Collaborators:

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Talk based on:

Related publications:
Ratio of identified hadrons in small to large systems...

...but what is small?

Small systems can have large multiplicities too...
Ratio of identified hadrons in small to large systems...

...but what is small?

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Ratio of identified hadrons in small to large systems...

...but what is **small**?

**Small** systems can have **large** multiplicities too...

Where does the quark-gluon plasma start in **multiplicity**?
Non-extensive statistics – summary:

\[ S_q = \frac{1}{q-1} \left( 1 - \frac{1}{T} \sum_{i=1}^{n} p_i^q \right) \]

\[ \lim_{q \to 1} S_q = S_{BG} \]

Thermodynamical consistency:

\[ P = T_s + \mu n - \epsilon \]

\[ P = g \int d^3 p \left( \frac{2}{\pi^2} T^2 \right) \]

\[ N = gV \int d^3 p \left( \frac{2}{\pi^2} f \right) \]

\[ \epsilon = g \int d^3 p \left( \frac{2}{\pi^2} E \right) \]

Final size effects:

\[ T = E \langle n \rangle \]

\[ T = E \left[ \delta^2 - \left( q - 1 \right) \frac{\langle n \rangle}{q-1} \right] \]

\[ q = 1 - \frac{1}{\langle n \rangle} + \Delta n^2 \frac{\langle n \rangle^2}{2} \]

\[ \frac{d^2 N}{2 \pi p_T dp_T dy} = A m_T \left[ 1 + \frac{q-1}{T} (m_T - m) \right]^{-\frac{q}{q-1}} \]

Non-extensive statistics – summary:

\( q \)-entropy:

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S_q = \frac{1}{q - 1} \left( 1 - \sum_{i=1}^{W} p_i^q \right)
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\[ P = g \int \frac{d^3p}{(2\pi)^3} T f \]

\[ s = g \int \frac{d^3p}{(2\pi)^3} \left[ \frac{E - \mu}{T} f^q + f \right] \]

\[ N = nV = gV \int \frac{d^3p}{(2\pi)^3} f^q \]

\[ \epsilon = g \int \frac{d^3p}{(2\pi)^3} Ef^q \]

\[ \frac{d^2N}{2\pi p_T dp_T dy} = Am_T \left[ 1 + \frac{q-1}{T} (m_T - m) \right]^{-\frac{q}{q-1}} \]

Motivation

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The $q$ and $T$ parameters can track down the size evolution!
Motivation

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The $q$ and $T$ parameters can track down the size evolution!

**Motivation**

**Phenomenological approach:**
Map the thermodynamically consistent non-extensive parameter space of the available experimental data and compare it with theoretical QCD calculations

- 11 identified hadron species: from $\pi^\pm$ to $\Omega$
- Various collision systems: proton-proton, proton-nucleus, nucleus-nucleus
- Wide range of multiplicities: $2.2 \leq \langle dN_{ch}/d\eta \rangle \leq 2047$
- Wide range of CM energies: $130 \leq \sqrt{s_{NN}} \leq 13000$ GeV
- **More than 30** published experimental datasets

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**Goal: calibrate the Tsallis-thermometer**
\textbf{Results}

\textbf{Parametrizations:}

\[ A = A_0 + A_1 \ln \frac{\sqrt{S_{NN}}}{m} + A_2 \langle dN_{ch}/d\eta \rangle \]

\[ T = T_0 + T_1 \ln \frac{\sqrt{S_{NN}}}{m} + T_2 \ln \ln \langle dN_{ch}/d\eta \rangle \]

\[ q = q_0 + q_1 \ln \frac{\sqrt{S_{NN}}}{m} + q_2 \ln \ln \langle dN_{ch}/d\eta \rangle \]

1. The \textbf{A, q} and \textbf{T} parameters characterize the collision

2. Strong \textbf{grouping:} \( T_{eq} \approx 0.144 \text{ GeV}, \ q_{eq} \approx 1.156 \)
**RESULTS**

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\[ q = q_0 + q_1 \ln \frac{\sqrt{s_{NN}}}{m} + q_2 \ln \ln \langle dN_{ch}/d\eta \rangle \]

**Radial flow:**

\[ T = T_{fro} + m \langle u_t \rangle^2 \]

\[ \langle v_t \rangle = \frac{\langle u_t \rangle}{\sqrt{1 + \langle u_t \rangle^2}} \]

1. The \( A, q \) and \( T \) parameters characterize the collision
2. Strong **grouping**: \( T_{eq} \approx 0.144 \text{ GeV}, q_{eq} \approx 1.156 \)
3. **Test**: results are comparable with experiments (Phys. Rev. C 83 (2011), 064903)
**Thermodynamical consistency: ✓**

\[ P = Ts + \mu n - \varepsilon \]

Comparison of the thermodynamical variables with theoretical calculations

Interpretation of the grouping phenomenon in the \( T - (q - 1) \) parameter space:
Thermodynamical consistency: ✓

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Comparison of the thermodynamical variables with theoretical calculations

Interpretation of the grouping phenomenon in the \( T \cdot (q - 1) \) parameter space:

1. Overlapping region with theoretical QCD calculations → **presence of hot QCD matter** just before the hadronization?
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Interpretation of the grouping phenomenon in the \( T - (q - 1) \) parameter space:

1. Overlapping region with theoretical QCD calculations \( \rightarrow \text{presence of hot QCD matter} \) just before the hadronization?

2. Hadron spectra of colliding systems with \( T \approx 0.144 \) GeV and \( q \approx 1.156 \): originates from a previous quark-gluon plasma state
Results

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3. This QGP does certainly not follow an equilibrium Boltzmann–Gibbs statistics
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With the parametrizations: \( \sqrt{s} \) and \( \langle dN_{ch}/d\eta \rangle \) regions:

- \( \sqrt{s} \gtrsim 7000 \text{ GeV}: \langle dN_{ch}/d\eta \rangle \gtrsim 130 \)
- \( \sqrt{s} \gtrsim 13000 \text{ GeV}: \langle dN_{ch}/d\eta \rangle \gtrsim 90 \)
Summary

- Consistent non-extensive analysis of a very large set of experimental data
- \( q \neq 1 \) for all hadron spectra: dependency on the size of the collisional system through multiplicity fluctuations
- Various checks of the non-extensive framework
- Grouping of the \( T \) and \( q \) parameters, comparison with theoretical QCD calculations
- Tsallis-thermometer: final state hadrons may originate from a previously present strongly interacting QCD matter at event multiplicities as low as \( \langle dN_{ch}/d\eta \rangle \sim 100 \)

Support


Thank you for your attention!
Backup
<table>
<thead>
<tr>
<th>System, $\sqrt{s_{NN}}$ (GeV)</th>
<th>$\eta$ or $y$</th>
<th>Hadron Mult. classes</th>
<th>$p_T$ range (GeV/$c$)</th>
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<tbody>
<tr>
<td>AuAu, 130</td>
<td>$</td>
<td>\eta</td>
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<td>$\Phi, \Xi^\pm, \Omega^\pm$</td>
<td>[0.4; 20.0] [0.8; 8.0]</td>
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<td>$\Phi, \Xi^\pm, \Omega^\pm$</td>
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