Lattice determination of the heavy quark diffusion constant

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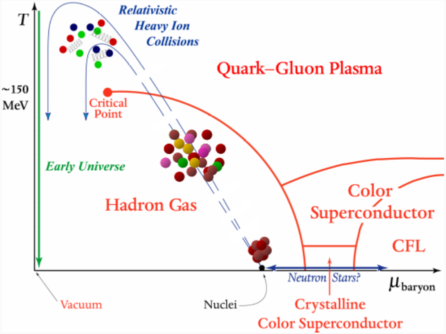
Introduction: QCD and QGP

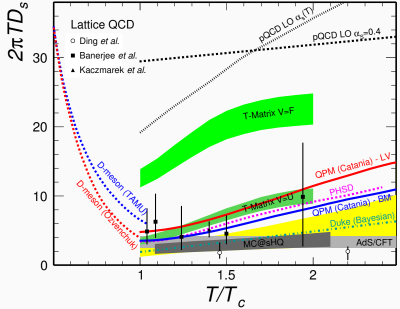
- We aim to understand the strongly coupled Quark Gluon Plasma (QGP)
- QGP generated at particle accelerators such as LHC/RHIC

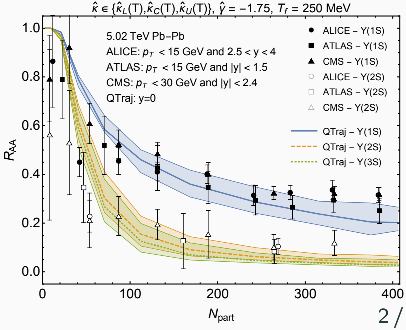


- The QGP can be described in terms of transport coefficients
- In this talk we focus on the heavy quark momentum diffusion coefficient κ
- κ related to experimental quantities nuclear modification factor R_{AA} and elliptic flow ν_2

Need for spatial diffusion coefficient

- R_{AA} and v_2 described by spatial diffusion coefficient D_x
- Observed v_2 is larger than expected from kinetic models but agrees more with hydrodynamic models
- Multiple theoretical models predicting wide range of values
- Non-perturbative lattice simulations needed
- κ dominant source of variation in R_{AA}





UP: X. Dong CIPANP (2018)

DOWN: N. Brambilla, M. Escobedo, M. Strickland, A. Vairo, P. Vander Griend and J. Weber, hep-ph/2012.01240

Heavy Quark in medium

- Heavy quark energy doesn't change much in collision with a thermal quark

$$E_k \sim T\,, \quad p \sim \sqrt{MT} \gg T$$

- HQ momentum is changed by random kicks from the medium
  $\rightarrow$ Brownian motion; Follows Langevin dynamics

$$\frac{dp_i}{dt} = -\frac{\kappa}{2MT} p_i + \xi_i(t)\,, \quad \langle \xi(t)\xi(t') \rangle = \kappa\delta(t-t')$$

- Heavy quark momentum diffusion coefficient κ related to many interesting phenomena

  Such as: Spatial diffusion coefficient $D_{\mathrm{s}} = 2T^2/\kappa$,
  Drag coefficient $\eta_{\mathrm{D}} = \kappa/(2MT)$,
  Heavy quark relaxation time $\tau_{\mathrm{Q}} = \eta_{\mathrm{D}}^{-1}$

Moore and Teaney PRC71 (2005), Caron-Huot and Moore JHEP02 (2008)

Quarkonium in medium

- Quarkonium in QGP (environment energy scale πT)

$$M \gg \frac{1}{a_0} \gg \pi T \gg E\,, \quad \tau_R \gg \tau_E \sim 1/\pi$$

- HQ mass M, Bohr radius a_0, binding energy E, correlation time τ_E
- Quarkonium in fireball can be described by Limbland equation

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} = -i[H,\rho] + \sum_{n,m} h_{nm}\left(L_i^n \rho L_i^{m\dagger} - \frac{1}{2}\{L_i^{m\dagger} L_i^n, \rho\}\right)$$

- All terms depend on two free parameters κ and γ
- κ turns out to be the heavy quark diffusion coefficient and related to thermal width $\Gamma(1S) = 3a_0^2\kappa$
- γ is correction to the heavy quark-antiquark potential and related to mass shift $\delta M(1S) = 2a_0^2\gamma/3$
- Unquenched lattice measurements of $\Gamma(1S)$ and $\delta M(1S)$ available

Brambilla *et.al.*PRD96 (2017), Brambilla *et.al.*PRD97 (2018)

κ from perturbation theory

- From kinetic theory we can derive

$$\kappa^{\rm LO} = \frac{g^4 C_{\rm F}}{12\pi^3}\int_0^\infty q^2 {\rm d}q \int_0^{2q} \frac{p^3 {\rm d}p}{(p^2+\Pi_{00}^2)^2} \times N_{\rm c}\, n_{\rm B}(q)(1+n_{\rm B}(q))\left(2-\frac{p^2}{q^2}+\frac{p^4}{4q^4}\right)$$

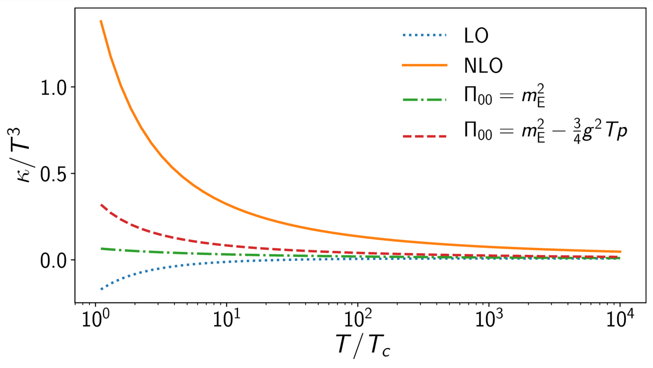
- Solution depends on assumptions
  - Is $m_{\rm E} \ll T$
  - How to expand the temporal gluon self-energy $\Pi_{00} \simeq m_{\rm E}$
- Alternatively from Kubo formula:

$$\frac{\kappa}{T^3} = \frac{g^4 C_f N_c}{18\pi}\left[\left(\ln\frac{2T}{m_{\rm E}}+\xi\right)+\frac{m_{\rm E}}{T}C\right], \quad \xi \simeq -0.64718$$

- Truncated LO: $C = 0$, NLO: $C \simeq 2.3302$
- $N_f = 0$ assumed here

Moore and Teaney PRC71 (2005), Caron-Huot and Moore JHEP02 (2008)

κ from perturbation theory



- Clearly $m_{\mathrm{E}} \ll T$ is too strict assumption on small T
- Huge perturbative variation
  $\Rightarrow$ needs non-perturbative measurements
- Also huge scale dependence trough $m_{\mathrm{E}} = g(\mu)T$
- Here we have scale from NLO EQCD $\mu \sim 2\pi T$

Moore and Teaney PRC71 (2005), Caron-Huot and Moore JHEP02 (2008)

Heavy quark diffusion from lattice

- Traditional approach using current correlators has transport peak
- HQEFT inspired Euclidean correlator free of transport peaks

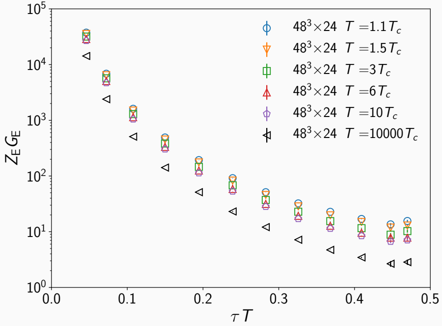
$$G_{\mathrm{E}}(\tau) = -\sum_{i=1}^{3} \frac{\langle \mathrm{Re}\,\mathrm{Tr}\,[U(1/T,\tau)E_i(\tau,0)U(\tau,0)E_i(0,0)]\rangle}{3\,\langle \mathrm{Re}\,\mathrm{Tr}\,U(1/T,0)\rangle}$$

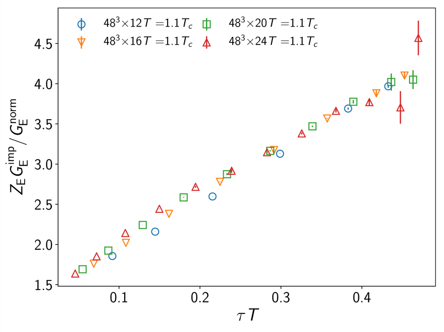
- To get momentum diffusion coefficient κ, a spectral function $\rho(\omega)$ needs to be reversed:

$$G_{\mathrm{E}}(\tau) = \int_0^\infty \frac{\mathrm{d}\omega}{\pi} \rho(\omega, T) K(\omega, \tau T)\,, \qquad K(\omega, \tau T) = \frac{\cosh\left(\frac{\omega}{T}\left(\tau T - \frac{1}{2}\right)\right)}{\sinh\left(\frac{\omega}{2T}\right)}$$

$$\kappa = \lim_{\omega\to 0} \frac{2T\rho(\omega)}{\omega}$$

- Measure using the multilevel algorithm in pure gauge (quenched)
- Compared to earlier studies we measure extremely wide range of temperatures
- Create model $\rho(\omega)$ by matching to perturbation theory at high T
- Invert the spectral function equation by varying the model ρ





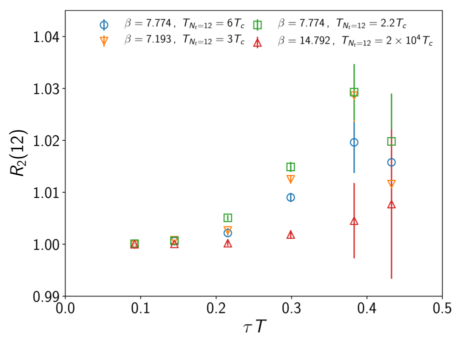
- Normalize lattice data with the LO Perturbative result:

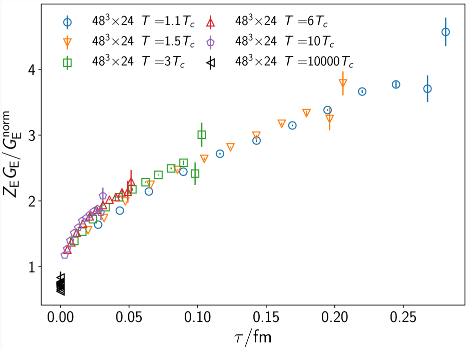
$$G_{\mathrm{E}}^{\mathrm{norm}} = \pi^2 T^4 \left[\frac{\cos^2(\pi\tau T)}{\sin^4(\pi\tau T)} + \frac{1}{3\sin^2(\pi\tau T)} \right]$$

- Perform tree-level improvement by matching lattice and continuum perturbation theories

Caron-Huot *et.al.*JHEP04 (2009), Francis *et.al.*PoSLattice (2011)

When do thermal effects start





$$R_2(N_t) = \frac{G_{\mathrm{E}}(N_t, \beta)}{G_{\mathrm{E}}^{\mathrm{norm}}(N_t)} \Big/ \frac{G_{\mathrm{E}}(2N_t, \beta)}{G_{\mathrm{E}}^{\mathrm{norm}}(2N_t)} .$$

- On small physical separation every T shares a scaling (apart from finite size effects)
- Thermal effect nonexistent for $\tau < 0.10$, then grow

Spectral function basics: LO

$$G_{\rm E}(\tau) = \int_0^\infty \frac{{\rm d}\omega}{\pi}\rho(\omega)\frac{\cosh\left(\frac{\omega}{T}[\tau T - \frac{1}{2}]\right)}{\sinh\frac{\omega}{2T}}$$

$$\kappa = \lim_{\omega\to 0}\frac{2T}{\omega}\rho(\omega)\,, \qquad \gamma = -\frac{1}{3N_c}\int_0^\infty \frac{d\omega}{2\pi}\frac{\rho(\omega)}{\omega}$$

- Assume simple behavior on IR ($\omega \ll T$):

$$\rho_{\rm IR}(\omega) = \frac{\kappa\omega}{2T}$$

- Perturbative behavior in UV in LO ($\omega \gg T$):

$$\rho_{\rm UV}^{\rm LO}(\omega) = \frac{g^2(\mu_\omega)C_F\omega^3}{6\pi}\,, \quad \mu_\omega = \max(\omega, \pi T)$$

- Use 5-loop running for the coupling

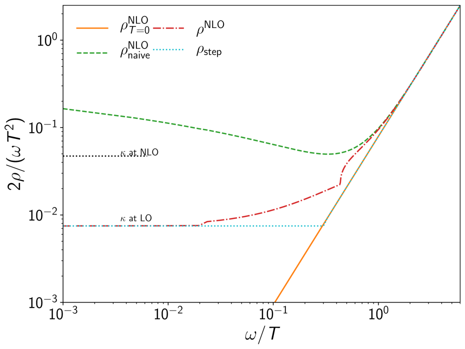
Spectral function: NLO

- NLO $\rho(\omega)$ known from (Burnier *et.al.*JHEP08 (2010))
- Full HTL resummed NLO ρ over corrects and gives negative κ at small T
- Naive QCD NLO $\rho(\omega)$ diverges logarithmically:

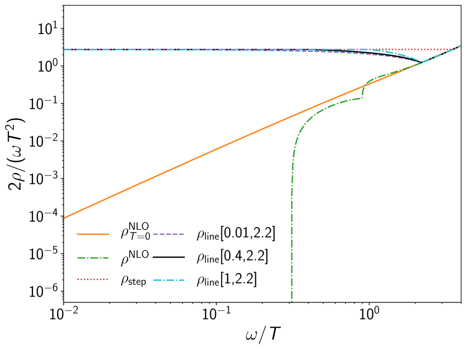
$$\begin{aligned}\rho_{\rm QCD,naive}(\omega) &= \frac{g^2 C_F \omega^3}{6\pi}\left\{1+\frac{g^2}{(4\pi)^2}\left[N_{\rm c}\left(\frac{11}{3}\ln\frac{\mu^2}{4\omega^2}+\frac{149}{9}-\frac{8\pi^2}{3}\right)\right]\right\} \\ &+ \frac{g^2 C_F}{6\pi}\frac{g^2}{2\pi^2}\left\{N_{\rm c}\int_0^\infty {\rm d}q\, n_{\rm B}(q)\left[(q^2+2\omega^2)\ln\left|\frac{q+\omega}{q-\omega}\right|+q\omega\left(\ln\frac{|q^2-\omega^2|}{\omega^2}-1\right)\right.\right. \\ &\left.\left. + \frac{\omega^4}{q}\mathbb{P}\left(\frac{1}{q+\omega}\ln\frac{q+\omega}{\omega}+\frac{1}{q-\omega}\ln\frac{\omega}{|q-\omega|}\right)\right]\right\},\end{aligned}$$

- We will use the $T=0$ naive QCD $\rho(\omega)$ as our normalization
- set scale: UV: $\rho^{\rm LO}_{T=0}=\rho^{\rm NLO}_{T=0}$, IR: from NLO EQCD

Spectral function behavior



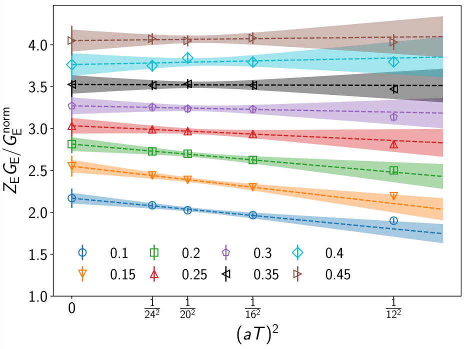
$T = 10\,T_c$

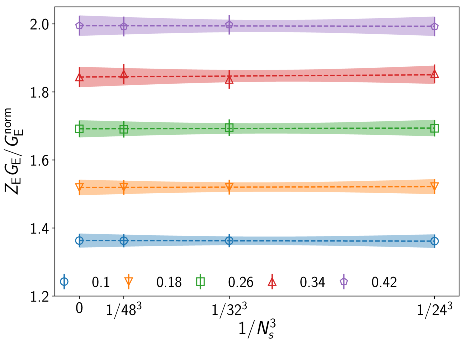


$T = 1.1\,T_c$

- NLO spectral function works only at very high temperatures
- Try different models for $\omega \sim T$ behavior
- Instead of inverting integral equation, compare to ansatz

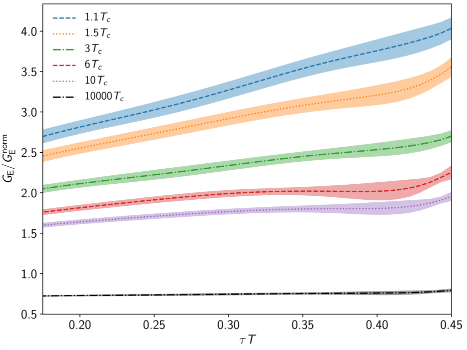
Continuum limit and finite size effects



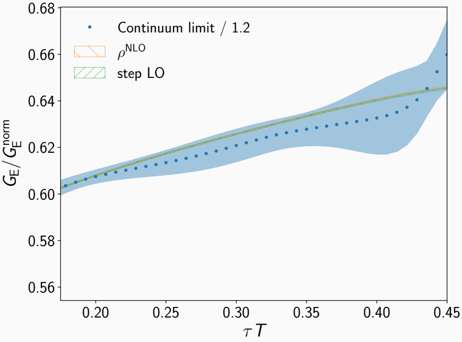


- Use 3 largest lattices for continuum limit
- Check systematics by including the $N_t = 12$ point
- $\chi^2/\mathrm{d.o.f.} < 5$ for $\tau T > 0.20$ when using 3 largest lattices (< 10 with $N_t = 12$)
- Finite size effects are negligible

Normalization of Continuum limit



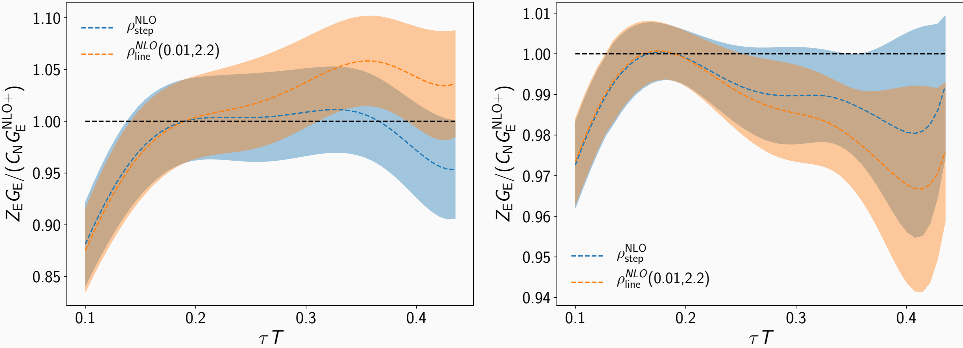
$T = 1.1\,T_c$



$T = 10^4\,T_c$

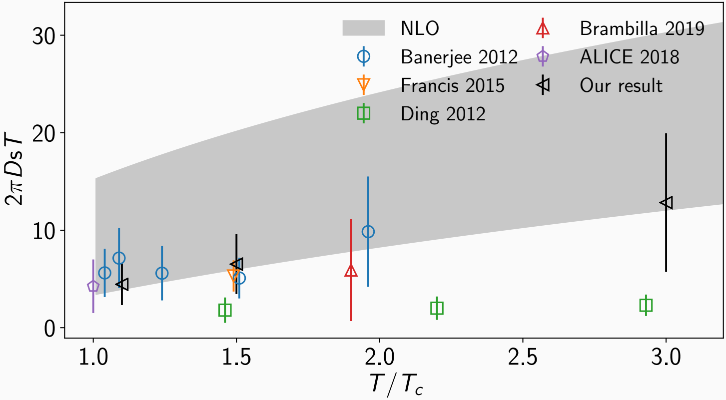
- Data needs additional normalization, do this at $\tau T = 0.19$
- Great agreement to perturbation theory at very high temperatures

κ extraction



- Take continuum limit of the lattice data
- Normalize with different models for spectral function
- Extract κ as all values that normalize to unity in $0.19 \leq \tau T \leq 0.45$

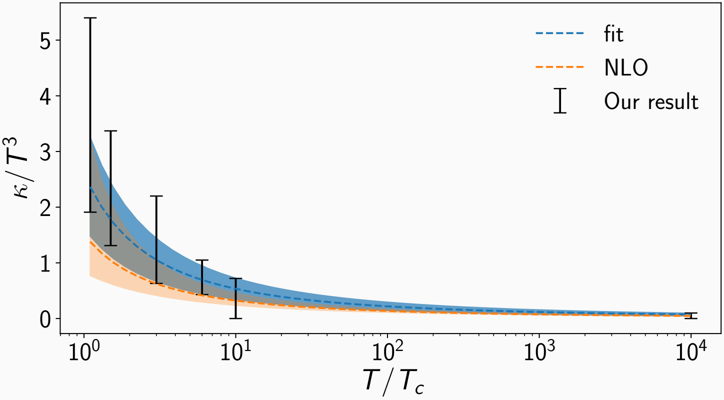
Lattice results for D_s



- On low temperature close to T_c, agreement with other results, including ALICE

Brambilla et al 2020: Accepted to PRD, hep-lat/2007.10078

Lattice results for κ



- Unprecedented temperature range: $T = 1.1 - 10^4\, T_{\rm c}$

$$\frac{\kappa^{\rm NLO}}{T^3} = \frac{g^4 C_{\rm F} N_{\rm c}}{18\pi}\left[\ln \frac{2T}{m_{\rm E}} + \xi + C\frac{m_{\rm E}}{T}\right].$$

- Can fit temperature dependence $C = 3.81(1.33)$

Brambilla et al 2020: Accepted to PRD, hep-lat/2007.10078

Conclusions and Future prospects

- We have measured κ in wide range of temperatures and fitted the temperature dependence
- Observe κ/T^3 decreasing when T increases, similar to perturbation theory
- Future prospects:
  - Measure γ
  - Implement gradient flow to go un-quenched
  - $1/M$ corrections
  - Other operators?

Thank You