Phase diagram and dualities in two color QCD

Roman N. Zhokhov

IZMIRAN, IHEP

20th ZIMÁNYI SCHOOL Winter Workshop on Heavy Ion Physics

December 7-11, 2020, Budapest, Hungary

Russian Science Foundation

БАЗИС

Фонд развития теоретической физики и математики

K.G. Klimenko, IHEP
T.G. Khunjua, University of Georgia, MSU

in the broad sense our group stems from
Department of Theoretical Physics, Moscow State University
Prof. V. Ch. Zhukovsky

details can be found in

Eur.Phys.J.C 80 (2020) 10, 995 arXiv:2005.05488 [hep-ph]
JHEP 06 (2020) 148 arXiv:2003.10562 [hep-ph]
Phys.Rev. D100 (2019) no.3, 034009 arXiv: 1904.07151 [hep-ph]
JHEP 1906 (2019) 006 arXiv:1901.02855 [hep-ph]
Eur.Phys.J. C79 (2019) no.2, 151, arXiv:1812.00772 [hep-ph],
Phys.Rev. D98 (2018) no.5, 054030 arXiv:1804.01014 [hep-ph],
Phys.Rev. D97 (2018) no.5, 054036 arXiv:1710.09706 [hep-ph]
Phys.Rev. D95 (2017) no.10, 105010 arXiv:1704.01477 [hep-ph]

The work is supported by

- Russian Science Foundation (RSF)

under grant number 19-72-00077

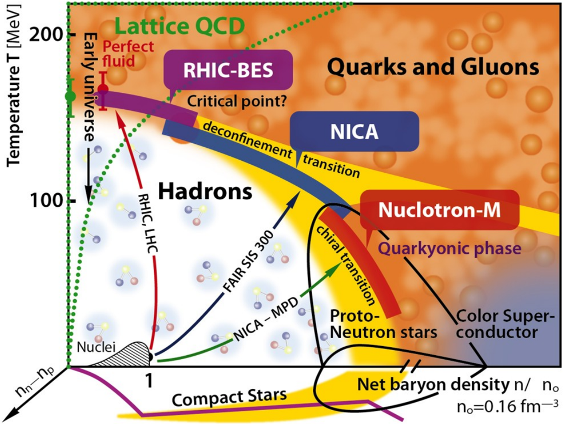
Russian Science Foundation

- Foundation for the Advancement of Theoretical Physics and Mathematics

БАЗИС

Фонд развития теоретической физики и математики

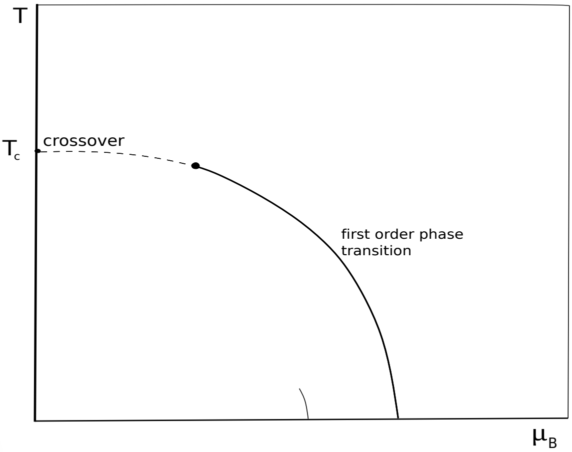
QCD Phase Diagram



Two main phase transitions

- confinement-deconfinement
- chiral symmetry breaking phase—chriral symmetric phase

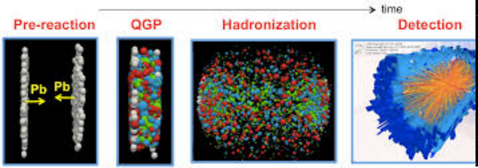
QCD Phase Diagram

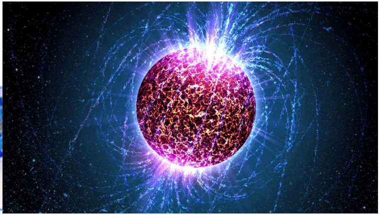


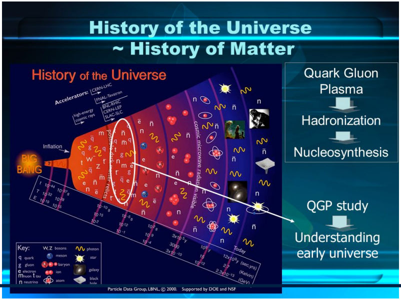
Two main phase transitions

- confinement-deconfinement
- chiral symmetry breaking phase—chriral symmetric phase

QCD at extreme conditions

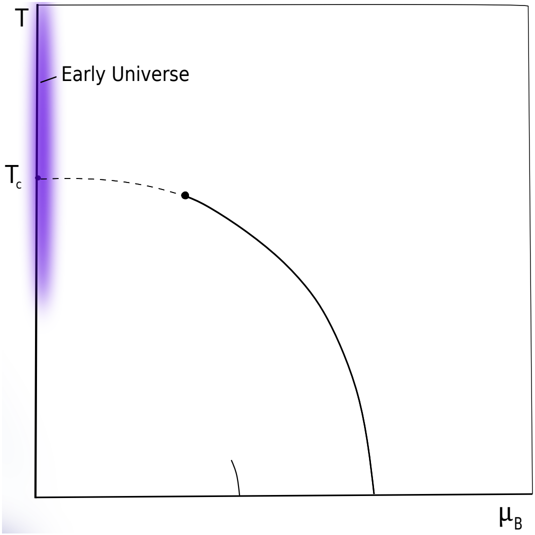






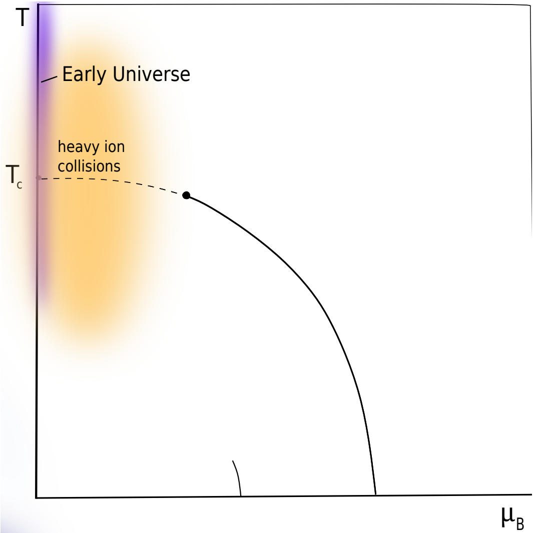
QCD at T and μ
(QCD at extreme conditions)

- Early Universe



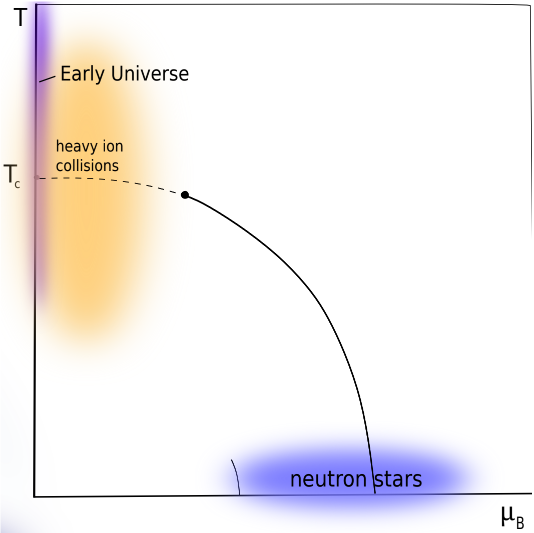
QCD at T and μ
(QCD at extreme conditions)

- Early Universe
- heavy ion collisions



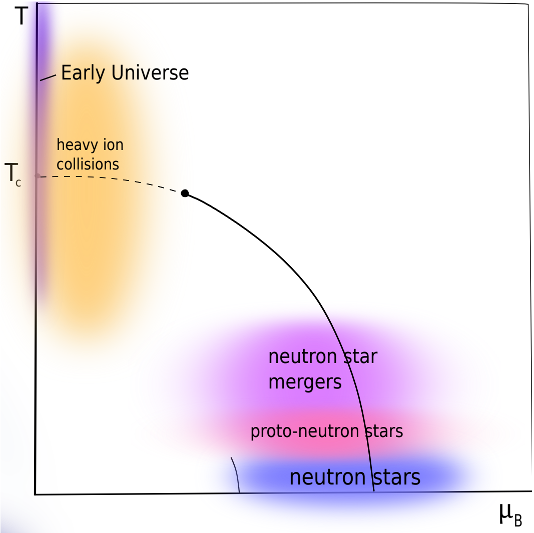
QCD at T and μ
(QCD at extreme conditions)

- Early Universe
- heavy ion collisions



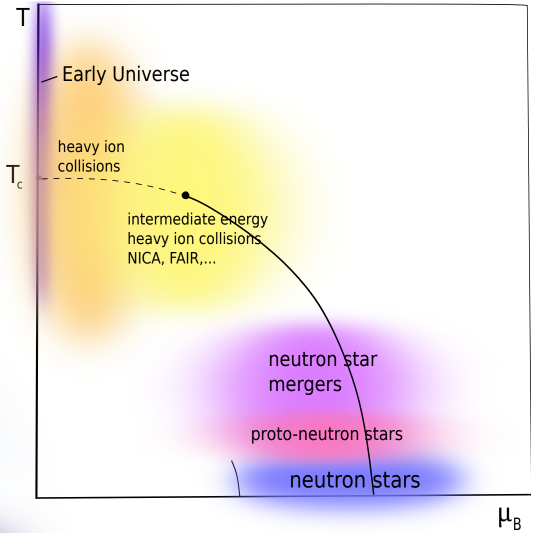
QCD at T and μ
(QCD at extreme conditions)

- Early Universe
- heavy ion collisions
- neutron stars
- proto- neutron stars
- neutron star mergers



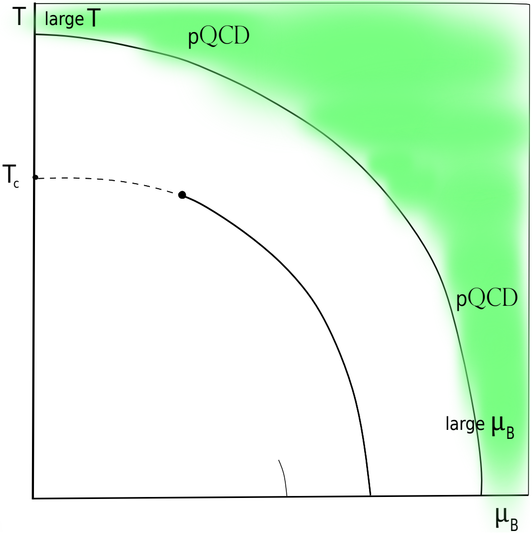
QCD at T and μ
(QCD at extreme conditions)

- Early Universe
- heavy ion collisions
- neutron stars
- proto- neutron stars
- neutron star mergers



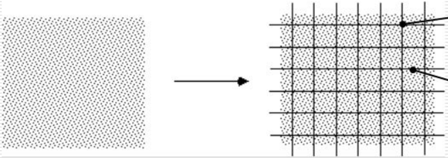
Methods of dealing with QCD

- Perturbative QCD



QCD on a space-time lattice

K. G. Wilson 1974



- Feynman path integral
  - Action $S_{QCD} = \frac{1}{g_s^2}\sum_P tr(UUUU) + \sum_f \bar{q}_f(\gamma\cdot U + m_f)q_f$
  - Physical quantities as integral averages

$$\langle O(U,\bar{q},q)\rangle = \frac{1}{Z}\int \prod_{n\mu} dU_{n\mu} \prod_n d\bar{q}_n dq_n\, O(U,(U,\bar{q},q))e^{-S_{QCD}}$$

→ *Monte Carlo Evaluation of the path integral*

lattice QCD at non-zero baryon chemical potential μ_B

$$Z = \int D[\text{gluons}]\, D[\text{quarks}]\, e^{-S^E_{QCD}}$$

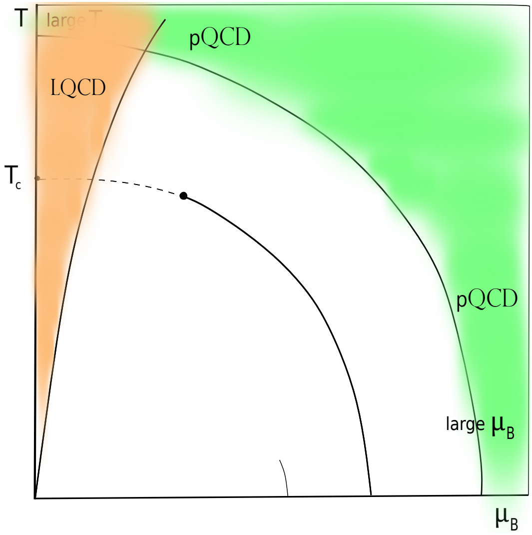
$$Z = \int D[\text{gluons}]\, \text{Det}\, \mathbb{D}(\mu)\, e^{-S^E_{\text{gluons}}}$$

It is well known that **at non-zero baryon chemical potential μ_B lattice simulation** is quite challenging due to the **sign problem**
complex determinant

$$(Det(D(\mu)))^{\dagger} = Det(D(-\mu^{\dagger}))$$

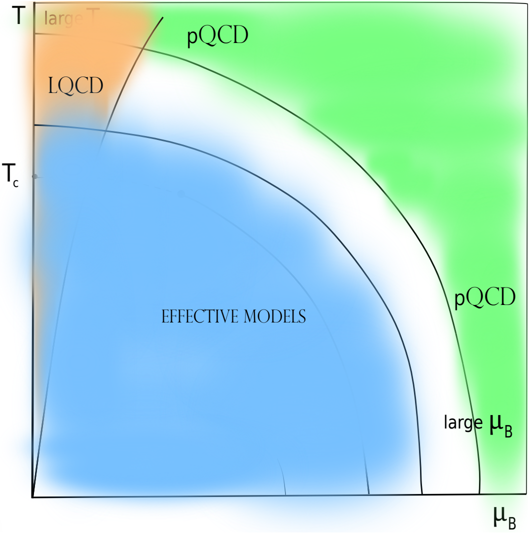
Methods of dealing with QCD

- Perturbative QCD
- First principle calculation – lattice QCD



Methods of dealing with QCD

- Perturbative QCD
- First principle calculation – lattice QCD
- Effective models
- DSE, FRG
- .....



Nambu–Jona-Lasinio model

$$\mathcal{L} = \bar{q}\gamma^{\nu}\mathrm{i}\partial_{\nu}q + \frac{G}{N_c}\Big[(\bar{q}q)^2 + (\bar{q}\mathrm{i}\gamma^5 q)^2\Big]$$

$$q \to e^{i\gamma_5\alpha}q$$

continuous symmetry

$$\widetilde{\mathcal{L}} = \bar{q}\Big[\gamma^{\rho}\mathrm{i}\partial_{\rho} - \sigma - \mathrm{i}\gamma^5\pi\Big]q - \frac{N_c}{4G}\Big[\sigma^2 + \pi^2\Big].$$

Chiral symmetry breaking

$1/N_c$ expansion, leading order

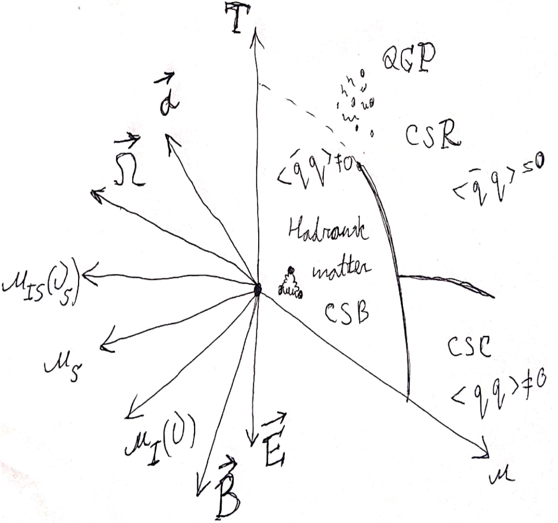
$$\langle\bar{q}q\rangle \neq 0$$

$$\langle\sigma\rangle \neq 0 \quad \longrightarrow \quad \widetilde{\mathcal{L}} = \bar{q}\Big[\gamma^{\rho}\mathrm{i}\partial_{\rho} - \langle\sigma\rangle\Big]q$$

More external conditions to QCD

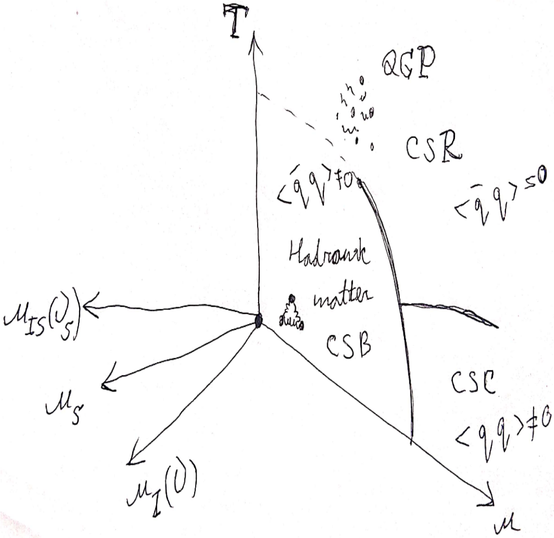
More than just QCD at (μ, T)

- more chemical potentials μ_i
- magnetic fields
- rotation of the system $\vec{\Omega}$
- acceleration $\vec{a}$
- finite size effects (finite volume and boundary conditions)



More than just QCD at (μ, T)

- **more chemical potentials** μ_i
- magnetic fields
- rotation of the system
- acceleration
- finite size effects (finite volume and boundary conditions)



Baryon chemical potential μ_B

Allow to consider systems with non-zero baryon densities.

$$\frac{\mu_B}{3}\bar{q}\gamma^0 q = \mu\bar{q}\gamma^0 q, \qquad n_B = \frac{1}{3}(n_u + n_d)$$

Baryon chemical potential μ_B

Allow to consider systems with non-zero baryon densities.

$$\frac{\mu_B}{3}\bar{q}\gamma^0 q = \mu\bar{q}\gamma^0 q, \qquad\qquad n_B = \frac{1}{3}(n_u + n_d)$$

Isotopic chemical potential μ_I

Allow to consider systems with isospin imbalance ($n_n \neq n_p$).

$$\frac{\mu_I}{2}\bar{q}\gamma^0\tau_3 q = \nu\left(\bar{q}\gamma^0\tau_3 q\right)$$

$$n_I = n_u - n_d \quad \longleftrightarrow \quad \mu_I = \mu_u - \mu_d$$

chiral (axial) chemical potential

Allow to consider systems with chiral imbalance (difference between densities of left-handed and right-handed quarks).

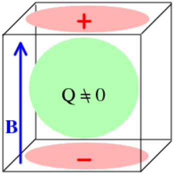
$$n_5 = n_R - n_L \quad \longleftrightarrow \quad \mu_5 = \mu_R - \mu_L$$

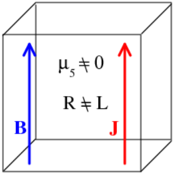
The corresponding term in the Lagrangian is

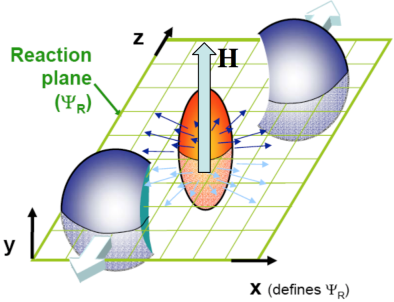
$$\mu_5 \bar{q} \gamma^0 \gamma^5 q$$

see talk of Soeren Schlichting and Roy Lacey

Chiral magnetic effect



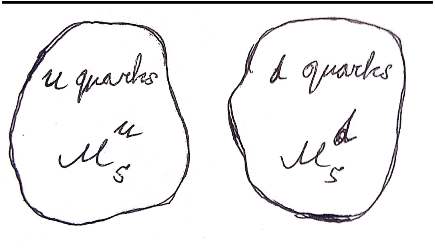




$$\vec{J} \sim \mu_5 \vec{B},$$

see talk of Soeren Schlichting and Roy Lacey

A. Vilenkin, PhysRevD.22.3080,
K. Fukushima, D. E. Kharzeev and H. J. Warringa, Phys. Rev. D **78** *(2008) 074033*



$$\mu_5^u \neq \mu_5^d \qquad \text{and} \qquad \mu_{I5} = \mu_5^u - \mu_5^d$$

Term in the Lagrangian — $\frac{\mu_{I5}}{2}\bar{q}\tau_3\gamma^0\gamma^5 q = \nu_5(\bar{q}\tau_3\gamma^0\gamma^5 q)$

$$n_{I5} = n_{u5} - n_{d5}, \qquad n_{I5} \longleftrightarrow \nu_5$$

$\mu_B \neq 0$ impossible on lattice due to the **sign problem**

- **QCD at non-zero** μ_5 — no sign problem: (μ_5, T)

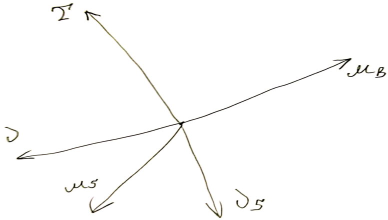
  (V. Braguta, A. Kotov et al, ITEP lattice group)

- **QCD at non-zero** μ_I — no sign problem: (μ_I, T)

  (G. Endrodi, B. Brandt et al, Emmy Noether junior research group, Goethe-University Frankfurt, Institute for Theoretical Physics ()

Different chemical potentials and matter content

$$\mu = \frac{\mu_B}{3}, \qquad \nu = \frac{\mu_I}{2}, \qquad \mu_5, \qquad \nu_5 = \frac{\mu_{I5}}{2}$$



Dualities

It is not related to holography or gauge/gravity duality

it is the dualities of the phase structures of different systems

Auxiliary fields

$$\sigma(x) = -2\frac{G}{N_c}(\bar{q}q); \quad \pi_a(x) = -2\frac{G}{N_c}(\bar{q}\mathrm{i}\gamma^5\tau_a q).$$

Condansates ansatz $\langle\sigma(x)\rangle$ and $\langle\pi_a(x)\rangle$ do not depend on spacetime coordinates

$$\langle\sigma(x)\rangle = M, \quad \langle\pi_1(x)\rangle = \pi, \quad \langle\pi_2(x)\rangle = 0, \quad \langle\pi_3(x)\rangle = 0.$$

$M = \langle\sigma(x)\rangle \sim \langle\bar{q}q\rangle,$ CSB phase: $M \neq 0,$

$\pi_1 = \langle\pi_1(x)\rangle = \langle\bar{q}\gamma^5\tau_1 q\rangle,$ PC phase: $\pi_1 \neq 0,$

The TDP

$\Omega(T, \mu, \mu_i, ..., \langle \bar{q}q \rangle, ...)$

The TDP

$$\Omega(T, \mu, \mu_i, ..., \langle \bar{q}q \rangle, ...) \qquad \Omega(T, \mu, \nu, \nu_5, ..., M, \pi, ...)$$

The TDP

$$\Omega(T, \mu, \mu_i, ..., \langle \bar{q}q \rangle, ...) \qquad \Omega(T, \mu, \nu, \nu_5, ..., M, \pi, ...)$$

The TDP (phase daigram) is invariant under

Interchange of - condensates - matter content

$$\Omega(M, \pi, \nu, \nu_5)$$

$$M \longleftrightarrow \pi, \qquad \nu \longleftrightarrow \nu_5$$

$$\Omega(M, \pi, \nu, \nu_5) = \Omega(\pi, M, \nu_5, \nu)$$

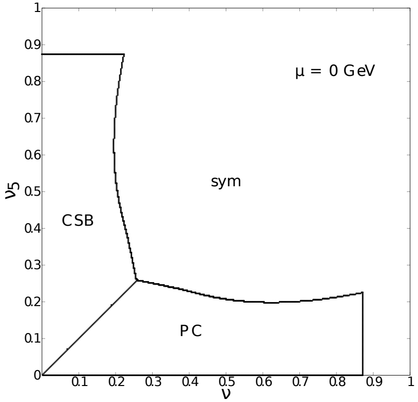


Figure: NJL model results

$$\Omega(M, \pi, \nu, \nu_5) = \Omega(\pi, M, \nu_5, \nu)$$

$$\mathcal{D}: \; M \longleftrightarrow \pi, \quad \nu \longleftrightarrow \nu_5$$

Duality between chiral symmetry breaking and pion condensation

$$\text{PC} \longleftrightarrow \text{CSB} \quad \nu \longleftrightarrow \nu_5$$

Duality was found in

- In the framework of effective model, namely NJL model
- In the leading order of **large** N_c approximation or in **mean field**

Dualities on the lattice

$(\mu_B, \mu_I, \mu_{I5}, \mu_5)$

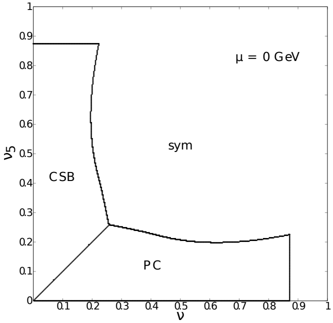
$\mu_B \neq 0$ impossible on lattice but if $\mu_B = 0$

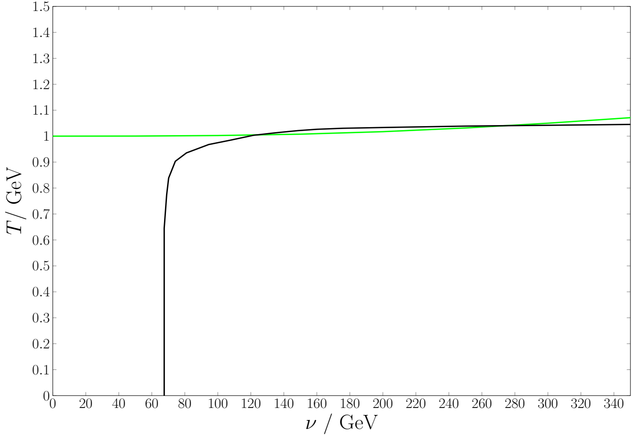
- **QCD at μ_5** —- (μ_5, T)

  V. Braguta, A. Kotov et al, ITEP lattice group

- **QCD at μ_I** —- (μ_I, T)

  G. Endrodi, B. Brandt et al, Emmy Noether junior research group, Goethe-University Frankfurt, Institute for Theoretical Physics ()





T_c^M as a function of μ_5 (green line) and $T_c^\pi(\nu)$ (black)

Uses of Dualities

How (if at all) it can be used

Let us discuss only Inhomogeneous phases (case)

discussed in Particles 2020, 3(1), 62-79

schematic (ν_5, μ)-phase diagram

$$(\mu, \nu) \longrightarrow (\mu, \nu_5)$$

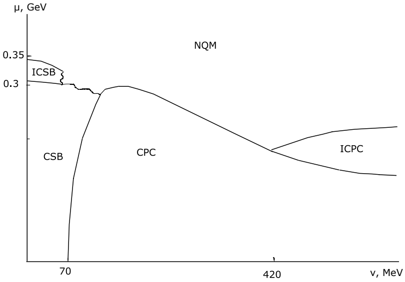


Figure: (ν, μ)-phase diagram.

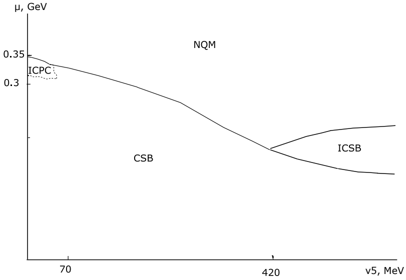


Figure: (ν_5, μ)-phase diagram

M. Buballa, S. Carignano, J. Wambach, D. Nowakovski, Lianyi He et al.

Two colour QCD case

QC_2D

There are a lot similarities:

- similar phase transitions:

  confinement/deconfinement, chiral symmetry breaking/restoration at large T and μ

- A lot of physical quantities coincide up to few dozens percent

  Critical temperature $T_c/\sqrt{\sigma}$, topological susceptibility $\chi^{\frac{1}{4}}/\sqrt{\sigma}$ shear viscosity η/s

There are **no sign problem** in SU(2) case

$$(Det(D(\mu)))^{\dagger} = Det(D(\mu))$$

and lattice simulations at non-zero baryon density are possible

It is a great playground for studying dense matter

Instead of chiral symmetry

$$SU_L(2) \times SU_R(2)$$

one has Pauli-Gursey flavor symmetry

$$SU(4)$$

Two colour NJL model

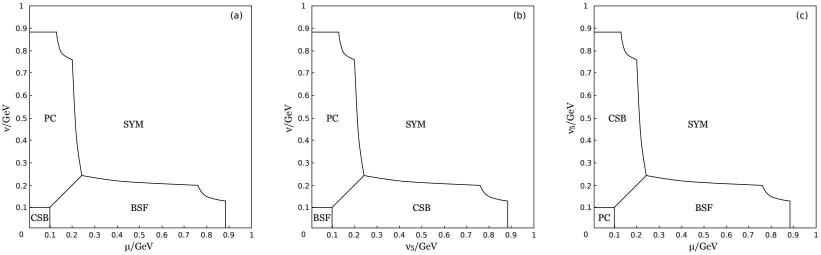
$$L = \bar{q}\Big[i\hat{\partial} - m_0\Big]q + H\Big[(\bar{q}q)^2 + (\bar{q}i\gamma^5\vec{\tau}q)^2 + \left(\bar{q}i\gamma^5\sigma_2\tau_2 q^c\right)\left(\overline{q^c}i\gamma^5\sigma_2\tau_2 q\right)\Big]$$

Condensates and phases

$$M = \langle \sigma(x) \rangle \sim \langle \bar{q}q \rangle, \qquad \text{CSB phase:} \quad M \neq 0,$$

$$\pi_1 = \langle \pi_1(x) \rangle = \langle \bar{q}\gamma^5 \tau_1 q \rangle, \qquad \text{PC phase:} \quad \pi_1 \neq 0,$$

$$\Delta = \langle \Delta(x) \rangle = \langle qq \rangle = \langle q^T C \gamma^5 \sigma_2 \tau_2 q \rangle, \qquad \text{BSF phase:} \quad \Delta \neq 0.$$



$$(a) \qquad \mathcal{D}_1 : \qquad \mu \longleftrightarrow \nu, \quad \pi_1 \longleftrightarrow |\Delta|, \qquad \mathrm{PC} \longleftrightarrow \mathrm{BSF}$$

J. Andersen, T. Brauner, D. T. Son, M. Stephanov, J. Kogut, ...

$$(b) \qquad \mathcal{D}_3 : \qquad \nu \longleftrightarrow \nu_5, \quad M \longleftrightarrow \pi_1, \qquad \mathrm{PC} \longleftrightarrow \mathrm{CSB}$$

$$(c) \qquad \mathcal{D}_2 : \qquad \mu \longleftrightarrow \nu_5, \quad M \longleftrightarrow |\Delta|, \qquad \mathrm{CSB} \longleftrightarrow \mathrm{BSF}$$

Dualities $\mathcal{D}_1$, $\mathcal{D}_2$ and $\mathcal{D}_3$ were found in

- In the framework of NJL model

- In the mean field approximation

Dualities are connected with Pauli-Gursey group

Dualities were found in

- In the framework of NJL model beyond mean field

- In QC_2D non-pertubartively (at the level of Lagrangian)

Duality $\mathcal{D}$ is a remnant of chiral symmetry

Duality was found in

- In the framework of NJL model beyond mean field or at all orders of N_c approximation
- In QCD non-pertubartively (at the level of Lagrangian)

- $(\mu_B, \mu_I, \nu_5, \mu_5)$ phase diagram was studied in two color color case
- It was shown that there exist dualities in QCD and QC_2D

  Richer structure of Dualities in the two colour case
- There have been shown ideas how dualities can be used

  Duality is not just entertaining mathematical property but an instrument with very high predictivity power
- Dualities have been shown non-perturbetively in the two colour case
- Duality has been shown non-perturbarively in QCD