Scaling properties of jets in high-energy pp collisions

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Outline

- Scaling of jet-momentum profiles with multiplicity

- KNO-like scaling within a jet in pp collisions
  - arXiv:2012.01132
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Motivation

- **Collectivity in small systems with high-multiplicity at LHC**

- **Current understanding:**
  - QGP is not necessary for collectivity
  - Vacuum-QCD effects at the soft-hard boundary: for instance *multiple-parton interactions (MPI)*
    eg. Schlichting, arXiv:1601.01177
  - and *color reconnection (CR)* [model element]
Motivation

- **Collectivity in small systems with high-multiplicity at LHC**
  - Substantial $\nu_n$, eg. Yan-Ollitrault, PRL 112, 082301 (2014).

- **Current understanding:**
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- **Jets:**
  - **A-A**: sensitive probe of nuclear modification.
  - **pp**: No suppression expected.
    However: soft and hard processes are related by MPI
    => jets can serve to study this connection
Radial jet profiles

- Differential jet shape

\[ \rho(r) = \frac{1}{\delta r} \frac{1}{p_T^{\text{jet}}} \sum_{r_d < r_i < r_b} p_T^i \]

\[ r_i = \sqrt{(\phi_i - \phi_{\text{jet}})^2 + (\eta_i - \eta_{\text{jet}})^2} \]

- CMS@LHC pp collisions, \( \sqrt{s} = 7 \text{ TeV} \)
- \( R=0.7 \) jets, \( 50 < p_T^{\text{jet}} < 60 \text{ GeV/c}, |y|<1 \)
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- $R=0.7$ jets, $50 < p_T^{\text{jet}} < 60$ GeV/$c$, $|y| < 1$

- Currently available LHC data are either multiplicity or transverse-momentum inclusive
More multiplicity classes

- **PYTHIA 8.2 simulations (HardQCD)**
  - pp collisions at $\sqrt{s} = 7$ TeV
  - $R=0.7$ jets, $50<p_T^\text{jet}<60$ GeV/c, $|y|<1$

- **7 multiplicity classes:**
  jet profile curves intersect at a given point $R_{\text{fix}}$
in any given $p_T^\text{jet}$ window

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Z. Varga, R.V, G.G.B,
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- $R_{\text{fix}}$ independent of
  - **generator**: Pythia, Hijing++
  - **tune**: 4C, Monash, Monash*
  - **nPDF sets**
  - **CR scheme or MPI**
  - **jet algorithm**: anti-$k_T$, C/A, $k_T$

- **Is it a scaling behavior?**

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Parametrizing the jet profiles

- Detailed PYTHIA 8 simulations (4C)
  - Jet radius: 12 bins up to $r=0.6$
  - Multiplicity 6 bins up to $N=100$
  - Momentum: 20 bins up to $p_T^{\text{jet}}=400$
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- Statistically motivated distributions:
  - Poissonian distribution
    \[ \rho(r) = Cr^\gamma e^{-\alpha r} \]
  - NBD (Negative binomial distribution)
    \[ \rho(r) = C \frac{\Gamma(rk+a)}{\Gamma(a)\Gamma(rk+1)} p^{rk} (1-p)^a \]

  *Note*: both in the wide-jet ($p \to 1$) and narrow-jet limit ($\gamma \to -1$), NBD reduces to Poissonian

- Simultaneous fit with a $\sim br$ background

Parameters of the Poissonian fits

- Poissonian with background

\[ \rho(r) = Cr^\gamma e^{-\alpha r} + br \]

- Monotonic trends observable

- Exception: lowest \( p_T \)
  - Underdetermined background fit (mostly affects \( b \) and \( C \))
  - Leakage of jet outside \( R=0.7 \) (affects \( C \))


Zimányi '20
R. Vértesi - Scaling properties of jets
Scaling of the jet profiles

- Scaling assumption: profiles at all multiplicities collapse into a single distribution,
  \[ \rho_N(r) = \lambda(N) f \left( \frac{r}{\kappa(N)} \right) \]

- Scaling is determined based on Poissonian fits
  - Chosen “good” mid-multiplicity fits, then others scaled to it minimizing \( \chi^2 \)

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- The scaling works within 5-10% in the peak region

The scaling parameter $\kappa$ is approximately linear with multiplicity.

Ideally, $\lambda \kappa \sim 1$. This is fulfilled on the 10% level except for the lowest-$p_T$ bin.

- Low-$p_T$ increase is because leakage increases $\lambda$.
- Slight high-$p_T$ decrease is because background determination.
How good are the Poisson fits?

- Poissonian mean:
  \[
  \bar{\rho}(r) = \frac{\gamma + 1}{\alpha}
  \]

- Ideally, it should scale:
  \[
  \frac{\kappa}{\bar{\rho}'} \sim 1
  \]
  where \( \bar{\rho}' \) is the rescaled mean
How good are the Poisson fits?

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  \[ \bar{\rho}(r) = \frac{\gamma + 1}{\alpha} \]

- Ideally, it should scale:
  \[ \frac{\kappa}{\bar{\rho}'} \sim 1 \]
  where \( \bar{\rho}' \) is the rescaled mean

- The mean approximately scales linearly with multiplicity
- Except for the lowest \( p_T \) bin, \( \frac{\kappa}{\bar{\rho}'} \sim 1 \) within 5%
- Hence,
  - Radial profiles scale with multiplicity
  - Poissonian is an adequate description
Based on the Poisson distribution parametrization, $R_{\text{fix}}$ is an approximate consequence of the scaling.

Note: $R_{\text{fix}}$ would be exact if $\rho(r)$ fell linearly in the given region.
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KNO-scaling and its violation

- KNO scaling: the multiplicity distribution scales with $\sqrt{s}$
  
  Koba-Nielsen-Olesen, NPB 40, 317 (1972); Polyakov, Sov.Phys.JETP 32, 296 (1971)

- The KNO scaling breaks down at high $\sqrt{s}$

- KNO may be violated by the presence of multiple-parton interactions or overlapping color strings
  
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- Is KNO-scaling valid within a single jet?
- How is affected by MPI and CR?
- Is there a connection of KNO to radial scaling?
KNO within jet: multiplicity scaling with $p_T^{\text{jet}}$

- Multiplicity (dominated by the jet multiplicity) vs. jet momentum $p_T^{\text{jet}}$
KNO within jet: multiplicity scaling with $p_{T}^{\text{jet}}$

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- Parametrized with a NBD

\[ P_N = \frac{\Gamma(Nk + a)}{\Gamma(a)\Gamma(Nk + 1)} p^{Nk} (1 - p)^a \]
KNO within jet: multiplicity scaling with $p_T^{\text{jet}}$

- Multiplicity (dominated by the jet multiplicity) vs. jet momentum $p_T^{\text{jet}}$
- Parametrized with a NBD
  $$P_N = \frac{\Gamma(Nk + a)}{\Gamma(a)\Gamma(Nk + 1)} p^{Nk}(1 - p)^a$$
- Distributions at all $p_T^{\text{jet}}$ fit well on a single NBD curve
- **KNO-like scaling observed within a jet**
  - In the following we quantify how well it is fulfilled
Multiplicity vs. $p_T^{\text{jet}}$: moments

- $q^{\text{th}}$ statistical moment

$$\langle N^q \rangle = \sum_{N=1}^{\infty} P_N N^q$$

- sensitive to goodness of scaling
- insensitive to fluctuations
- no need to parametrize and fit

- Scaling:

$$\langle N^q(p_T^{\text{jet}}) \rangle = \lambda^q(p_T^{\text{jet}}) \langle N^q(p_0) \rangle \quad \lambda(p_0) = 1$$
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  \]
  - $\log N^q/q$ vs. $\log <N>$ is a straight line with $\sim$unity slope
    - up to the 9th moment
  => scaling is fulfilled in the whole $p_T^{\text{jet}}$ range
Moments: Role of MPI and CR

- No multiple-parton interactions: scaling is present
  - "possible physical" scenario producing low-activity events
- No color reconnection: no scaling
  - color-flow not handled, non-physical scenario
- Physical case (Monash): All 9 moments are consistent with unity, slope within ~1%
  - *Note*: scaling holds for different tunes & nPDFs (Monash, 4C, Monash*) and also for different jet algos (anti-\(k_T\), C/A and \(k_T\))
- No CR: Scaling is broken by ~15%
- No MPI (also no CR by construction): Scaling is fulfilled to ~2%.
- All fits are statistically good (\(\chi^2/\text{NDF}<8\), ~proportional to the order of moment)
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- All fits are statistically good ($\chi^2$/NDF<8, ~proportional to the order of moment)
- The emerging picture is different from that of radial profile scaling, which holds for CR=off as well
We observed scaling behavior in jets from 7 TeV pp collisions using MC

Radial jet-momentum profiles scale with multiplicity
- Profiles can be parametrized with a Poissonian, and scale with event multiplicity
- Scaling is present in a broad model class, regardless of settings (nPDF, CR, MPI settings, jet reconstruction, and even MC generator)
- Fundamental statistical / thermodinamical property of jet development?
- Cross-check with real data would be essential
Summary

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- **KNO-like scaling within a jet:** scaling of multiplicities with jet momentum
  - Multiplicity distributions are NBD and can be collapsed into a single distribution.
  - This scaling holds without MPI but breaks down without CR.
  - KNO scaling is likely violated by complex QCD processes outside the jet development, such as single and double-parton scatterings or softer MPI.
  - This statement holds as long as the multiplicities are described. Testing for this scaling behavior can be an important element in model development.
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Thank you!

Special thanks to Sándor Hegyi for fruitful discussions and guidance
Scaling of the jet profiles - log scale

- Scaling assumption: profiles at all multiplicities collapse into a single distribution,

\[ \rho_N(r) = \lambda(N) f\left(\frac{r}{\kappa(N)}\right) \]

**Note:** Ideally, \( \lambda = 1/\kappa \), however... “leakage” (distribution is cut-off at high \( r \) before normalization)

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  - Chosen “good” mid-multiplicity fits, then others scaled to it minimizing \( \chi^2 \)

- The scaling works within 5-10% in the peak region
Heavy-flavor jets also show KNO-like scaling
Hijing++ does not exhibit the scaling
KNO-like scaling: summary
Statistical moments of jet profiles
(Monash with MPI and CR)

The gradients are not 1, but it could be explained with the binning.
Effects of finite-size bins (jet profiles)

Dotted lines: effect of binning on analytical curves. Qualitatively explains the behavior seen in the simulations.