Curvature masses for (axial) vector mesons in Extended linear sigma model

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Outline

- Current state of Extended linear sigma model (ELSM)
- Improvements of the model
- Fermionic contribution to (axial) vector curvature masses

ELSM

Vector and axial vector meson extended Polyakov linear sigma model Phys. Rev. D **93**, no. 11, 114014 (2016)

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- Linear sigma model with full Scalar, Pseudoscalar, Vector and Axial vector nonets. Isospin symmetric case: 16 mesonic degrees of freedom.
- The Lagrangian build up from the fields

$$L^\mu = \sum_a (V_a^\mu + A_a^\mu)\, T_a, \quad R^\mu = \sum_a (V_a^\mu - A_a^\mu)\, T_a, \quad M = \sum_a (S_a + iP_a)\, T_a, \tag{1}$$

  with terms up to fourth order, taking care of the symmetry properties.
- $\mathcal{L}_m$ contains the dynamical, the symmetry braking, and the meson-meson interaction terms.
- Constituent quarks (2+1 flavors) are included,

$$\mathcal{L}_{Yukawa} = \bar{\psi}\,(i\gamma^\mu \partial_\mu - g_F(S - i\gamma_5 P)))\,\psi, \tag{2}$$

ELSM

- SSB with nonzero vacuum expectation value for scalar-isoscalar sector $\phi_N,\ \phi_S$.
  Polyakov loop variables $\Phi,\ \bar{\Phi}$. $\Rightarrow$ 4 order parameters.
- In the Yukawa term after SSB $m_{u,d} = \frac{g_F}{2}\phi_N,\ m_s = \frac{g_F}{\sqrt{2}}\phi_S$ fermion masses appear.
- S-V and P-A mixing in the quadratic part of the Lagrangian $\Rightarrow$ Shift in the A/V fields
  $\Rightarrow$ The S/P tree level masses get an extra factor $m^2 \to Z^2 m^2$.
- Parametrization with χ^2 method with 30 physical quantities for 15 parameters in the Lagrangian based on the (pseudo)scalar curvature masses and the (axial) vector tree level masses.
- Good match with the lattice results at $\mu_B = 0$.
  CEP at $T = 52.7\ MeV,\ \mu_B = 885\ MeV$.

ELSM

Thermodynamics: **Mean field level** effective potential:
classical potential + fermionic one loop correction with vanishing fluctuating mesonic fields + Polyakov term

$$\Omega(T,\mu_q) = U_{Cl}(\langle M\rangle) + \mathrm{Tr}\log\left(iS_0^{-1}\right) + U(\Phi,\bar{\Phi}) \tag{3}$$

Field equations (FE):

$$\frac{\partial\Omega}{\partial\bar{\Phi}} = \frac{\partial\Omega}{\partial\Phi} = \frac{\partial\Omega}{\partial\phi_N} = \frac{\partial\Omega}{\partial\phi_S} = 0 \tag{4}$$

Mesonic one loop corrections ($\pi,\ K,\ f_L^0$): taken into account only in the pressure!

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Possible steps to improve the model:

Including (axial) vector-fermion interaction in the Yukawa Lagrangian

Including mesonic one loop correction in the potential and the FE

(Axial) vector-fermion interaction

Yukawa term in the Lagrangian

$$\mathcal{L}_Y = \bar{\psi}\left(i\gamma^\mu\partial_\mu - g_F(S - i\gamma_5 P) - g_V\gamma^\mu(V_\mu + \gamma_5 A_\mu)\right)\psi \quad . \tag{5}$$

So the action $S_Y = \int d^4x\bar{\psi}i\mathcal{S}^{-1}[\varphi]\psi$ (with $\varphi = \{S,\ P,\ V,\ A\}$)
For he partition function

$$\begin{gathered}\int \mathcal{D}\bar{\psi}\mathcal{D}\bar{\psi}\exp\left(iS_Y\right) = \int \mathcal{D}\bar{\psi}\mathcal{D}\bar{\psi}\exp\left(i\int d^4x\bar{\psi}i\mathcal{S}^{-1}\psi\right) = \mathrm{Det}i\mathcal{S}^{-1} \\ \Downarrow \\ Z = \int \mathcal{D}\varphi\mathcal{D}\bar{\psi}\mathcal{D}\bar{\psi}e^{i(S_m[\varphi]+S_Y[\varphi,\ \psi])} = \int \mathcal{D}\varphi e^{iS_m[\varphi]+\log \mathrm{Det}i\mathcal{S}^{-1}[\varphi]}\end{gathered} \tag{6}$$

SSB shift $\varphi \to \phi + \varphi$ and changing to momentum space

$$U_f(\phi,\varphi) = i\int_k \log \mathrm{Det}i\mathcal{S}^{-1}(k;\phi,\varphi) \tag{7}$$

The effective potential: $U_{MF}(\phi) = U_{Cl}(\phi) + U_f(\phi,\ \varphi = 0)$ remains the same!

Going further in approximation for the thermodynamics

Now: Mean-field level:

$$U(\phi) = U_{Cl}(\phi) + U_f(\phi,\ \varphi = 0) \tag{8}$$

Next step: Including one loop mesonic correction:

$$U(\phi) = U_{Cl}(\phi) + U_f(\phi,\ \varphi = 0) - \frac{i}{2}\mathrm{tr}\int_k \log\left(i\mathcal{D}^{-1}(k) - \Pi(k)\right) \tag{9}$$

with

$$i\mathcal{D}^{-1}(\phi; x, y) = \left.\frac{\delta^2 S_m}{\delta\varphi(x)\delta\varphi(y)}\right|_{\varphi=\phi}, \qquad -\Pi(\phi; x, y) = \left.\frac{\delta^2 S_Y}{\delta\varphi(x)\delta\varphi(y)}\right|_{\varphi=\phi}. \tag{10}$$

Expanding the logarithm: $\log(i\mathcal{D}^{-1}(1 + i\mathcal{D}\Pi)) = \log(i\mathcal{D}^{-1}) + \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}(i\mathcal{D}\Pi)^n$, one can see that this leads to a ring resummation.

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Some further approximations:

- Local approximation: $\Pi(p) \to \Pi(0)$,
- In the thermodynamics $g_V = 0$, ie. no fermion-(axial) vector interaction.

Currently under work.

Curvature masses

$$
\begin{aligned}
U_f(\phi,\varphi) =& i\int_k \log \mathrm{Det} i\mathcal{S}^{-1}(k;\phi,\varphi) \\
i\mathcal{S}^{-1}(k;\phi,\varphi) =& \gamma_0\big(i\gamma^\mu\partial_\mu + \mathbb{1}\mathrm{diag}(m_u,m_d,m_s) - g_F(\mathbb{1}S^a\lambda^a - i\gamma_5 P^a\lambda^a) \\
& - g_V\gamma^\mu(V_\mu^a\lambda^a + \gamma_5 A_\mu^a\lambda^a)\big)
\end{aligned} \tag{11}
$$

$i\mathcal{S}^{-1}$ can be represented as a 12×12 matrix (3 in flavor, 4 in Dirac space)
The fermionic contribution for the curvature masses for $X = S,\ P$ and $Y_\mu = V_\mu,\ A_\mu$

$$
\delta\hat{m}^2_{(ab)} = \left.\frac{d^2U_f(\phi,\varphi)}{dX_a dX_b}\right|_{\varphi=0} \qquad \delta\hat{M}^2_{(ab)\mu\nu} = -\left.\frac{d^2U_f(\phi,\varphi)}{dY_a^\mu dY_b^\nu}\right|_{\varphi=0} \tag{12}
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Curvature masses

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$$\delta\hat{m}^2_{(ab)} = \frac{d^2 U_f(\phi,\varphi)}{dX_a dX_b}\bigg|_{\varphi=0} \qquad \delta\hat{M}^2_{(ab)\mu\nu} = -\frac{d^2 U_f(\phi,\varphi)}{dY_a^\mu dY_b^\nu}\bigg|_{\varphi=0} \tag{12}$$

- $\delta\hat{m}^2_{(ab)}$ was already calculated using the method of Schaefer and Wagner [PRD **79**, 014018 (2009)]
  - Dirac and flavor structure can be totally separated; one can get rid of the momentum
  - Can be reduced to the derivative of fermionic masses
- $\delta\hat{M}^2_{(ab)\mu\nu}$ cannot be calculated this way
  the mass has to be a scalar: $\delta M^2_{Y_a} = \delta\hat{M}^2_{(aa)\mu\nu} g^{\mu\nu}$ (No mixing in flavor)

Brut force calculation

For $a = 1-2, 4-7$ in flavor space

$$\mathrm{Det}\, i\mathcal{S}^{-1} = \left[\frac{g_V^4}{16}(Y_a^2)^2 + \frac{g_V^2}{2}\left(Y_a^2(K^2 \mp m_i m_j) - 2(Y_a \cdot K)^2\right) + (m_i^2 - K^2)(m_j^2 - K^2)\right]^2 \left[m_k^2 - K^2\right]^2 \tag{13}$$

with $ij, k = \begin{cases} ud, s & \text{for } a = 1-2 \\ su, d & \text{for } a = 4-5 \\ ds, u & \text{for } a = 6-7 \end{cases}$ with $\mp$ for vectors and axials resp.

After the derivation:

$$\frac{2g_V^2}{(m_i^2 - K^2)(m_j^2 - K^2)}\left[g_{\mu\nu}(K^2 \mp m_i m_j) - 2K_\mu K_\nu\right] \tag{14}$$

$$\begin{aligned} \frac{2g_V^2}{(m_j^2 - m_i^2)}\Big(&(m_i(m_i + m_j)g_{\mu\nu} - 2K_\mu K_\nu)\frac{1}{m_i^2 - K^2} \\ &- (m_j(m_j \mp m_i)g_{\mu\nu} - 2K_\mu K_\nu)\frac{1}{m_j^2 - K^2}\Big) \end{aligned} \tag{15}$$

The fermionic contribution can be expressed in terms of tadpoles and derivated tadpoles.

One loop self energy

Connection to one loop (fermionic) self energy (for the mesons):

$$\Pi_{ab}(\phi; x, y) = \left.\frac{d^2 S_f(\phi, \varphi)}{d\varphi_a(x) d\varphi_b(y)}\right|_{\varphi=0} \to \Pi_{ab}(\phi; p) \longrightarrow \Pi_{ab}(\phi; 0) = \delta m^2_{(ab)} \tag{16}$$

The fermionic contribution to the mean field level curvature mass is the one loop self energy at vanishing external momentum.

E.g.: $\mathbf{N_f = 1}$:

$$i\Pi^V_{ab}(p=0) = -(-ig_V)^2 \mathrm{tr} \int_k \gamma_\mu \mathcal{S}_f(k) \gamma_\nu \mathcal{S}_f(k) \tag{17}$$

with $S_f = (\not{k} + m_f^2)\dfrac{i}{k^2 - m_f^2} = i(\not{k} + m_f^2) G_f(k)$. The trace leads to

$$-4 g_{\mu\nu} G_f(k) + 8 k_\mu k_\nu G_f^2(k) \tag{18}$$

Using $\int_k k_\mu k_\nu f(k^2) = \dfrac{g_{\mu\nu}}{4} \displaystyle\int_k k^2 f(k^2)$

The mass, defined via the Dirac trace

$$\delta m^{2(V)} = -2 g_V^2 \left[1 - m_f^2 \frac{d}{dm_f^2}\right] \mathcal{T}(m_f) \tag{19}$$

Curvature masses for (axial)vectors

For $\mathbf{N_f = 2 + 1}$ the **vacuum part of the self energy**
vector $1-3$, $N-S$ example:

$$\delta\hat{M}^2_{(ab)\mu\nu} \propto g_{\mu\nu}\left(1 - m_f^2\frac{d}{dm_f^2}\right)\mathcal{T}_f \tag{20}$$

One can use the same forms for masses.

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At finite temperature: **matter part of the self energy**. k_0 and k_i should be handled differently

$$\rightarrow \delta \hat{M}^2_{(ab)00} \neq \delta \hat{M}^2_{(ab)ij}$$

Lorentz symmetry is violated.

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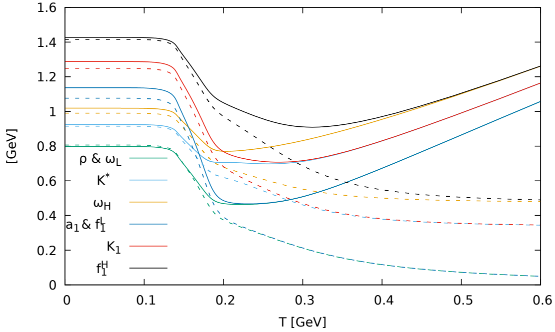
New parametrization with the fermionic one loop corrected curvature masses.
Using the same forms of masses for the finite temperature case:

$$\begin{aligned}\hat{m}^2_{S/P} =& Z^2_{S/P}(\hat{M}^2_{V/A}, \phi_N, \phi_S)\left(\hat{m}_{cl}{}^2_{S/P} + \delta\hat{m}^2_{S/P}\right)\\ \hat{M}^2_{V/A} =& \hat{M}_{cl}{}^2_{V/A} + \delta\hat{M}^2_{V/A}\end{aligned} \tag{21}$$

Curvature masses



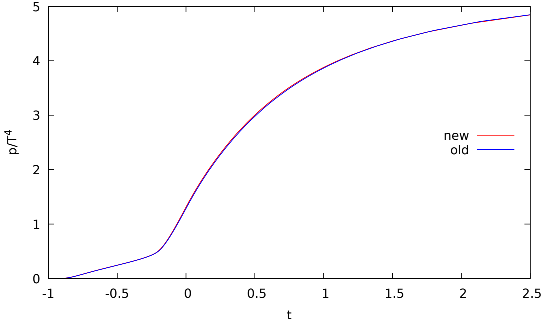


Figure: Vector and axial vector curvature masses (left). The dashed line shows the results with only tree level masses. Pressure as a function of reduced temperature (right).