Estimating nuclear matter parameters from compact star observables

G.G. Barnaföldi, D. Alvarez-Castillo, A. Ayrian, H. Grigorian, A. Jakovác, P. Pósfay, B. Szigeti


Support: Hungarian OTKA grants, NK123815, K135515 Wigner GPU Laboratory, the PHAROS MP16214 and THOR CA15213 COST actions.

20th Zimányi Wigner School, Budapest, 11th December 2020
Unveiling the strong interaction among hadrons at the LHC

ALICE Collaboration

Nature 588, 232–238(2020) | Cite this article

\[ C(k^*) = \int S(r^*) |m(k^*, r^*)|^2 d^3r^* = \xi(k^*) \otimes N_{\text{mixing}}(k^*) \]
Why is this so important?

**EoS experiment & theory**

**Application in compact stars**

**Constraints by astrophysical observations**

G.G. Barnafoldi: Zimányi Winter School 2020
Face the masquerade problem!

EoS experiment & the

Constraints by astrophysical observations
Face the masquarade problem!

Uuups, which EoS I should choose?

Constraints by astrophysical observations
(De)motivation...


Optimal neutron-star mass ranges to constrain the equation of state of nuclear matter with electromagnetic and gravitational-wave observations

L. R. Weih, E. R. Most, AND L. REZZOLLA

Institut für Theoretische Physik, Goethe Universität Frankfurt am Main, Germany

ABSTRACT

Exploiting a very large library of physically plausible equations of state (EOSs) containing more than $10^7$ members and yielding more than $10^9$ stellar models, we conduct a survey of the impact that a neutron-star radius measurement via electromagnetic observations can have on the EOS of nuclear matter. Such measurements are soon to be expected from the ongoing NICER mission and will complement the constraints on the EOS from gravitational-wave detections. Thanks to the large statistical range of our EOS library, we can obtain a first quantitative estimate of the commonly made assumption that the high-density part of the EOS is best constrained when measuring the radius of the most massive, albeit rare, neutron stars with masses $M \gtrsim 2.1 M_\odot$. At the same time, we find that radius measurements of neutron stars with masses $M \simeq 1.7 - 1.85 M_\odot$ can provide the strongest constraints on the low-density part of the EOS. Finally, we quantify how radius measurements by future missions can further improve our understanding of the EOS of matter at nuclear densities.
The sad reality is...


Optimal neutron-star mass ranges to be tested by future missions and...
The sad reality is...


Optimal neutron-star mass ranges to 2.6--3.0\ M_{\odot} with a 250-80-250 Gzatta model matter with electromagnetic and nuclear contributions. Our results indicate that a neutron star containing more than 10^{53} ergs is unlikely. The study of the impact of the model on the EOS of neutron star mass and nuclear radius measurements will complement the existing statistical ranges. The assumption that the most massive neutron stars are the most massive known will be confirmed. We find that neutron star mass constraints on the EOS of matter at nuclear densities can provide the strongest constraints on the EOS of neutron star matter.
Let’s explore the uncertainties...

...in a traditional way

Investigate this with extended $\sigma$-$\omega$ model in mean field

$$\mathcal{L}_{MF} = \sum_{i=1,2} \bar{\psi}_i \left( i\gamma^\mu \left( -m_N + g_\sigma \sigma - g_\omega \gamma^0 \omega_0 \right) \psi_i \right)$$

- Nucleon effective mass

- $-\frac{1}{2} m_\sigma^2 \sigma^2 - \lambda_3 \sigma^3 - \lambda_4 \sigma^4$

- $+ \frac{1}{2} m_\omega^2 \omega_0^2$

- $+ \frac{1}{2} m_\rho^2 \rho_\mu^a \rho_\mu^a$

- $+ \bar{\Psi}_e \left( i\gamma^\mu \left( -m_e \right) \right) \Psi_e$

Proton and neutron

Scalar meson self interaction terms

Extra terms

Vector meson

Tensor meson

Electron in $\beta$-equilibrium

$$\mu_n = \mu_p + \mu_e$$
Investigate this with extended $\sigma$-$\omega$ model in mean field

$$\mathcal{L}_{MF} = \sum_{i=1,2} \bar{\psi}_i \left( i\not\!\!E - m_N + g_\sigma \overline{\sigma} - g_\omega \gamma^0 \overline{\omega}_0 \right) \psi_i$$

Proton and neutron

Nucleon effective mass

- $-\frac{1}{2} m_\sigma^2 \overline{\sigma}^2 - \lambda_3 \overline{\sigma}^3 - \lambda_4 \overline{\sigma}^4$

Scalar meson self interaction terms

$p - n$ Nuclear force

Extra terms

+ $\frac{1}{2} m_\omega^2 \overline{\omega}_0^2$

Vector meson

+ $\frac{1}{2} m_\rho^2 \rho^a_\mu \rho^\mu_a$

Tensor meson

+ $\bar{\Psi}_e \left( i\not\!\!E - m_e \right) \Psi_e$

Electron in $\beta$-equilibrium

$$\mu_n = \mu_p + \mu_e$$
Investigate this with extended $\sigma$-$\omega$ model in mean field

$$\mathcal{L}_{MF} = \sum_{i=1,2} \bar{\psi}_i \left( i\gamma^0 \partial_0 - m_N + g_\sigma \sigma - g_\omega \gamma^0 \omega_0 \right) \psi_i$$

- Proton and neutron
  - Nucleon effective mass

  \[ - \frac{1}{2} m_\sigma^2 \sigma^2 - \lambda_3 \sigma^3 - \lambda_4 \sigma^4 \]

  \[ + \frac{1}{2} m_\omega^2 \omega_0^2 \]

- Scalar meson self interaction terms

- Vector meson

- Tensor meson

- Isospin asymmetry

- Electron in $\beta$-equilibrium

\[ \mu_n = \mu_p + \mu_e \]
Investigate this with extended $\sigma$-$\omega$ model in mean field

\[ \mathcal{L}_{MF} = \sum_{i=1,2} \bar{\psi}_i \left( i\not\!\partial - m_N + g_\sigma \bar{\sigma} - g_\omega \gamma^0 \bar{\omega}_0 \right) \psi_i \]

- Proton and neutron
  \[ -\frac{1}{2} m_\sigma^2 \bar{\sigma}^2 - \lambda_3 \bar{\sigma}^3 - \lambda_4 \bar{\sigma}^4 \]

- Scalar meson self interaction terms
  \[ + \frac{1}{2} m_\omega^2 \bar{\omega}_0^2 \]

- Vector meson
  \[ + \frac{1}{2} m_\rho^2 \rho^a_\mu \rho^\mu a \]

- Tensor meson
  \[ + \bar{\Psi}_e \left( i\not\!\partial - m_e \right) \Psi_e \]

- Electron in $\beta$-equilibrium
  \[ \mu_n = \mu_p + \mu_e \]
Investigate this with extended $\sigma$-$\omega$ model in mean field

$$\mathcal{L}_{MF} = \sum_{i=1,2} \bar{\psi}_i \left( i\not\!\!\!\!\!\partial - m_N + g_\sigma \sigma - g_\omega \gamma^0 \omega_0 \right) \psi_i$$

- Nucleon effective mass
  - $- \frac{1}{2} m_\sigma^2 \sigma^2 - \lambda_3 \sigma^3 - \lambda_4 \sigma^4$
  - Extra terms
    - $+ \frac{1}{2} m_\omega^2 \omega_0^2$
    - $+ \frac{1}{2} m_\rho^2 \rho_\mu^a \rho^{\mu a}$
    - $+ \bar{\Psi}_e (i\not\!\!\!\!\!\partial - m_e) \Psi_e$

Proton and neutron
Scalar meson self interaction terms
Vector meson
Tensor meson
Electron in $\beta$-equilibrium

$$\mu_n = \mu_p + \mu_e$$
Modified $\sigma$-$\omega$ model in mean field

- **Theoretical mean field model:**
  - Symmetric case: 3 combinations with the higher-order scalar meson self-interaction terms to original Walecka:
  - Asymmetric case: tensor force is added to the interaction in addition to the electrons, for $\beta$-equilibrium.

- **Parameters of the theoretical model**
  - Fit couplings/masses/etc. according to the Rhoades-Ruffini theorem in agreement with experimental data.
  - Parameters are usually non-independent: optimalization of the parameters need to perform $\rightarrow$ similar EoS

- **Cross check the consistency with the the existing EM, GR, HIC, etc data + errors $\rightarrow$ Theoretical uncertainties**
Parameters to fit normal nuclear matter

<table>
<thead>
<tr>
<th>Model</th>
<th>$n_s$ [fm$^{-3}$]</th>
<th>$B$ [MeV]</th>
<th>$K$ [MeV]</th>
<th>$S_0$ [MeV]</th>
<th>$m^*$ [$m_N$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>NL$\rho$</td>
<td>0.1459</td>
<td>-16.062</td>
<td>203.3</td>
<td>30.8</td>
<td>0.603</td>
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<tr>
<td>DBHF</td>
<td>0.1810</td>
<td>-16.150</td>
<td>230.0</td>
<td>34.4</td>
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<tr>
<td>DD</td>
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<td>-16.021</td>
<td>240.0</td>
<td>32.0</td>
<td>0.565</td>
</tr>
<tr>
<td>D3C</td>
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<td>-15.981</td>
<td>232.5</td>
<td>31.9</td>
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<tr>
<td>KVR</td>
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<td>KVOR</td>
<td>0.1600</td>
<td>-16.000</td>
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<td>0.800</td>
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<td>DD-F</td>
<td>0.1469</td>
<td>-16.024</td>
<td>223.1</td>
<td>31.6</td>
<td>0.556</td>
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Parameters to fit normal nuclear matter

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**Incompressibility**

\[ K = k_F^2 \frac{\partial^2 (\epsilon/n)}{\partial k_F^2} = 9 \frac{\partial p}{\partial n} \]

**Landau mass**

\[ m_L = \frac{k_F}{v_F} = \sqrt{k_F^2 + m_{N,eff}^2} \]
Parameters to fit normal nuclear matter

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The effective mass and Landau mass are NOT independent!
The can not be fitted simultaneously!
The EoS & M-R of different model fits

- Landau mass fit $m_{\text{Eff}} = 0.83 \, m_N$
- Effective mass fit $m_{\text{Eff}} = 0.6 \, m_N$

M-R diagram with these nuclear matter EoS
Cases with extra $x^3$ and/or $x^4$ terms provide similar band structures
→ Landau mass fits provide lower $M_{\text{max}}$ closer to the observations
The EoS & M-R of different model fits

Landau mass fit $m_{\text{Eff}} = 0.83 m_N$

Effective mass fit $m_{\text{Eff}} = 0.6 m_N$

Original Walecka model

Realistic nuclear EoS: WFF1, AP4 (SQM) support the Landau mass

Cases with extra $x^3$ and/or $x^4$ terms provide similar band structures

→ Landau mass fits provide lower $M_{\text{max}}$ closer to the observations

M-R diagram with these nuclear matter EoS

P. Pósfay, GGB, A. Jakovác: 2004.08230
The EoS & M-R of different model fits

- Landau mass fit $m_{\text{Eff}} = 0.83 m_N$
- Effective mass fit $m_{\text{Eff}} = 0.6 m_N$
- Original Walecka model
- Realistic nuclear EoS: WFF1, AP4 (SQM) support the Landau mass
- Assymetry (electrons) is weak effect

M-R diagram with these nuclear matter EoS

Cases with extra $x^3$ and/or $x^4$ terms provide similar band structures

→ Landau mass fits provide lower $M_{\text{max}}$ closer to the observations

→ Nuclear ASYMMETRY result in 10-20% lower $M_{\text{max}}$
The EoS & M-R of different model fits

Landau mass fit $m_{\text{Eff}} = 0.83 m_N$

Effective mass fit $m_{\text{Eff}} = 0.6 m_N$

Original Walecka model

Realistic nuclear EoS: WFF1, AP4 (SQM) support the Landau mass

Assymetry (electrons) is weak effect

Crust (BPS) make more realistic

M-R diagram with these nuclear matter EoS

Cases with extra $x^3$ and/or $x^4$ terms provide similar band structures

→ Landau mass fits provide lower $M_{\text{max}}$ closer to the observations

→ Nuclear ASYMMETRY result in 10-20% lower $M_{\text{max}}$

→ Adding CORE with BPS has no effect on $M_{\text{max}}$, only on R (~km)

G.G. Barnafoldi: Zimányi Winter School 2020
The M-R diagrams: EoS & Landau mass fit

Evolution/scaling in $M_{\text{max}}$ appears
- The $M_{\text{max}}$ is increasing as the Landau (effective) mass is decreasing

$\rightarrow$ Scaling by nuclear parameters
Scaling: maximum star mass vs. nuclear parameters

Maximal mass with Landau mass

\[ M_{\text{max}}(m_L)[M_{\odot}] = 5.418 - 0.00434 \, m_L[\text{MeV}] \]

P. Pósfay, GGB, A. Jakovác: 2004.08230
Scaling: maximum star mass vs. nuclear parameters

Maximal mass with Landau mass

\[ M_{\text{max}}(m_L)[M_\odot] = 5.418 - 0.00434 \, m_L[\text{MeV}] \]

with (in)compressibility

\[ M_{\text{max}}(K)[M_\odot] = 1.766 + 0.00110K[\text{MeV}] \]

\[ \Delta M_{\text{max}}(\delta m_L) \stackrel{10\times}{>} \Delta M_{\text{max}}(\delta K) \]
Scaling: maximum star mass vs. nuclear parameters

Maximal mass with Landau mass

\[ M_{\text{max}}(m_L) [M_\odot] = 5.418 - 0.00434 m_L [\text{MeV}] \]

with (in)compressibility

\[ M_{\text{max}}(K) [M_\odot] = 1.766 + 0.00110 K [\text{MeV}] \]

Combine these to a 2-parameter fit:

\[ M_{\text{max}}(m_L, K) [M_\odot] = 6.29 - 0.00574 m_L [\text{MeV}] - 0.00379 K [\text{MeV}] + 0.00000524 m_L \cdot K [\text{MeV}^2]. \]

P. Pósfay, GGB, A. Jakovác, B. Szigeti 2006.03710

G.G. Barnafoldi: Zimányi Winter School 2020
Scaling: maximum star mass vs. nuclear parameters

Maximal mass & its radius with Landau mass

\[
M_{\text{max}}(m_L)[M_\odot] = 5.418 - 0.00434 m_L[\text{MeV}]
\]

\[
R_{\text{max}}(m_L)[\text{km}] = 19.04 - 0.01040 m_L[\text{MeV}]
\]

Maximal mass & its radius with (in)compressibility

\[
M_{\text{max}}(K)[M_\odot] = 1.766 + 0.00110K[\text{MeV}]
\]

\[
R_{\text{max}}(K)[\text{km}] = 8.878 + 0.00767K[\text{MeV}]
\]
Scaling: maximum star mass vs. nuclear parameters

Maximal mass & its radius with Landau mass

\[ M_{\text{maxM}}(m_L)[M_\odot] = 5.418 - 0.00434 \ m_L[\text{MeV}] \]
\[ R_{\text{maxM}}(m_L)[\text{km}] = 19.04 - 0.01040 \ m_L[\text{MeV}] \]

with (in)compressibility

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\[ R_{\text{maxM}}(K)[\text{km}] = 8.878 + 0.00767K[\text{MeV}] \]

Calculation for maximal mass star

Measured: \( M_{\text{maxM}} \rightarrow (m_L \ & \ K) \rightarrow R_{\text{maxM}} \)

P. Pósfay, GGB, A. Jakovác, B. Szigeti 2006.03710

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Scaling: maximum star mass vs. nuclear parameters

Maximal mass & its radius with Landau mass

\[ M_{\text{max} M}(m_L)[M_\odot] = 5.418 - 0.00434 \ m_L[\text{MeV}] \]
\[ R_{\text{max} M}(m_L)[\text{km}] = 19.04 - 0.01040 \ m_L[\text{MeV}] \]

with (in)compressibility

\[ M_{\text{max} M}(K)[M_\odot] = 1.766 + 0.00110K[\text{MeV}] \]
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Calculation for maximal mass star

<table>
<thead>
<tr>
<th>Pulsar</th>
<th>( R_{\text{max} M}[\text{km}] )</th>
<th>( M_{\text{max} M}[M_\odot] )</th>
<th>( m_L[\text{MeV}] )</th>
<th>( K[\text{MeV}] )</th>
</tr>
</thead>
<tbody>
<tr>
<td>PSR J0740+0620</td>
<td>11.25( \pm )0.04</td>
<td>2.17( \pm )0.10 *</td>
<td>748.39( \pm )63.3</td>
<td>351.8( \pm )115</td>
</tr>
<tr>
<td>PSR J0348+0432</td>
<td>10.87( \pm )0.52</td>
<td>2.01( \pm )0.04 *</td>
<td>785.25( \pm )29.3</td>
<td>206.4( \pm )32.7</td>
</tr>
<tr>
<td>PSR J1614–2230</td>
<td>10.77( \pm )0.80</td>
<td>1.97( \pm )0.04 *</td>
<td>794.47( \pm )20.4</td>
<td>170.0( \pm )29.5</td>
</tr>
</tbody>
</table>

P. Pósfay, GGB, A. Jakovác, B. Szigeti 2006.03710

G.G. Barnafoldi: Zimányi Winter School 2020
From data: Maximum star mass vs. nuclear parameters

Maximal mass & its radius with Landau mass

\[ M_{\text{max}}(m_L) [M_\odot] = 5.418 - 0.00434 m_L [\text{MeV}] \]
\[ R_{\text{max}}(m_L) [\text{km}] = 19.04 - 0.01040 m_L [\text{MeV}] \]

with (in)compressibility

\[ M_{\text{max}}(K) [M_\odot] = 1.766 + 0.00110K [\text{MeV}] \]
\[ R_{\text{max}}(K) [\text{km}] = 8.878 + 0.00767K [\text{MeV}] \]

Combine these to a 2-parameter fit:

\[ M_{\text{max}}(m_L, K) [M_\odot] = 6.29 - 0.00574 m_L [\text{MeV}] - 0.00379 K [\text{MeV}] + 0.00000524 m_L \cdot K [\text{MeV}^2] \]
\[ R_{\text{max}}(m_L, K) [\text{km}] = 27.51 - 0.0239 m_L [\text{MeV}] - 0.0241 K [\text{MeV}] + 0.0000411 m_L \cdot K [\text{MeV}^2] \]
From data: Maximum star mass vs. nuclear parameters

Maximal mass & its radius with Landau mass

\[ M_{\text{max}}(m_L)[M_\odot] = 5.418 - 0.00434 m_L [\text{MeV}] \]
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Results from data using fit formulae:

\[ m_L = 776.0^{+38.5}_{-84.9} \text{ MeV} \text{ and } K = 242.7^{+57.2}_{-28.0} \text{ MeV} \]
Explore the uncertainties...

... using a the brute force

Brute force: Bayesian analysis

Data: $\vec{\pi}_q = \{m_{L(i)}, K_{0(j)}, S_{0(k)}\}$

Likelihood for given independent constraints:

$$P(E | \vec{\pi}_q) = \prod_w P(E_w | \vec{\pi}_q)$$

Posterior:

$$P(\vec{\pi}_q | E) = \frac{P(E | \vec{\pi}_q) P(\vec{\pi}_q)}{\sum_{p=0}^{N-1} P(E | \vec{\pi}_p) P(\vec{\pi}_p)}$$

Marginalization (1-parameter):

$$P(m_{L(i)} | E) = \sum_{j,k} P(m_{L(i)}, K_{0(j)}, S_{0(k)} | E)$$

Brute force: Bayesian analysis

Data: \( \pi_q = \{m_L(i), K_0(j), S_0(k)\} \)

Likelihood for given independent constraints:

\[
P(E | \pi_q) = \prod_w P(E_w | \pi_q)
\]

Posterior:

\[
P(\pi_q | E) = \frac{P(E | \pi_q) P(\pi_q)}{\sum_{p=0}^{N-1} P(E | \pi_p) P(\pi_p)}
\]

Marginalization (1-parameter):

\[
P(m_L(i) | E) = \sum_{j,k} P(m_L(i), K_0(j), S_0(k) | E)
\]

Likelihood for GW170817:

\[
P(E_{GW} | \pi_q) = \int \beta(\Lambda_1(n_c), \Lambda_2(n_c)) \, dn_c
\]

Likelihood for maximal mass

\[
P(E_M | \pi_q) = \Phi(M_q, \mu_C, \sigma_C) \times \Phi(M_q, \mu_A, \sigma_A) \times \mathcal{N}(M_q, \mu_U, \sigma_U)
\]

Likelihood for mass & radius

\[
P(E_{MR} | \pi_q) = 0.5 \int \mathcal{N}(\mu^{(1)}_M, \sigma^{(1)}_M, \mu^{(1)}_R, \sigma^{(1)}_R, \alpha^{(1)}) \, dn_c \\
+ 0.5 \int \mathcal{N}(\mu^{(2)}_M, \sigma^{(2)}_M, \mu^{(2)}_R, \sigma^{(2)}_R, \alpha^{(2)}) \, dn_c
\]


G.G. Barnafoldi: Zimányi Winter School 2020
Brute force: Bayesian analysis

Data with $m_L$ only

Brute force: Bayesian analysis

Data with $m_L$ only

Brute force: a Bayesian analysis

Data with $m_L$ & $K$

Summary:

- **Traditional way: mean field model**
  - In CORE approximation: maximal mass provide a unique message:
    - $\Delta M_{\text{max}}(\delta m_L) \gg \Delta M_{\text{max}}(\delta K) > \Delta M_{\text{max}}(\delta a_{\text{sym}})$.
  - Soft part of the EoS changes the CRUST, thus vary $R$
  - FRG: parameter 10-25% observables: 5-10%

- **Values & uncertainties - a cross check**
  - Traditional model: $m_L = 776.0^{+38.5}_{-84.5}$ MeV and $K = 242.7^{+57.2}_{-28.0}$ MeV
  - Bayesian model*: $m_L = 727.4 \pm 15$ MeV and $K = 232 \pm 20$ MeV