Chromo-electric screening and bottomonium properties from lattice QCD

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How chromo-electric screening works in QGP and how it is related to quarkonium properties at high T ?
Matsui and Satz, PLB 178 (1986) 416

Polyakov loop correlators and the Debye mass:
PP, Sebastian Steinbeißer, Johannes Weber, work in progress

Bottomonium properties:
Rasmus Larsen, Stefan Meinel, Swagato Mukherjee, PP, PRD100 (2019) 074506 ; PLB800 (2020) 135119 ; arXiv:2008.00100

20th Zimányi School, December 7-11, 2020

Deconfinement and color screening

Onset of color screening is described in terms of the free energy of static quark anti-quark pair

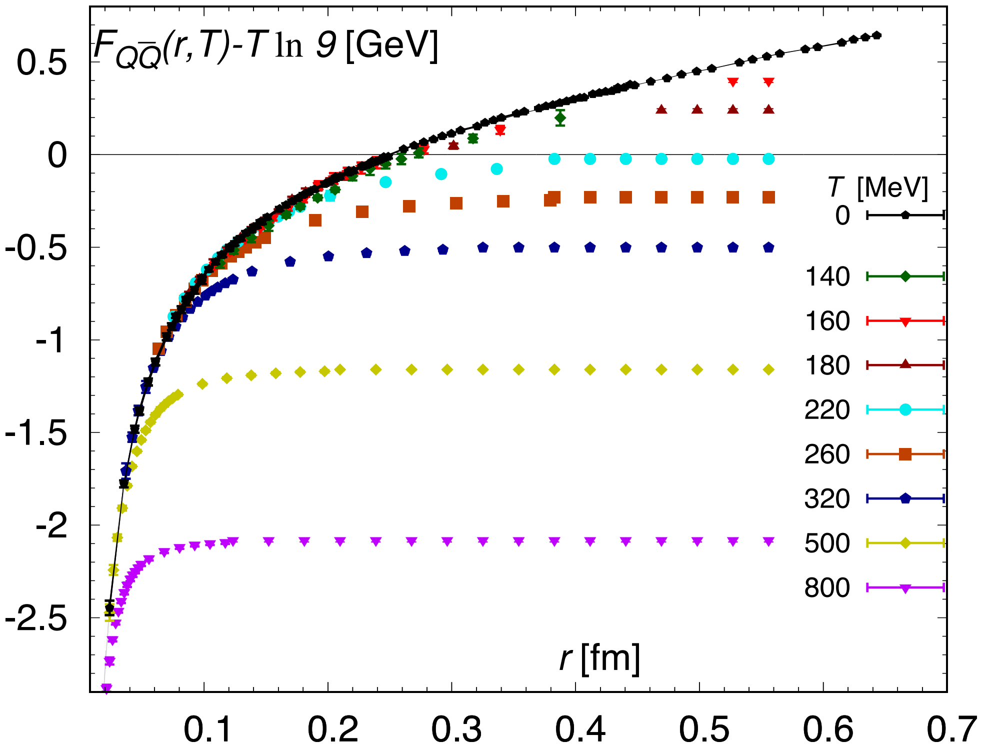
$$L(x) = \mathcal{P} \exp\left(-ig \int_0^{1/T} d\tau A_0(x,\tau)\right)$$

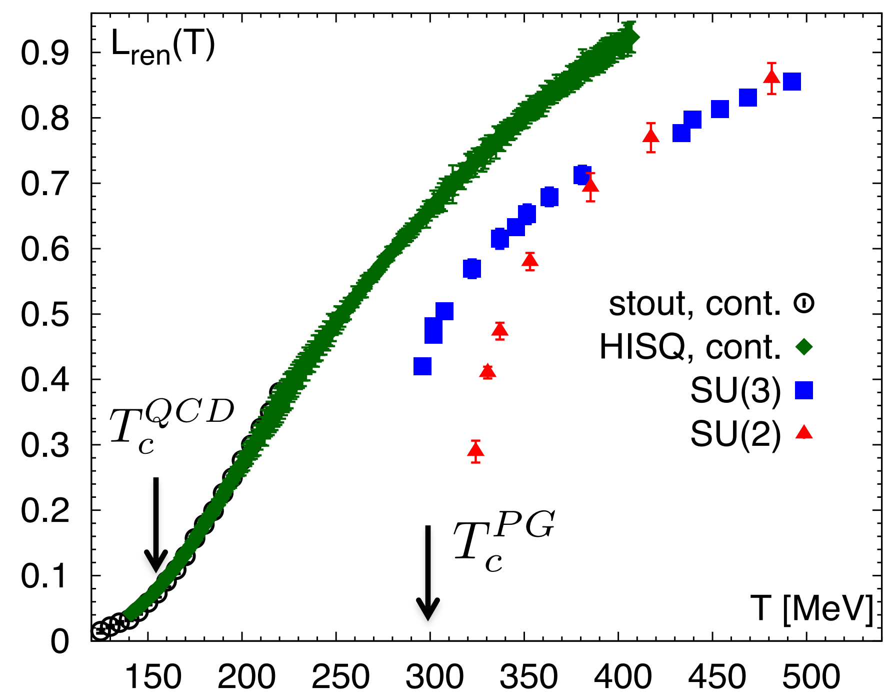
$$\exp(-F_{Q\bar{Q}}(r,T)/T) = \frac{1}{9}\langle \mathrm{tr} L(r) \mathrm{tr} L^\dagger(0)\rangle$$

$$L_{ren} = \exp(-F_Q(T)/T)$$

$$F_{Q\bar{Q}}(r \to \infty, T) = 2F_Q(T)$$

TUMQCD, PRD 98 (2018) 054511





free energy of static quark anti-quark pair shows Debye screening at high temperatures

At what distance color screening sets in ? What is the screening mass (length) ?

Correlators of real and imaginary part of the Polyakov loop

$$C_{PL}(r,T) = C^R_{PL}(r,T) + C^I_{PL}(r,T),$$

$$C^R_{PL}(r,T) = \langle \mathrm{Re}L(r)\cdot \mathrm{Re}L(0)\rangle, C^I_{PL}(r,T) = \langle \mathrm{Im}L(r)\cdot \mathrm{Im}L(0)\rangle.$$

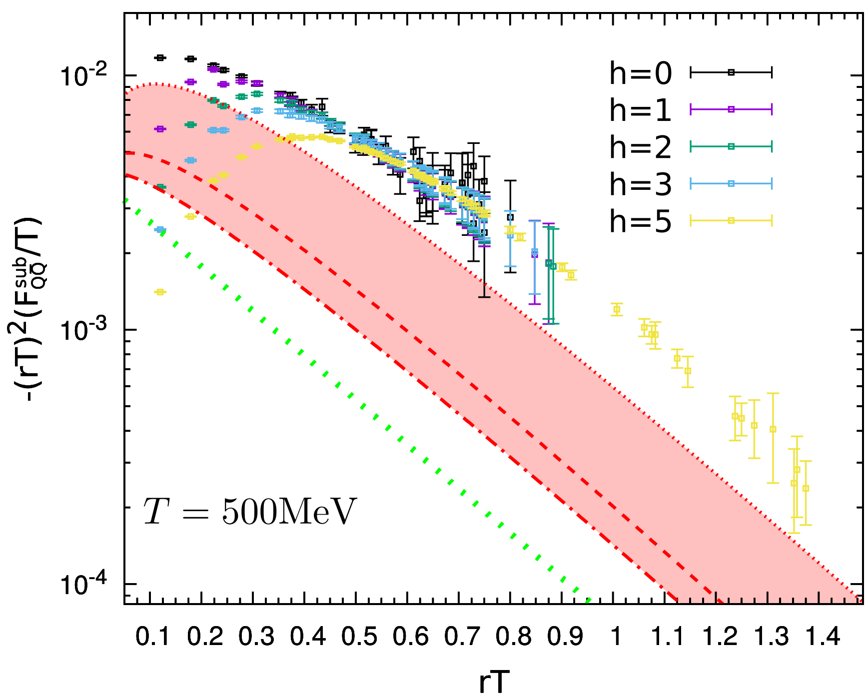
LO perturbative result:

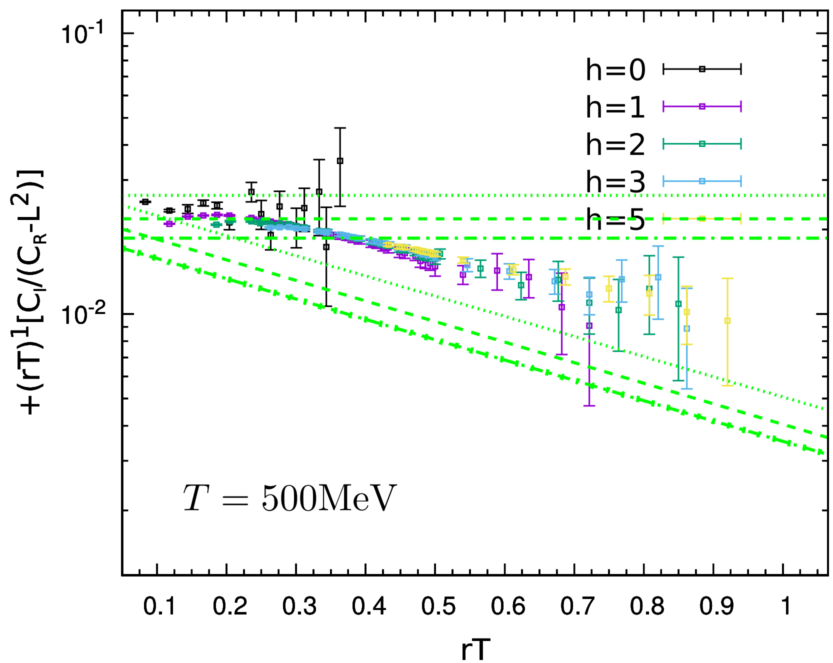
$$C^R_{PL}(r,T) = 1 + 2F_Q^{(0)} + g^4\frac{N^2-1}{8N^2}\left(\frac{e^{-m_D r}}{4\pi r T}\right)^2, \quad C^I_{PL}(r,T) = g^6\frac{N^2-1}{8N^2}\frac{N^2-4}{12N}\left(\frac{e^{-m_D r}}{4\pi r T}\right)^3$$

2-gluon exchange

3-gluon exchange

Hypercubic (HYP) smearing of the gauge files to improve the signal on the lattice





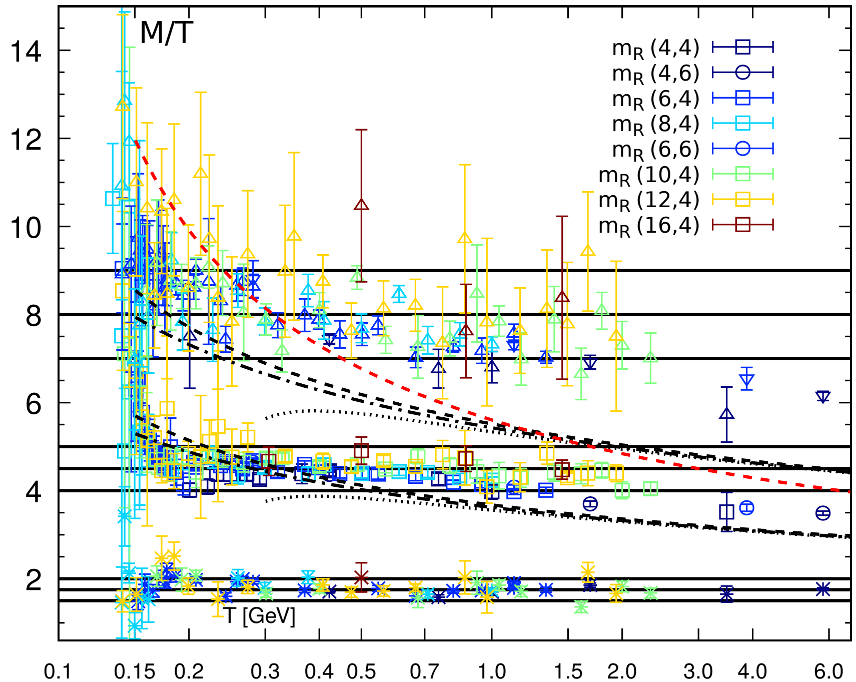
Qualitative but not quantitative agreement with the perturbative result

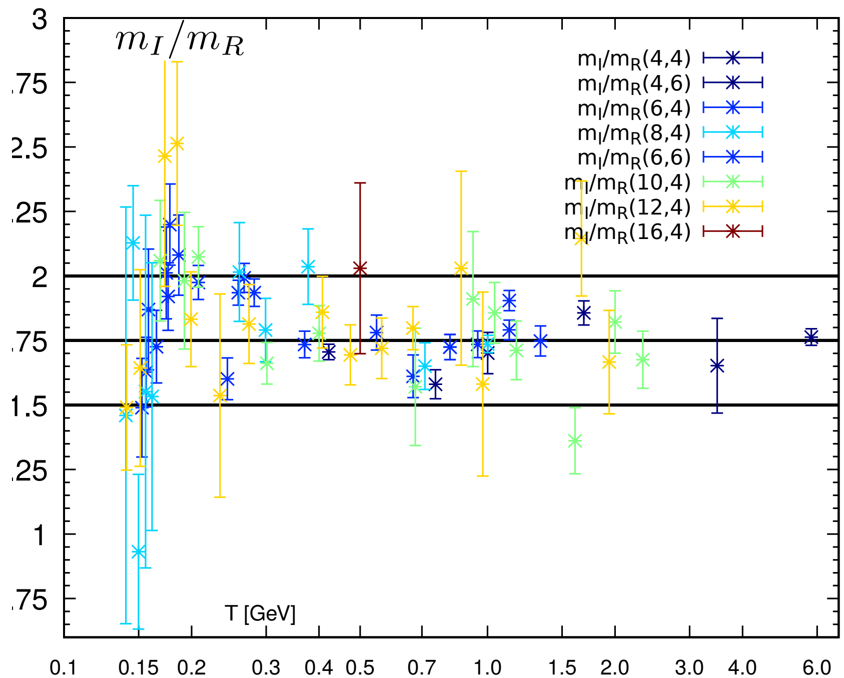
Electric screening masses from the Polyakov loop correlators

The large distance behavior of the correlation functions is governed by screening masses:

$$C_R(r,T) \sim \exp(-m_R(T)r),\ \ C_I \sim \exp(-m_I(T)r)$$

In the perturbative picture $m_I/m_R \simeq 3/2$ and $m_R \simeq 2m_D, m_I \simeq 3m_D$





No quantitative agreement between the lattice and the perturbative results but the lattice Results are consistent with the picture of chromo-electric gluon exchange picture; the non-perturbative corrections are not very large

Why NRQCD ?

Quarkonia to a good approximation are non-relativistic bound state

$$p_Q \sim M_Q v \ll M_Q$$

EFT approach: integrate the physics at scale of the heavy quark mass

NRQCD is the EFT at scale << M_Q; Heavy quark fields are Pauli spinors:

$$L_{NRQCD} = \psi^\dagger \left(D_\tau - \frac{D_i^2}{2M_Q} \right) \psi + \chi^\dagger \left(D_\tau + \frac{D_i^2}{2M_Q} \right) \chi + \ldots + \frac{1}{4} F_{\mu\nu}^2 + \bar{q}\gamma_\mu D_\mu q$$

Advantages:

- the large quark is not a problem for lattice calculations, lattice study of bottomonium is feasible (usually $a M_Q \ll 1$, which is challenging)
- The the spectral function is less UV dominant => more sensitivity to the bound state properties
- Quarkonium correlators are not periodic and can be studied at larger time extent (=$1/T$) => more sensitivity to bound state properties

$$C(\tau, T) = \int_{-\infty}^{+\infty} d\omega \rho(\omega, T) e^{-\tau\omega}$$

NRQCD on the Lattice

Inverse lattice spacing provides a natural UV cutoff for NRQCD provided $a^{-1} \leq 2M_Q$ (lattices cannot be too fine)

Quark propagators are obtained as initial value problem:

$$G_\psi(\mathbf{x},t) = \langle \psi(\mathbf{x},t)\, \psi^\dagger(\mathbf{0},0)\rangle \qquad G_\chi(\mathbf{x},t) = -G_\psi^\dagger(\mathbf{x},t)$$

$$G_\psi(t) = K(t) G_\psi(t-1),$$

$$K(t) = \left(1 - \frac{a\delta H|_t}{2}\right)\left(1 - \frac{aH_0|_t}{2n}\right)^n U_4^\dagger(t) \times \left(1 - \frac{aH_0|_{t-1}}{2n}\right)^n \left(1 - \frac{a\delta H|_{t-1}}{2}\right),$$

$$t = \tau/a,\ \ H_0 = \frac{-\Delta^{(2)}}{2M_b},\ \ \delta H \sim v^4,\ \ v^6(\text{ spin} - \text{dep.})$$

Meinel, PRD 82 (2010) 114502

masses are defined up to *a*-dependent shift: $M_{\Upsilon(1S)} = E_{\Upsilon(1S)} + C_{\text{shift}}(a)$

Light d.o.f (gluons, u,d,s quarks) are represented by gauge configurations from HotQCD, $m_s = m_s^{phys}$, $m_{u,d} = m_s/20 \leftrightarrow m_\pi = 161$ MeV
$T > 0$: $48^3 \times 12$ lattices, $T_c = 159$ MeV, the temperature is varied by varying $a \leftrightarrow \beta = 10/g^2$ Bazavov et al, PRD85 (2012) 054503

$\Rightarrow 140\text{MeV} \leq T \leq 334\text{MeV}$

NRQCD meson correlators

Point correlators:

Aarts et al (FASTUM) , Kim, PP, Rothkopf

$$C_p(t) = \sum_{\mathbf{x}} \langle O_p(t,\mathbf{x}) O_p(0,\mathbf{0}) \rangle,$$

$$O_p(t,\mathbf{x}) = \chi^\dagger(t,\mathbf{x}) \Gamma \psi(t,\mathbf{x}))$$

State	Irrep Λ^{PC}	Γ
η_b	A_1^{-+}	1
Υ	T_1^{--}	σ_j
h_b	T_1^{+-}	∇_j
χ_{b0}	A_1^{++}	$\boldsymbol{\sigma} \cdot \boldsymbol{\nabla}$
χ_{b1}	T_1^{++}	$(\boldsymbol{\sigma} \times \boldsymbol{\nabla})_j$
χ_{b2}	T_2^{++}	$\sigma_j \nabla_k + \sigma_k \nabla_j$

Extended correlators:

$$O_p(t,\mathbf{x}) \to O(t,\mathbf{x}) = \sum_{\mathbf{r}} \Psi(\mathbf{r}) \chi^\dagger(t,\mathbf{x}) \Gamma \psi(t,\mathbf{x}+\mathbf{r}))$$

$\Psi(\mathbf{r}) \sim e^{-|\mathbf{r}|^2/\sigma^2}$
or realistic wave-function

Optimized correlators: use several different extended meson operators with realistic wave functions and form orthogonal combinations

$$O_i \to \tilde{O}_\alpha = \Omega_{\alpha j} O_j,\ \langle \tilde{O}_\alpha(t) \tilde{O}^\dagger_\beta(0) \rangle \propto \delta_{\alpha,\beta},\ i = 1,2,3,\ldots$$

Mixed correlators (Bethe-Salpeter amplitudes):

Bethe-Salpeter amplitude

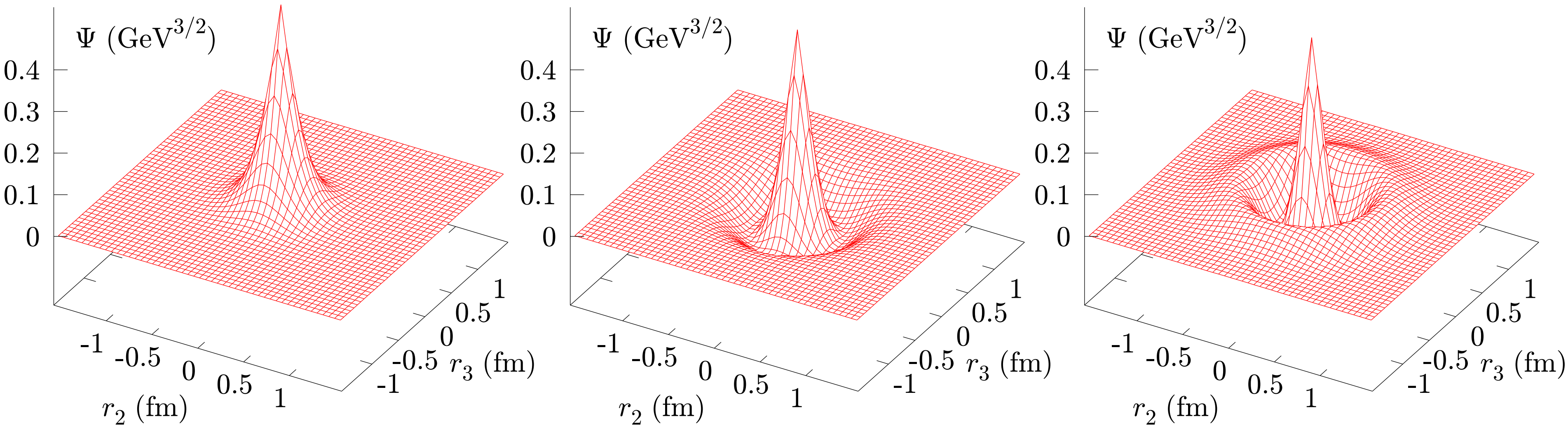
$$\tilde{C}^r_\alpha(t) = \sum_{\mathbf{x}} \langle O^r_{qq}(t,\mathbf{x}) \tilde{O}_\alpha(0,\mathbf{0}) \rangle \sim \phi_\alpha(r) e^{-E_\alpha t},\ t \to \infty$$

$$O^r_{qq}(t,\mathbf{x}) = \chi^\dagger(t,\mathbf{x}) \Gamma \psi(t,\mathbf{x}+\mathbf{r}))$$

Optimized Meson Operators

$$O_i(\mathbf{x},t) = \sum_{\mathbf{r}} \Psi_i(\mathbf{r})\chi^\dagger(\mathbf{x}+\mathbf{r},t)\Gamma\psi(\mathbf{x},t)$$ $\Psi_i(\mathbf{r})$ from potential model with Cornell potential

Meinel, PRD 82 (2010) 114502



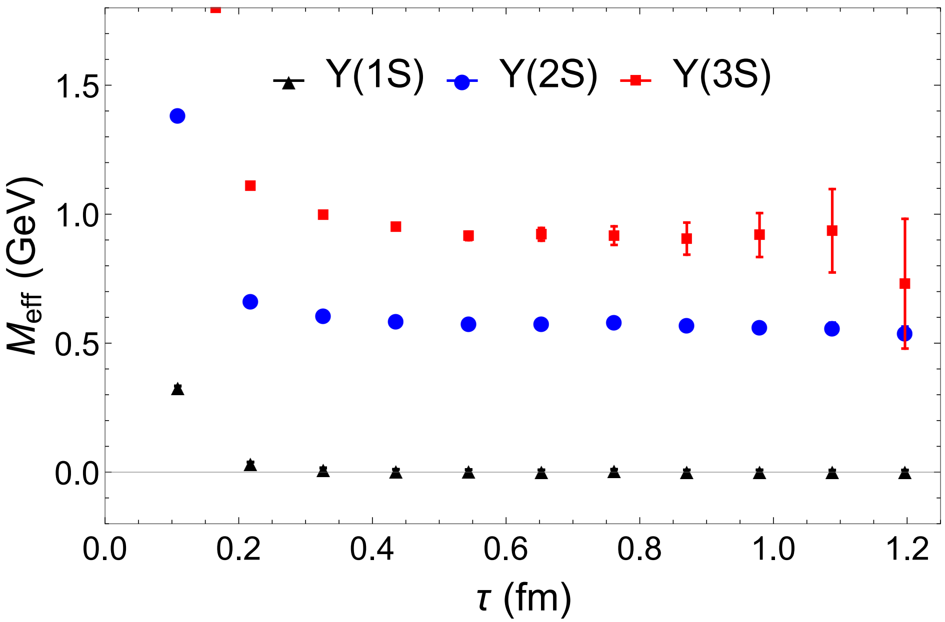
Good overlap with bottomonium states but

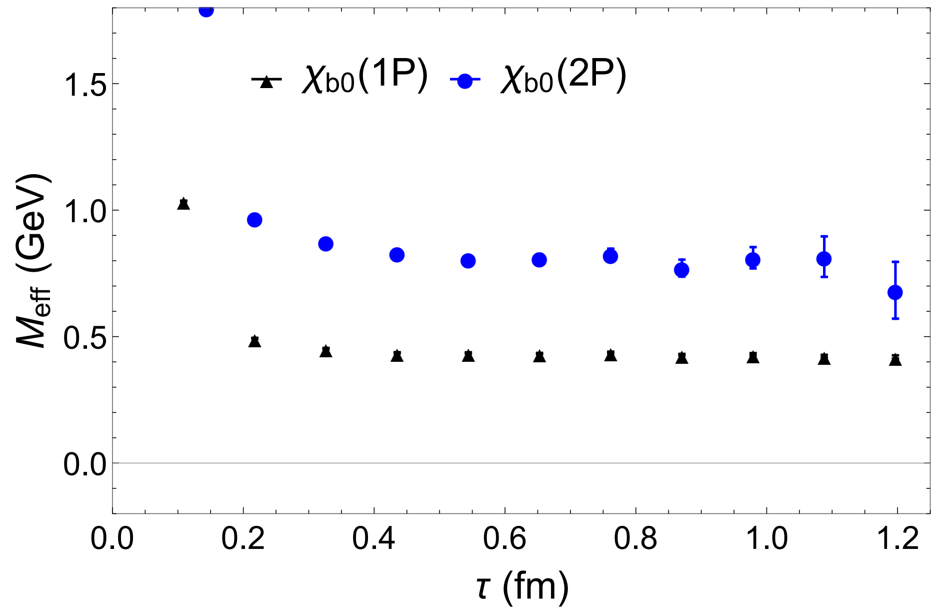
$C_{ij}(t) = \langle O_i(t) O_j^\dagger(0)\rangle \neq 0$ for $i \neq j$

$O_i \to \tilde{O}_\alpha = \Omega_{\alpha j} O_j$ such that
$\langle \tilde{O}_\alpha(t) \tilde{O}^\dagger_\beta(0)\rangle \propto \delta_{\alpha,\beta}$
$\Omega_{\alpha j}$ can be obtained as
$G_{ij}(t)\Omega_{\alpha j} = \lambda_\alpha(t,t_0) G_{ij}(t_0)\Omega_{\alpha j}$.

Correlators of Optimized Meson Operators at T=0

$$aM_{\rm eff}(t) = \ln[C_\alpha(t)/C_\alpha(t+1)]$$



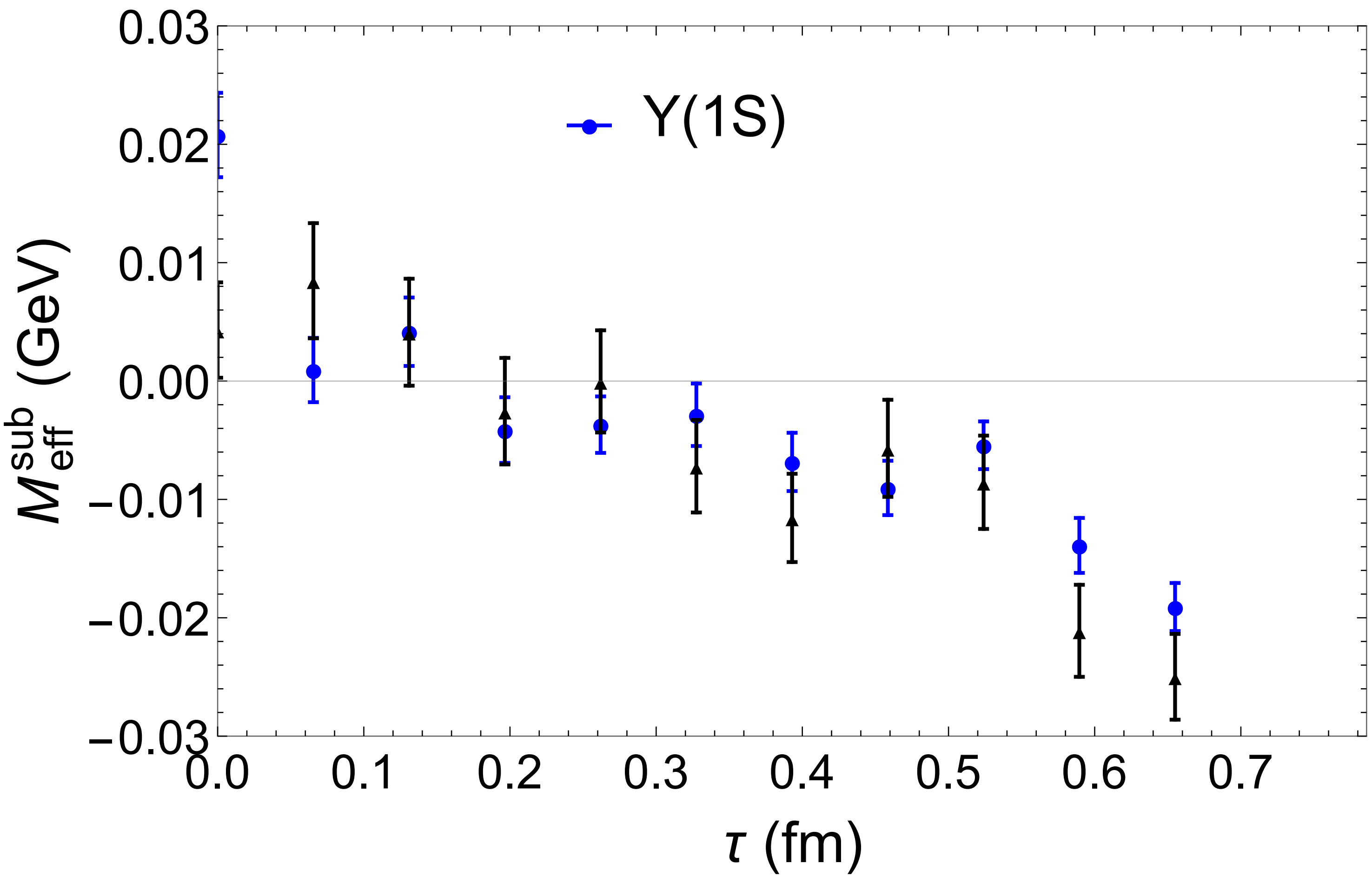


$$C_\alpha(\tau,T) = \int_{-\infty}^{\infty} d\omega \rho_\alpha(\omega,T) e^{-\omega\tau} \qquad \rho_\alpha(\omega,T) = \rho_\alpha^{\rm med}(\omega,T) + \rho_\alpha^{\rm high}(\omega)$$

$$\rho_\alpha^{\rm med}(\omega,T=0) = A_\alpha \delta(\omega - M_\alpha) \Rightarrow C_\alpha(\tau,T=0) = A_\alpha e^{-M_\alpha\tau} + C_\alpha^{\rm high}(\tau)$$

Determine A_α,M_α from single exponential fit for $\tau > 0.6$fm and then $C_\alpha^{\rm high}(\tau)$

Comparison of different Meson Operators

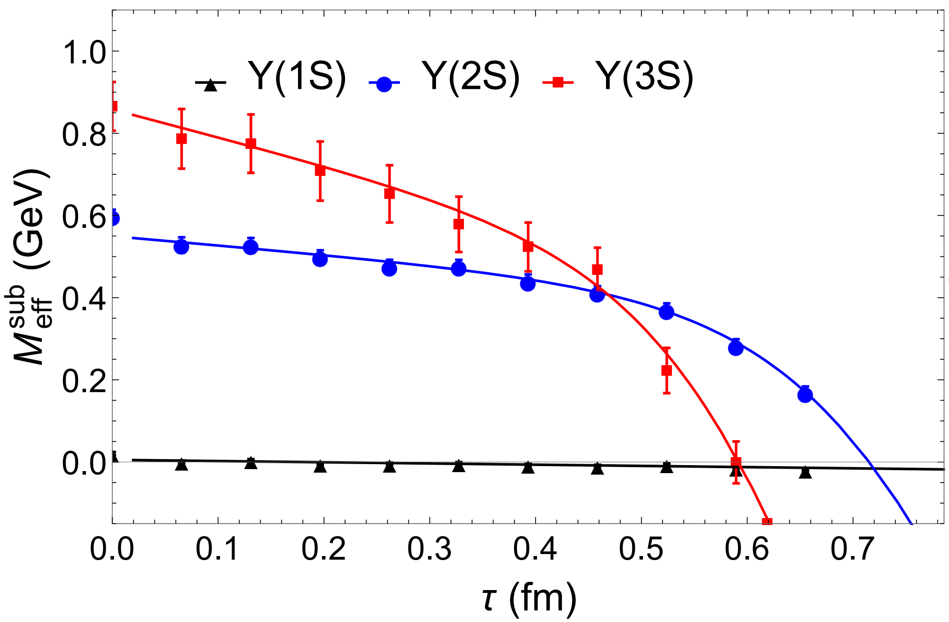


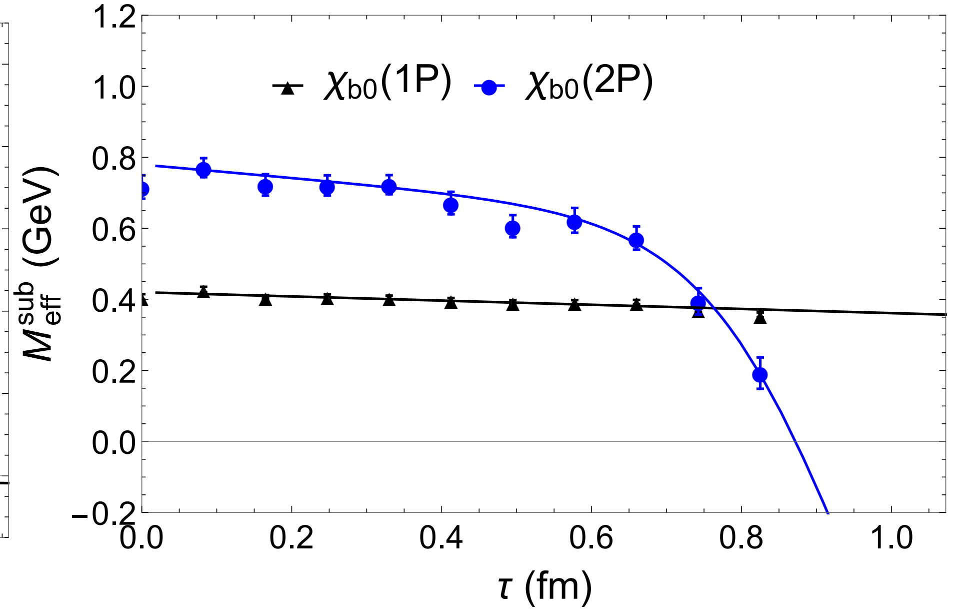
Blue circles: optimized operators

Black triangles: extended operators with Gaussian smearing

Correlators of Extended Meson Operators at T>0

$$C_\alpha^{\rm sub}(\tau,T)=C_\alpha(\tau,T)-C_\alpha^{\rm high}(\tau)\Rightarrow aM_{\rm eff}^{\rm sub}(\tau,T)=\ln\left(C_\alpha^{\rm sub}(\tau,T)/C_\alpha^{\rm sub}(\tau+a,T)\right)$$





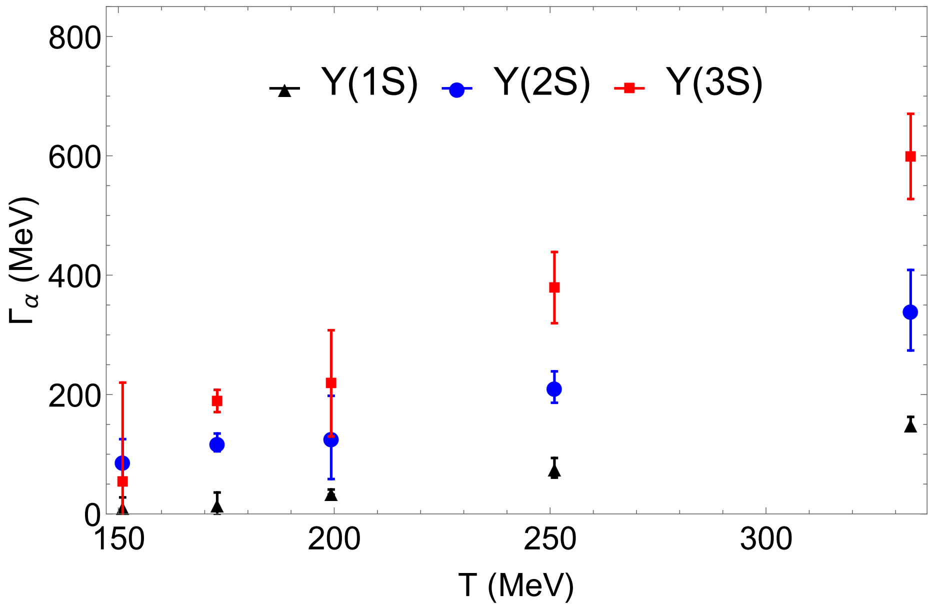
Fit $M_{\rm eff}^{\rm sub}(\tau,T)$ using a simple Ansatz:

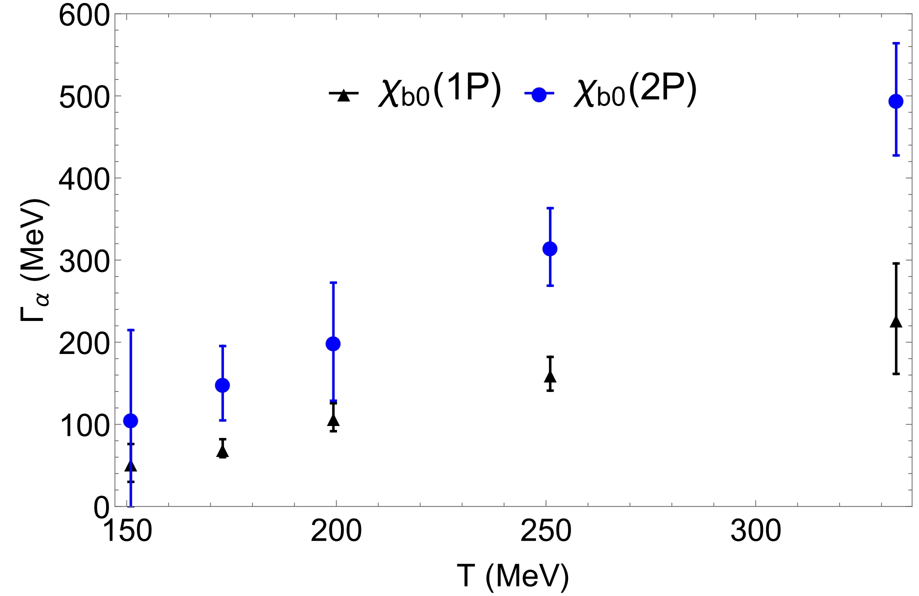
$$\rho_\alpha^{\rm med}(\omega,T)=A_\alpha^{\rm cut}(T)\,\delta\left(\omega-\omega_\alpha^{\rm cut}(T)\right)+A_\alpha(T)\exp\left(-\frac{[\omega-M_\alpha(T)]^2}{2\Gamma_\alpha^2(T)}\right)$$

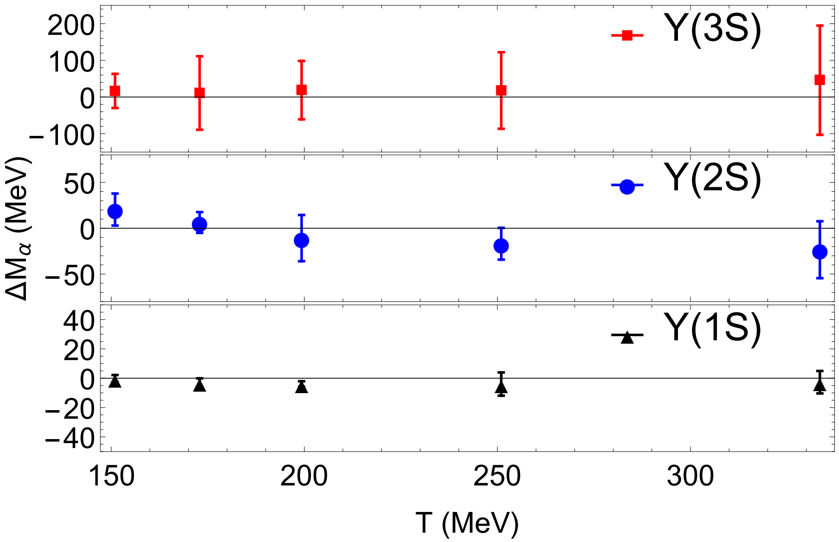
Low energy tail

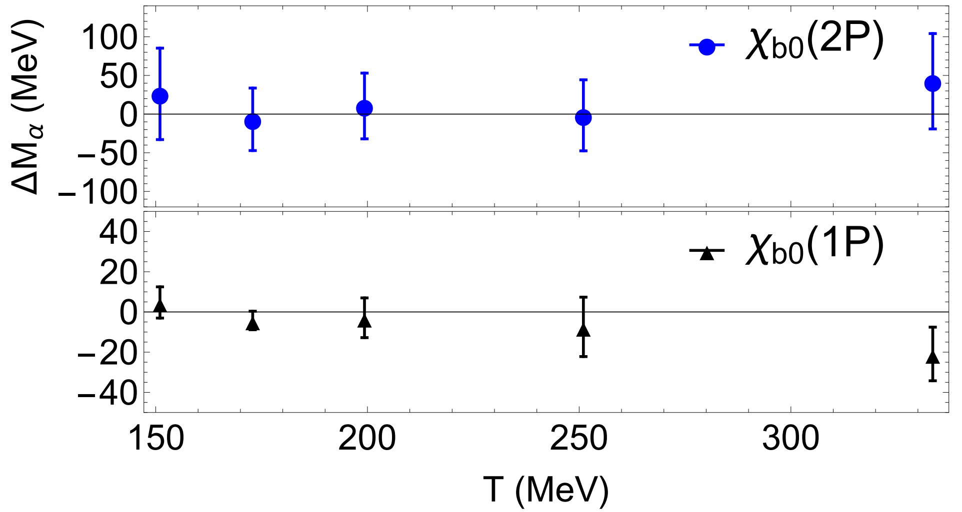
$$\Rightarrow M_\alpha(T),\Gamma_\alpha(T)$$

Thermal width and mass shift of bottomonium









Bottomonium Bethe-Salpeter amplitude at T=0

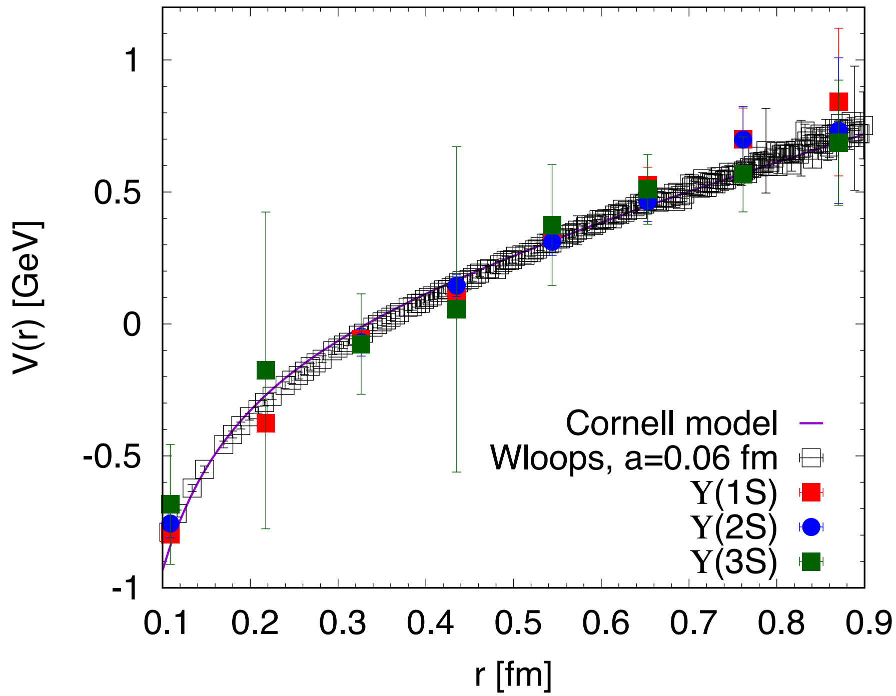
$$\tilde{C}^r_\alpha(\tau) = \sum_{\mathbf{x}} \langle O^r_{qq}(\tau,\mathbf{x}) \tilde{O}_\alpha(0,0)\rangle, \;\; O^r_{qq}(\tau,\mathbf{x}) = \chi^\dagger(\tau,\mathbf{x})\Gamma\psi(\tau,\mathbf{x}+r))$$

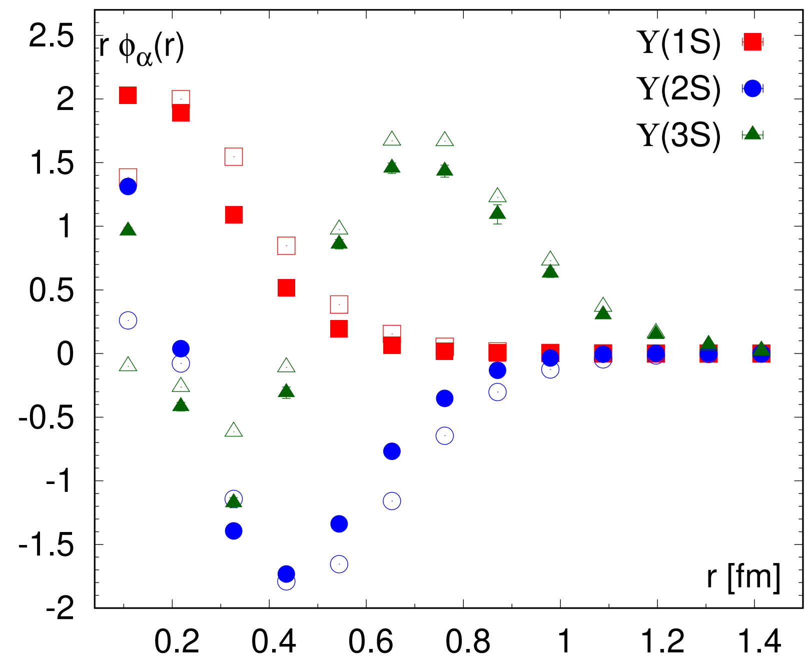
$$\tilde{C}^r_\alpha(\tau) = \sum_n \langle 0|O^r_{qq}(0)|n\rangle\langle n|\tilde{O}_\alpha(0)|0\rangle e^{-E_n\tau}|_{\tau\to\infty} \sim \langle 0|O^r_{qq}(0)|\alpha\rangle e^{-E_\alpha\tau}$$

$\phi_\alpha(r)$- Bethe-Salpeter amplitude

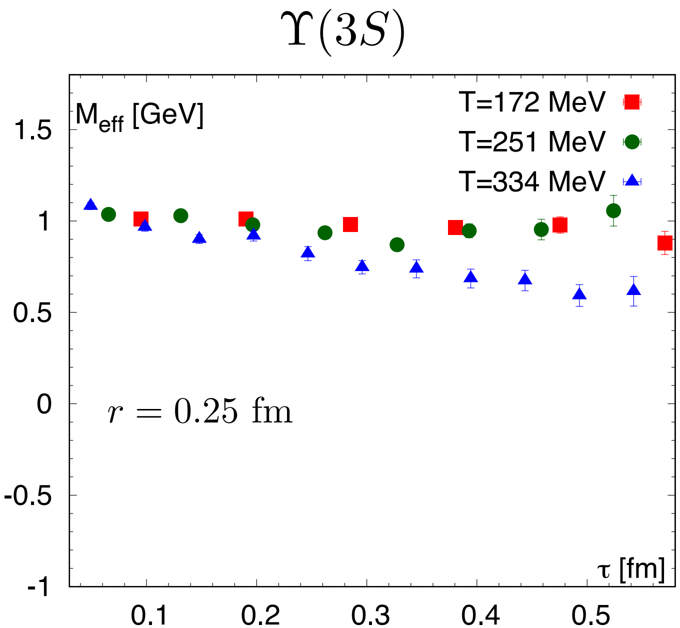
$$\left(\frac{-\nabla^2}{m_b} + V(r)\right)\phi_\alpha = E_\alpha\phi_\alpha$$

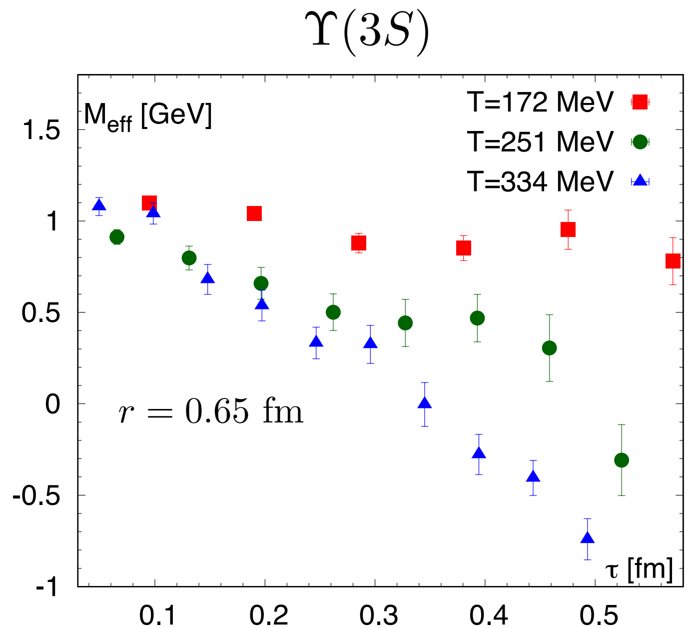
$$m_b = 5.52 \pm 0.33 \text{ GeV}$$





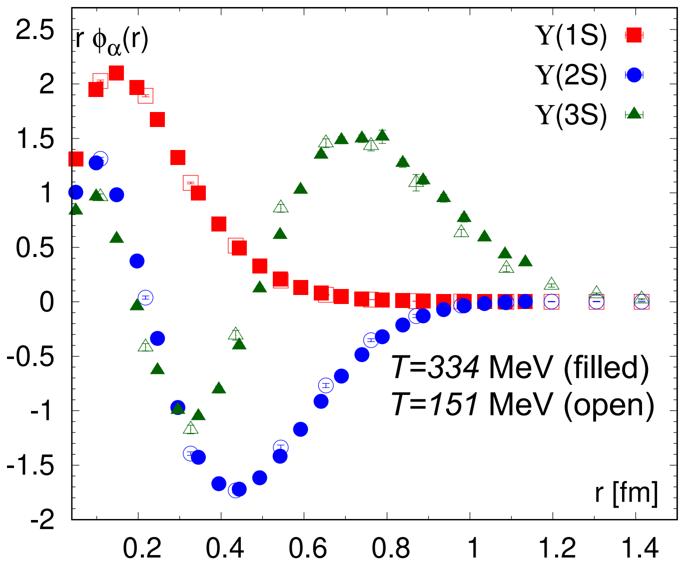
Bottomonium Bethe-Salpeter amplitude at T>0

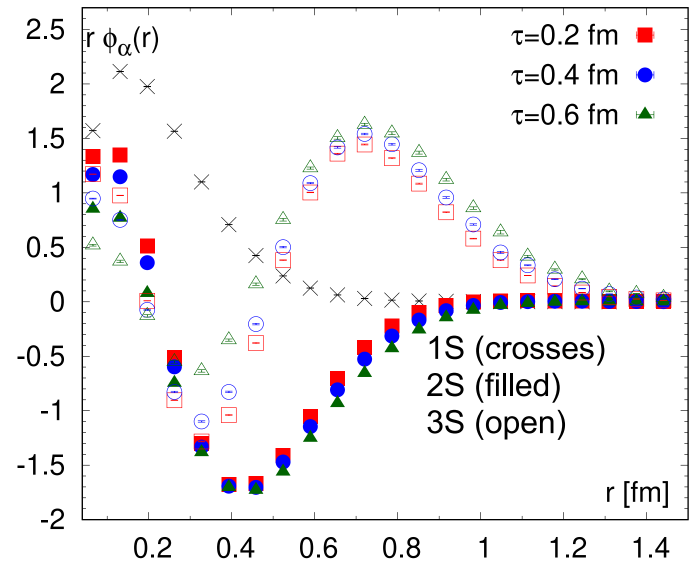




M_{eff} shows similar thermal effects similar to one obtained from optimized correlators;

Thermal effects are larger for larger r





Thermal effects can be seen but ϕ_α is similar to the $T = 0$ result at qualitative level

Summary

- The correlators of the Polyakov loop obtained on the lattice are consistent with the picture of chromo-electric screening inferred from the weak coupling calculations, and the non-perturbative corrections to the Debye mass are not very large.

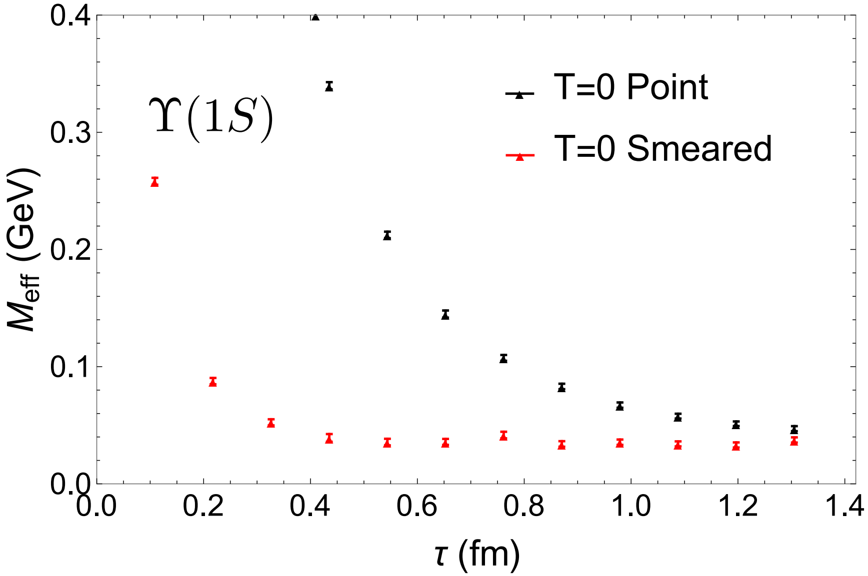
- Using a simple Ansatz for the spectral function we extracted the thermal width of $\Upsilon(1S)$, $\Upsilon(2S)$, $\Upsilon(3S)$, $\chi_{b1}(1P)$ and $\chi_{b2}(2P)$ states and found that the value of the thermal width follows the hierarchy of the bottomium sizes, as expected.

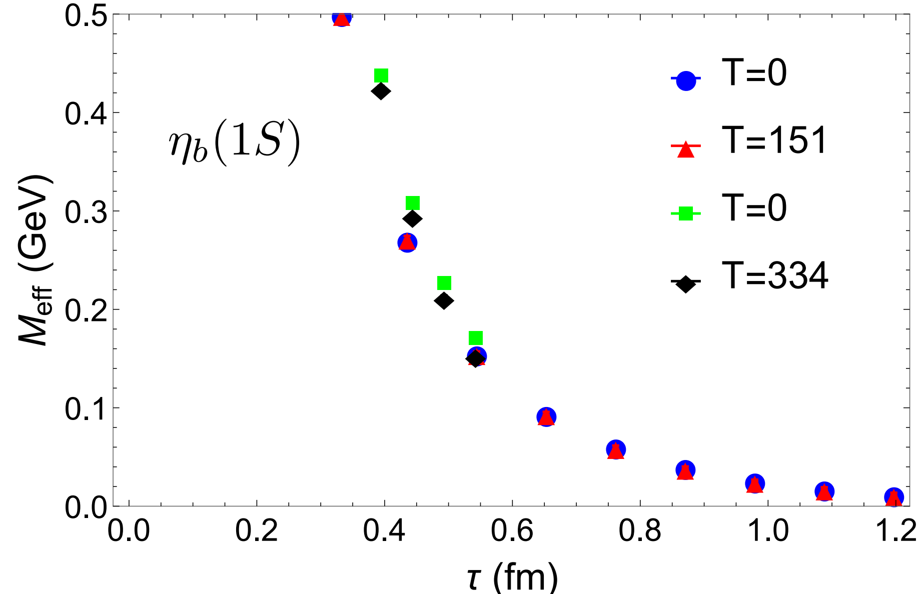
- No significant thermal modification of bottomonium masses have been found in contrast with the expectations based on potential models with screened potential.

- The lattice study of Bethe-Salpeter amplitudes confirms the potential model description of bottomonium at $T = 0$, but does not support the potential picture with screened potential at high temperatures.

Point operators vs. extended operators

$$aM_{\text{eff}}(t) = \ln[C_\alpha(t)/C_\alpha(t+1)]$$





- The effective masses of point correlators do not show a plateau for $\tau < 1.2$ fm and have very small temperature dependence
- The small τ behavior of the effective masses is well described by perturbation theory for P-wave bottomonia
- The correlators of extended operators approach a plateau for $\tau < 1$ fm.