Radiation reaction: charge distributions or point charges?

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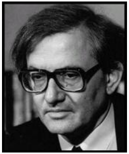


What is radiation reaction?

- Classical problem of electrodynamics. Lorentz, 1892, 27 years after Maxwell equations, 5 years before electron.
- Accelerating charges radiate, radiated fields make accelerate.
- Field-matter interaction.
- Lorentz-Abraham-Dirac equation:

$$\boxed{m\dot{u}^\mu = -eF^{\mu\nu}_{ext}u_\nu - \frac{2\alpha}{3}(\ddot{u}^\mu + \dot{u}^\nu \dot{u}_\nu u^\mu)}$$

  Coupled Maxwell-Newton equations. Special relativistic.
  $c = 1, \alpha = 1/137, \eta^{\mu\nu} = diag(-1,1,1,1)$.
- It is fundamental for accelerator design. New experiments.

Is it a problem or not?

- Mathematical? Runaway solutions, but not only.
- Conceptual? Infinite energies. Point charges?
- Lorentz, Abraham, Dirac, Born, Feynmann, Landau, ..., Wald, Rafelski, ...

What is the problem?

Maxwell+Newton $\rightarrow$ Lorentz-Abraham-Dirac equation

Electrodynamics is not about interaction

Given electromagnetic field $\rightarrow$ particle motion.
Given worldline of a charged particle $\rightarrow$ electromagnetic field.
Maxwell equations are not interpreted along the wordline of a point charge.

Various strategies:

1. Wrong interpretation of the LAD equation. E.g. Dirac - particular solutions. Spohn - second order kernel.
2. Problems with Maxwell equations. E.g. Born-Infeld. Feynmann.
3. Problems with Newton equation. Dissipative forces. Landau-Lifshitz, Sokolov. New experiments.
4. Problems with point charges. Infinite energy. Various regularisations, charge distributions. Lorentz-Abraham-Dirac equation follows.
5. Problems with classical approach. Quantum electrodynamics. Statistical physics.

From continuum to point

Bild–Deckert–Ruhl, Phys. Rev. D, 99:096001, 2019.

- Dirac's derivation of LAD equation: unjustified Gauss-Stokes theorem, Taylor expansion, limit to a point.
- Rigid spherical shell, radius ϵ, uniform continuous charge distribution. Lorentz, Abraham, Dirac, Parrott.
- Improved equation of motion with time delay ϵ.

Basic assumption: the electromagnetic fields outside the world tube of the shell and the point charge at the centre are equal.
We have shown (Matolcsi–PV, arXiv:2010.04940) that

1. the world tube breaks down for accelerations higher than $1/\epsilon$,
2. the basic assumption is not valid, proved for uniformly accelerated rigid spherical shell by Distribution Theory.

Frame independent short calculation. Uniformly accelerated charged sphere – world tube. A surface charge density – vector measure. The related potential - a Distribution - cannot be equal by the potential of any effective point charge at the centre.

distribution and Distribution

x spacetime event, φ test function, Distributions: function space duals,
Charge distributions are Distributions.

		Distribution
measure λ	$\rightarrow$	$(\lambda\vert\varphi) := \int \varphi(x) d\lambda(x)$
locally integrable function f	$\rightarrow$	$(f\vert\varphi) := \int f(x)\varphi(x)dx$
locally non-integrable function f	$\rightarrow$	pole taming!

- Dirac measure: $\phi(0) = \int \varphi(x) d\delta(x) \left(= \int \varphi(x) d\delta(x)\right)$, (Dirac, 1930).
- non-integrable function: $\frac{1}{|x|^m}, \quad 2 < m \in \mathbb{N}$.
- pole taming: $\left(tm \frac{1}{|x|^m} | \varphi\right) := \int \frac{\varphi(x) - T_{\varphi}^{m-1}(x)}{|x|^m} dx$.

Electromagnetic potential, field, energy-momentum are best represented by Distributions.

see e.g. Horváth: Topological vector spaces and distributions, 2012.

Electrostatic energies (e.g. Jackson)

ρ charge density, ϕ electrostatic potential, $\mathbf{E}$ electric field

$$\rho\phi = (\nabla \cdot \mathbf{E})\phi = \nabla \cdot (\phi \mathbf{E}) - \mathbf{E} \cdot \nabla\phi = \nabla \cdot (\phi \mathbf{E}) + |\mathbf{E}|^2.$$
$$\nabla \cdot \mathbf{E} = \rho, \quad \mathbf{E} = -\nabla\phi$$

Total (self?) energy:

$$\frac{1}{2}\int \rho\phi = \frac{1}{2}\int |\mathbf{E}|^2$$

Electric energy density:

$$w = \frac{1}{2}|\mathbf{E}|^2$$

For a point charge e at zero:

$$\phi(\mathbf{x}) = \frac{e}{4\pi|\mathbf{x}|}, \qquad w_{pch}(\mathbf{x}) = \frac{1}{32\pi^2}\frac{e^2}{|\mathbf{x}|^4}$$

Electrostatic point charge quantities

(self)Energy density:

$$w = \frac{1}{2}|\mathbf{E}|^2$$

(self)Energy:

$$\boxed{W = \left(tm \frac{|\mathbf{E}|^2}{2} \middle| 1 \right) = 0}, \qquad \left(\int \frac{|\mathbf{E}|^2}{2} = \infty \right)$$

It is not locally integrable, it is a Distribution.

(self)Pressure tensor field:

$$\mathbf{P} := \mathbf{E} \otimes \mathbf{E} - \frac{|\mathbf{E}|^2}{2}\mathbf{I}$$

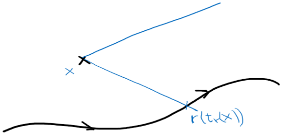
It is not locally integrable, it is a Distribution.

(self)Force density: $\mathbf{f} := \nabla \cdot \mathbf{P}$.

$$\boxed{\nabla \cdot (tm\,\mathbf{P}) = 0} \qquad (\nabla \cdot \mathbf{P} = \rho \mathbf{E})$$

Electrodynamic point charge quantities

world line function: r^μ proper time $\to$ flat spacetime
world line: $Ran\, r^\mu$ (one dimensional submanifold)
The Liénard-Wiechert (four)potential of a given (!) r^μ:



$$\phi^\mu_{LW}(x) = \frac{e}{4\pi} \frac{-\dot r^\mu(t_r(x))}{(x^\nu - r^\nu(t_r(x)))\dot r_\nu(t_r(x))}$$

Here $t_r(x)$ is the retarted proper time ($x^\mu - r^\mu(t_r(x))$ is future-lightlike)
The electromagnetic field, $F^{\mu\nu} = \partial^\nu \phi^\mu - \partial^\mu \phi^\nu$ and the energy-momentum:

$$T^{\mu\nu} = -F^{\mu\kappa} F_\kappa^{\ \nu} + \frac{1}{2} F^{\gamma\kappa} F_{\kappa\gamma} g^{\mu\nu}$$

- $T^{\mu\nu}$ is not locally integrable,
- $T^{\mu\nu}$ is not differentiable on the world line,
- $T^{\mu\nu}$ has a pole in radial distance along the world line.

$$\boxed{\partial_\nu(tm\, T^{\mu\nu}) = \frac{2}{3}\frac{e^2}{4\pi} \dot r^{[\mu} \dddot r^{\nu]} \dot r_\nu \lambda_{Ranr} = -\frac{2\alpha}{3}(\ddot u^\mu + \dot u^\nu \dot u_\nu u^\mu)\lambda_{Ranr}}, \quad u^\mu := \dot r^\mu .$$

Lorentz-Abraham-Dirac equation (?)

Self-force:

$$\boxed{\partial_\nu (tm T^{\mu\nu}) = \frac{2}{3}\frac{e^2}{4\pi}\dot{r}^{[\mu}\dddot{r}^{\nu]}\dot{r}_\nu \lambda_{Ranr} = -\frac{2\alpha}{3}(\ddot{u}^\mu + \dot{u}^\nu \dot{u}_\nu u^\mu)}$$

Therefore it seems reasonable to put together Newton equation with two different kind of electromagnetic forces like this

$$m\dot{u}^\mu = F^{\mu\nu}_{ext} u_\nu - \frac{2\alpha}{3}(\ddot{u}^\mu + \dot{u}^\nu \dot{u}_\nu u^\mu).$$

However,

it is NOT a differential equation.

(Is is not even an equation.)

Lorentz-Abraham-Dirac formula

LAD:

$$\boxed{m\dot{u}^\mu = F^{\mu\nu}_{ext} u_\nu} + \boxed{\frac{2\alpha}{3}\left(\ddot{u}^\mu + \dot{u}^\nu \dot{u}_\nu u^\mu\right)}.$$

Newton equation: $m\ddot{r} = f(r, \dot{r}) \to r(t)$
Liénard-Wiechert potential: GIVEN a world line one calculates the fields.

A differential equation is a definition, it is not.

A possible interpretations:

- First correction: Landau-Lifshitz
- Landau-Lifshitz is a hidden submanifold: Spohn
- A self consistent procedure? (see Matolcsi-Fülöp-Weiner, MPL 2017.)

Does continuum better?

ρ charge density, ϕ electrostatic potential, $\mathbf{E}$ electric field

$$\rho\phi = (\nabla \cdot \mathbf{E})\phi = \nabla \cdot (\phi \mathbf{E}) - \mathbf{E} \cdot \nabla\phi = \nabla \cdot (\phi \mathbf{E}) + |\mathbf{E}|^2.$$
$$\nabla \cdot \mathbf{E} = \rho, \quad \mathbf{E} = -\nabla\phi$$

Previous –usual– conclusion (e.g. Jackson):

$$\frac{1}{2}\int \rho\phi = \frac{1}{2}\int |\mathbf{E}|^2 \quad \stackrel{?}{\Rightarrow} \quad w = \frac{1}{2}\rho\phi \stackrel{?}{=} \frac{1}{2}|\mathbf{E}|^2$$

Energy and energy density of continuum charge distributions?
Problems:

- Positive-negative charges?
- Different domains.
- Self or extraneous?

$$w_{ext} = \rho\phi_{ext}, \qquad w_{self} = \frac{1}{2}\rho\phi_{self}$$

See also the recent seminar of Tamás Matolcsi.

Summary

- Radiation reaction : physical and mathematical problems
- Continuum to point is problematic. Bild-Decker-Ruhl (2019)
- The problem is not with point charges (wild Distributions can be tamed)
- Lorentz-Abraham-Dirac is not an equation for defining the possible world lines
- Self and extraneous fields ?

Electrodynamics is not about interaction

Given electromagnetic field $\rightarrow$ motion of charge distribution.
Given charge distribution flow $\rightarrow$ electromagnetic field.

Other strategies ? ? QED ? Quantum ? Renormalisation ?

Thank you for the attention!

About the importance of this kind of porblems

I mean old, simple looking, out of fashion ones, that somehow are annoying.

There are two singular opinions:

- Importance $= 0$. We know much more. It is just mathematics....
- Importance $= \infty$. Physics is meaningless. End of world.

Height: 109m

AI, what do you think?