambda_2 E_{\mathrm{BEC}_2}(R_2,\, Q)]$$

$$\mathrm{LRC}_{(\delta)} = c(1.0 + \delta Q)$$

$$E_{\mathrm{BEC}} = \exp(-RQ)\ (\text{Exponential})\ \text{or}\ \exp(-(RQ)^2)\ (\text{Gaussian})$$

- Two processes from quarks jet to pions:

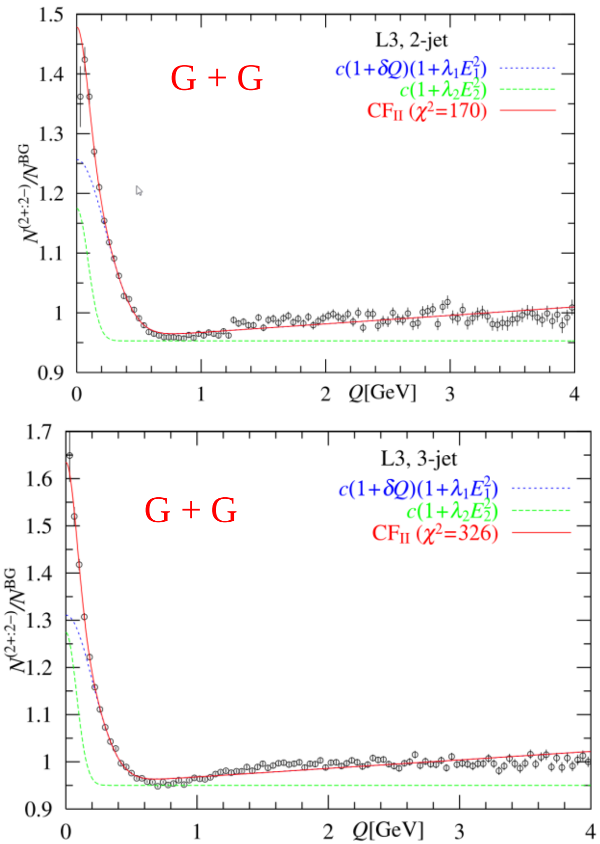
$$e^+ + e^- \to \bar{q}q\text{-jet} \begin{cases} u+\bar{u},\ d+\bar{d}, & (\bullet) \\ s+\bar{s},\ c+\bar{c},\ b+\bar{b}, & (\bullet\bullet) \end{cases}$$

$$(\bullet)\ u + \bar{u} + \sum(q_i\bar{q}_i) \to \sum \text{many direct } \pi\text{'s},$$

$$(\bullet\bullet)\ s + \bar{s} + \sum(q_i\bar{q}_i) \to \sum \text{many resonances}$$

$$\to \sum \text{many decayed } \pi\text{'s}$$

Thus, we expect that $\mathrm{CF_{II}}$ works for BEC in e^+e^- collisions at Z-pole.



Analysis of L3 BEC at Z−pole by $\mathrm{CF_{II}}$ (G + G) × $\mathrm{LRC}_{(\delta)}$

event	R_1 (fm)	R_2 (fm)	λ_1	λ_2	δ (GeV^{-1})	c	χ^2/dof
2-jet	0.61±0.01	1.54±0.12	0.32±0.01	0.23±0.02	0.015±0.001	0.953±0.002	170/94
3-jet	0.69 ±0.01	1.73 ±0.11	0.38±0.02	0.34±0.03	0.019±0.001	0.950±0.001	326/94
all-jet	0.67 ±0.01	1.64 ±0.09	0.35±0.01	0.30±0.02	0.018±0.001	0.951±0.001	633/94

P.Achard et al. [L3 Collab.], Eur. Phys. J. C 71 (2011) 1648, to the L3 paper and W.J. Metzger and T. Novak, private communication. There is, unfortunately, no data base of L3 data.

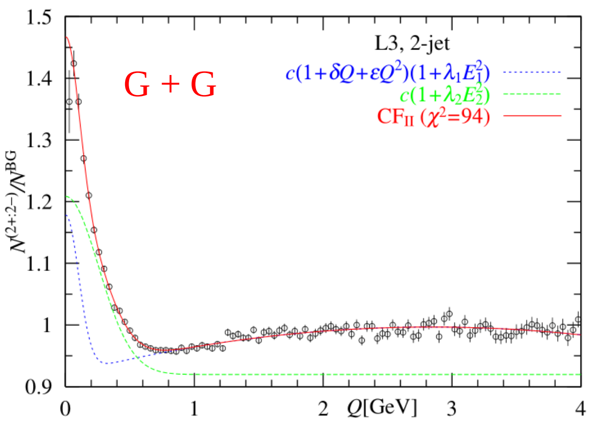
Analysis of the same data by $\mathrm{CF_{II}} \times \mathrm{LRC}_{(\delta,\varepsilon)}$

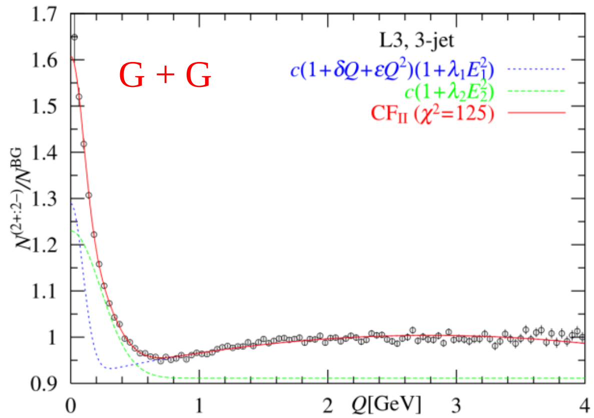
- To improve the χ^2 values, we use $\mathrm{CF_{II}} \times \mathrm{LRC}_{(\delta,\varepsilon)}$.

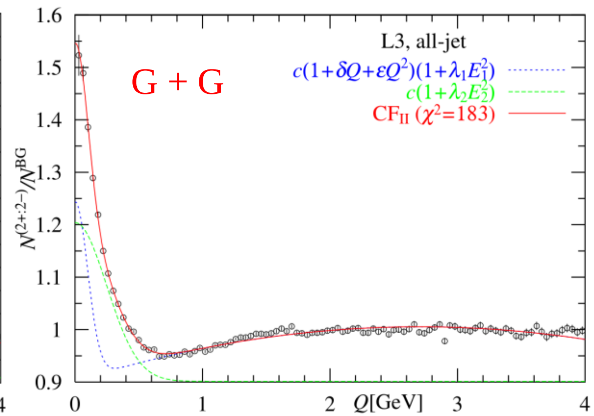
$$\mathrm{CF_{II}} = [1.0 + \lambda_1 E_{\mathrm{BEC_1}}(R_1,\ Q) + \lambda_2 E_{\mathrm{BEC_2}}(R_2,\ Q)]$$

$$\mathrm{LRC}_{(\delta,\varepsilon)} = c(1.0 + \delta Q + \varepsilon Q^2)$$

This one is suggested by OPAL (1991).







Analysis of L3 BEC at Z-pole by $\mathrm{CF_{II}}$(G+G) $\times$ $\mathrm{LRC}_{(\delta,\varepsilon)}$

event	R_1 (fm)	R_2 (fm)	λ_1	λ_2	δ (GeV^{-1})	ε (GeV^{-2})	c	χ^2/dof
2-jet	1.38±0.10	0.54±0.02	0.28±0.02	0.31±0.02	0.059±0.006	−0.010±0.001	0.920±0.005	94/93
3-jet	1.49±0.08	0.60±0.01	0.41±0.02	0.35±0.02	0.073±0.004	−0.013±0.001	0.911±0.003	125/93
all-jet	1.43±0.06	0.56±0.01	0.38±0.02	0.34±0.01	0.086±0.004	−0.016±0.001	0.901±0.003	183/93

Comparison of some results by $\mathrm{CF}_{\mathrm{II}} \times \mathrm{LRC}_{(\delta,\varepsilon)}$ and τ-model

$$\mathrm{CF}_{\mathrm{II}} \times \mathrm{LRC}_{(\delta,\varepsilon)} = [1.0 + \lambda_1 E_{\mathrm{BEC}_1}(R_1,\, Q) + \lambda_2 E_{\mathrm{BEC}_2}(R_2,\, Q)] \times \mathrm{LRC}_{(\delta,\varepsilon)}$$

$$F_\tau(e^+e^-) = \left[1 + \lambda \cos(R_a Q)^{2\alpha_\tau} \exp(-(RQ)^{2\alpha_\tau})\right] \times \mathrm{LRC}_{(\delta)}$$

Introduction of $R_{\mathrm{eff}} = R_1\lambda_1 + R_2\lambda_2$ in $\mathrm{CF}_{\mathrm{II}}$ (G+G). Comparison of R's.

	$\mathrm{CF}_{\mathrm{II}}$		τ-model	
event	R_{eff} (fm)	$\sigma = \pi R_{\mathrm{eff}}^2$ (mb)	R (fm)	σ (mb)
2-jet	$1.38 \times 0.281 + 0.54 \times 0.314 = 0.56 \pm 0.04$	9.8	0.78 ± 0.04	19.1
3-jet	$1.49 \times 0.414 + 0.60 \times 0.350 = 0.83 \pm 0.05$	21.6	0.99 ± 0.04	30.8
all-jet	$1.43 \times 0.379 + 0.56 \times 0.336 = 0.73 \pm 0.04$	16.7	0.86 ± 0.03	23.2

Introduction of $\lambda_{\mathrm{eff}} = \lambda_1 + \lambda_2$ in $\mathrm{CF}_{\mathrm{II}}$ (G+G). Comparison of λ's.

event	$\mathrm{CF}_{\mathrm{II}}$: λ_{eff}	τ-model: λ
2-jet	$0.28 + 0.31 = 0.59 \pm 0.03$	0.61 ± 0.03
3-jet	$0.41 + 0.35 = 0.76 \pm 0.03$	0.85 ± 0.04
all-jet	$0.38 + 0.34 = 0.72 \pm 0.02$	0.71 ± 0.02

Density distributions in Euclidean space by Fourier transformation

Bessel transformation in Euclidean space : $\xi = (x^2 + y^2 + z^2 + (ct)^2)^{1/2}$

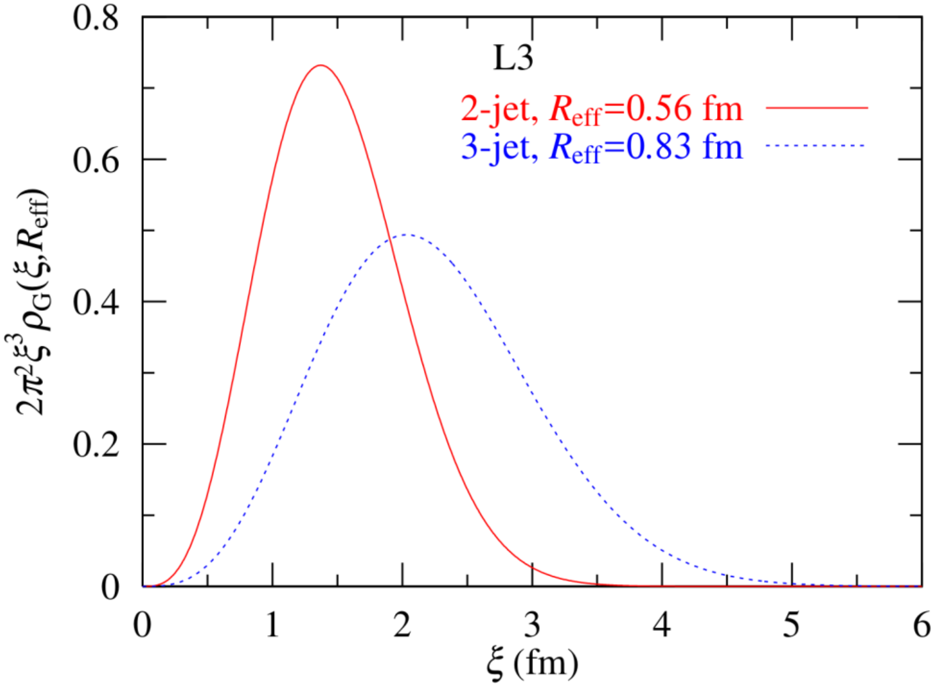
$$\rho_{\rm BEC}(\xi,\, R) = \frac{1}{(2\pi)^2 \xi} \int_0^\infty Q^2 E_{\rm BEC(LRC-1.0)}(Q,\, R) J_1(Q\xi) dQ$$

$$\int_0^\infty 2\pi^2 \xi^3 \rho_{\rm BEC}(\xi,\, R) d\xi = 1.0$$

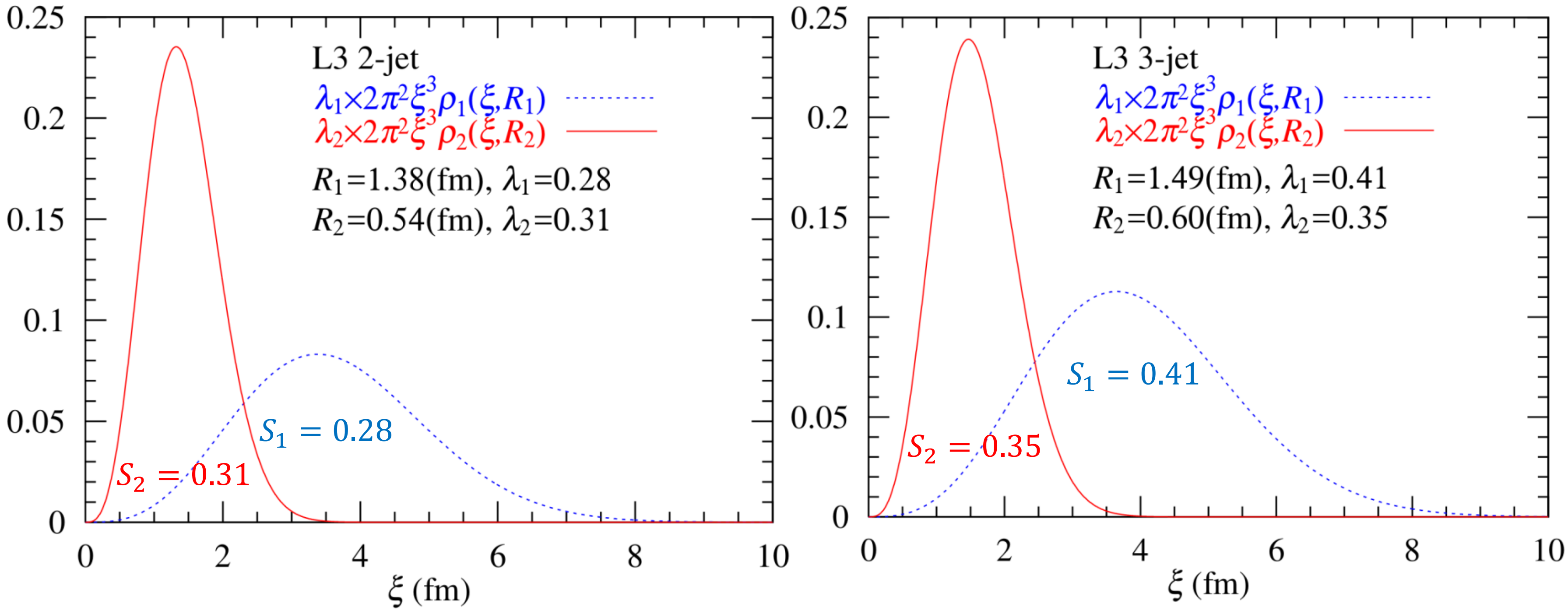
$E_{\rm BEC}(R,\, Q)$	$\rho(\xi)$
$\exp(-R^2Q^2)$	$\frac{1}{16\pi^2 R^4} \exp\left(-\frac{\xi^2}{4R^4}\right)$
$\exp(-RQ)$	$\frac{3}{4\pi^2 R^4} \frac{1}{(1 + (\xi/R)^2)^{5/2}}$

R. Shimoda, MB, N. Suzuki, Prog. Theor. Phys. 89 (1993) 697

Density distributions by $CF_{\rm II}$ (G+G)× $LRC_{(\delta,\varepsilon)}$



Density distributions of two sources by CF_{II} (G+G)× $LRC_{(\delta,\varepsilon)}$



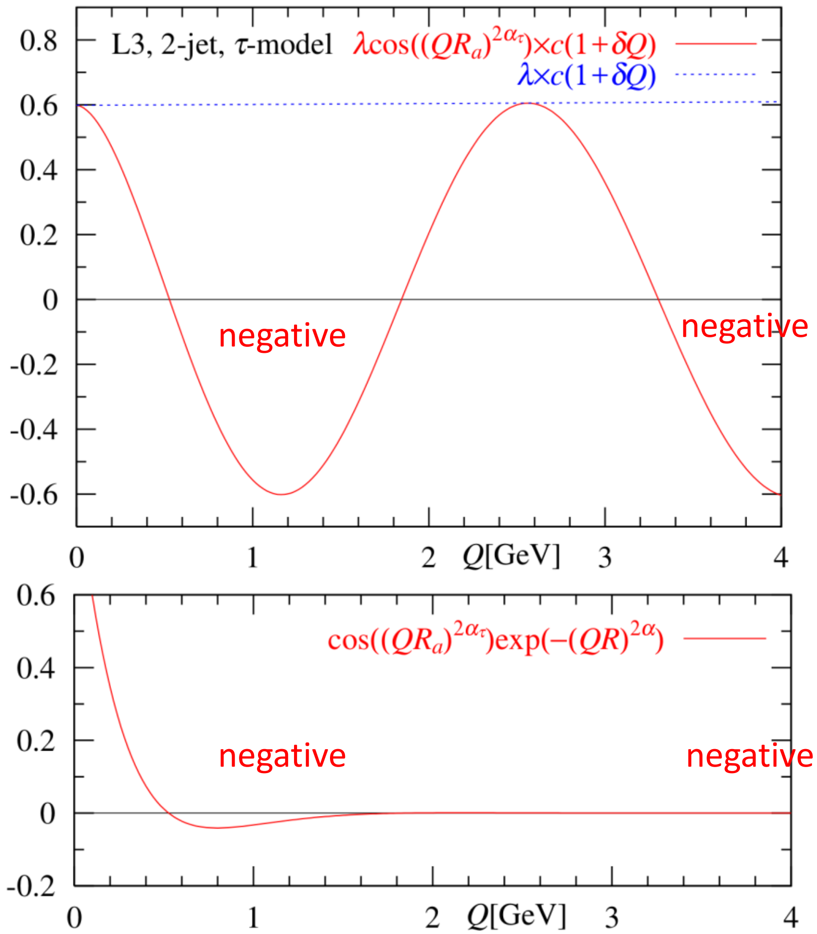
Density distributions of 1st and 2nd sources for L3 BEC. The degree of coherence ($\lambda_1$and λ_2) are multiplied.

Long-range correlations in τ-model

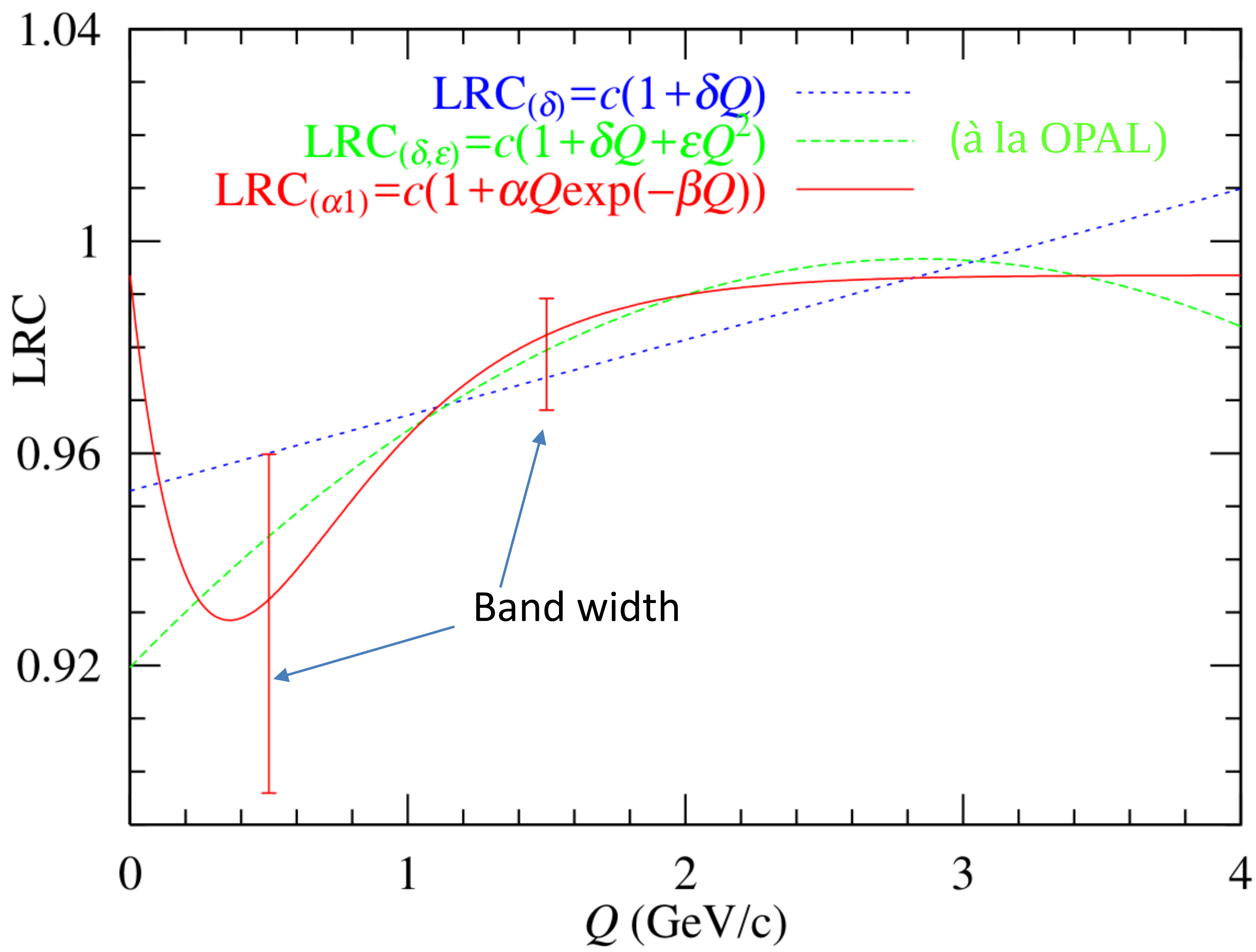
- In the τ model, $\lambda\cos\left((QR_a)^{2\alpha_\tau}\right) \times \mathrm{LRC}_{(\delta)}$ has the effect of long-range correlation. This is oscillating.

$$F_\tau(e^+e^-) = \left[1 + \lambda\cos(R_aQ)^{2\alpha_\tau}\exp(-(RQ)^{2\alpha_\tau})\right] \times \mathrm{LRC}_{(\delta)}$$

- Thus, the correlation function $\cos\left((QR_a)^{2\alpha_\tau}\right)\exp(-(QR)^{2\alpha_\tau})$ becomes to be negative.



Comparison of LRC's in $\mathrm{CF_{II}} \times$ LRC for L3 2-jet BEC

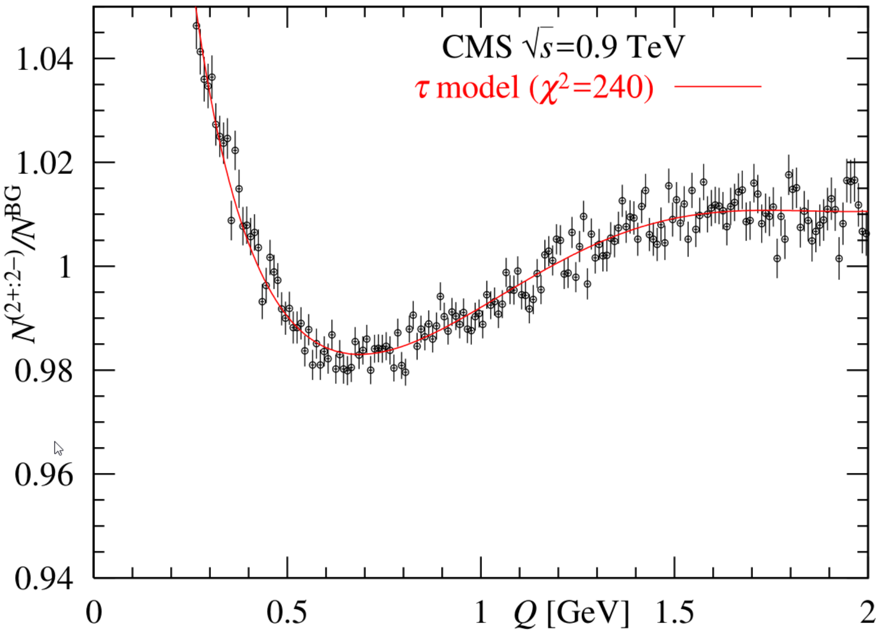


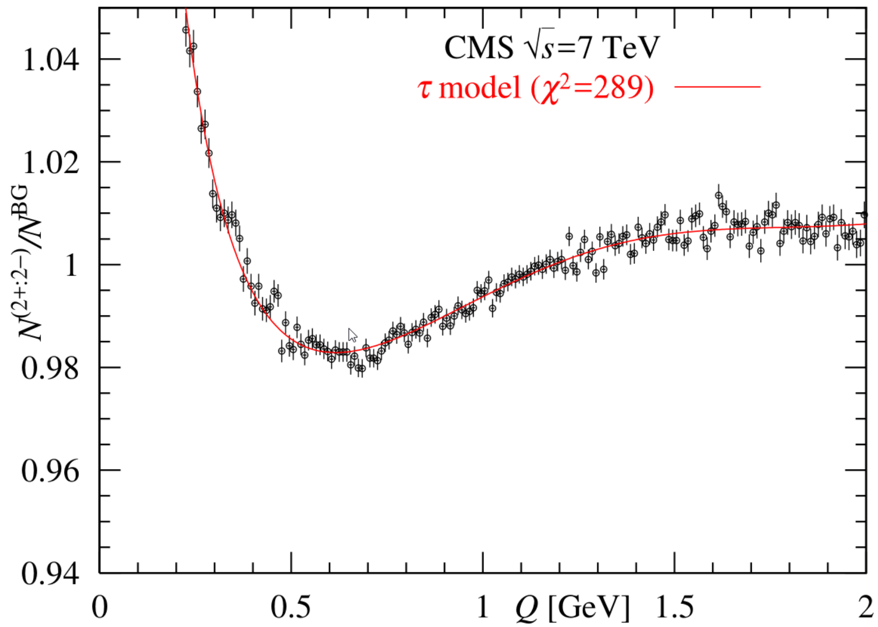
The third $\mathrm{LRC}_{(\alpha 1)}$ will be discussed later.

3. Analysis of CMS BEC at 0.9 and 7 TeV

- CMS collaboration used a τ-model formula for analysis of BEC.

$$F_\tau = \left\{1.0 + \lambda \cos\left[(R_0 Q)^2 + \tan\left(\frac{\alpha_\tau \pi}{4}\right)(RQ)^{\alpha_\tau}\right] \exp(-(RQ)^{\alpha_\tau})\right\} \times \mathrm{LRC}_{(\delta)}$$





Analysis of CMS BEC by τ-model formula with $\lambda \le 1$.

$\sqrt{s}$ (TeV)	R_0 (fm)	R (fm)	λ	c	δ (GeV^{-1})	α_τ	χ^2/n.d.f.
0.9	0.22±0.01	2.98±0.06	1.0	0.994±0.001	0.0082±0.0006	0.56±0.01	240/192
7.0	0.22±0.00	3.46±0.05	1.0	0.992±0.000	0.0081±0.0004	0.56±0.00	289/192

These parameter values is obtained by us and are not published in the paper by CMS.

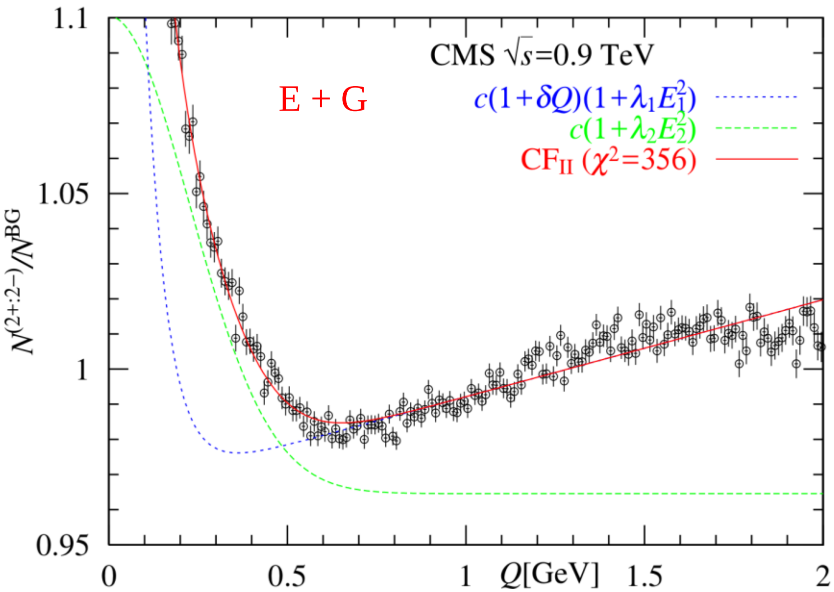
Analysis of CMS BEC by $\mathrm{CF_{II}} \times \mathrm{LRC}_{(\delta)}$

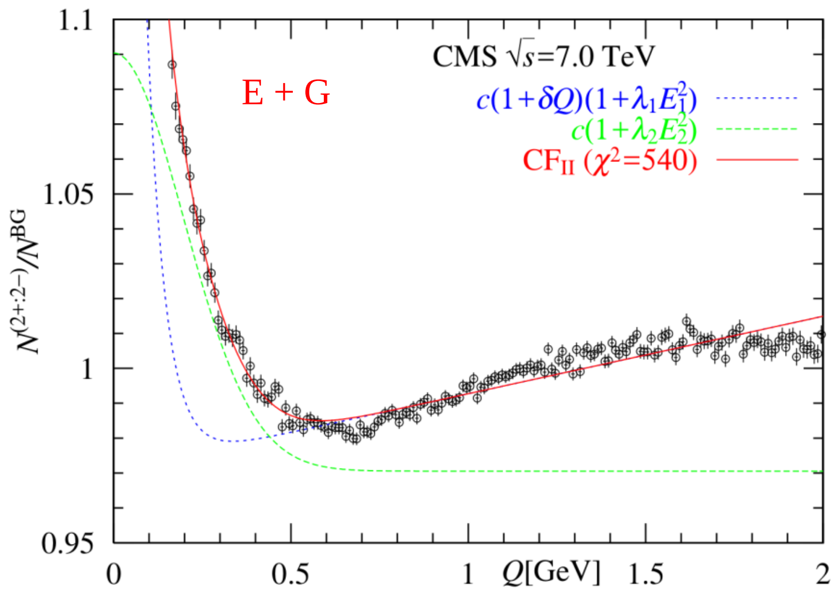
- $\mathrm{CF_{II}} \times \mathrm{LRC}_{(\delta)}$

$$\mathrm{CF_{II}} = [1.0 + \lambda_1 E_{\mathrm{BEC}_1}(R_1,\, Q) + \lambda_2 E_{\mathrm{BEC}_2}(R_2,\, Q)]$$

$$\mathrm{LRC}_{(\delta)} = c(1.0 + \delta Q)$$

$$E_{\mathrm{BEC}} = \exp(-RQ)\ (\text{Exponential})\ \text{or}\ \exp(-(RQ)^2)\ (\text{Gaussian})$$





Our results with $\mathrm{LRC}_{(\delta)}$

$\sqrt{s}$	R_1 (fm)	λ_1	R_2 (fm)	λ_2	c	δ (GeV^{-1})	χ^2/n.d.f.
0.9	3.37±0.19 (E)	0.80±0.04	0.62±0.01 (G)	0.14±0.01	0.965±0.001	0.029±0.001	356/192
	2.06±0.07 (G)	0.38±0.02	0.65±0.01 (G)	0.17±0.01	0.965±0.001	0.028±0.001	384/192
7.0	3.88±0.18 (E)	0.84±0.03	0.71±0.01 (G)	0.12±0.01	0.971±0.001	0.023±0.000	540/192
	2.39±0.07 (G)	0.40±0.01	0.76±0.01 (G)	0.16±0.00	0.971±0.001	0.022±0.000	600/192

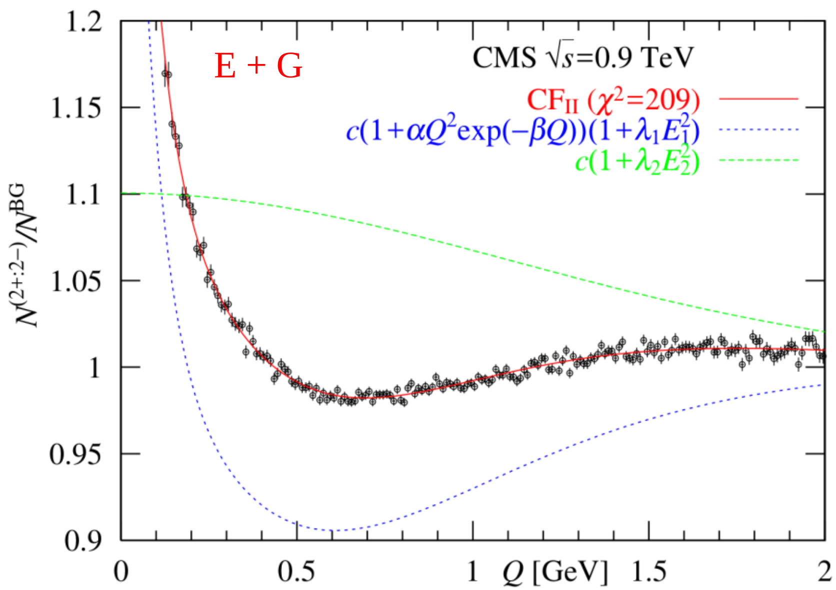
These χ^2' s are larger than τ-model.

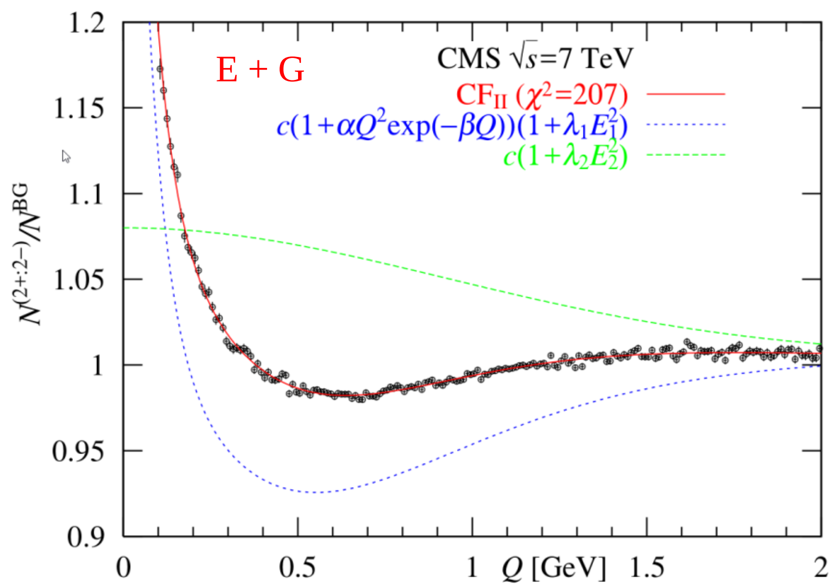
Analysis of CMS BEC by $\mathrm{CF_{II}} \times \mathrm{LRC}_{(\alpha\ 2)}$

- To improve χ^2's , we adopt the form: $\mathrm{LRC}_{(\alpha\ 2)}$

$$\mathrm{CF_{II}} = [1.0 + \lambda_1 E_{\mathrm{BEC}_1}(R_1,\ Q) + \lambda_2 E_{\mathrm{BEC}_2}(R_2,\ Q)]$$

$$\mathrm{LRC}_{(\alpha\ 2)} = c(1.0 + \alpha Q^2 \exp(-\beta Q))$$





Our results with $\mathrm{LRC}_{(\alpha 2)}$. These combinations are E+G.

$\sqrt{s}$	R_1 (fm)	λ_1	R_2 (fm)	λ_2	c	α (GeV^{-2})	β (GeV^{-1})	χ^2/n.d.f.
0.9	3.07±0.16	0.71±0.03	0.13±0.02	0.10±0.01	1.000±0.009	−1.90±0.18	3.29±0.09	209/191
7.0	3.42±0.14	0.75±0.03	0.15±0.01	0.07±0.00	1.005±0.003	−1.92±0.17	3.63±0.08	207/191

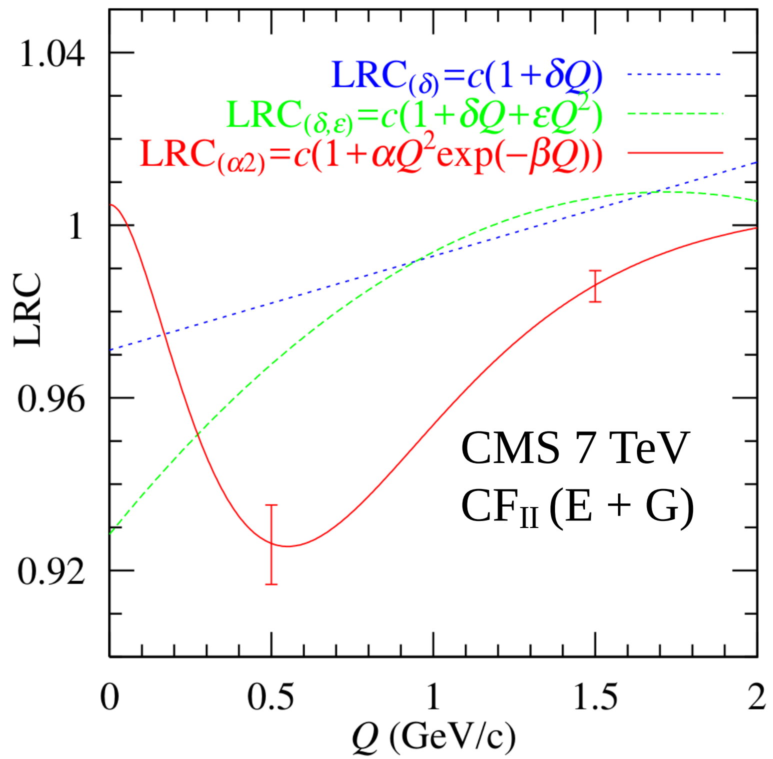
cf. $\mathrm{LRC}_{(\alpha 2)}$ is too drastic for L3 BEC, in spite of good χ^2 values.

Comparison of some results for 7 TeV

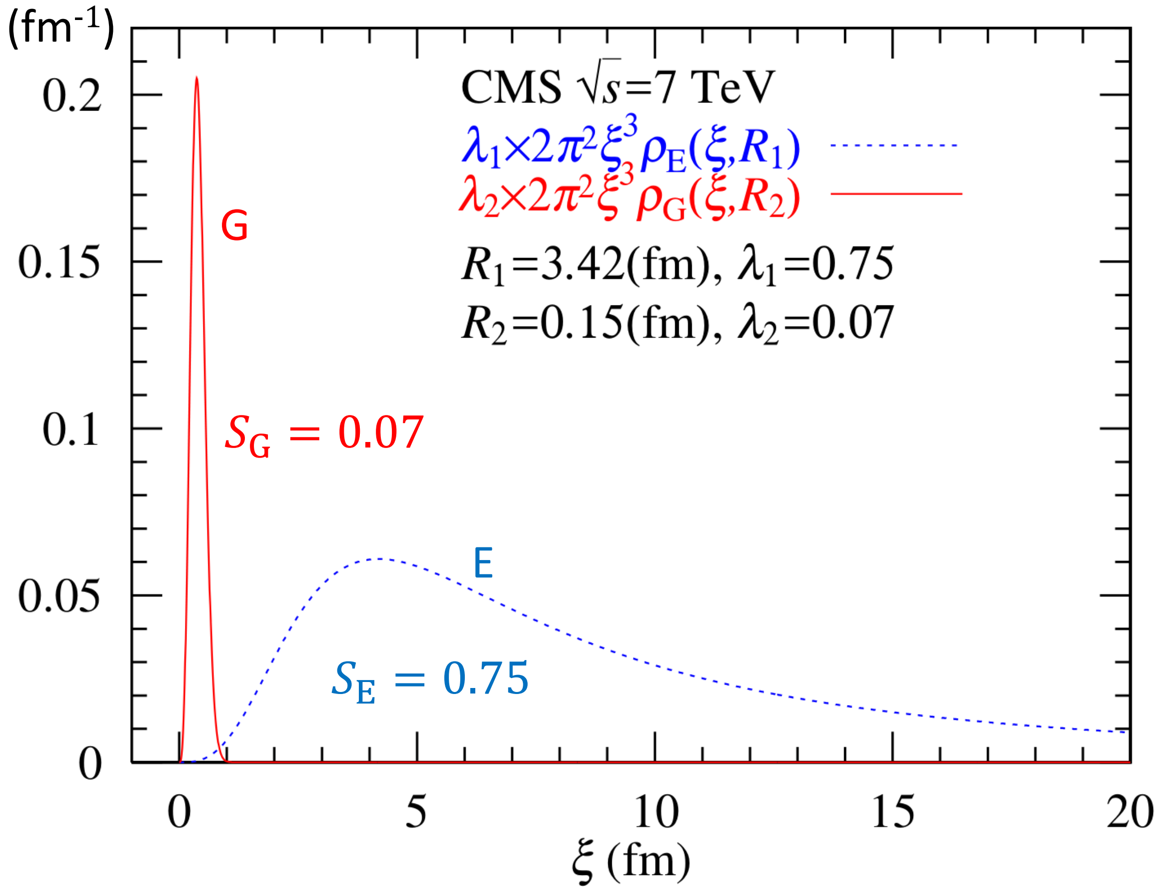
$$F_\tau = \left\{1.0 + \lambda \cos\left[(R_0 Q)^2 + \tan\left(\frac{\alpha_\tau \pi}{4}\right)(RQ)^{\alpha_\tau}\right]\exp(-(RQ)^{\alpha_\tau})\right\} \times \mathrm{LRC}_{(\delta)}$$

$\mathrm{CF_{II}} \times \mathrm{LRC}_{(\delta,\ \varepsilon)}$	$\mathrm{CF_{II}} \times \mathrm{LRC}_{(\alpha 2)}$	F_τ (τ-model)
$R_1 = 3.15$ fm, $\lambda_1 = 0.83$ (E)	$R_1 = 3.42$ fm, $\lambda_1 = 0.747$ (E)	$R = 3.46$ fm
$R_2 = 0.53$ fm, $\lambda_2 = 0.12$ (G)	$R_2 = 0.15$ fm, $\lambda_2 = 0.075$ (G)	$R_0 = 0.22$ fm
$\lambda_1 + \lambda_2 = 0.95$	$\lambda_1 + \lambda_2 = 0.822$	$\lambda = 1.0$ ($\alpha_\tau = 0.56$)
$\delta = 0.020$ fm, $\varepsilon = -0.0011$ fm^2	$\alpha = -0.075$ fm^2, $\beta = 0.72$ fm	
χ^2/n.d.f. = 208/191	χ^2/n.d.f. = 207/191	χ^2/n.d.f. = 289/192

Comparison of LRC's



Density distributions of two sources by $\mathrm{CF}_{\mathrm{II}} \times \mathrm{LRC}_{(\alpha\ 2)}$



$E_{\mathrm{BEC}}(R,\,Q)$	$\rho(\xi)$
$\exp(-R^2Q^2)$	$\frac{1}{16\pi^2R^4}\exp\left(-\frac{\xi^2}{4R^4}\right)$
$\exp(-RQ)$	$\frac{3}{4\pi^2R^4}\frac{1}{(1+(\xi/R)^2)^{5/2}}$

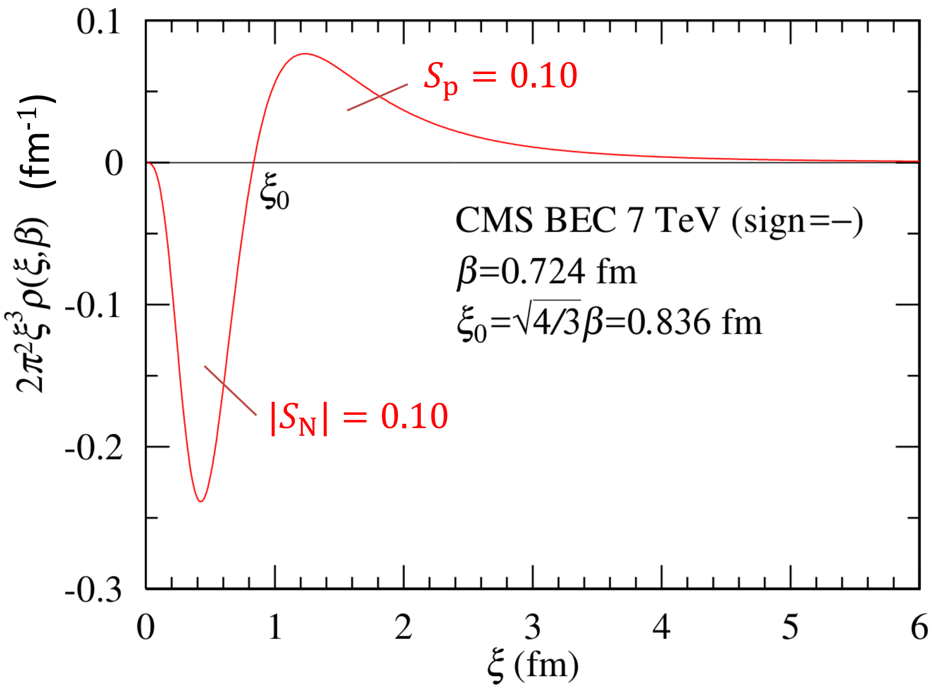
Density distributions of Gaussian and exponential forms for CMS BEC at 7 TeV. The degree of coherence (λ_1 and λ_2) are multiplied.

Density distribution of $(\mathrm{LRC}_{(\alpha 2)} - 1) = \alpha Q^2 \exp(-\beta Q)$

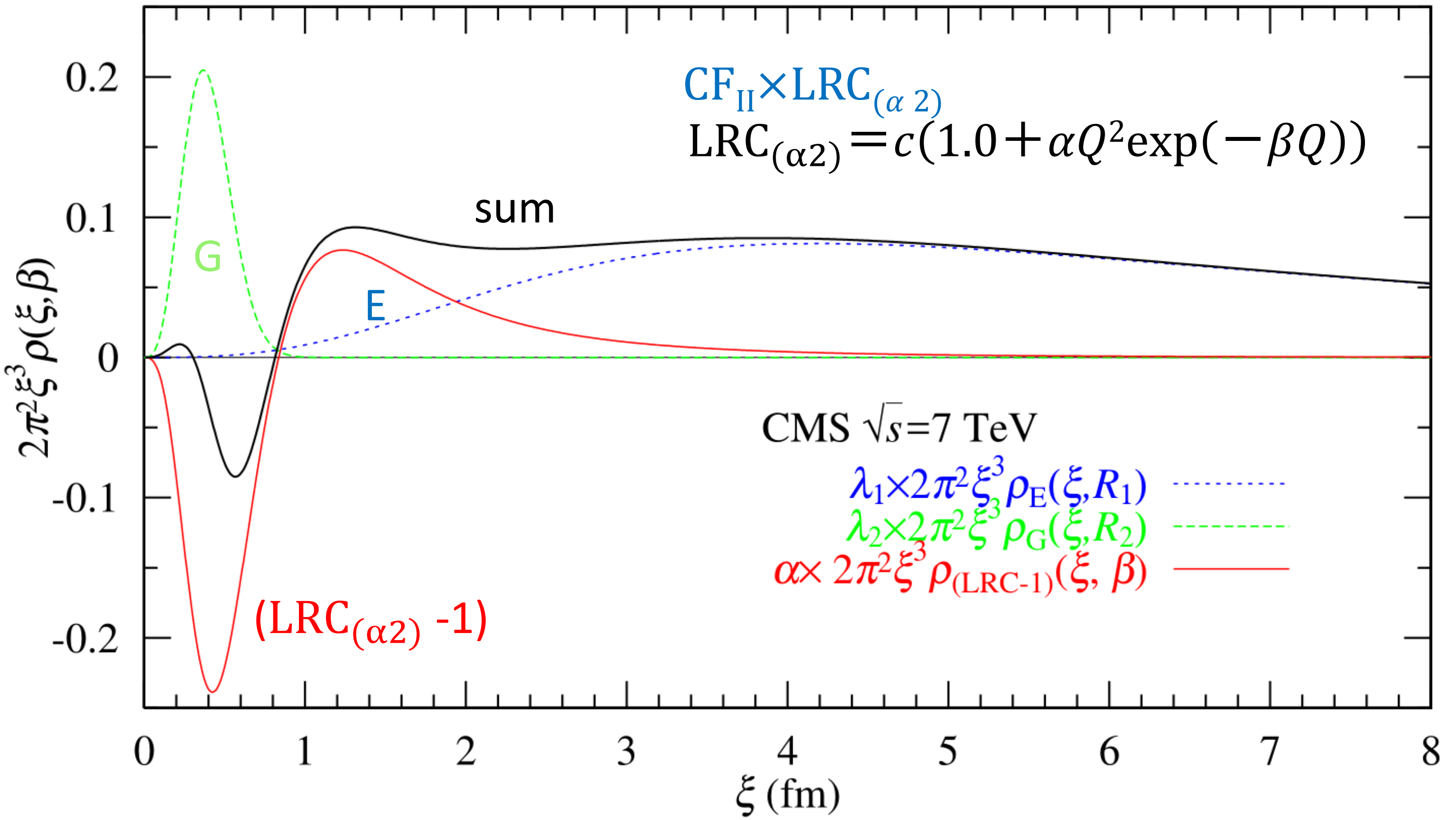
Using Bessel transformation, we derive the density distribution :

$$\rho(\xi,\,\beta) = \frac{\alpha}{(2\pi)^2\xi}\frac{\Gamma(6)}{(\beta^2+\xi^2)^{5/2}}\mathrm{P}_4^{-1}\left(\frac{\beta}{\sqrt{\beta^2+\xi^2}}\right)$$

where $\alpha = -1.92\ \mathrm{GeV}^{-2} = -0.0763\ \mathrm{fm}^2$.
$\mathrm{P}_4^{-1}(\ )$ is the associated Legendre function.



Sum of density distributions



- Crossed contribution can be omitted because of smallness $O(\alpha \cdot \lambda) \sim (10^{-3})$.

4. Concluding remarks

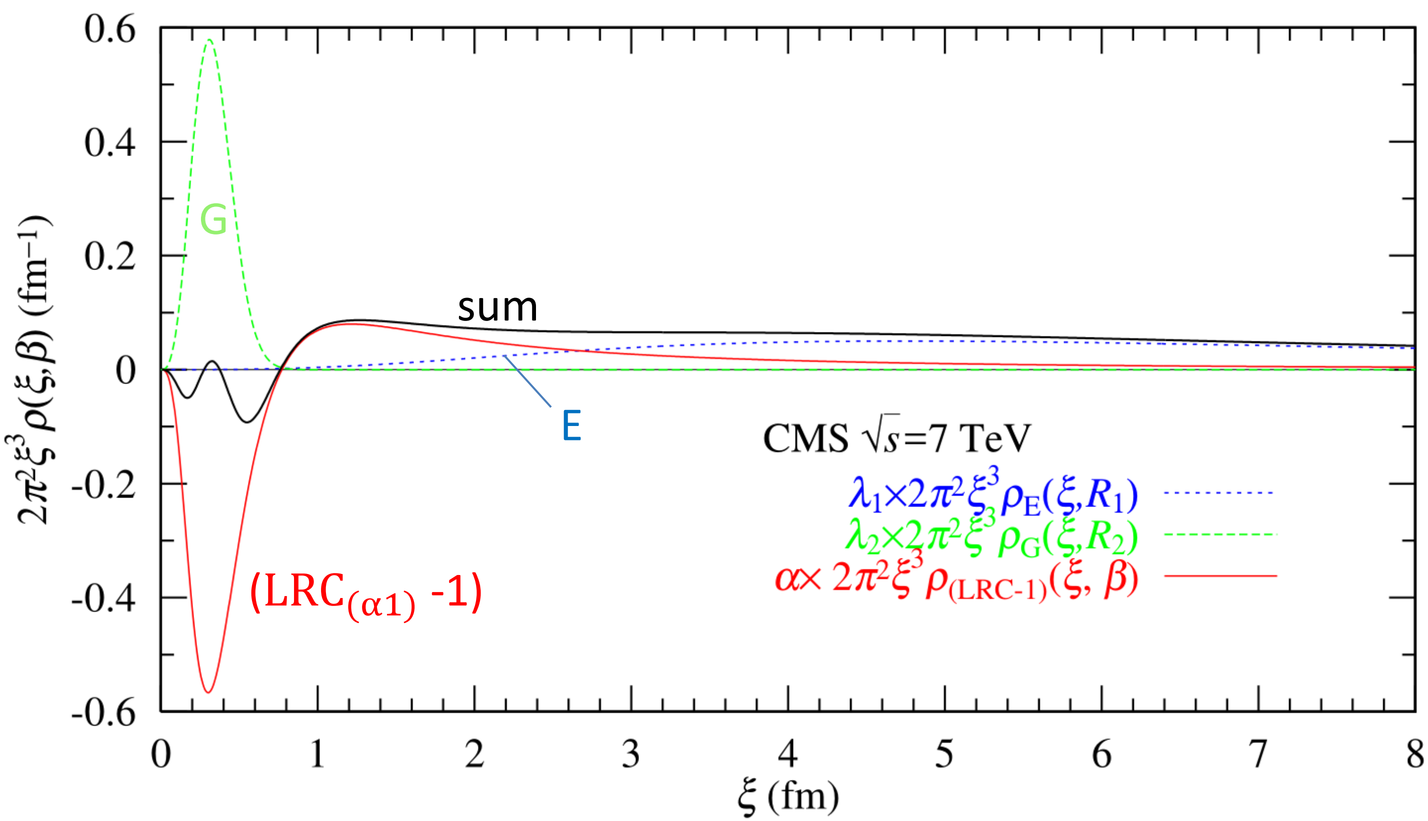
- From analysis of L3 BEC:
  - The determination of LRC seems to be an important work.
  - R_{eff}'s in the $\mathrm{CF_{II}}$ are almost the same as R's in the τ-model.
  - $\lambda_{\mathrm{eff}} = \lambda_1 + \lambda_2$ in the $\mathrm{CF_{II}}$ are almost the same as the degree of coherence in the τ-model.

- From analysis of CMS BEC:
  - The similar results are obtained from analysis of CMS BEC at 0.9 and 7 TeV.
  - R_1 (Exponential) in $[\mathrm{CF_{II}} \times \mathrm{LRC}_{(\alpha 2)}] \cong R$ in τ-model.
  - R_2 (Gaussian) in $[\mathrm{CF_{II}} \times \mathrm{LRC}_{(\alpha 2)}] \cong R_0$ in τ-model.
  - LRC's are important. Provided that LRC is an analytic function, we can probably draw some physical information from LRC.

Density distributions by $CF_{II}\times LRC_{(\alpha\ 1)}$

Density distribution of $(LRC_{(\alpha\ 1)}$-1)=$\alpha Q\exp(-\beta Q)$:

$$\rho(\xi,\,\beta)=\frac{\alpha}{(2\pi)^2\xi}\frac{\Gamma(5)}{(\beta^2+\xi^2)^2}\mathrm{P}_3^{-1}\left(\frac{\beta}{\sqrt{\beta^2+\xi^2}}\right)$$



Thank you for your attention.