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Investigating the structure of compact stars with degenerate Fermi gas having many degree of freedom

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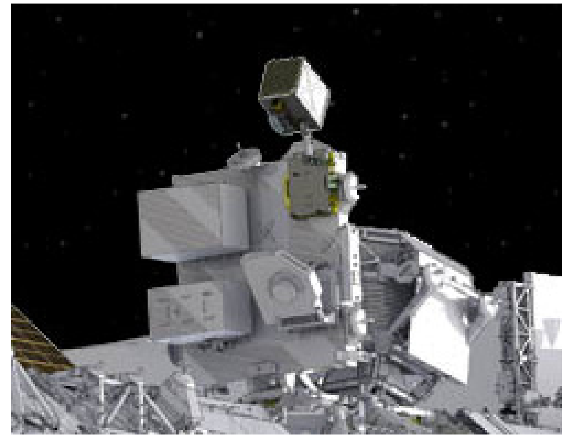
Motivation and objective

- Compact stars – focus of many investigations, extreme laboratories
- Close to 3000 pulsars are already catalogued
- New data is available from instruments like NICER (Neutron Star Interior Composition Explorer), gravitational wave detectors, future Athena

- Opportunity to evaluate **new compact star models** by comparing **model prediction** with physical data
- Testing particles beyond the standard model before experimental setup (e.g. Future Circular Collider for ~100 TeV)



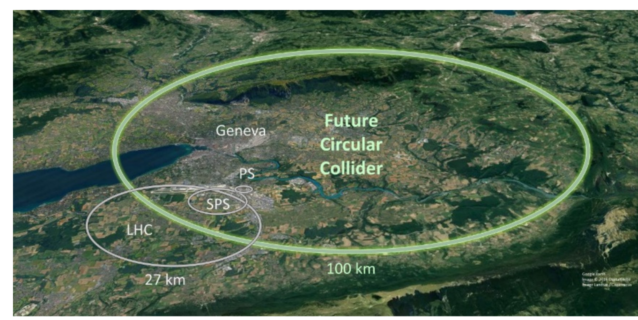


Image source: https://heasarc.gsfc.nasa.gov/docs/nicer/
https://home.cern/science/accelerators/future-circular-collider, https://www.ligo.caltech.edu

Work covered

- Defined various particle compositions (linear ladder, Kaluza Klein) of the compact star – Equation of State, its explicit analytical calculation, root finding, etc.
- Compact star model – integration based on the TOV model, surface definition
- Detailed analysis of the numerical behavior of the numerical model
- Analysis of two defined multiparticle models: Mass-Radius diagrams, maximum mass, internal composition, etc.

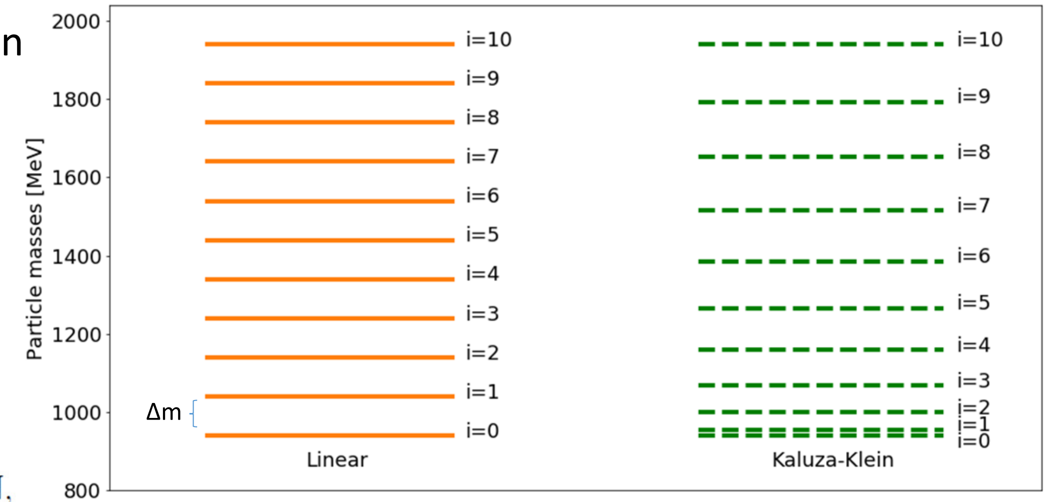
Modelled particles

- Multi-particle, hadron *(i – exication number*), non interacting models
- Built on the neutron (m_0) with mass gap (Δm)
- Constant gap, linear ladder

$$m_i = m_0 + i \cdot \Delta m \text{ , where } \; i \in \mathbb{N},$$

- Kaluza – Klein ladder

$$m_i = \sqrt{m_0^2 + (i \cdot \Delta m)^2} \text{ , where } \; \Delta m = \frac{\hbar c}{R_C} \; \text{ and } \; i \in \mathbb{N},$$



- Equation of State was defined

$$p_i = -\frac{\partial \Omega|_{T=0}}{\partial V} = \frac{g_i}{24\pi^2} \times \left[\mu_i \sqrt{\mu_i^2 - m_i^2} \left(\mu_i^2 - \frac{5}{2} m_i^2 \right) - \frac{3}{2} m_i^4 \ln \frac{m_i}{\mu_i + \sqrt{\mu_i^2 - m_i^2}} \right],$$

$$\epsilon_i = \frac{g_i}{(2\pi)^3} \int_0^{k_F} \varepsilon_i \mathrm{d}^3\mathbf{k}|_{T=0} = \frac{g_i}{8\pi^2} \times \left[\mu_i \sqrt{\mu_i^2 - m_i^2} \left(\mu_i^2 - \frac{1}{2} m_i^2 \right) + \frac{m_i^4}{2} \ln \frac{m_i}{\mu_i + \sqrt{\mu_i^2 - m_i^2}} \right].$$

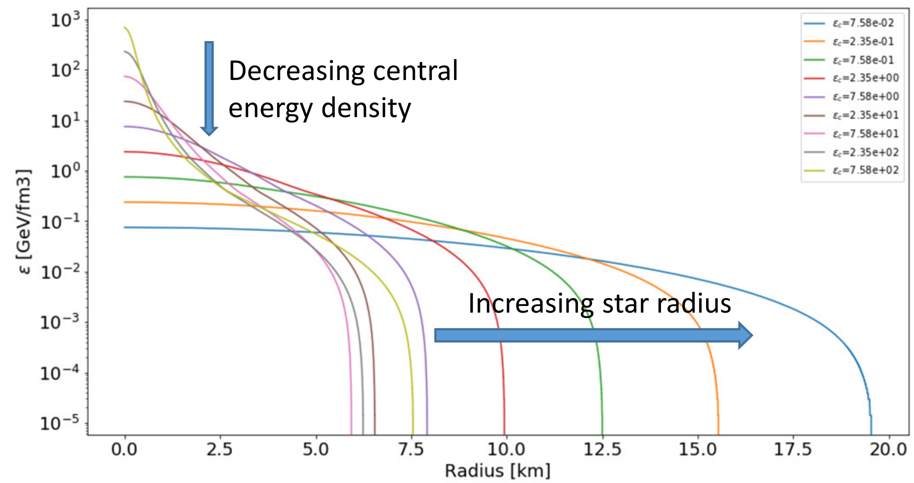
Compact star integrator

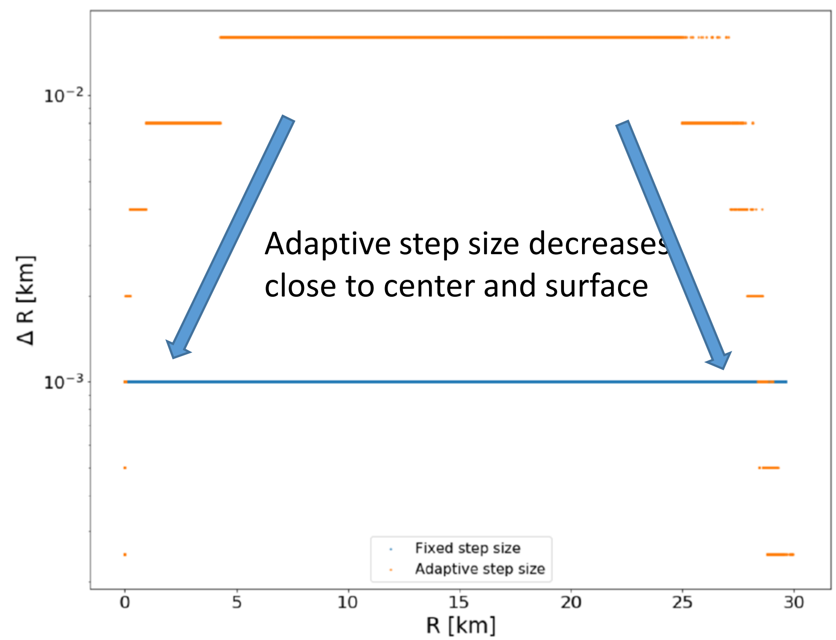
- TOV equations

$$\frac{\mathrm{d}p(r)}{\mathrm{d}r} = -\frac{GM(r)\varepsilon(r)}{r^2} \times \left[1+\frac{p(r)}{\varepsilon(r)}\right]\left[1+\frac{4\pi r^3 p(r)}{M(r)}\right]\left[1-\frac{GM(r)}{r}\right]^{-}$$

$$M(r) = \int_0^r \mathrm{d}r' 4\pi r'^2 \varepsilon(r') \ .$$

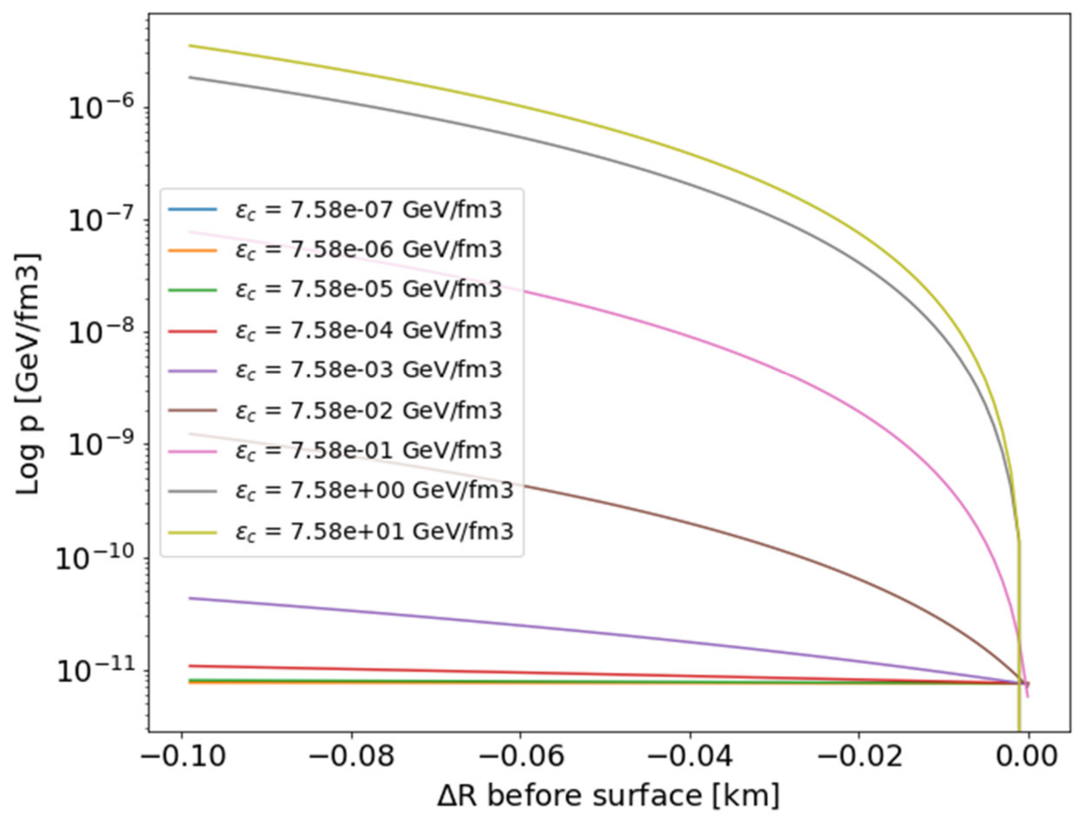
- Initial condition is given as the central energy density
- Euler, Runge-Kutta, Runge-Kutta-Merson and its adaptive version were tested
- Various root finding algorithms (Python, own developed) were analyzed
- Physical parameters of pressure, energy density, particle composition, density, chemical potential are tracked along the star radius
- Cross check against the Kodama relationship





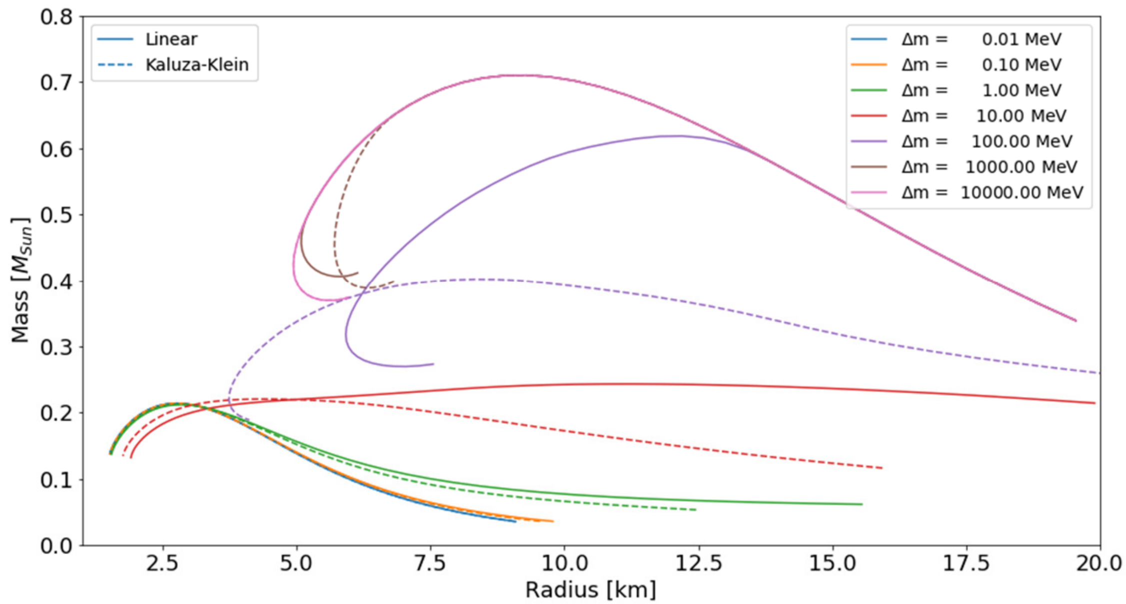
Analysis of identifying the surface

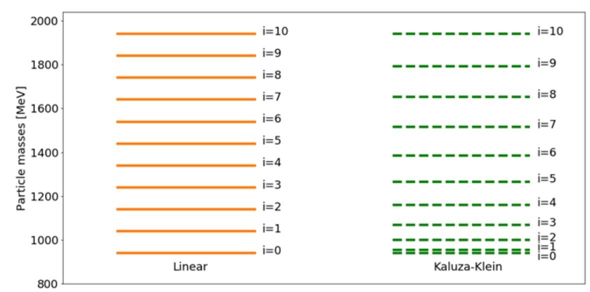
- Various surface detection criteria was implemented and evaluated
  - Cut off pressure
  - Pressure gradient cut off
  - Enthalpy cut off



Mass – Radius curve results

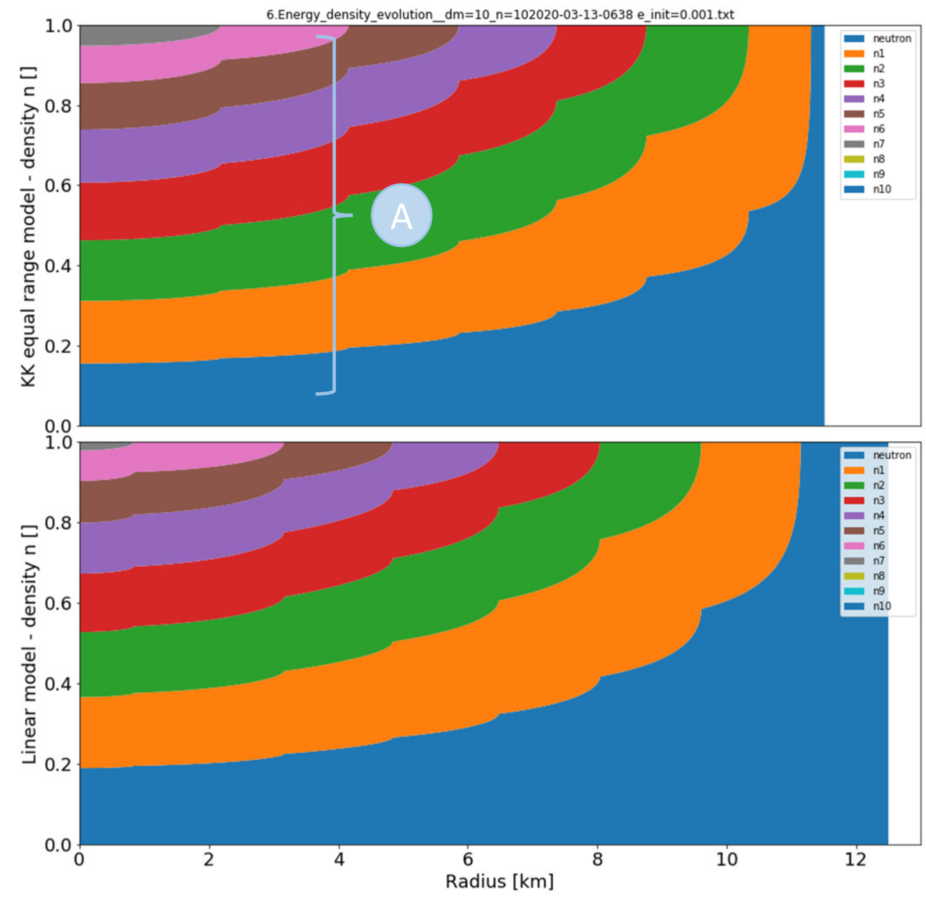
- Given EoS, varying initial condition of central energy density
- Objective: identifying the resulting star sizes and the maximum star size
- A 10 particle mix is show on the figure including the linear ladder and Kaluza Klein ladder model results
- Maximum star size found was 0.7 M_{Sun}





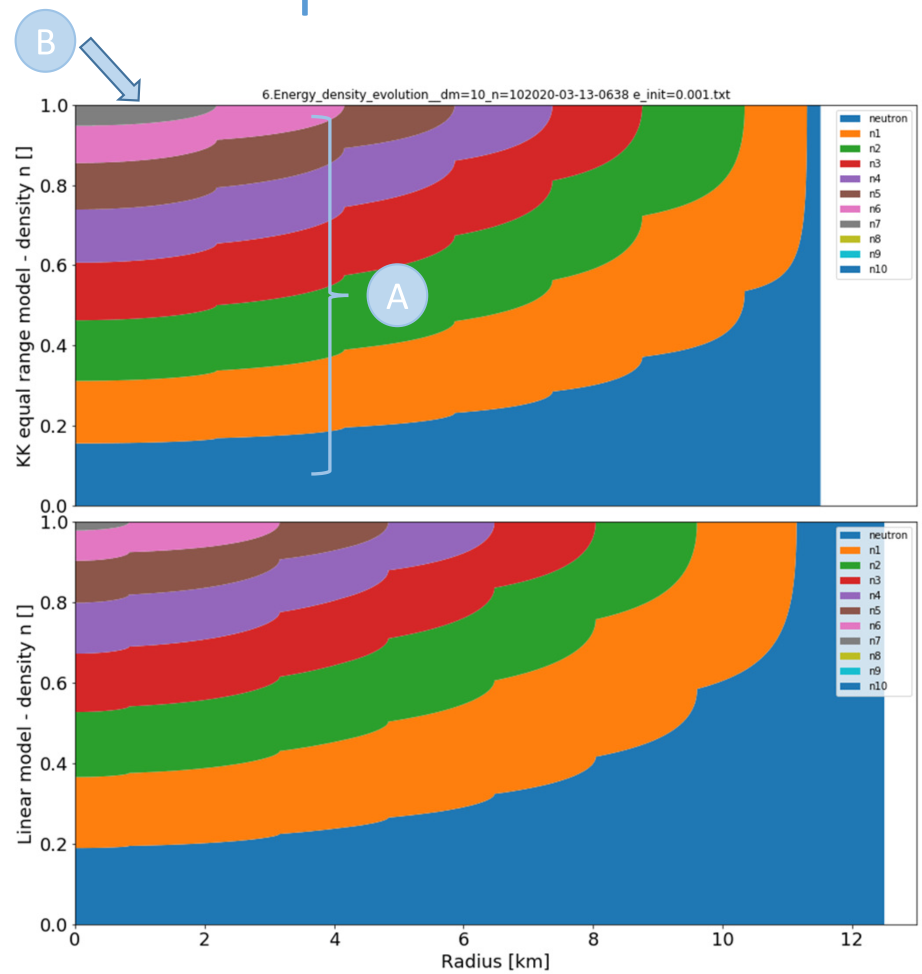
Internal composition of the compact star

- A Lake of particles with varying composition along the star radius
- Linear ladder, Kaluza Klein compared



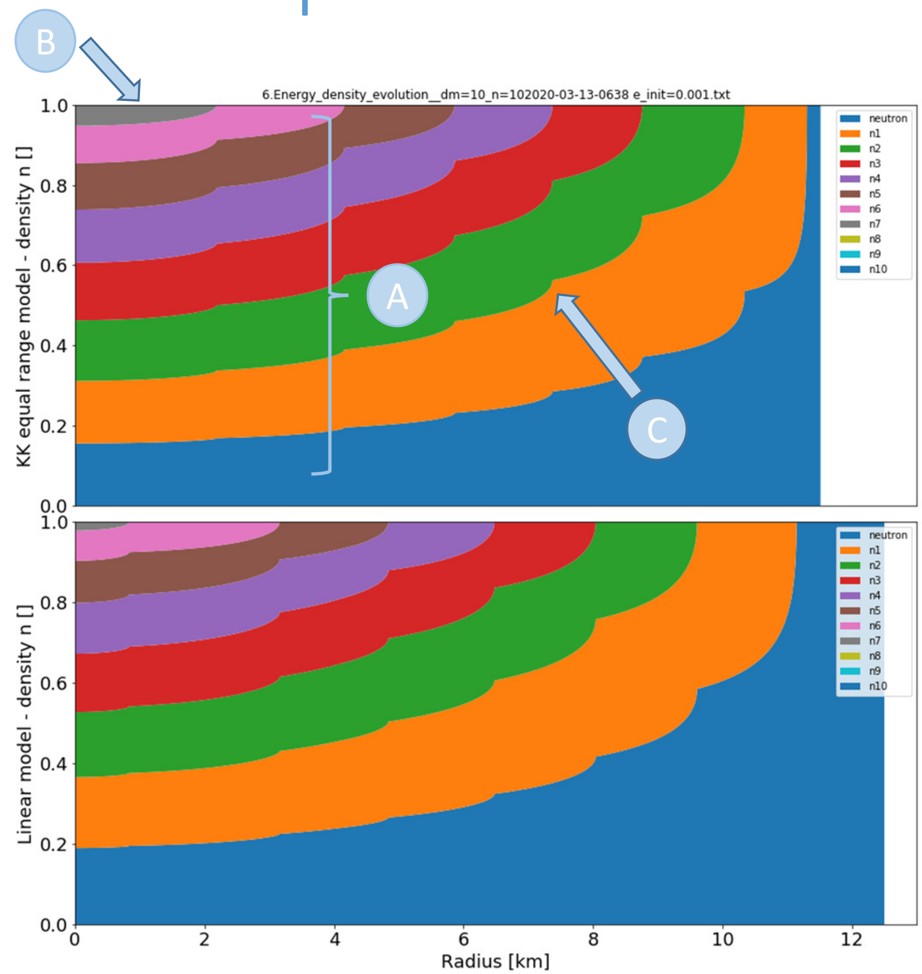
Internal composition of the compact star

- (A) Lake of particles with varying composition along the star radius
- Linear ladder, Kaluza Klein compared
- (B) Initial condition of central energy density influences the number of particles activated and star radius



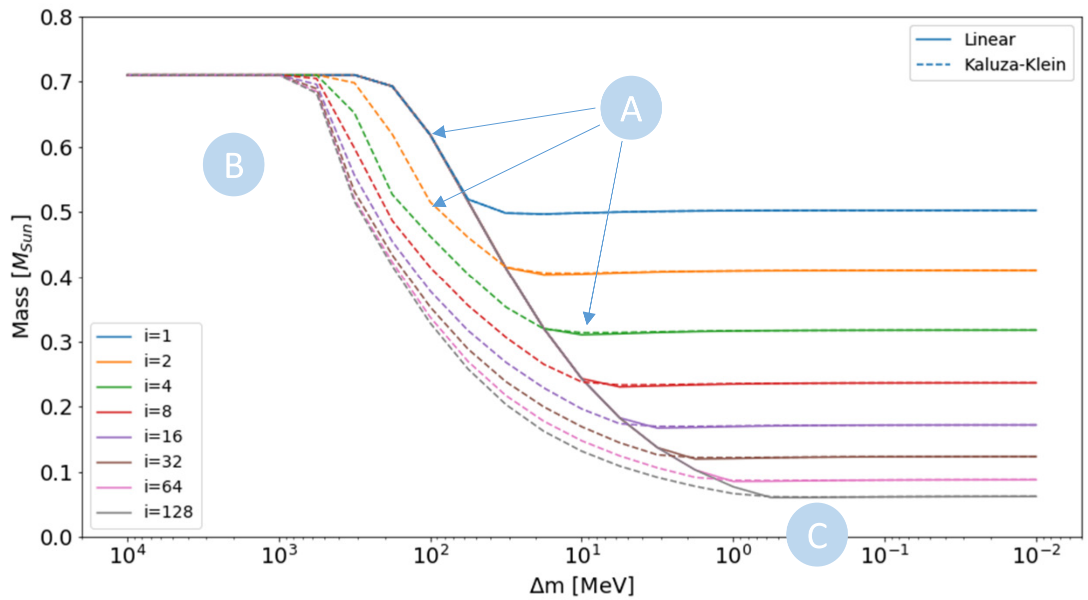
Internal composition of the compact star

- A Lake of particles with varying composition along the star radius
- Linear ladder, Kaluza Klein compared
- B Initial condition of central energy density influences the number of particles activated and star radius
- C Bumps indicate when a new particle is activated and energy is needed first to build up its mass



Exploring the maximum and minimum size of resulting compact stars

- (A) Each data point represents the maximum value on the M-R curve of given model parameters
- (B) At high Δm values, only small number of excitations can switch on, hence result is close to the single neutron model
- (C) This model cannot give a minimum value for the compact star size



Conclusion

- Built up a compact star model based on various EoS and the TOV equation.
- Performed numerical investigation on the model behavior
- Modelled two multi-particle setup: linear ladder and Kaluza-Klein models
- Found that the maximum star masses can reach 0.7 M_{Sun} while minimum size could not be identified
- Next step is to introduce interacting models