

ZIMÁNYI SCHOOL 2020



J. E.: From darkness, the light

20th ZIMÁNYI SCHOOL
WINTER WORKSHOP
ON HEAVY ION PHYSICS

December 7-11, 2020

Budapest, Hungary



József Zimányi (1931 - 2006)

Relativistic Hydrodynamics –Then and Now

presented at

Zimányi School 2020
Winter Workshop on Heavy Ion Physics
December 11th, 2020

W.A. Zajc
Columbia University

Thanks to Jamie Nagle and Ron Belmont for various slides in this presentation,
and to Krista Smith and Jorge Noronha

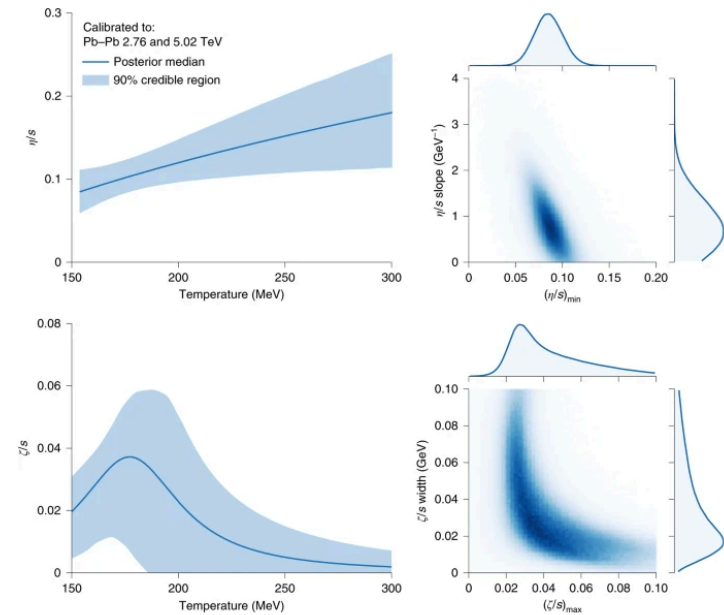
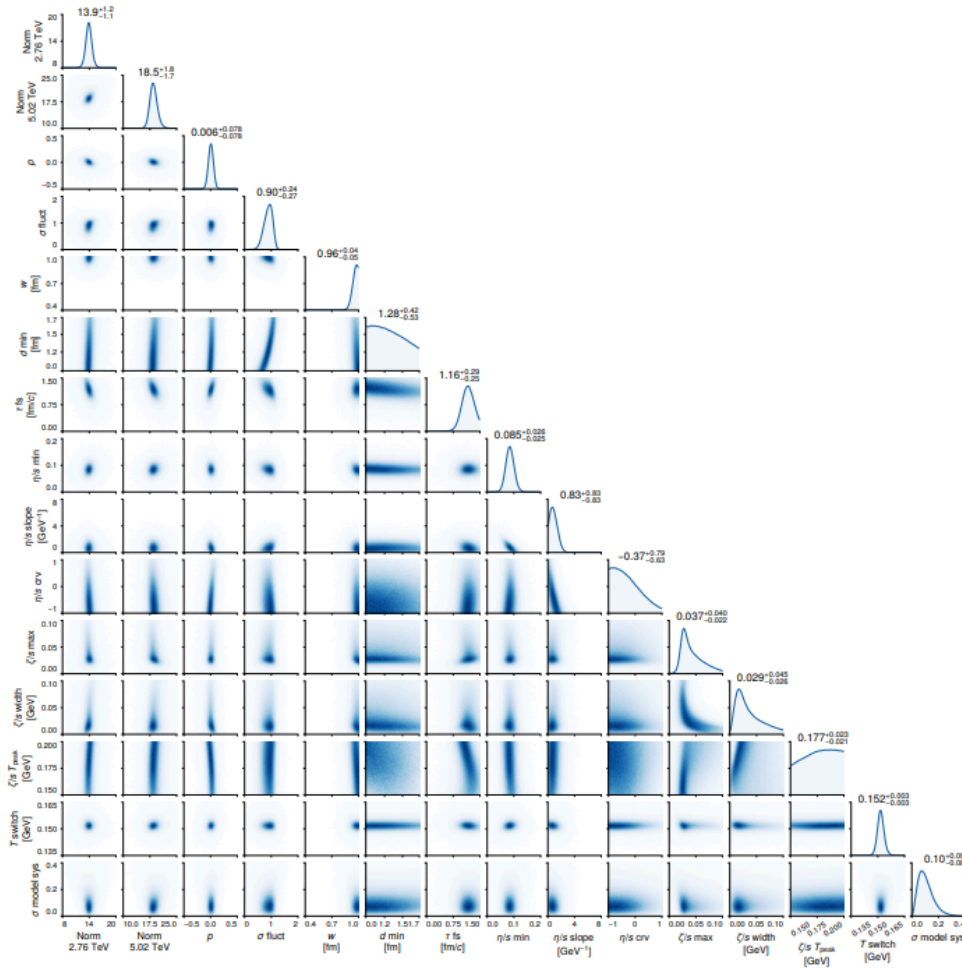
This work was supported by the United States Department
of Energy Grant DOE-FG02-86ER-40281

Introduction (of Caveats)

- This talk is intended for the *students* at this workshop
- Largely historical review (along with a few PHENIX results)
- I will review developments, not credit
- very likely to (inadvertently) offend
- It's history but not necessarily *accurate* history
- Rather, it's my "narrative"

See talk by Panos Christakoglou for
an authoritative overview
of the current status

Present Day: Golden Era of Relativistic Hydrodynamics



It didn't used to be this way !

Bayesian estimation of the specific shear and bulk viscosity of quark-gluon plasma, Bernhard, Moreland and Bass, Nature Physics 15, 1113 (2019)

The Simplified Version of My Talk

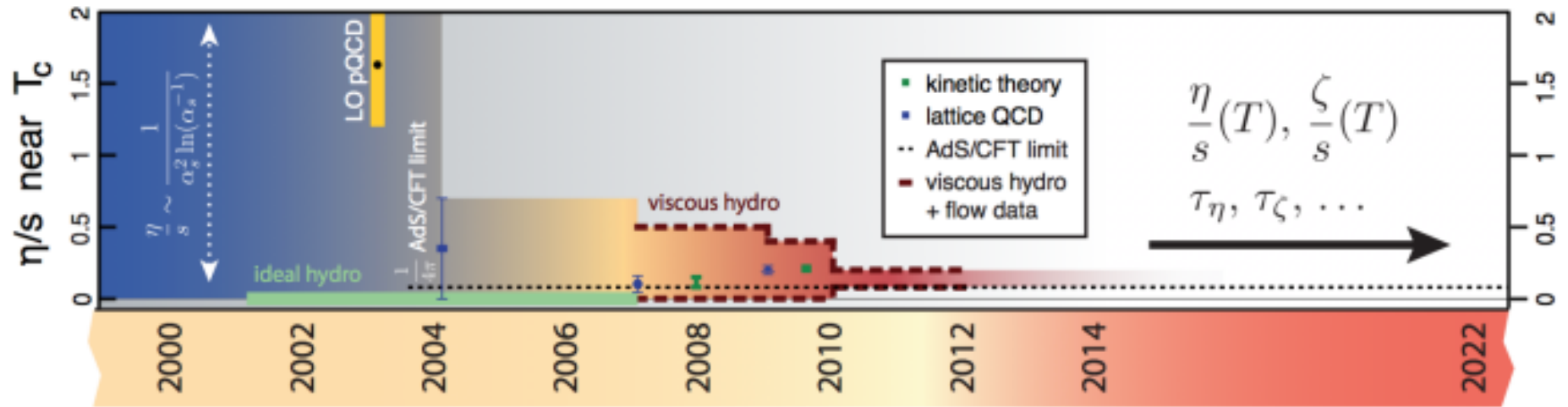


Figure by Björn Schenke

History

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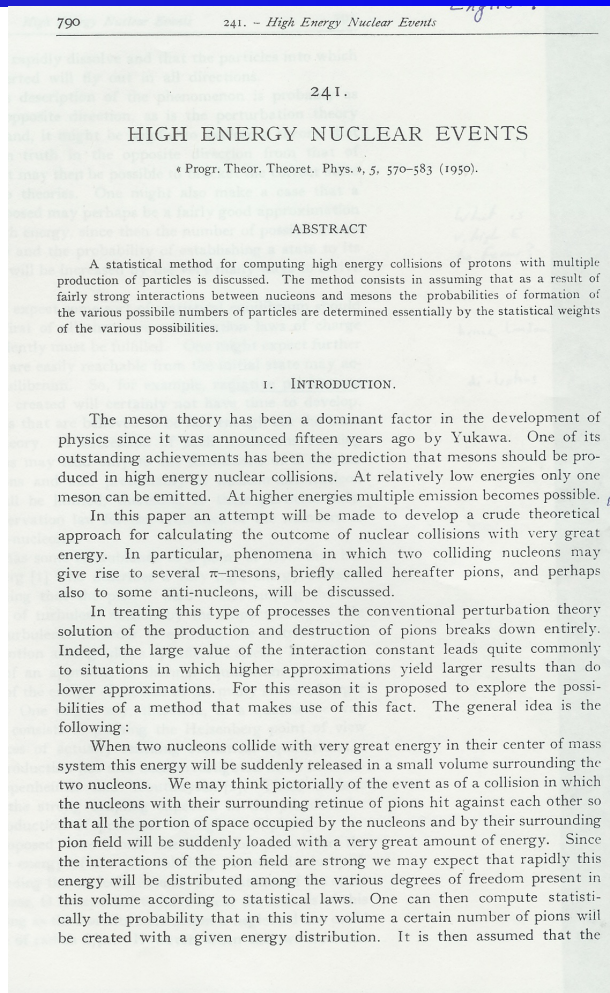
- I will address ~~four~~ (OK, five) epochs:
 - ▶ Pre-historic
 - ▶ 2000-2005: Exploration
 - ▶ 2005-2010: Consolidation
 - ▶ 2010-2015: Quantification
 - ▶ 2015-2020: Validation (and interrogation)

Pre-History

Fermi (1950)

"High Energy Nuclear Events",
Prog. Theor. Phys. 5, 570 (1950)

- "Since the interactions of the pion field are **strong**, we may expect that rapidly this energy will be distributed among the various degrees of freedom present in this volume according to statistical laws."



Landau (1955)

- "The defects of Fermi's theory arise mainly because the expansion of the compound system is not correctly taken into account... (The) expansion of the system can be considered on the basis of **relativistic hydrodynamics**."

88. A HYDRODYNAMIC THEORY OF MULTIPLE FORMATION OF PARTICLES

1. INTRODUCTION

Experiment shows that in collisions of very fast particles a large number of new particles are formed in multi-prong stars. The energy of the particles which produce such stars is of the order of 10^{12} eV or more. A characteristic feature is that such collisions occur not only between a nucleon and a nucleus but also between two nucleons. For example, the formation of two mesons in neutron-proton collisions has been observed at comparatively low energies, of the order of 10^9 eV, in cosmotron experiments¹.

Fermi^{2,3} originated the ingenious idea of considering the collision process at very high energies by the use of thermodynamic methods. The main points of his theory are as follows.

(1) It is assumed that, when two nucleons of very high energy collide, energy is released in a very small volume V in their centre of mass system. Since the nuclear interaction is very strong and the volume is small, the distribution of energy will be determined by statistical laws. The collision of high-energy particles may therefore be treated without recourse to any specific theories of nuclear interaction.

(2) The volume V in which energy is released is determined by the dimensions of the meson cloud around the nucleons, whose radius is $\hbar/\mu c$, μ being the mass of the pion. But since the nucleons are moving at very high speeds, the meson cloud surrounding them will undergo a Lorentz contraction in the direction of motion. Thus the volume V will be, in order of magnitude,

$$V = \frac{4\pi}{3} \left(\frac{\hbar}{\mu c} \right)^3 \frac{2M c^2}{E'} \quad (1.1)$$

where M is the mass of a nucleon and E' the nucleon energy in the centre of mass system.

(3) Fermi assumes that particles are formed, in accordance with the laws of statistical equilibrium, in the volume V at the instant of collision. The particles formed do not interact further with one another, but leave the volume in a "frozen" state.

С. З. Бельский и Л. Д. Ландау, Гидродинамическая теория множественного образования частиц, *Успехи Физических Наук*, 56, 309 (1955).

S. Z. Belenkiy and L. D. Landau, Hydrodynamic theory of multiple production of particles, *Nuovo Cimento*, Supplement, 8, 15 (1956).

Long Interregnum . . .

- Birth of QCD in early 1970's
⇒ little or no interest in hydrodynamic models for elementary particle collisions

- Work by

- ▶ Carruthers, Cooper, Feinberg, Frye, Koba
Duong-van Minh, Hagedorn, McLerran, Schonberg,

maintained despite active anti-interest

“...even mentioning a thermodynamic or hydrodynamic theory of subhadronic physics looked vulgar and even indecent” E.L. Feinberg (1988)

In contrast: active development for many workers in applications to nucleus+nucleus collisions

PHYSICAL REVIEW D

VOLUME 27, NUMBER 1

1 JANUARY 1983

Highly relativistic nucleus-nucleus collisions: The central rapidity region

J. D. Bjorken
Fermi National Accelerator Laboratory,* P.O. Box 500, Batavia, Illinois 60510
(Received 13 August 1982)

The space-time evolution of the hadronic matter produced in the central rapidity region in extreme relativistic nucleus-nucleus collisions is described. We find, in agreement with previous studies, that quark-gluon plasma is produced at a temperature ≥ 200 – 300 MeV, and that it should survive over a time scale ≥ 5 fm/c. Our description relies on the existence of a flat central plateau and on the applicability of hydrodynamics.

I. INTRODUCTION

Collisions of highly relativistic nuclei offer the possibility of producing quasimacroscopic systems of dense nucleonic and/or quark-gluon matter at relatively high temperature. In principle this seems to be an interesting way to explore the question of phase transitions between ordinary (confined) matter and (unconfined) quark-gluon plasma. It is also of interest to historians of the early universe. At some early epoch, of order 10^{-6} sec after the big bang, the conditions in the universe were probably rather similar.

On the other hand, interpretation of these complex collisions poses a major problem. What are the experimental signatures and how can one deduce what is going on? Is there information which unambiguously teaches us about the state of the matter formed during and immediately after the collision?

All these problems are under active investigation nowadays.¹ There seems to be a consensus that enough initial kinetic energy is converted into heat so that quark-gluon plasma is created. Less understood is the question of how the system evolves. Furthermore, most (but not all) of the work has concentrated on the system of leading particles which carries the baryon number of the incident nucleus. This system is especially interesting since it is essentially compressed nuclear matter and carries with it not only a heritage of nuclear physics but also of nuclear astrophysics, e.g., the question of neutron-star composition. However, the remaining phase space, the so-called central rapidity region, is of interest in its own right.² It may well be that this region of phase space is easier to study experimentally.

It is our purpose in this paper to sketch out a picture of the space-time evolution³ of the collision process in this “central” region of phase space. We

shall treat the problem in the context of the Landau hydrodynamic model,⁴ but with a different initial boundary condition. We shall assume that at sufficiently high energy there is a “central-plateau” structure for the particle production as a function of the rapidity variable, be it in nucleon-nucleon, nucleon-nucleus, or nucleus-nucleus collisions. The essence of this assumption is the assertion that the space-time evolution of the system looks essentially the same in all center-of-mass-like frames, i.e., in all frames where the emergent excited nuclei are, shortly after the collision, highly Lorentz-contracted pancakes receding in opposite direction from the collision point at the speed of light.

This assertion implies a symmetry property of the system. We will impose this symmetry as an initial condition. However, the hydrodynamic equations respect the symmetry as well. This leads to simple solutions of the hydrodynamic equations.⁵ In particular, for central collisions of large nuclei, the fluid expansion near the collision axis is longitudinal and homogeneous. The fluid midway between the receding pancakes remains at rest, while the fluid a longitudinal distance z from that midpoint moves with longitudinal velocity z/t , where t is the time elapsed since the pancakes collided. This picture is modified at large transverse distances, comparable to the nuclear radii. In that region there will be a rarefaction front moving inward at the velocity of sound of the fluid. For transverse distances larger than the rarefaction front, the fluid will expand radially outward, cooling more rapidly than the fluid in the interior.

The initial energy density produced in the collision is very roughly estimated to be ~ 3 GeV/fm³ with an uncertainty of at least a factor of 3 in either direction. The estimate is based simply on the energy release per unit of rapidity in nucleon-nucleon collisions. This energy density (and consequent en-

A Long Time Ago (1985)

- Miklos Gyulassy and Pawel Danielewicz:

▶ *Dissipative Phenomena
In Quark-Gluon Plasmas*
P. Danielewicz, M. Gyulassy
Phys.Rev. D31, 53, 1985.



noted restrictions on smallest allowed η :

- Most restrictive:
 - $\lambda > \hbar / \langle p \rangle \Rightarrow \eta > \sim n / 3$
 - But recall $s = 3.6 n$ for the quanta they were considering
 - $\Rightarrow \eta/s > 1 / (3.6 \times 3) \sim \hbar / (4 \pi)$
 $\sim 0.1 \hbar$

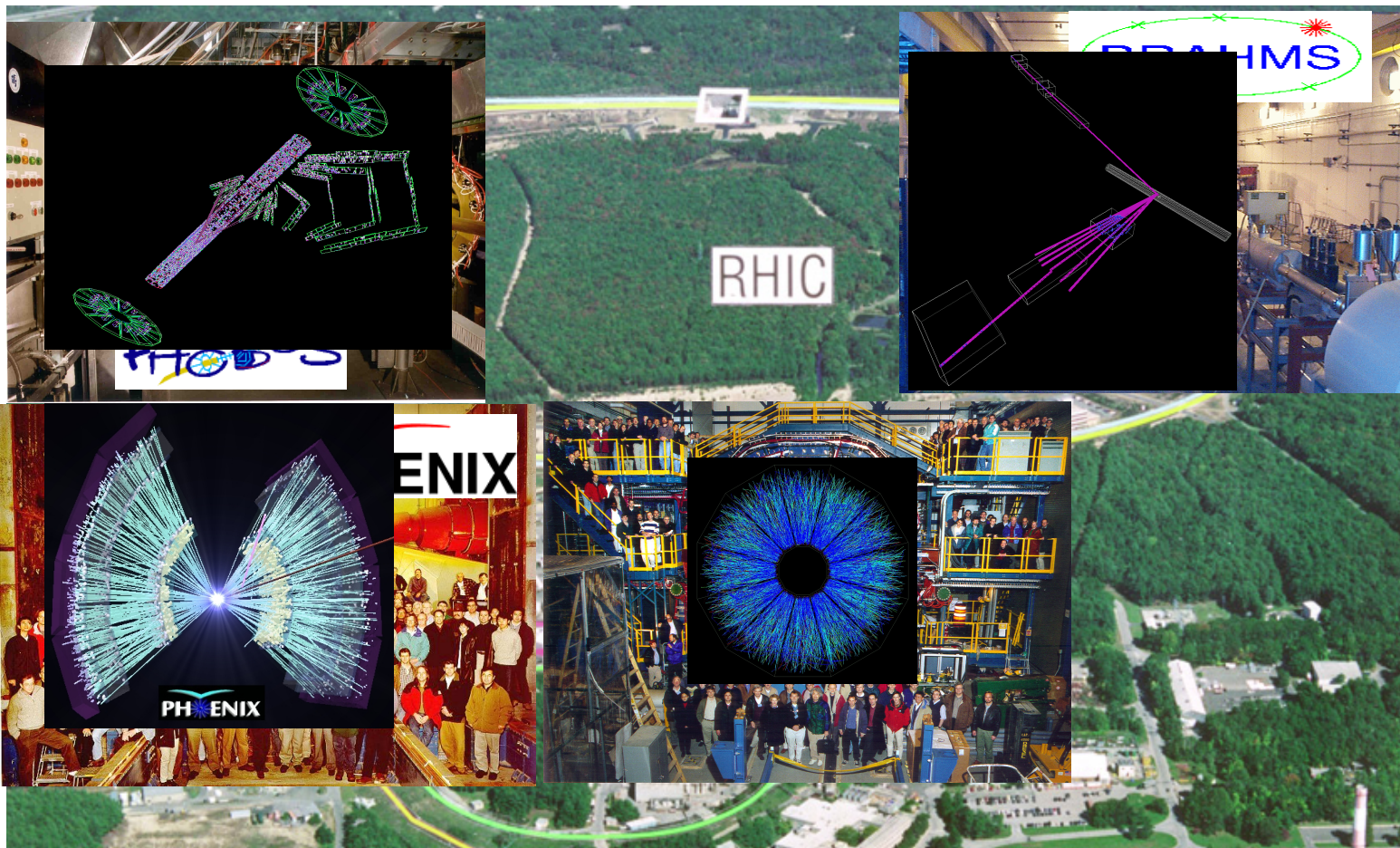
Before estimating λ_i via Eq. (3.2) we note several physical constraints on λ_i . First, the uncertainty principle implies that quanta transporting typical momenta $\langle p \rangle$ cannot be localized to distances smaller than $\langle p \rangle^{-1}$. Hence, it is meaningless to speak about mean free paths smaller than $\langle p \rangle^{-1}$. Requiring $\lambda_i \geq \langle p \rangle_i^{-1}$ leads to the lower bound

$$\eta \geq \frac{1}{3} n, \quad (3.3)$$

where $n = \sum n_i$ is the total density of quanta. What seems amazing about (3.3) is that it is independent of dynamical details. There is a finite viscosity regardless of how large is the free-space cross section between the quanta. See Refs. 21 and 22 for examples illustrating how the thermalization rate of many-body systems is limited by the uncertainty principle.

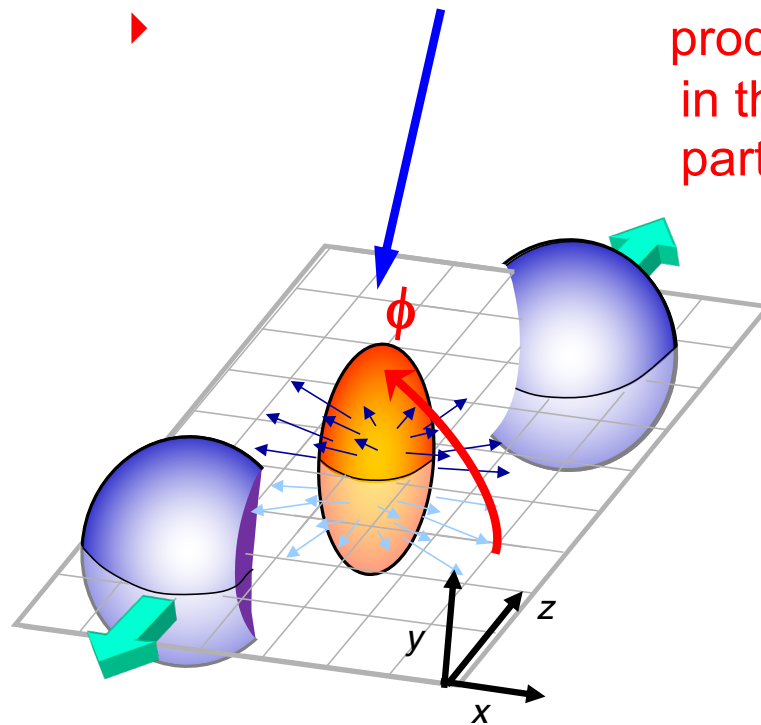
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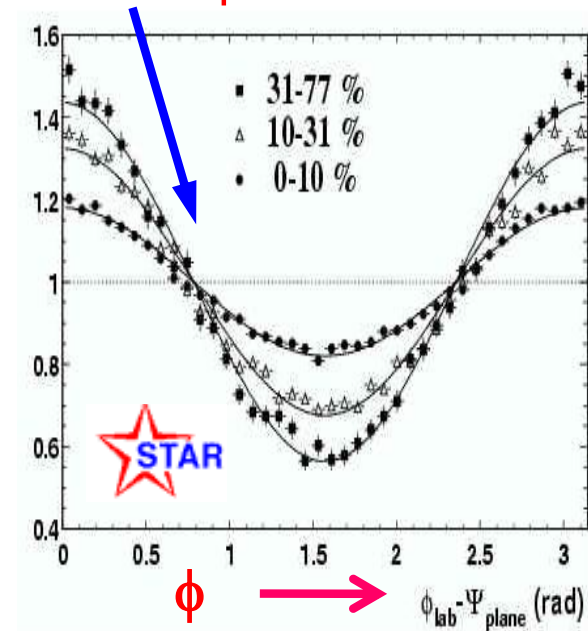


Bulk Motion Is *Hydrodynamic*

- Is the produced matter strongly coupled?
 - ▶ The initial state azimuthal asymmetry



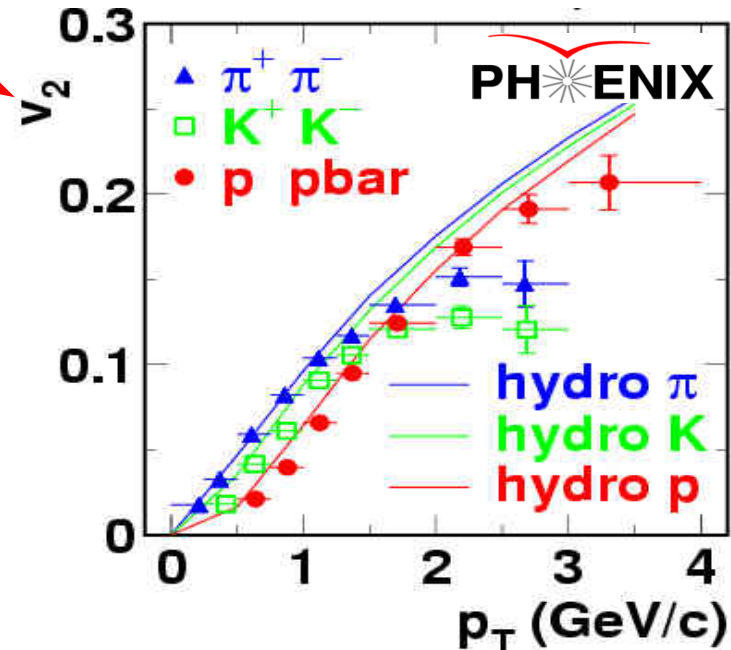
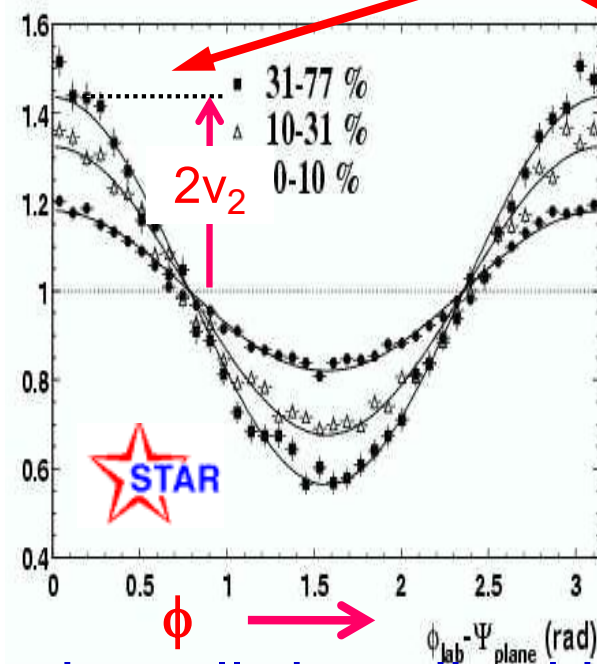
produces a **strong** signal
in the final state
particle emission pattern



Bulk Motion Is *Hydrodynamic*

- The final state azimuthal asymmetry

$$dn/d\phi \sim 1 + 2 v_2(p_T) \cos (2 \phi) + \dots$$



is well-described by \sim *ideal* hydrodynamics

Landau on Viscosity

- 1) Use of hydro relies on $R/\lambda \gg 1$
- 2) Negligible viscosity η equivalent to large Reynolds number $Re \equiv \rho VR / \eta \gg 1$

$$\rho VR / \eta \sim V R / v_{th} \lambda$$

but for a *relativistic* system

$$V \sim v_{th}$$

so

$$Re \gg 1 \Rightarrow R / \lambda \gg 1 ; \text{ see \#1}$$

666

(4) Fermi calculations in collisions. I momentum collisions a obtained.

Fermi's study of these invo

The assu mined by t collision is particles ar of particles the initial i with the a leave the

In reality only when move away calculated energy dist with the th on collision i.e. to a di This meanu derably ex

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Qualitat

(1) Whe is released verse direc

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and statistical equilibrium is set up.

(2) The second stage of the collision consists in the expansion of the system. Here the hydrodynamic approach must be used, and the expansion may be regarded as the motion of an ideal fluid (zero viscosity and zero thermal con-

† The conditions of applicability of thermodynamics and hydrodynamics are comprised in the requirement $l/L < 1$, where l is the "mean free path" and L the least dimension of the system.

ARTICLES 667

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(2.1)

a particle in

(2.2)

(2.3)

† This may be made clear by the following qualitative arguments. If viscosity and thermal conductivity are to be negligible, the Reynolds number $L V / l v$ must be much greater than unity. Here L is the least dimension of the system, V the "macroscopic" velocity, v the "molecular" velocity and l the mean free path. Since V and v are of the order of c , the condition $R \gg 1$ corresponds to $l/L < 1$.

How Close to Ideal ??

- In 2003-4 a new estimate (bound?) appeared from the AdS/CFT correspondence in string theory (!):
 - ▶ *A Viscosity Bound Conjecture*,
P. Kovtun, D.T. Son, A.O. Starinets,
[hep-th/0405231](http://arxiv.org/abs/hep-th/0405231)

$$\frac{\eta}{s} \geq \frac{\hbar}{4\pi} \sim 0.08\hbar$$

in a rigorous calculation with no (apparent) appeal to the uncertainty principle.

From Discovery to Precision(?) - Forlorn Hope in 2004 . . .

- From a talk at the May 15, 2004 RBRC Workshop

“New Discoveries at RHIC”

PHENIX

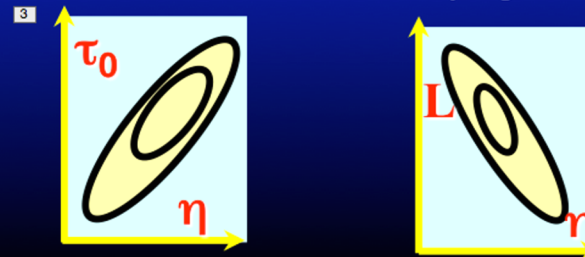
On Estimating Errors

- ~All of data analysis effort is expended on understanding systematic errors:

- Example taken from (required) Analysis Note prior to release of even Preliminary Data

	p_T indep.	2 GeV	6 GeV	type
peak extraction	5.0%(5.0%)			A
geometric acc.		3.0%(3.0%)	2.0%(2.0%)	B
π^0 reconstr. eff.		5.0%(5.0%)	5.0%(5.0%)	B
energy scale		4.0%(4.0%)	9.0%(9.0%)	B
Conversion corr.	3.0%(3.0%)			C
Total error		9.1%(9.1%)	12%(12%)	

- Would like to see this (and more) from those theory analyses dedicated to extraction of physical parameters



2005 – “Perfect Liquid” Discovery

Brookhaven National Laboratory search DOE ENERGY

RHIC

Brookhaven National Laboratory's Relativistic Heavy Ion Collider

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Contacts: [Karen McNulty Walsh](#), (831) 344-8350 or [Peter Gancz](#), (831) 344-3174

RHIC Scientists Serve Up 'Perfect' Liquid

New state of matter more remarkable than predicted — raising many new questions

Monday, April 18, 2005

TAMPA, FL — The four detector groups conducting research at the [Relativistic Heavy Ion Collider \(RHIC\)](#) — a giant atom “smasher” located at the U.S. Department of Energy’s Brookhaven National Laboratory — say they’ve created a new state of hot, dense matter out of the quarks and gluons that are the basic particles of atomic nuclei, but it is a state quite different and even more remarkable than had been predicted. In [peer-reviewed papers](#) summarizing the first three years of RHIC findings, the scientists say that instead of behaving like a gas of free quarks and gluons, as was expected, the matter created in RHIC’s heavy ion collisions appears to be more like a liquid.

“Once again, the physics research sponsored by the Department of Energy is producing historic results,” said Secretary of Energy Samuel Bodman, a trained chemical engineer. “The DOE is the principal federal funder of basic research in the physical sciences, including nuclear and high-energy physics. With today’s announcement we see that investment paying off.”


“The truly stunning finding at RHIC that the new state of matter created in the collisions of gold ions is more like a liquid than a gas gives us a profound insight into the earliest moments of the universe,” said Dr. Raymond L. Orbach, Director of the DOE Office of Science.

Also of great interest to many following progress at RHIC is the emerging connection between the collider’s results and calculations using the methods of string theory, an approach that attempts to explain fundamental properties of the universe using 10 dimensions instead of the usual three spatial dimensions plus time.


“The possibility of a connection between string theory and RHIC collisions is unexpected and exhilarating,” Dr. Orbach said. “String theory seeks to unify the two great intellectual achievements of twentieth-century physics, general relativity and quantum mechanics, and it may well have a profound impact on the physics of the twenty-first century.”

The papers, which the four RHIC collaborations ([BRAHMS](#), [PHENIX](#), [PHOBOS](#), and [STAR](#)) have been working on for nearly a year, will be published simultaneously by the journal *Nuclear Physics A*, and will also be compiled in a [special Brookhaven report](#), the Lab announced at the April 2005 meeting of the American Physical Society in Tampa, Florida.

These summaries indicate that some of the observations at RHIC fit with the theoretical predictions for a quark-gluon plasma (QGP), the type of matter postulated to have existed just microseconds after the Big Bang. Indeed, many theorists have concluded that RHIC has already demonstrated the creation of quark-gluon plasma. However, all four collaborations note that there are discrepancies between the experimental data and early theoretical predictions based on simple models of quark-gluon plasma formation.



Secretary of Energy Samuel Bodman



Dr. Raymond L. Orbach

Other RHIC News

[Energy Secretary Moniz Announces 2014 Ernest Orlando Lawrence Award Winners](#)

[U.S.-CERN Agreement Paves Way for New Era of Scientific Discovery](#)

[Sengry Balomastnykh Receives Particle Accelerator Science & Technology Award](#)

[Into the Depths of the Electromagnetic Spectrum](#)


[Giant Electromagnet Arrives at Brookhaven Lab to Map Melted Matter](#)

[Explorations of Quarks and Gluons in Scientific American](#)

[Relativistic Heavy Ion Collider Smashes Record for Polarized Proton Luminosity at 200 GeV Collision Energy](#)

[A Tale of Two Colliders, One Thesis, Two Awards—and a Physics Mystery](#)

“We know that we’ve reached the temperature [up to 150,000 times hotter than the center of the sun] and energy density [energy per unit volume] predicted to be necessary for forming such a plasma,” said Sam Aronson, Brookhaven’s Associate Laboratory Director for High Energy and Nuclear Physics. But analysis of RHIC data from the start of operations in June 2000 through the 2003 physics run reveals that the matter formed in RHIC’s head-on collisions of gold ions is more like a liquid than a gas.



Sam Aronson

That evidence comes from measurements of unexpected patterns in the trajectories taken by the thousands of particles produced in individual collisions. These measurements indicate that the primordial particles produced in the collisions tend to move collectively in response to variations of pressure across the volume formed by the colliding nuclei. Scientists refer to this phenomenon as “flow,” since it is analogous to the properties of fluid motion.

However, unlike ordinary liquids, in which individual molecules move about randomly, the hot matter formed at RHIC seems to move in a pattern that exhibits a high degree of coordination among the particles — somewhat like a school of fish that responds as one entity while moving through a changing environment.

“This is fluid motion that is nearly ‘perfect,’” Aronson said, meaning it can be explained by equations of hydrodynamics. These equations were developed to describe theoretically “perfect” fluids — those with extremely low viscosity and the ability to reach thermal equilibrium very rapidly due to the high degree of interaction among the particles. While RHIC scientists don’t have a direct measure of viscosity, they can infer from the flow pattern that, qualitatively, the viscosity is very low, approaching the quantum mechanical limit.

Together, these facts present a compelling case: “In fact, the degree of collective interaction, rapid thermalization, and extremely low viscosity of the matter being formed at RHIC make this the most nearly perfect liquid ever observed,” Aronson said.

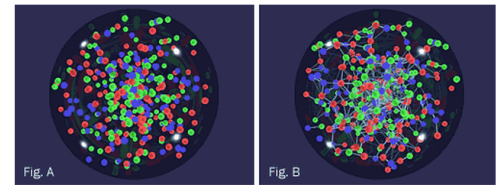


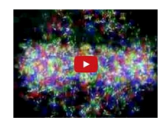
Fig. A Fig. B

ENLARGE These images contrast the degree of interaction and collective motion, or “flow,” among quarks in the predicted gaseous quark-gluon plasma state (Figure A, see [video animation](#)) vs. the liquid state that has been observed in gold-gold collisions at RHIC (Figure B, see [video animation](#)). The green “force lines” and collective motion (visible on the animated version only) show the much higher degree of interaction and flow among the quarks in what is now being described as a nearly “perfect” liquid. (Click images for larger version.) [An updated video comparing the expected gas with the observed “perfect” liquid is available.](#)

In results [reported earlier](#), other measurements at RHIC have shown “jets” of high-energy quarks and gluons being dramatically slowed down as they traverse the hot fireball produced in the collisions. This “jet quenching” demonstrates that the energy density in this new form of matter is extraordinarily high — much higher than can be explained by a medium consisting of ordinary nuclear matter.

“The current findings don’t rule out the possibility that this new state of matter is in fact a form of the quark-gluon plasma, just different from what had been theorized,” Aronson said. Many scientists believe this to be the case, and detailed measurements are now under way at RHIC to resolve this question.

Theoretical physicists, whose standard calculations cannot incorporate the strong coupling observed between the quarks and gluons at RHIC, are also revisiting some of their early models and predictions. To try to address these issues, they are running massive numerical simulations on some of the world’s most powerful computers. Others are attempting to incorporate quantitative measures of viscosity into the equations of motion for fluid moving at nearly the speed of light. One subset of calculations uses the methods of string theory to predict the viscosity of the liquid being created at RHIC and to explain some of the other surprising findings. Such studies will provide a more quantitative understanding of how “nearly perfect” the liquid is.

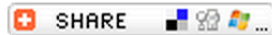


[See an updated version of the “perfect” liquid animation.](#)

The unexpected findings also introduce a wide range of opportunity for new scientific discovery regarding the properties of matter at extremes of temperature and density previously inaccessible in a laboratory.

“The finding of a nearly perfect liquid in a laboratory experiment recreating the conditions believed to have existed a few

RHIC Scientists Serve Up 'Perfect' Liquid



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... the scientists say that instead of behaving like a gas of free quarks and gluons, as was expected, the matter created in RHIC's heavy ion collisions appears to be more like a *liquid*.

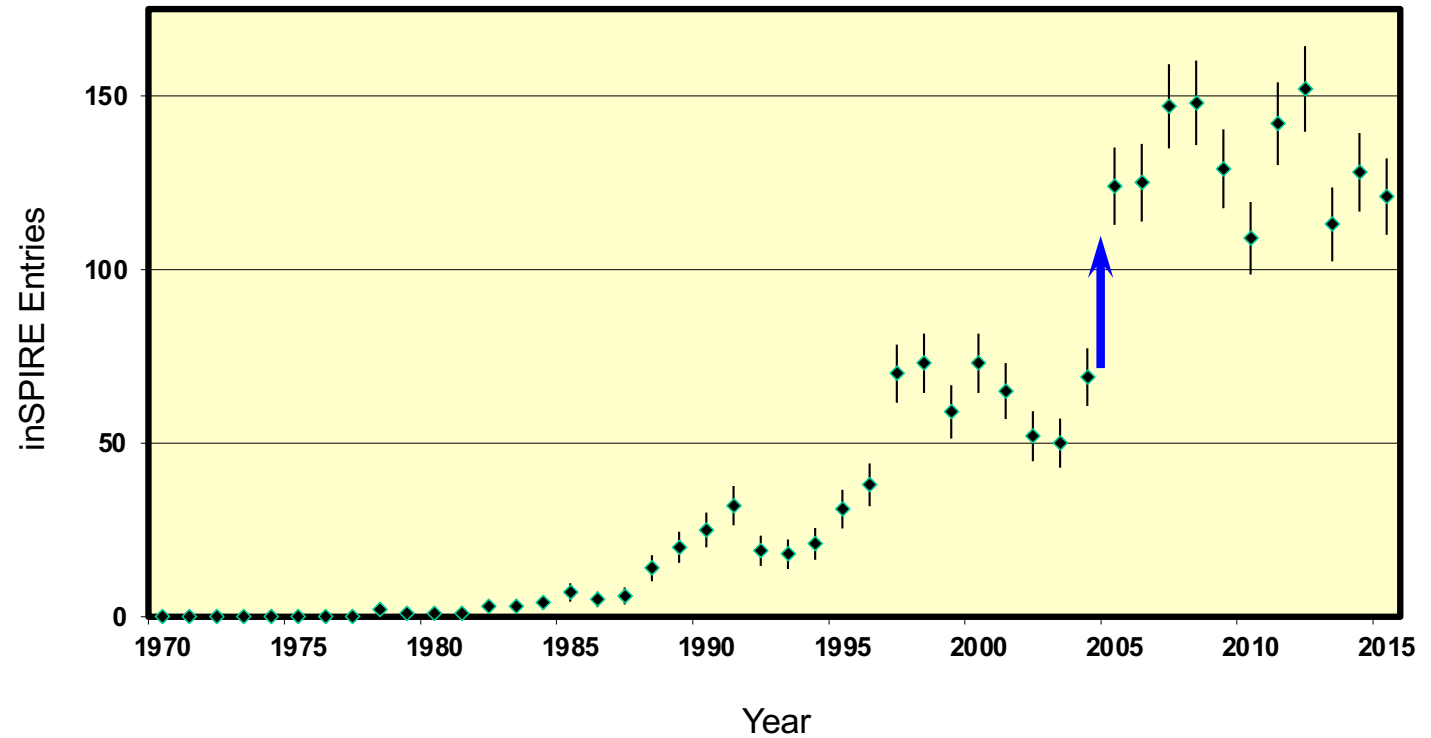
Paradigm Shift

- inSPIRE query for all publications with

QGP, SQGP, QUARK-GLUON PLASMA, QCD PLASMA,
STRONGLY COUPLED PLASMA,
STRONGLY-COUPLED PLASMA,
RHIC PLASMA

in their title:

"Quark Gluon Plasma" Publications Versus Time



From the Press Release

- *Once again, the physics research sponsored by the Department of Energy is producing historic results*
 - ▶ Samuel Bodman, Secretary of Energy
- *In fact, the degree of collective interaction, rapid thermalization, and extremely low viscosity of the matter being formed at RHIC make this the most nearly perfect liquid ever observed...The current findings don't rule out the possibility that this new state of matter is in fact a form of the quark-gluon plasma, just different from what had been theorized.*
 - ▶ Sam Aronson, Associate Laboratory Director for High Energy and Nuclear Physics
- *The possibility of a connection between string theory and RHIC collisions is unexpected and exhilarating.*
 - ▶ Raymond L. Orbach, Director of the DOE Office of Science

2011 - A Nice Surprise

13

Jul 2011

THE TOPCITED HEP PAPER OF ALL TIME.

by INSPIRE

For as long as the annual [topcited papers](#) lists have been around, the all-time champion has been Weinberg's "[A model of leptons](#)", the 1967 paper that laid the foundation stone for the Standard Model. 30 years later, in November of 1997, the paper [The Large N limit of superconformal field theories and supergravity](#) by Maldacena appeared that established a connection between string theory and quantum field theory. It immediately set off a revolution in HEP and was the most highly cited paper ever since. Remarkably, its [highest citation count](#) was in 2010, where it received over 1,000 citations in a single year! One reason for this is the [heavy ion results](#) from Brookhaven that drew people to conclude that, based on Maldacena's work, the quark gluon plasma can be modeled using string theory techniques.

Posted in: [References and Citations](#)

First RHIC Experimental Paper Quantifying η/s



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Energy Loss and Flow of Heavy Quarks in Au+Au Collisions at $s(\text{NN})^{1/2} = 200\text{-GeV}$

PHENIX Collaboration (A. Adare (Colorado U.) *et al.*) [Show all 421 authors](#)

Nov 2006 - 6 pages

Phys.Rev.Lett. 98 (2007) 172301

DOI: [10.1103/PhysRevLett.98.172301](https://doi.org/10.1103/PhysRevLett.98.172301)

e-Print: [nucl-ex/0611018](#) | [PDF](#)

Experiment: [BNL-RHIC-PHENIX](#)

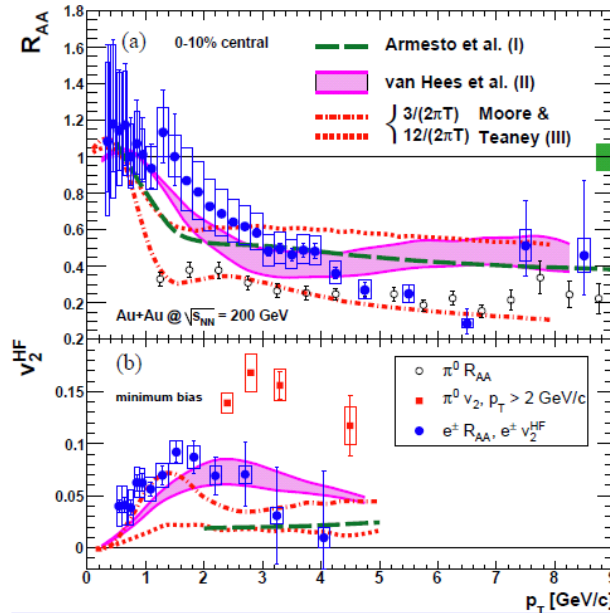
Abstract: The PHENIX experiment at the Relativistic Heavy Ion Collider (RHIC) has measured electrons from heavy flavor (charm and bottom) decays for $0.3 < p_T < 9$ GeV/c at midrapidity ($|\eta| < 0.35$) in Au+Au collisions at $\sqrt{s_{\text{NN}}} = 200$ GeV. The nuclear modification factor R_{AA} relative to p+p collisions shows a strong suppression in central Au+Au collisions, indicating substantial energy loss of heavy quarks in the medium produced at RHIC. A large azimuthal anisotropy, v_2 , with respect to the reaction plane is observed for $0.5 < p_T < 5$ GeV/c indicating non-zero heavy flavor elliptic flow. Both R_{AA} and v_2 show a p_T dependence different from those of neutral pions. A comparison to transport models which simultaneously describe $R_{\text{AA}}(p_T)$ and $v_2(p_T)$ suggests that the viscosity to entropy density ratio is close to the conjectured quantum lower bound, i.e., near a perfect fluid.

PACS: [25.75.Dw](#)

Keyword(s): INSPIRE: [nucleus nucleus: colliding beams](#) | [scattering: heavy ion](#) | [gold](#) | [charm](#) | [bottom](#) | [quark: hadroproduction](#) | [quark: decay](#) | [electron: yield](#) | [elliptic flow](#) | [multiple scattering](#) | [quark: energy loss](#) | [nuclear matter: effect](#) | [viscosity](#) | [entropy](#) | [model: fluid](#) | [PHENIX](#) | [experimental results](#) | [Brookhaven RHIC Coll](#) | [200 GeV-cms/nucleon](#)

Record created 2006-11-13, last modified 2012-10-24

2006 - The Real Surprise



● Heavy quark

▶ Energy loss

See talk by Jamie Nagle

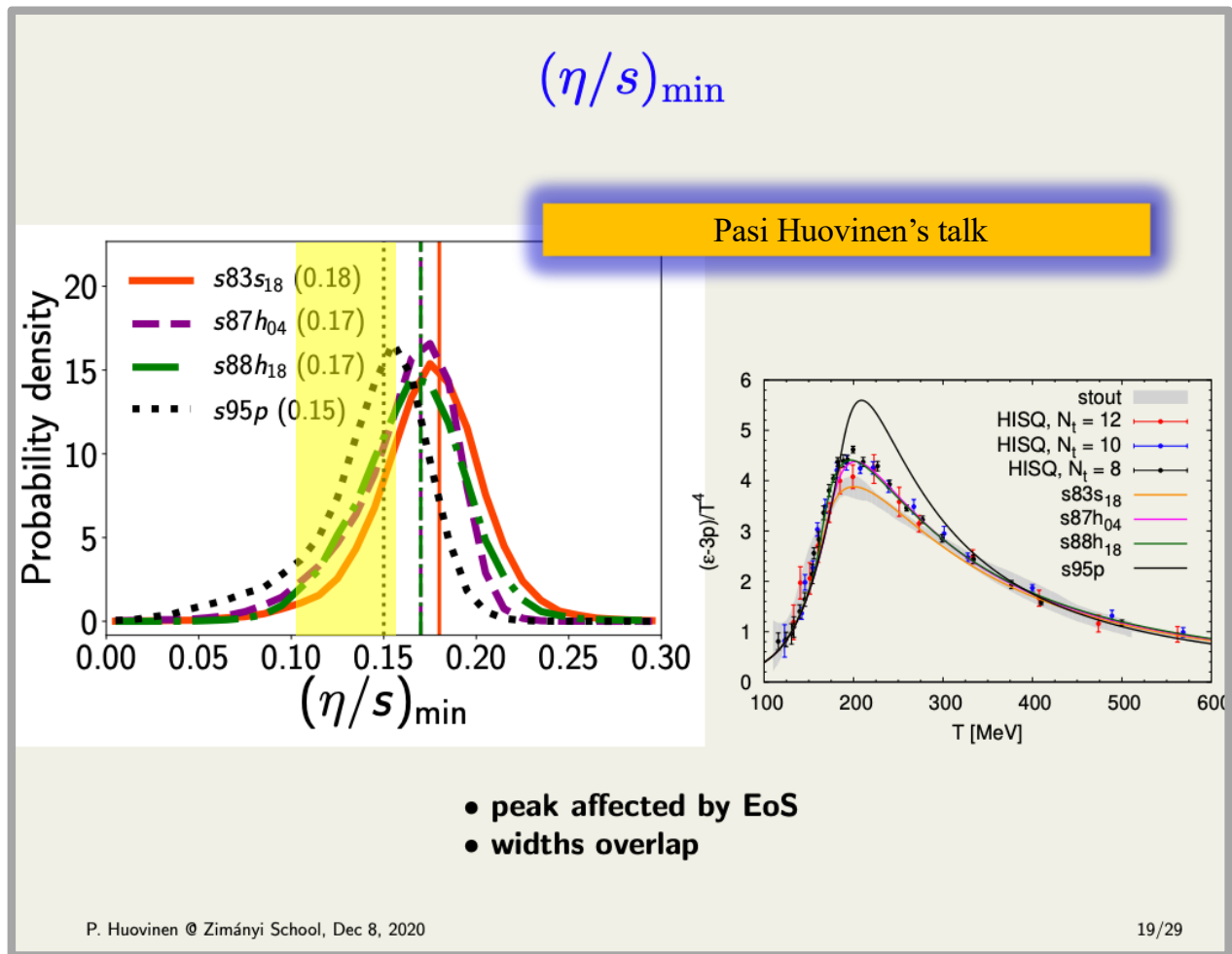
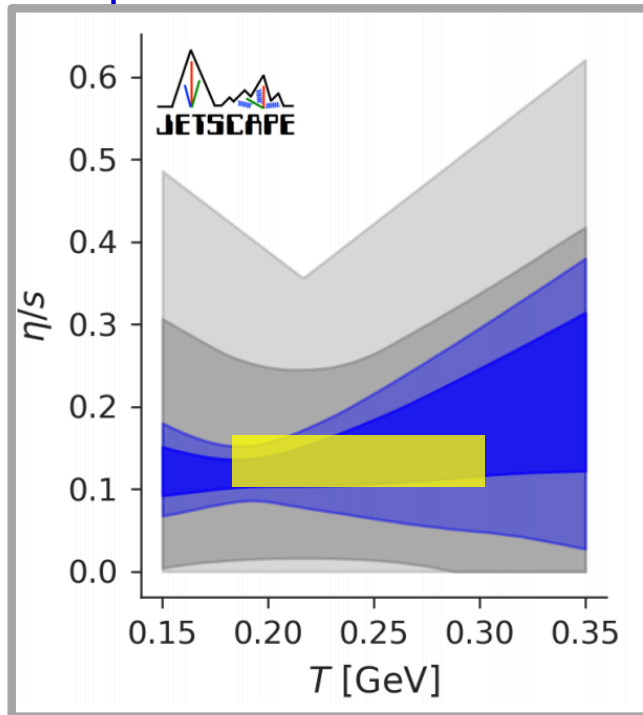
▶ Flow

along with

matter's diffusion coefficient D . Using the observation [32] that $D \approx 6 \times \eta / (\epsilon + p)$ with $\epsilon + p = Ts$ at $\mu_B = 0$ provides an estimate for the viscosity to entropy ratio $\eta/s \approx (\frac{4}{3} - 2)/4\pi$, intriguingly close to the conjectured quantum lower bound $1/4\pi$ [33]. This result is consistent with

2020 Interjection

- That estimate remains consistent with latest and greatest Bayesian extractions
- Examples at this conference



Circa 2006 - The Difficulties with Hydro

- Seemingly straightforward :

$$\partial_{\nu} T^{\mu\nu} = 0$$

Circa 2006 - The Difficulties with Hydro

- Seemingly insurmountable:

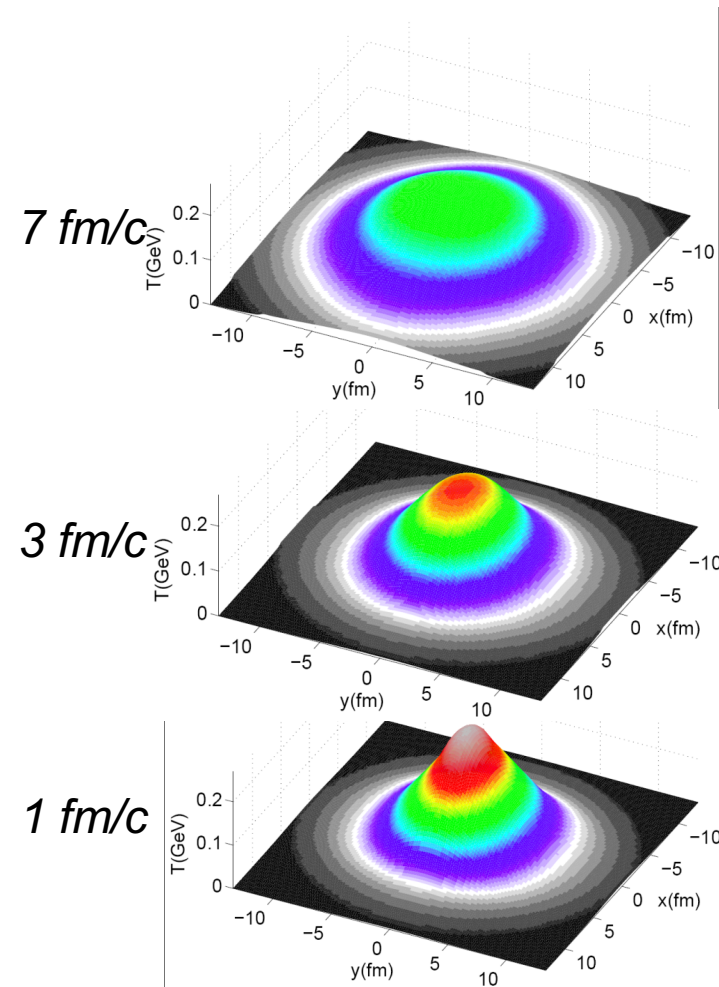
$$\begin{aligned}
 \tau_{\Pi} \dot{\Pi} + \Pi &= \Pi_{\text{NS}} + \tau_{\Pi q} q \cdot \dot{u} - \ell_{\Pi q} \partial \cdot q - \zeta \hat{\delta}_0 \Pi \theta \\
 &\quad + \lambda_{\Pi q} q \cdot \nabla \alpha + \lambda_{\Pi \pi} \pi^{\mu\nu} \sigma_{\mu\nu} \\
 \tau_q \Delta^{\mu\nu} \dot{q}_{\nu} + q^{\mu} &= q_{\text{NS}}^{\mu} - \tau_{q\Pi} \Pi \dot{u}^{\mu} - \tau_{q\pi} \pi^{\mu\nu} \dot{u}_{\nu} \\
 &\quad + \ell_{q\Pi} \nabla^{\mu} \Pi - \ell_{q\pi} \Delta^{\mu\nu} \partial^{\lambda} \pi_{\nu\lambda} + \tau_q \omega^{\mu\nu} q_{\nu} - \frac{\kappa}{\beta} \hat{\delta}_1 q^{\mu} \theta \\
 &\quad - \lambda_{qq} \sigma^{\mu\nu} q_{\nu} + \lambda_{q\Pi} \Pi \nabla^{\mu} \alpha + \lambda_{q\pi} \pi^{\mu\nu} \nabla_{\nu} \alpha \\
 \tau_{\pi} \dot{\pi}^{<\mu\nu>} + \pi^{\mu\nu} &= \pi_{\text{NS}}^{\mu\nu} + 2 \tau_{\pi q} q^{<\mu} \dot{u}^{\nu>} \\
 &\quad + 2 \ell_{\pi q} \nabla^{<\mu} q^{\nu>} + 2 \tau_{\pi} \pi_{\lambda}^{<\mu} \omega^{\nu>\lambda} - 2 \eta \hat{\delta}_2 \pi^{\mu\nu} \theta \\
 &\quad - 2 \tau_{\pi} \pi_{\lambda}^{<\mu} \sigma^{\nu>\lambda} - 2 \lambda_{\pi q} q^{<\mu} \nabla^{\nu>} \alpha + 2 \lambda_{\pi\Pi} \Pi \sigma^{\mu\nu}
 \end{aligned}$$

- ▶ Unknown Initial Conditions
- ▶ Eccentricity fluctuations
- ▶ Unknown equation of state
- ▶ Instabilities, acausal effects in relativistic viscous hydro
- ▶ Hadronic rescattering effects
- ▶ Bulk viscosity
- ▶ Numerical viscosity
- ▶ Finite size, core/corona effects

2007-2008: Implementing and Testing

● Examples

- ▶ [P. Romatschke and U. Romatschke, Phys. Rev. Lett. 99:172301, 2007](#)
- ▶ [H. Song and U. Heinz, Phys. Rev. C78, 024902, 2008](#)
- ▶ [M. Luzum and P. Romatschke, Phys. Rev. C78:034915, 2008](#)



2008 - Concordance

- BNL, April 2008:
 - ▶ [Workshop on Viscous Hydrodynamics and Transport Models in Heavy Ion Collisions](#)
 - ▶ [Workshop Summary](#)

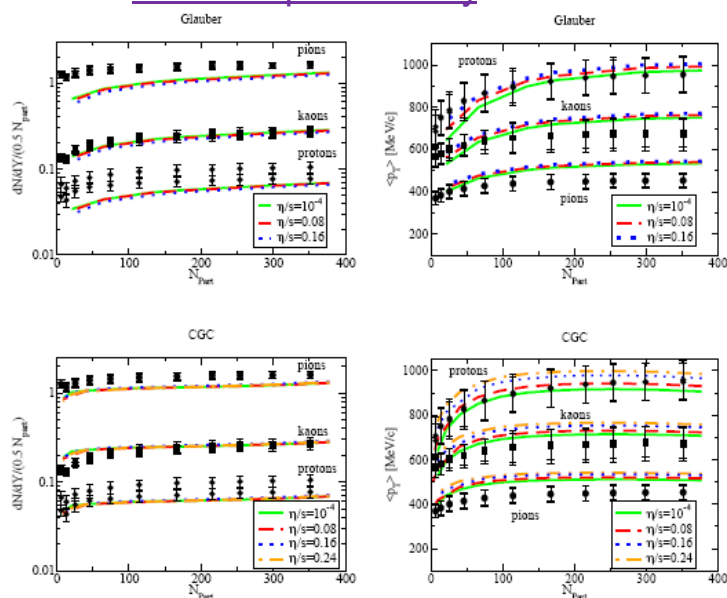


FIG. 7: (Color online) Centrality dependence of total multiplicity dN/dY and $\langle p_T \rangle$ for π^+ , π^- , K^+ , K^- , p and \bar{p} from PHENIX [84] for Au+Au collisions at $\sqrt{s} = 200$ GeV, compared to the viscous hydrodynamic model and various η/s , for Glauber initial conditions (from [22]) and CGC initial conditions. The model parameters used here are $\tau_0 = 1$ fm/c, $\tau_{II} = \theta\eta/s$, $\lambda_1 = 0$, $T_f = 150$ MeV and adjusted T_i (see text for details).

[Luzum and Romatschke, arXiv:0804.4015](#)

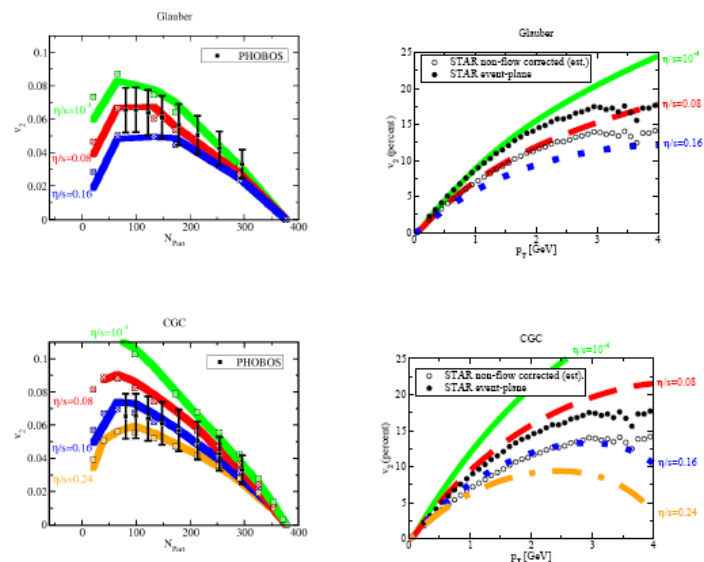
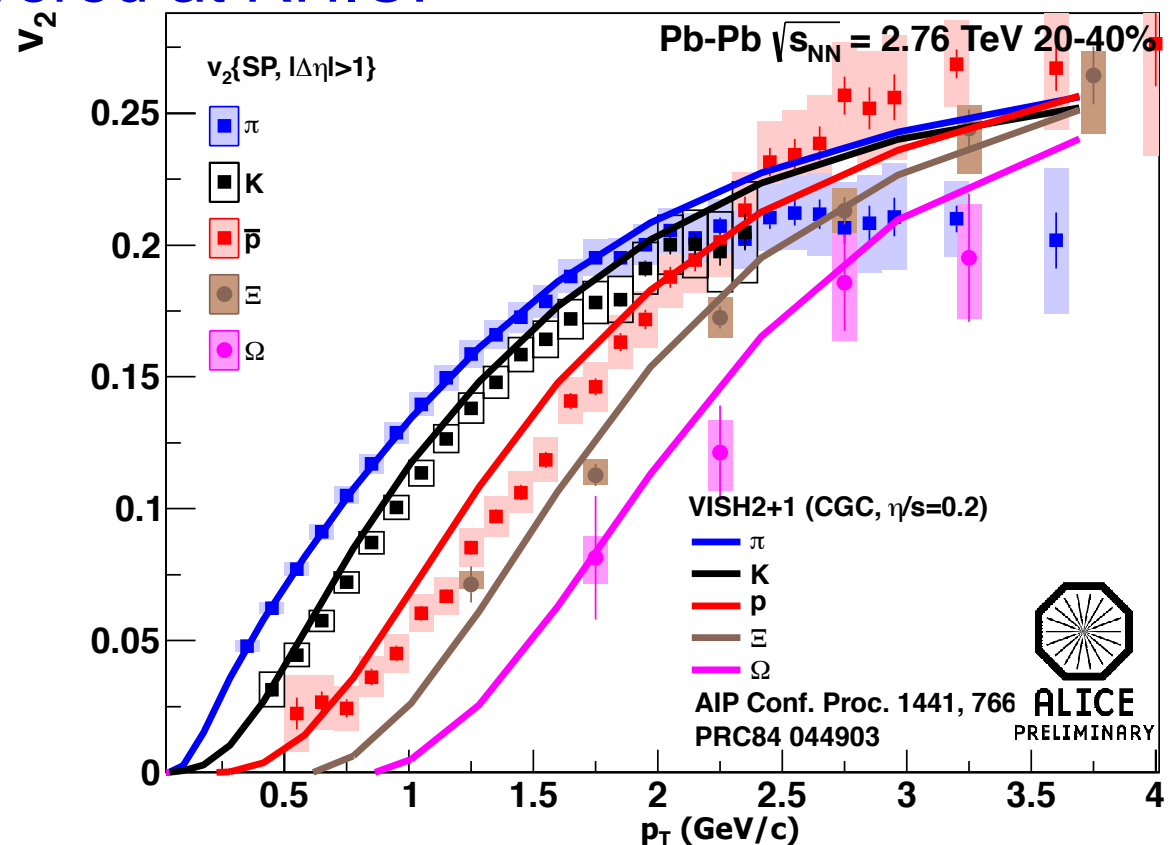


FIG. 8: (Color online) Comparison of hydrodynamic models to experimental data on charged hadron integrated (left) and minimum bias (right) elliptic flow by PHOBOS [85] and STAR [87], respectively. STAR event plane data has been reduced by 20 percent to estimate the removal of non-flow contributions [87, 88]. The line thickness for the hydrodynamic model curves is an estimate of the accumulated numerical error (due to e.g. finite grid spacing). The integrated v_2 coefficient from the hydrodynamic models (full lines) is well reproduced by $\frac{1}{2}e_p$ (dotted); indeed, the difference between the full lines and dots gives an estimate of the systematic uncertainty of the freeze-out prescription.

2010 – First LHC Heavy Ion Run

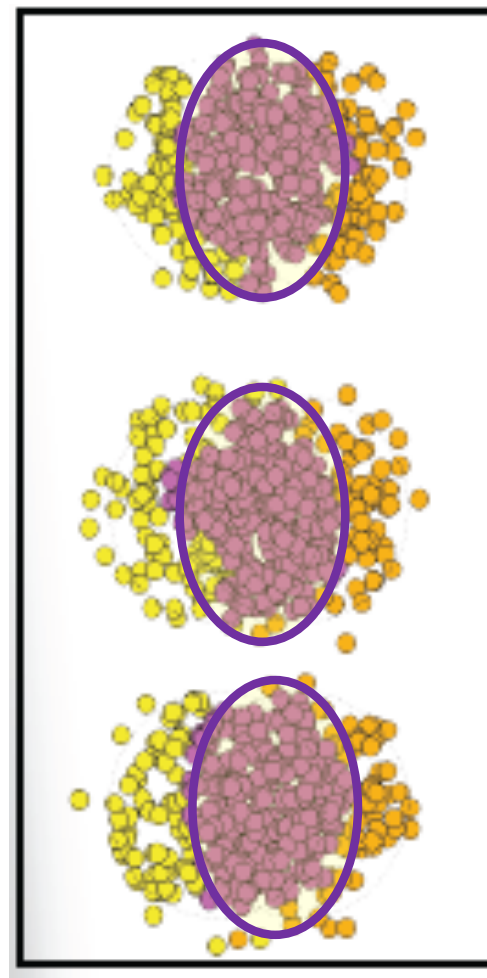
- The matter produced in LHC collisions exhibits the same qualitative features discovered at RHIC:

- ▶ Strong hydrodynamic flow
- ▶ RHIC *and* LHC data well-described by relativistic viscous hydrodynamics
- ▶ “Fine structure” (mass ordering) in hydrodynamic response *predicted* for π , K, p, Ξ , Ω :



2010: The Noise /s The Signal

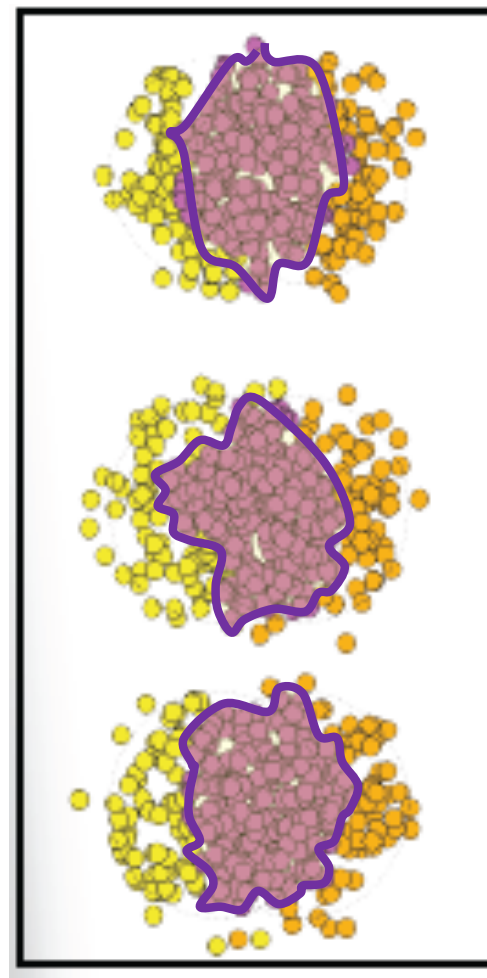
- Importance of higher harmonics
- $dn/d\phi \sim 1 + 2 v_2(p_T) \cos(2\phi) + \dots$



B. Alver and G. Roland, [Phys. Rev. C81, 054905 \(2010\)](#)

2010: The Noise /s The Signal

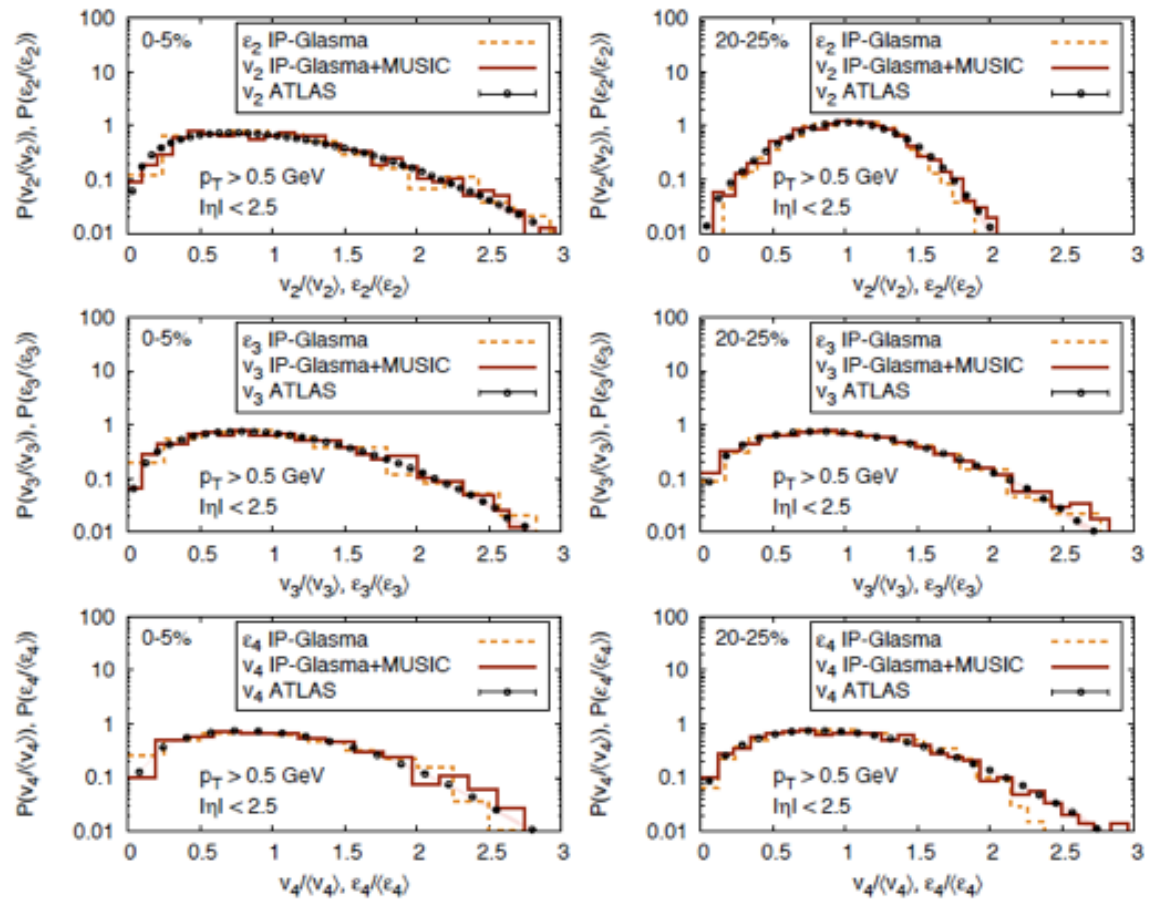
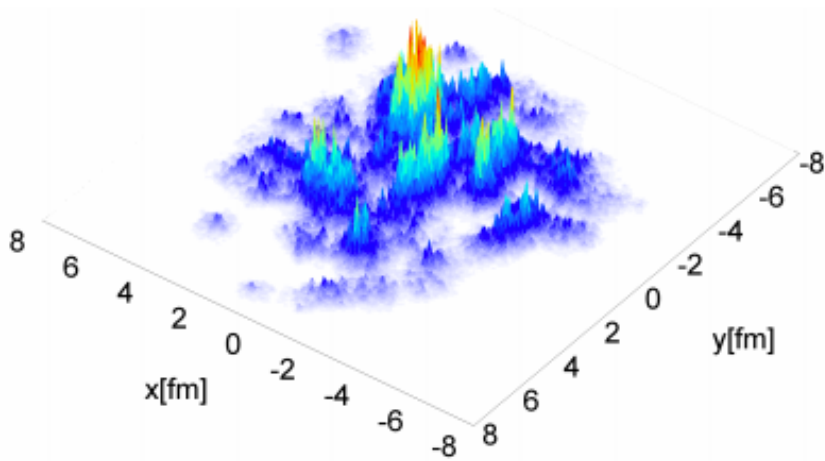
- Importance of higher harmonics
- $dn/d\phi \sim 1 + 2 v_2(p_T) \cos(2\phi) + 2 v_3(p_T) \cos(3\phi) + 2 v_4(p_T) \cos(4\phi) + \dots$
- Fluctuations critical for determining allowed range of η/s .
- ▣ Persistence of “bumps” \rightarrow small η/s !



B. Alver and G. Roland, [Phys. Rev. C81, 054905 \(2010\)](#)

2012 - Measuring and Predicting the Fluctuation Spectrum

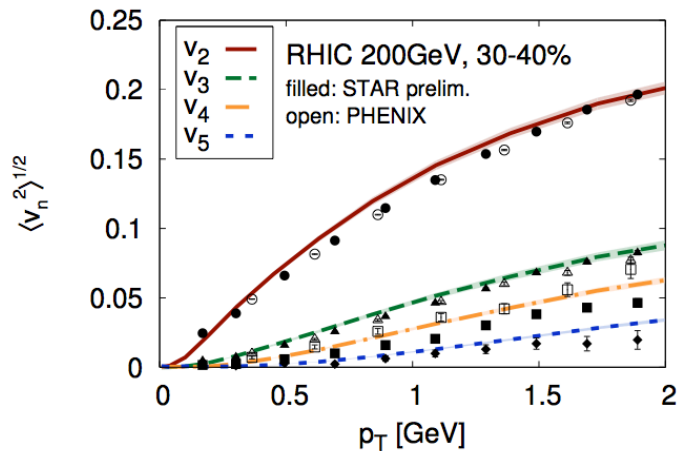
- Treatment of gauge field fluctuations at sub-nucleon scales:



- C. Gale et al., Phys. Rev. Lett. 110, 012302 (2013)
- B. Schenke, P. Tribedy and R. Venugopalan, Phys. Rev. Lett. 108, 252301 (2012)

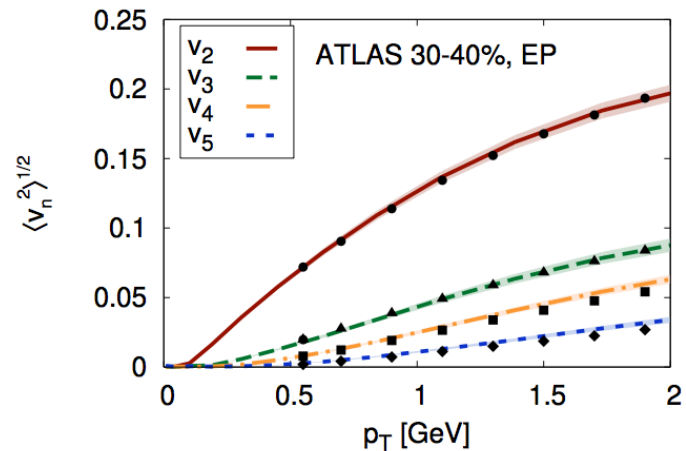
2013: Higher Harmonics Used to Determine η/s

- The *fundamental* matter formed at RHIC and the LHC is within a factor of 3 of KSS bound(!)



$$\eta/s \approx 0.12 \text{ at } \sqrt{s} = 0.2 \text{ TeV}$$

$\approx 1.5 \times$ KSS Bound

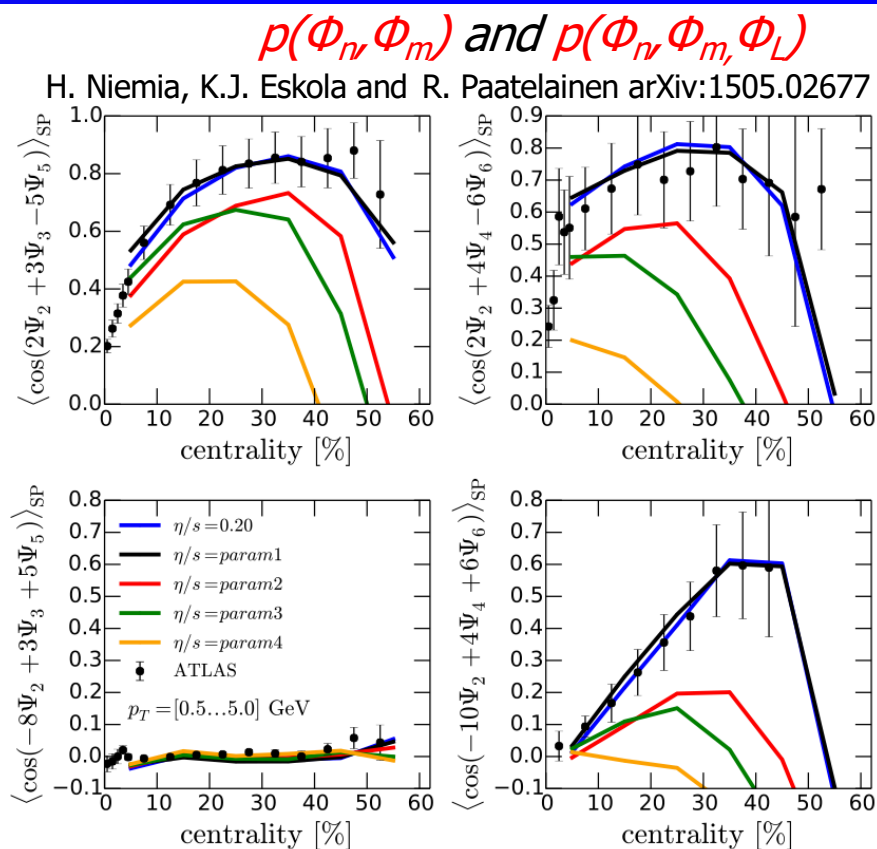


$$\eta/s \approx 0.2 \text{ at } \sqrt{s} = 2.76 \text{ TeV}$$

$\approx 2.5 \times$ KSS Bound

2015: - Then-Current State of the Art

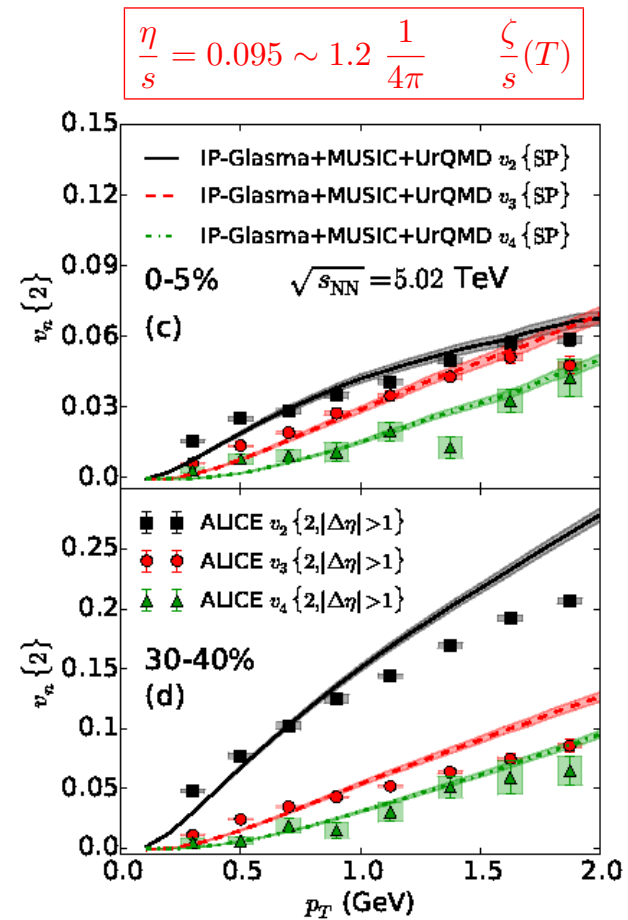
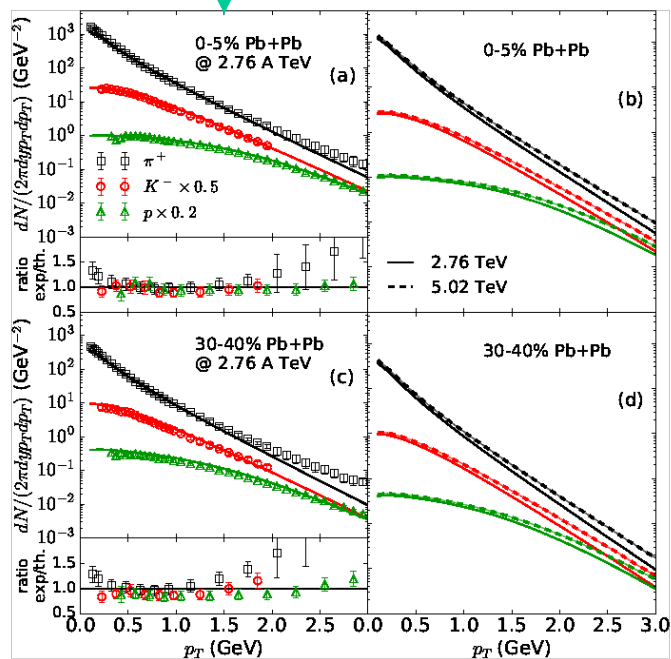
- Event-by-Event flow observables
- Event shape engineering
- Longitudinal fluctuations
- Event-Plane correlations
- Multi-particle observables



From June 9, 2015 Talk by Jiangyong Jia at Workshop on "Quantifying the Properties of the Perfect Fluid"

2017 – Refining Viscosity Estimates

- Describe over a broad range both spectra and v_n 's



S. McDonald et al., [Phys. Rev. C95, 064913 \(2017\)](#)

An A-historical Motivation

- *Small value of η/s :*

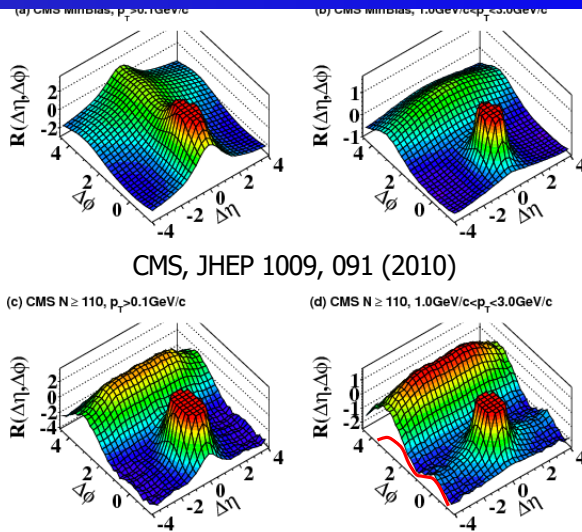
⇒ Persistence of v_n up to $n \sim 4-5$

⇒ Each $(1/n)$ -th *part* of the initial state flows

⇒ Test this by studying small systems

⇒ A-historical: in reality flow in small systems was discovered by experimentalists at the LHC

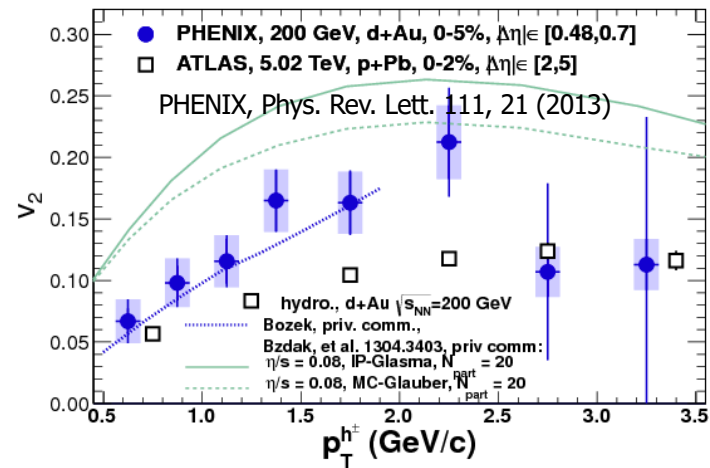
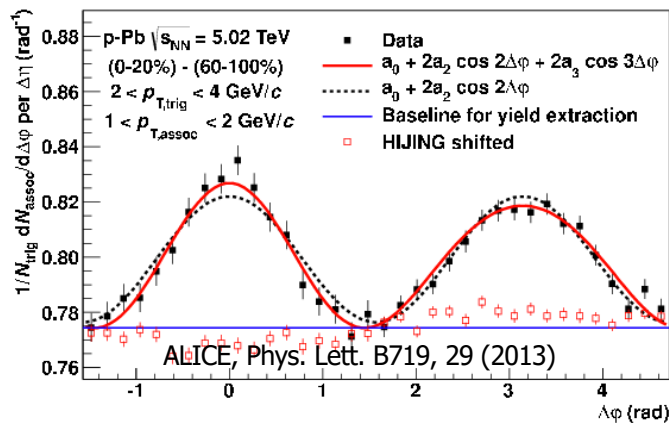
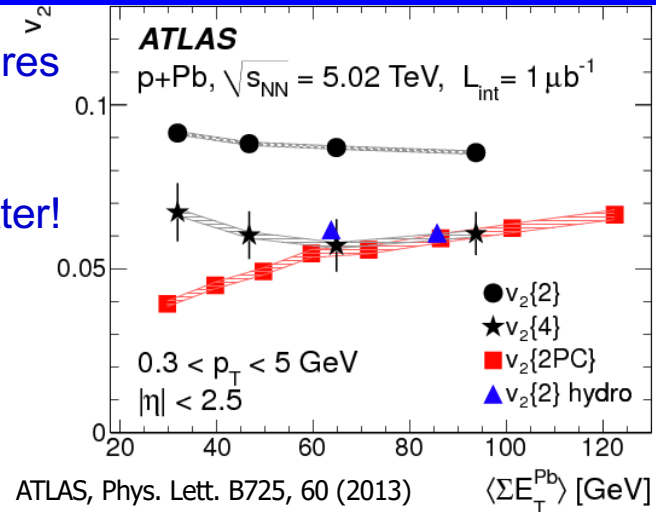
2010-2013: Hydrodynamic Ubiquity – An Embarrassment of Riches



CMS, JHEP 1009, 091 (2010)

Flow signatures
observed
in smallest
flecks of matter!
Collisions of:

- ▶ p+p
- ▶ p+Pb,
- ▶ d+Au



Initial Reaction – How Can This Possibly Be Hydrodynamic ?

- Any “reasonable” estimate suggests mean free paths comparable to system size:

$$\lambda \sim 2 \left(\frac{T_0}{T} \right)^3 \left(\frac{\sigma_1}{\sigma} \right) \quad \text{with } T_0 = 200 \text{ Mev and } \sigma_1 = 1 \text{ mb}$$

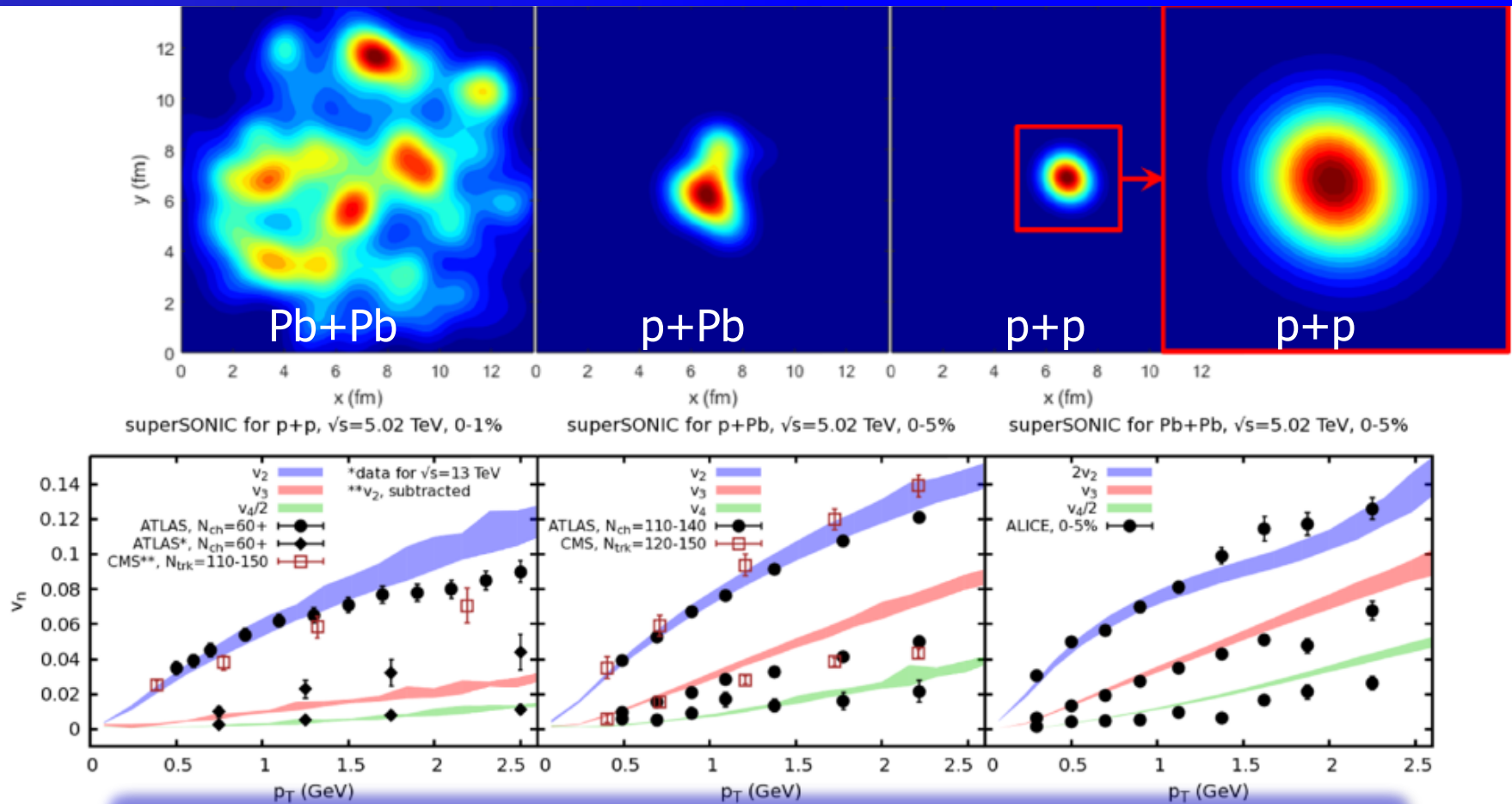
- *Eppure scorre*
(*And yet it flows*)

2015 - Challenges to the Paradigm?

Slide from WAZ talk
at Brookhaven Forum
“Celebrating Ten Years of
the Perfect Liquid”,
June 2015

- Does the appearance of “hydro-like” features in small “perturbative” systems call into question the “standard model” of heavy ion collisions?
 - ▶ We should keep an open mind....
 - ▶ While biasing our Bayesian prior on the enormous descriptive power of the current formalism
- My guess for small systems
 - {Confinement+Strong Fields+Color Recombination+ $\partial_\nu T^{\mu\nu} = 0$ }
 - will look a lot like hydro (see Fermi+Landau)
 - (Vaguely related: “Canonical Typicality”, S. Goldstein *et al.* Phys. Rev. Lett. **96**, 050403, 2006)
 - ▶ *Beware the tyranny of asymptotic freedom!*

2017 - One Fluid to Rule Them All



R. Weller and P. Romatschke, *One Fluid to Rule Them All*, Phys. Lett. **B774**, 561 (2017)

2014 – Geometry Scan

PHYSICAL REVIEW LETTERS

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Exploiting Intrinsic Triangular Geometry in Relativistic $^3\text{He} + \text{Au}$ Collisions to Disentangle Medium Properties

J. L. Nagle, A. Adare, S. Beckman, T. Koblesky, J. Orjuela Koop, D. McGlinchey, P. Romatschke, J. Carlson, J. E. Lynn, and M. McCumber

Phys. Rev. Lett. **113**, 112301 – Published 12 September 2014

2019 – Geometry Scan Results

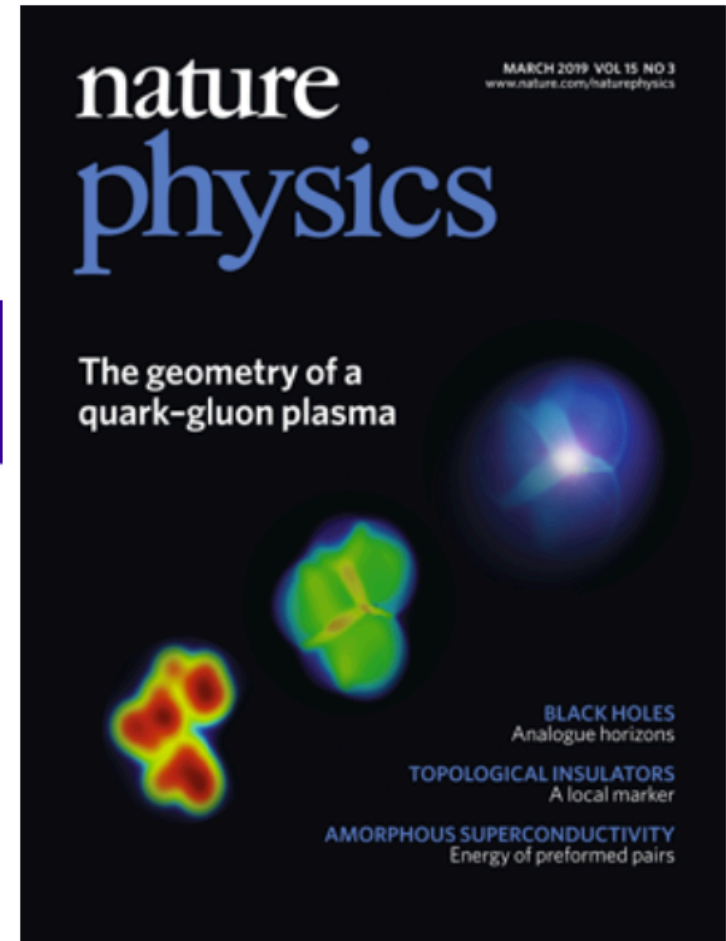
naturephysics

Letter | Published: 10 December 2018

Creation of quark–gluon plasma droplets with three distinct geometries

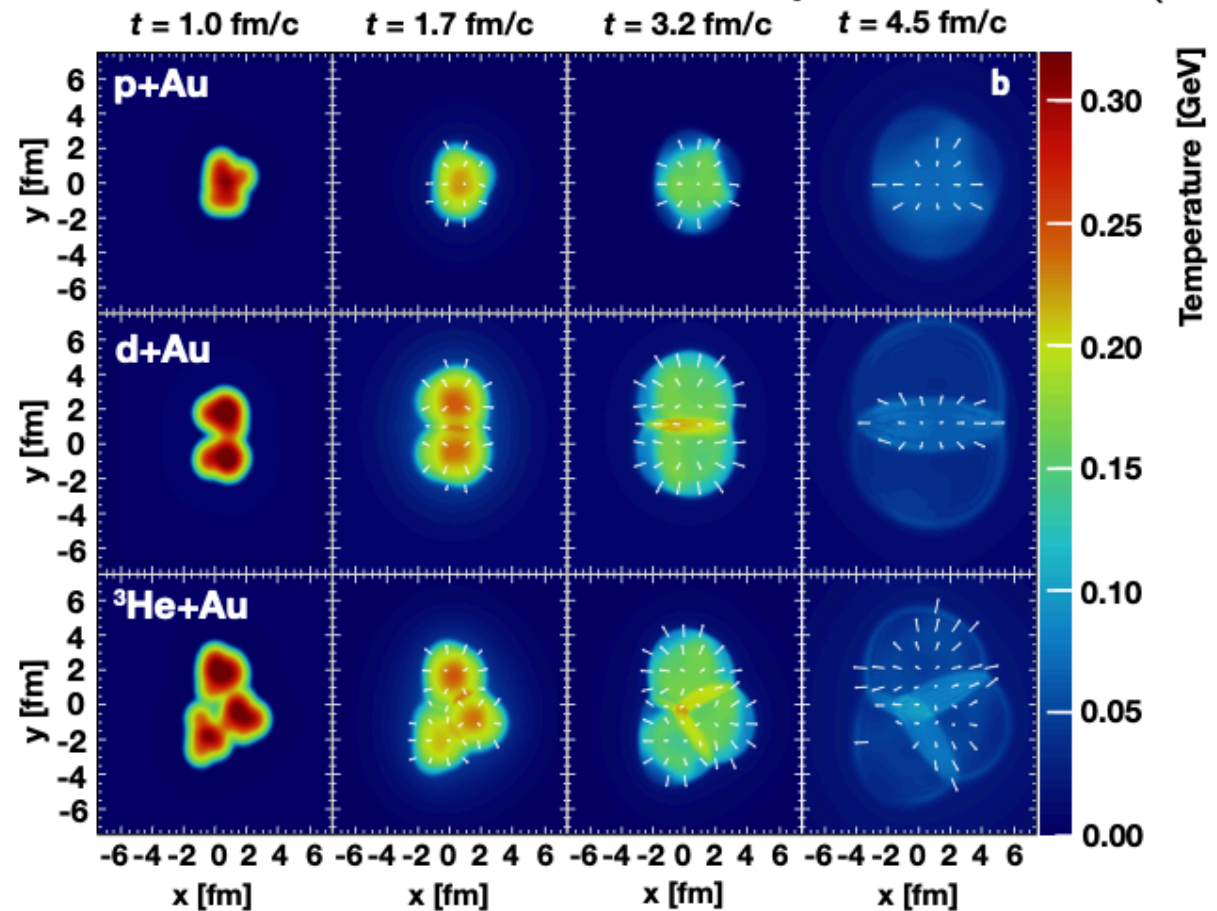
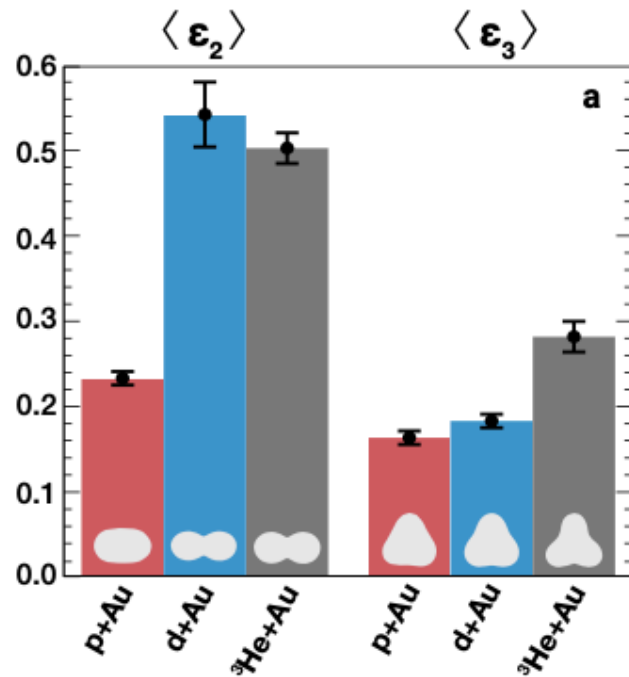
PHENIX Collaboration

Nature Physics **15**, 214–220(2019) | [Cite this article](#)



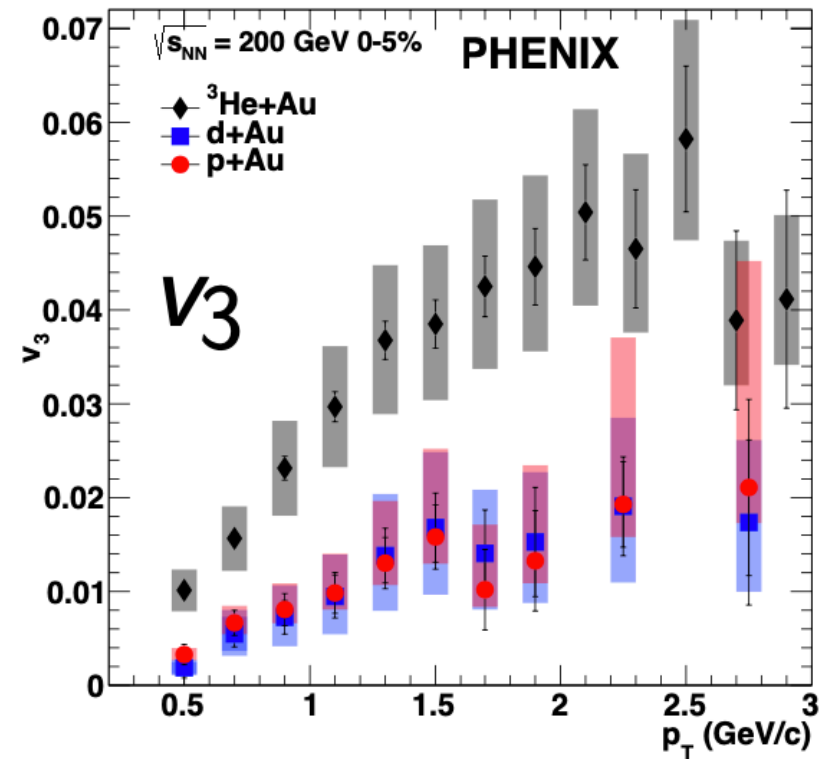
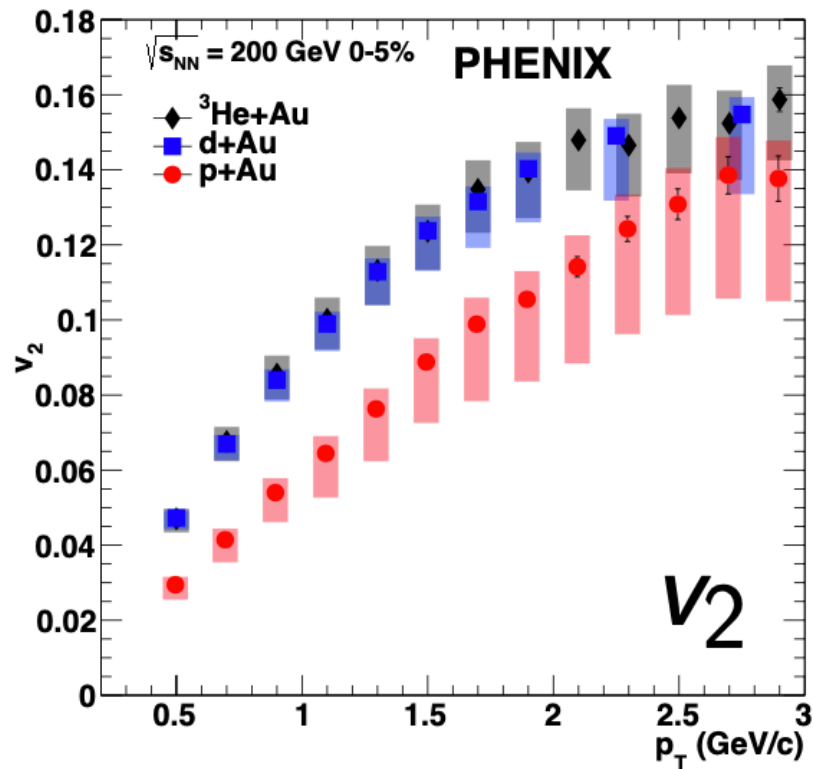
It's All About Geometry

Nature Physics 15, 214–220 (2019)



It's All About Geometry

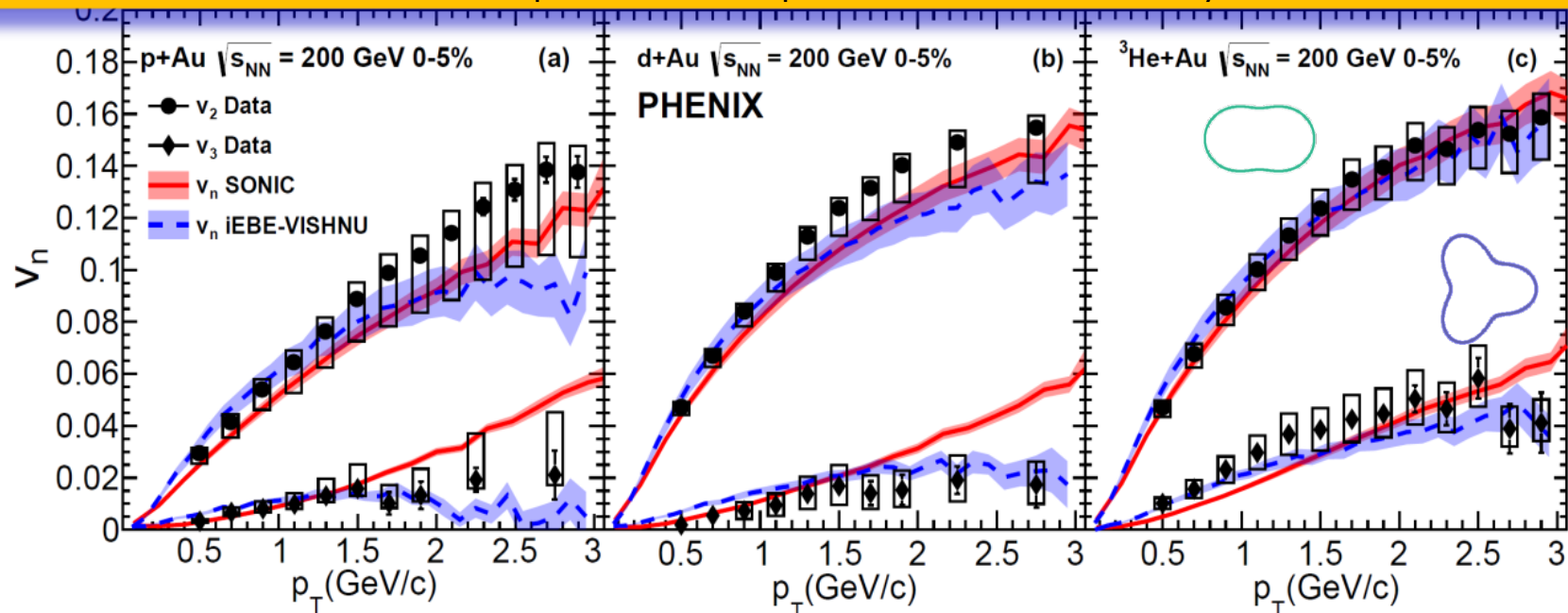
Nature Physics 15, 214–220 (2019)



- v_2 and v_3 ordering matches ε_2 and ε_3 ordering in all three systems
—Regardless of mechanism, the correlation is geometrical and thus collective

It's All About the Hydrodynamics (of a near-perfect fluid)

We find that initial-state momentum correlation models where color domains are individually resolved are ruled out as the dominant mechanism behind the observed collectivity. ... Further, we find that hydrodynamical models which include QGP formation provide a simultaneous and quantitative description of the data in all three systems.



- v_2 and v_3 vs p_T predicted or described very well by hydrodynamics in all three systems
 - All predicted (except v_2 in $d+Au$) in J.L. Nagle et al, PRL 113, 112301 (2014)
 - v_3 in $p+Au$ and $d+Au$ predicted in C. Shen et al, PRC 95, 014906 (2017)

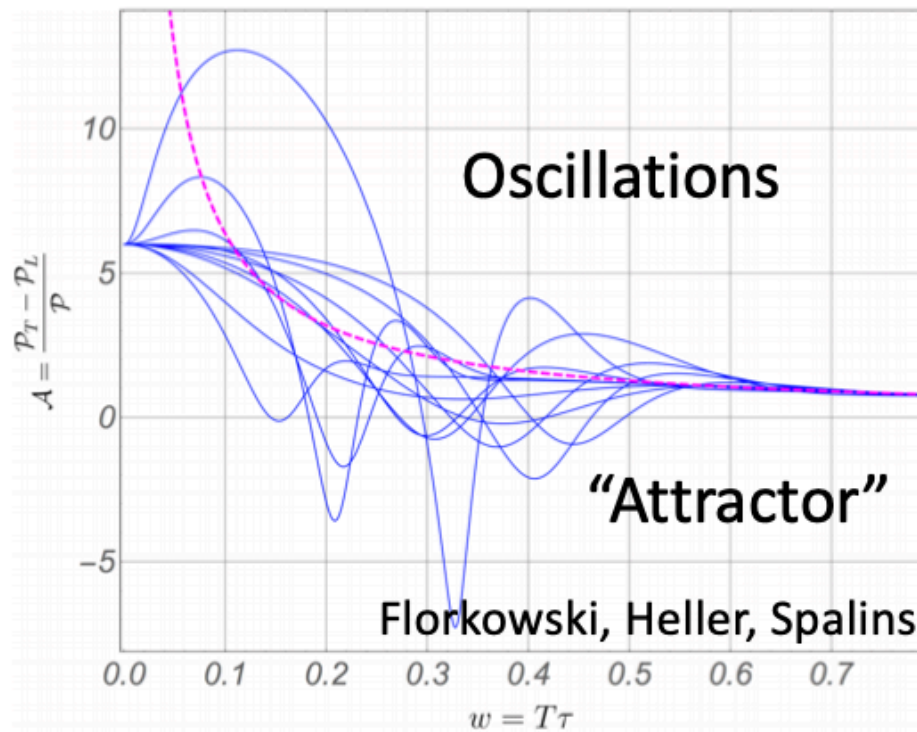
Remarkable Date: 14-Dec-07

- R. Baier, P. Romatschke, D. T. Son, A. O. Starinets, and M. A. Stephanov, *Relativistic viscous hydrodynamics, conformal invariance, and holography*, JHEP 04 (2008) 100, [arXiv:0712.2451](https://arxiv.org/abs/0712.2451)
 - “we find three (out of five) second-order transport coefficients in the strongly coupled N=4 supersymmetric Yang-Mills theory ... We point out that the Mueller-Israel-Stewart theory, often used in numerical simulations, does not contain all allowed second-order terms and, frequently, terms required by conformal invariance.”
- S. Bhattacharyya, V. E. Hubeny, S. Minwalla, and M. Rangamani, *Nonlinear Fluid Dynamics from Gravity*, JHEP 02 (2008) 045, [arXiv:0712.2456](https://arxiv.org/abs/0712.2456)
 - “Our work may be regarded as a derivation of the nonlinear equations of boundary fluid dynamics from gravity. As a concrete application we find an explicit expression for the expansion of this fluid stress tensor including terms up to second order in the derivative expansion.”

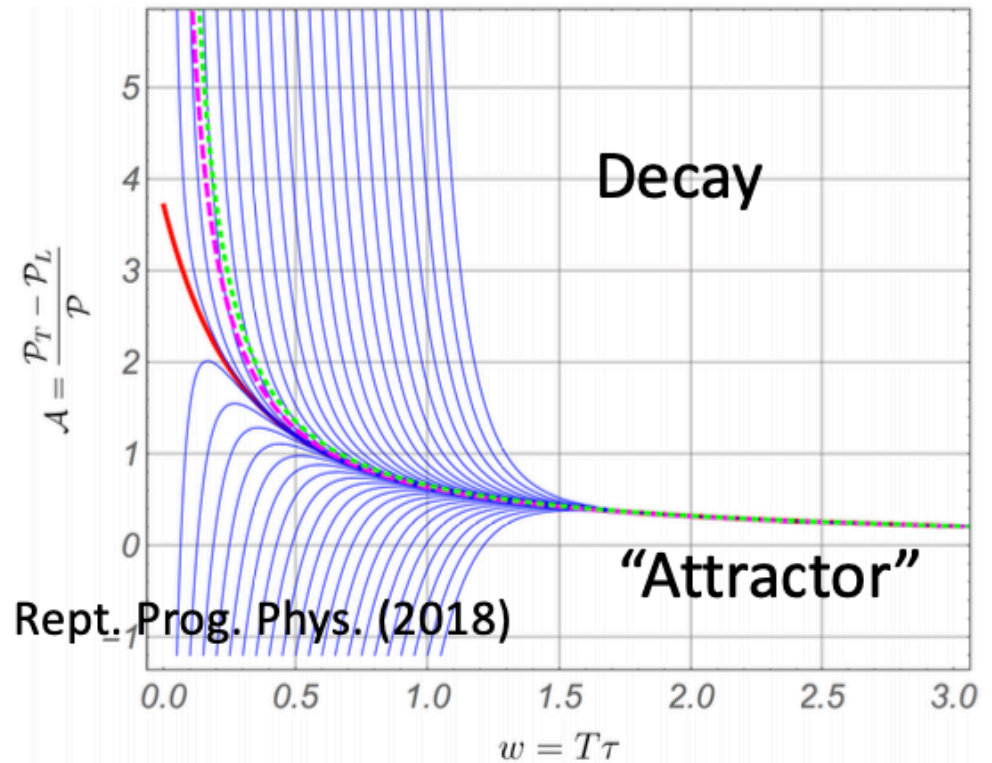
2007-Present: Understanding What's Going on "Under the hood"

- Gauge/gravity duality continues to inform...

AdS/CFT



Israel-Stewart



Circa 2017

- An emerging understanding drawing from
 - ▶ Kinetic theory
 - ▶ Linear response theory
 - ▶ Gauge/gravity duality
- Hydrodynamics does *not* require thermalization
- Rather, hydro modes should dominate non-hydro modes (at late times)
- “Hydrodynamization time” $T \tau \sim 1$ characterizes this condition
- Multiplicities as low as $dN_{\text{ch}}/dy \sim 4$ (!) may suffice for hydro description

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Home / Annual Review of Nuclear and Particle Science / Volume 68, 2018 / Nagle, pp 211-235

Small System Collectivity in Relativistic Hadronic and Nuclear Collisions

Annual Review of Nuclear and Particle Science
Vol. 68:211-235 (Volume publication date 19 October 2018)
First published as a Review in Advance on July 18, 2018
<https://doi.org/10.1146/annurev-nucl-101916-123209>

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Sections

ABSTRACT

KEYWORDS

INTRODUCTION

HISTORICAL PRELUDES

Abstract

The bulk motion of nuclear matter at the ultrahigh temperatures created in heavy ion collisions at the Relativistic Heavy Ion Collider and the Large Hadron Collider is well described in terms of nearly inviscid hydrodynamics, thereby establishing this system of quarks and

Two masterful reviews:

- W. Florkowski, M. P. Heller, and M. Spalinski, *New theories of relativistic hydrodynamics in the LHC era*, [1707.02282](#)
- P. Romatschke and U. Romatschke, *Relativistic Fluid Dynamics Out of Equilibrium – Ten Years of Progress in Theory and Numerical Simulations of Nuclear Collisions*, [1712.05815](#)

2019-2020: New Developments (I)

- 1st-order, *causal* formulations of relativistic hydrodynamics:
 - ▶ Nonlinear causality of general first-order relativistic viscous hydrodynamics, Bemfica, Disconzi and Noronha, [1907.12695](#)
 - "Our causality, existence, and uniqueness results hold in the full nonlinear regime, without symmetry assumptions, in four space-time dimensions, with or without coupling to Einstein's equations, and are mathematically rigorously established."
 - ▶ Stable and causal relativistic Navier-Stokes equations, Hout and Kovtun, [2004.04102](#)
 - "We show that the viscous-fluid equations are stable and causal if one adopts suitable non-equilibrium definitions of the hydrodynamic variables."
- Challenges lore that 2nd-order Müller-Israel-Stewart formalism required for causal solutions, suggests debate of Landau "versus" Eckart frames spurious, etc.

Earlier In This Conference

Tamás Csörgő :

It's always important to trouble the student's mind.

Questioning the Questioners

- It appears there are trade-offs between the (new) 1st order causal and stable solutions and the (now) traditional 2nd order Muller-Israel-Stewart theory

Masoud Shokri's talk

First-order hydro is stable!

We can make a first-order stable and causal theory by expanding the number of **transport parameters**, instead of introducing new dynamical variables. [Bemfica et al., 2017](#)–[Kovtun, 2019](#)–[Bemfica et al., 2019](#)

Let's assume an uncharged conformal fluid: $\epsilon = 3p = 3\underline{p}T^4$.

By adding any possible term at first order, not fixing the frame, and do not remove the **off-shell** terms, we end up with

$$\begin{aligned}
 T^{\mu\nu} = & \underline{p}T^3 \left[T + 16C_\eta C_p \left(\frac{u^\nu \partial_\nu T}{T} + \frac{\nabla \cdot u}{3} \right) \right] (g^{\mu\nu} + 4 u^\mu u^\nu) \\
 & + 16\underline{p}C_\eta C_Q T^3 \left[\left(u^\nu \nabla_\nu u^\mu + \frac{\Delta^{\mu\lambda} \partial_\lambda T}{T} \right) u^\nu + (\mu \leftrightarrow \nu) \right] \\
 & - 4\underline{p}C_\eta T^3 \sigma^{\mu\nu}.
 \end{aligned}$$

SC conditions:

$$C_\eta > 0 \quad C_p > 1 \quad C_Q > 1$$

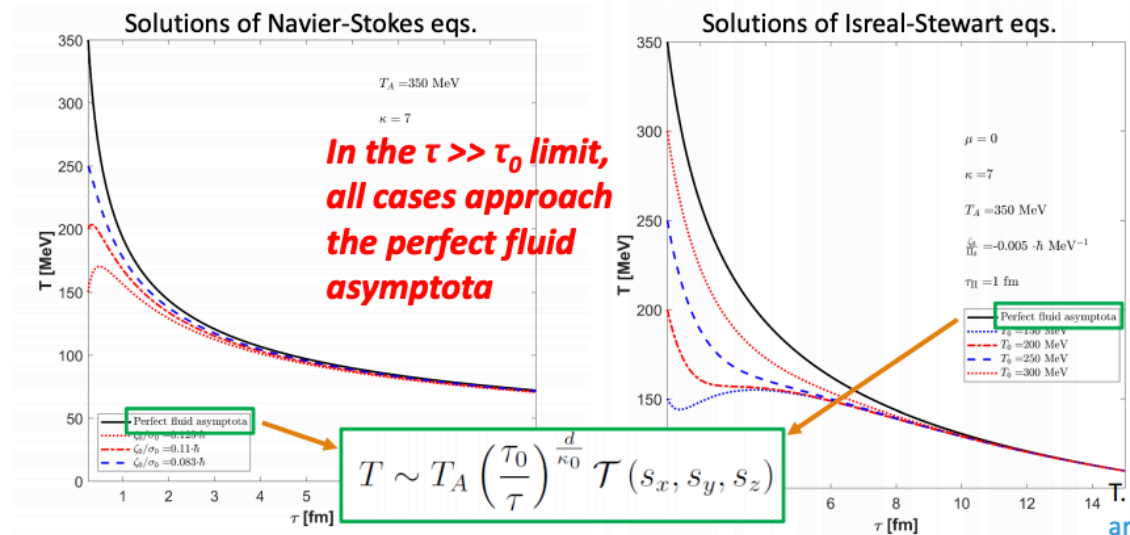
Yet Another Question

- “Thus no mathematically precise information can be gained about viscosity effects by studying only the hadronic final state of the asymptotically perfect fluids in relativistic heavy ion collisions, if the velocity field has a special, asymptotically Hubble form.” (!!)

☞ Are there degeneracies in real-world Bayesian parameter extraction?

Gábor Kasza's talk

Evolution of the temperature: same attractor



2004 → 2020

- Extraordinary progress !
- Essential feature: *open source code*

JETSCAPE, Multi-system Bayesian constraints on the transport coefficients of QCD matter, [2011.01430](#)

PHENIX On Estimating Errors

- ~All of data analysis effort is expended on understanding systematic errors:
 - Example taken from (required) Analysis Note prior to release of even Preliminary Data

	p_T indep.	2 GeV	6 GeV	type
peak extraction	5.0%(5.0%)			A
geometric acc.		3.0%(3.0%)	2.0%(2.0%)	B
π^0 reconstr. eff.		5.0%(5.0%)	5.0%(5.0%)	B
energy scale		4.0%(4.0%)	9.0%(9.0%)	B
Conversion corr.	3.0%(3.0%)			C
Total error		9.1%(9.1%)	12%(12%)	

- Would like to see this (and more) from those theory analyses dedicated to extraction of physical parameters

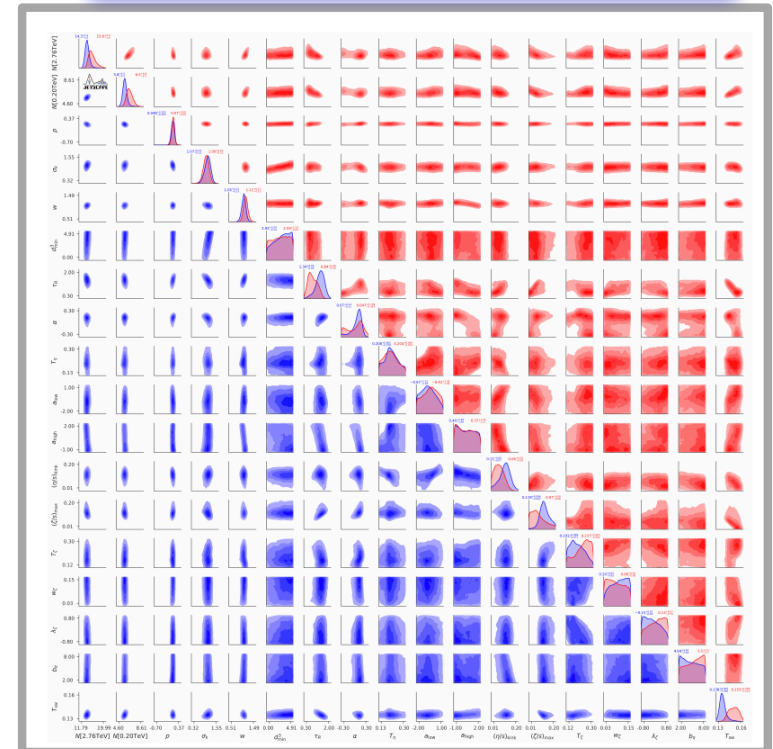
5-7Mar-08

Leadership by

Scott Pratt

Steffen Bass

Abhijit Majumder



The Question for All of Us



The Unreasonable Effectiveness of Mathematics in the Natural Sciences

Richard Courant Lecture in Mathematical Sciences delivered at New York University, May 11, 1959

EUGENE P. WIGNER
Princeton University

"and it is probable that there is some secret here which remains to be discovered." (C. S. Peirce)

There is a story about two friends, who were classmates in high school, talking about their jobs. One of them became a statistician and was working on population trends. He showed a reprint to his former classmate. The reprint started, as usual, with the Gaussian distribution and the statistician explained to his former classmate the meaning of the symbols for the actual population, for the average population, and so on. His classmate was a bit incredulous and was not quite sure whether the statistician was pulling his leg. "How can you know that?" was his query. "And what is this symbol here?" "Oh," said the statistician, "this is π ." "What is that?" "The ratio of the circumference of the circle to its diameter." "Well, now you are pushing your joke too far," said the classmate, "surely the population has nothing to do with the circumference of the circle."

Naturally, we are inclined to smile about the simplicity of the classmate's approach. Nevertheless, when I heard this story, I had to admit to an eerie feeling because, surely, the reaction of the classmate betrayed only plain common sense. I was even more confused when, not many days later, someone came to me and expressed his bewilderment¹ with the fact that we make a rather narrow selection when choosing the data on which we test our theories. "How do we know that, if we made a theory which focusses its attention on phenomena we disregard and disregards some of the phenomena now commanding our attention, that we could not build another theory which has little in common with the present one but which, nevertheless, explains just as many phenomena as the present theory." It has to be admitted that we have not definite evidence that there is no such theory.

The preceding two stories illustrate the two main points which are the

¹The remark to be quoted was made by F. Werner when he was a student in Princeton.

How can we understand

The Unreasonable Effectiveness of Hydrodynamics

in describing so many features of hadronic collisions?

- A fascinating work in progress
- To be answered by the students in 202x !

ZIMÁNYI SCHOOL 2020



J. E.: From darkness, the light

**20th ZIMÁNYI SCHOOL
WINTER WORKSHOP
ON HEAVY ION PHYSICS**

December 7-11, 2020

Budapest, Hungary



József Zimányi (1931 - 2006)

Thank You!

This work was supported by the United States Department
of Energy Grant DOE-FG02-86ER-40281

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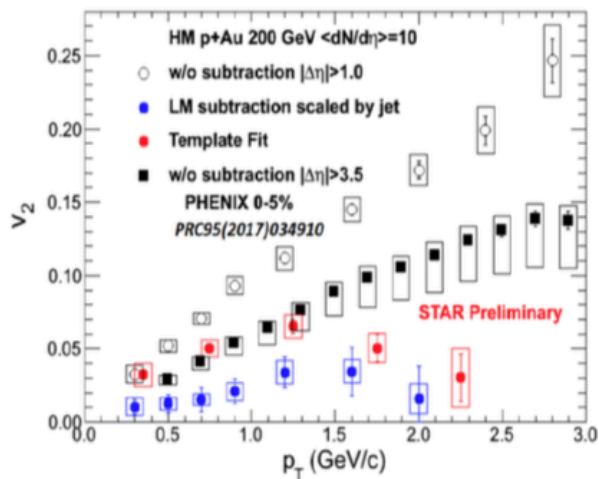
József Zimányi (1931 - 2006)

Back-Up Material

The Situation Regarding STAR Results on This Topic

STAR Preliminary v_2 #1, QM 2018 (Venice)

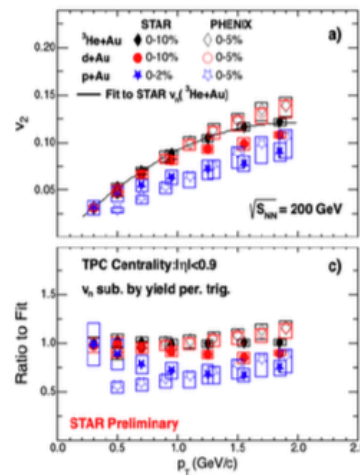
<https://indico.cern.ch/event/656452/contributions/2869833/>



STAR states that PHENIX result is “wrong” and has substantial non-flow not accounted for in uncertainties.

STAR Preliminary v_2 #2, QM 2019 (Wuhan)

<https://indico.cern.ch/event/792436/contributions/3535629/>

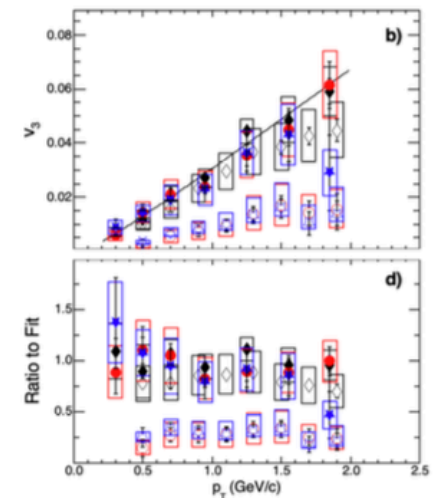


STAR states

“The STAR and PHENIX measurements for $v_2\{2\}$ are in reasonable agreement for all systems”
NO EXPLANATION FOR WHAT CHANGED!

STAR New Preliminary v_3 , QM 2019 (Wuhan)

<https://indico.cern.ch/event/792436/contributions/3535629/>



“The STAR and PHENIX $v_3\{2\}$ measurements for p/d+Au differ by more than a factor of 3”

Current Status

PHENIX takes the issue seriously, so we are doing our due diligence!

The published small systems results use the event plane method, where the resolution nominally follows

$$R(\chi) = \frac{\sqrt{\pi}}{2} \chi e^{-\frac{\chi^2}{2}} \left(I_0\left(\frac{\chi^2}{2}\right) + I_1\left(\frac{\chi^2}{2}\right) \right)$$

In small systems we're in the limiting case where $\chi \ll 1$ so $R \propto \chi$ (note that $\chi = v_n \sqrt{N_{ch}}$).

The set of PHENIX event plane resolutions do not follow the expected pattern.

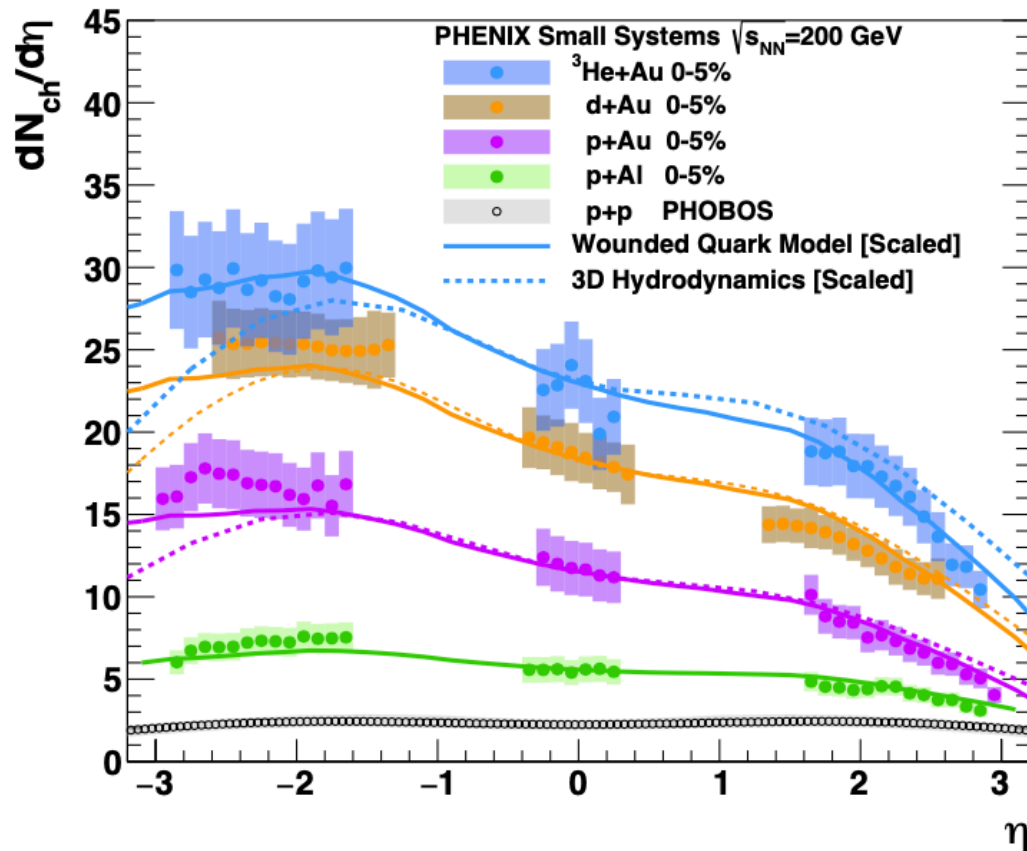
The origin of this effect appears to be the beam and angle offset relative to the detector and an additional offset of the PHENIX central carriage (all of these things vary between operational periods). The effect is qualitatively reproduced in toy simulation studies that utilize the full analysis procedure.

The three-subevent 2-particle correlation method uses event mixing, which appears to correct these effects quite well. Checks with the 3x2PC method show no such bias as seen in EP method for all systems, and all of these checks agree with published EP results within uncertainties.

Further checks on going as part of due diligence!

PHENIX Results in Pseudorapidity

Phys. Rev. Lett. 121, 222301 (2018)



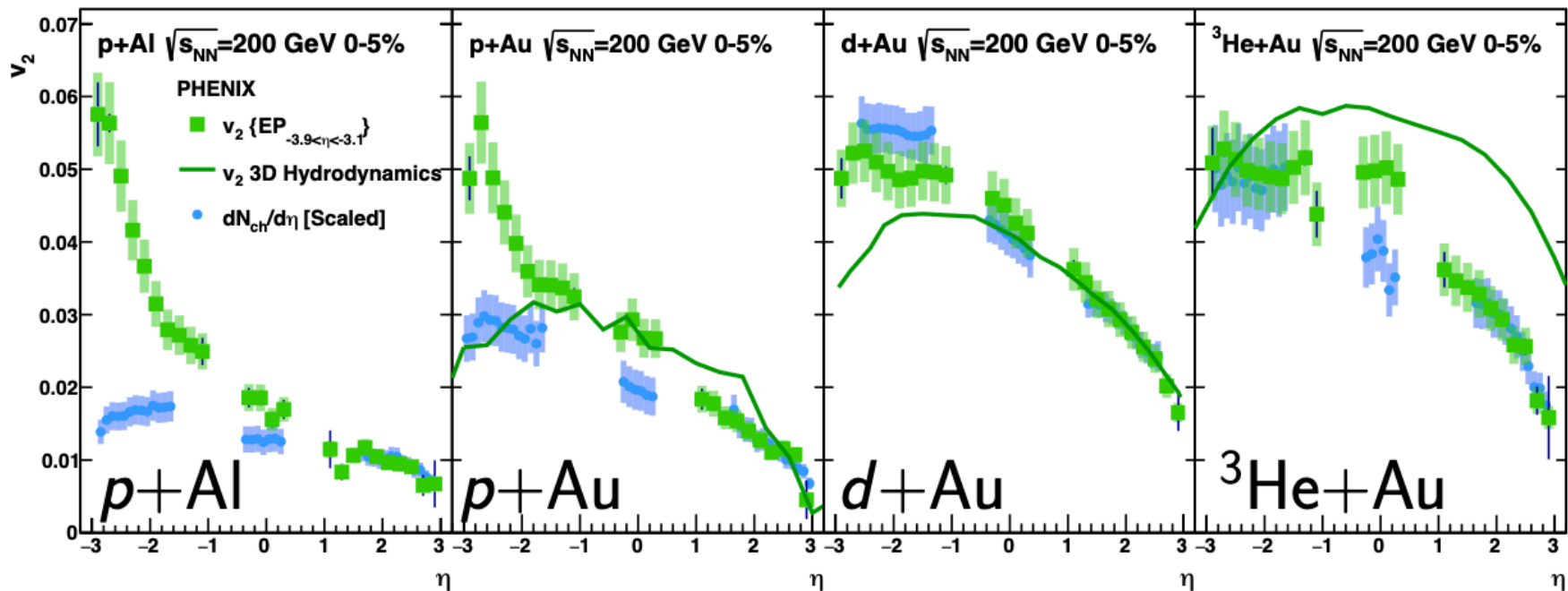
$p+\text{Al}$, $p+\text{Au}$, $d+\text{Au}$, $^3\text{He}+\text{Au}$

Good agreement with wounded quark model
(M. Barej et al, Phys. Rev. C 97, 034901 (2018))

Good agreement with 3D hydro
(P. Bozek et al, Phys. Lett. B 739, 308 (2014))

PHENIX Results in Pseudorapidity

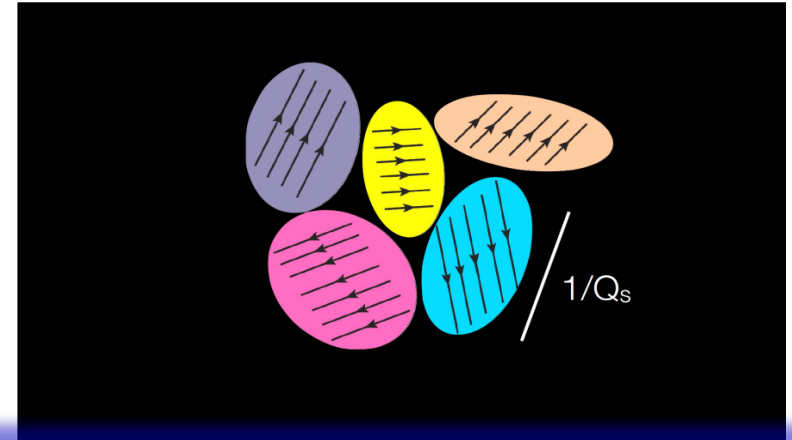
Phys. Rev. Lett. 121, 222301 (2018)



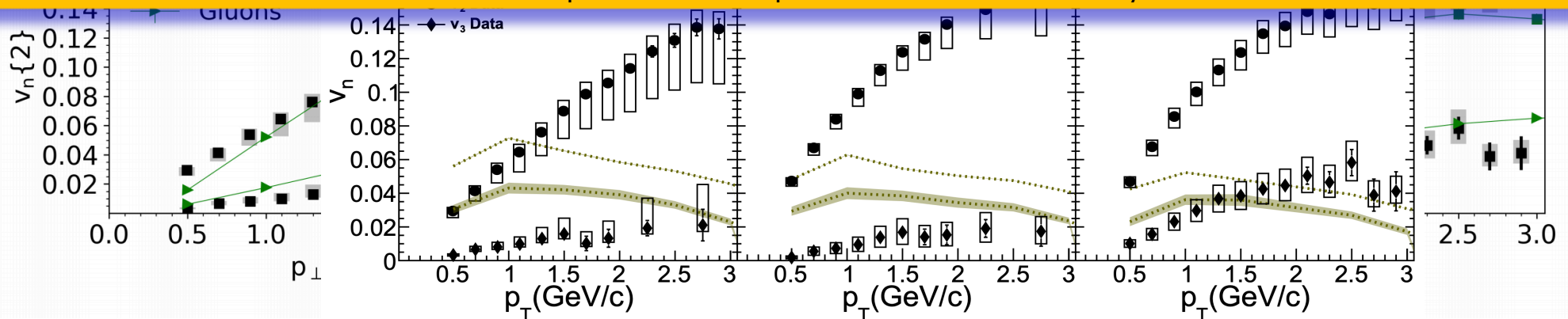
- v_2 vs η in $p+Al$, $p+Au$, $d+Au$, and ${}^3He+Au$
- Good agreement with 3D hydro for $p+Au$ and $d+Au$ (Bozek et al, PLB 739, 308 (2014))
- Prevalence of non-flow near the EP detector, decreases with increasing system size/multiplicity

A Challenge to Hydrodynamic Paradigm

- Azimuthal anisotropies are due to “*color domains*” from saturation of *initial state* gluon *fields*:
- (After correcting for error) fails to describe the data



We find that initial-state momentum correlation models where color domains are individually resolved are ruled out as the dominant mechanism behind the observed collectivity. ... Further, we find that hydrodynamical models which include QGP formation provide a simultaneous and quantitative description of the data in all three systems.



We Missed An Opportunity...

Important discovery in 2005

R. Andrade et al, Eur. Phys. J. A 29, 23-26 (2006)

NeXSPheRIO results on elliptic flow at RHIC and connection with thermalization

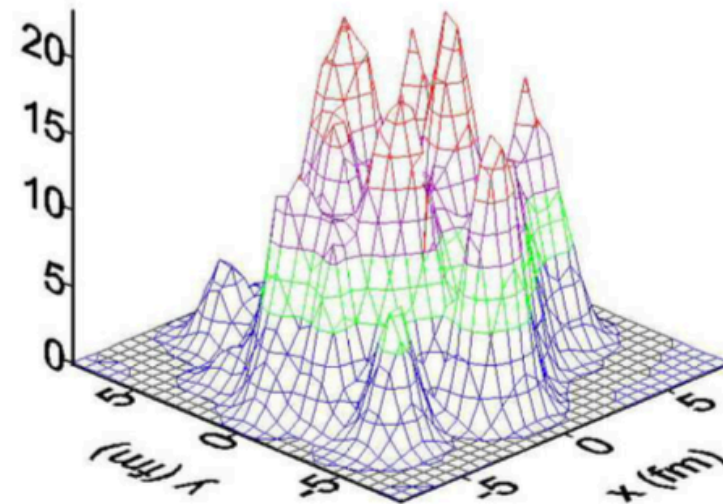
R.Andrade¹, F.Grassi¹, Y.Hama¹, T.Kodama², O.Socolowski Jr.³, and B.Tavares²

¹ Instituto de Física, USP,
C. P. 66318, 05315-970 São Paulo-SP, Brazil

² Instituto de Física, UFRJ,
C. P. 68528, 21945-970 Rio de Janeiro-RJ, Brazil

³ CTA/ITA,
Praça Marechal Eduardo Gomes 50, CEP 12228-900 São José dos Campos-SP,
Brazil

Received 1 January 2004

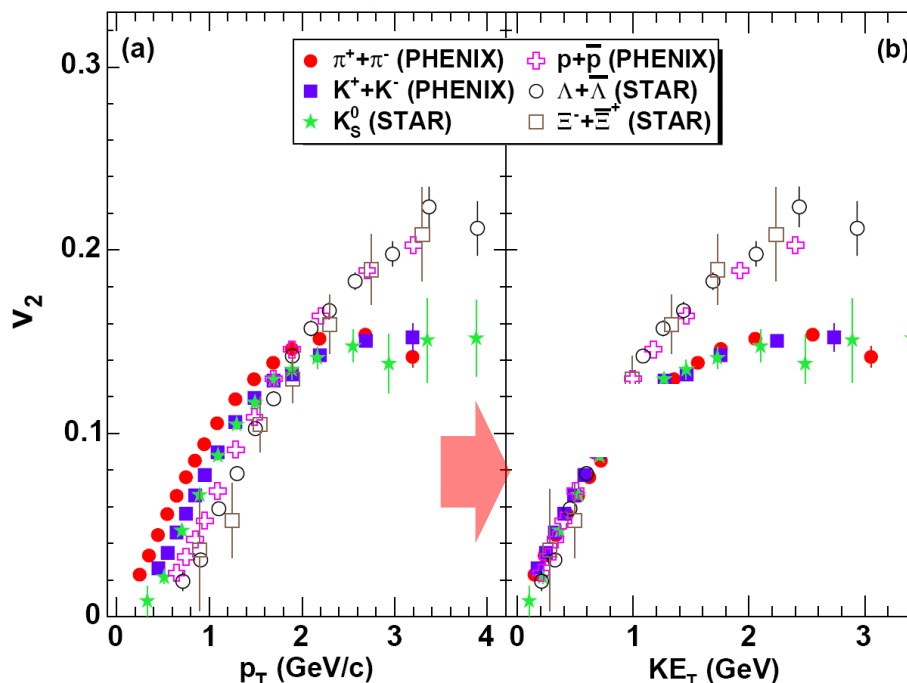


Worth noting that lumpy initial conditions were predicted some time in 2003

The Flow Is \sim Perfect

- The “fine structure” $v_2(p_T)$ for different mass particles in good agreement with ideal (“perfect fluid”) hydrodynamics

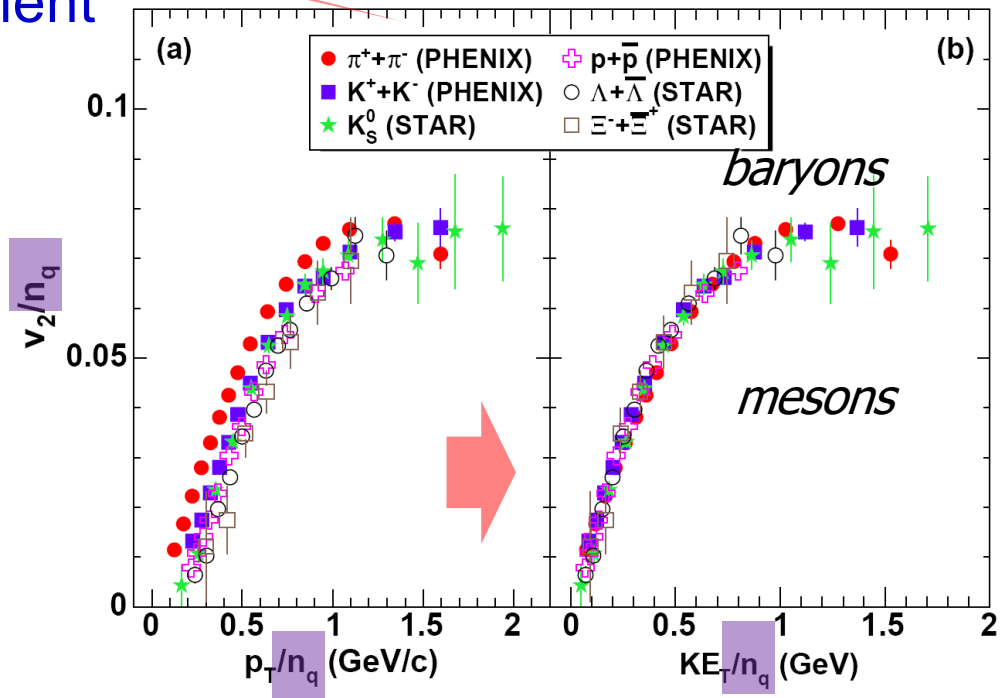
$$KE_T = \sqrt{m^2 + p_T^2}$$



- Roughly:
 $\partial_\nu T^{\mu\nu} = 0 \rightarrow$ Work-energy theorem
 $\rightarrow \int \nabla P d(\text{vol}) = \Delta E_K \cong m_T - m_0 \equiv \Delta KE_T$

The Flow Knows Quarks

- The “fine structure” $v_2(p_T)$ for different mass particles in good agreement with ideal (“perfect fluid”) hydrodynamics

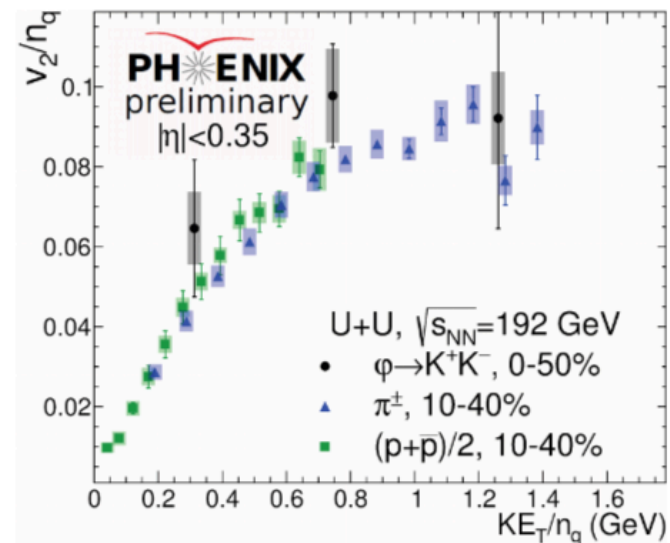
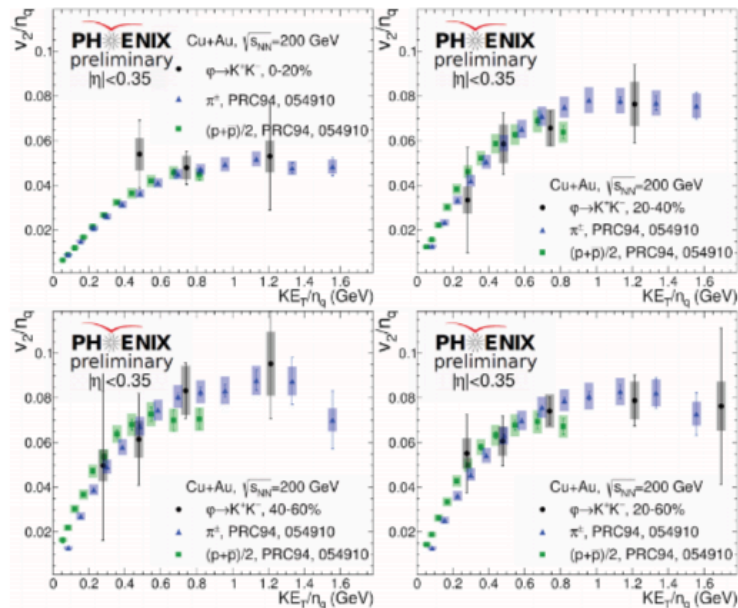


- Scaling flow parameters by quark content n_q resolves meson-baryon separation of final state hadrons

Recombination 2020

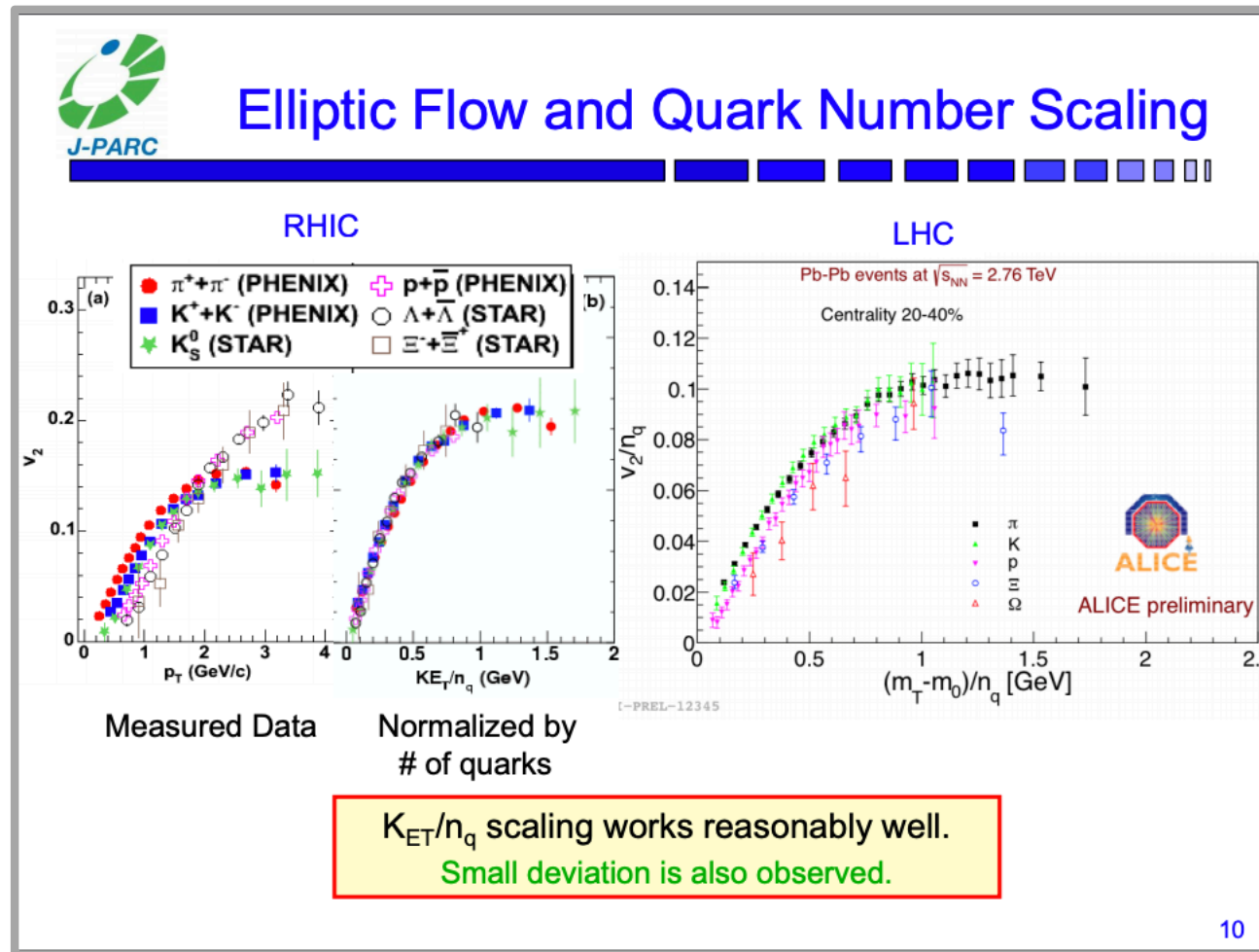
Flow for identified particles in Cu+Au, U+U and Au+Au

- KET scaling works for Cu+Au and U+U for phi, pi and Protons
- Recombination model is alive



Takao
Sakaguchi's
talk

Elliptic Flow and Quark Number Scaling



Shoji
Nagamiya's
talk

Summary

- Turbulence in QGP
 - Over-occupied system follows a **self-similar universal scaling**, not limited to pure Yang-Mills theory but also for QCD, even for moderately strongly coupled system
 - Under-occupied system follows a **bottom-up thermalization**
- Hydrodynamization of QGP
 - Ineffectiveness of quarks interaction in isotropization / equilibration
 - Kinetic theory provides **effective constitutive relation** far from equilibrium
 - Hydrodynamization ~ 1.5 Kinetic equilibration time \ll Isotropization time
 - Realistic matching to hydrodynamics at finite density with universal attractor and fixed certain scales from experiments (**charged particle multiplicity, baryon density, etc...**)

Condition and Scale Fixing

Fixed from experimental/lattice data

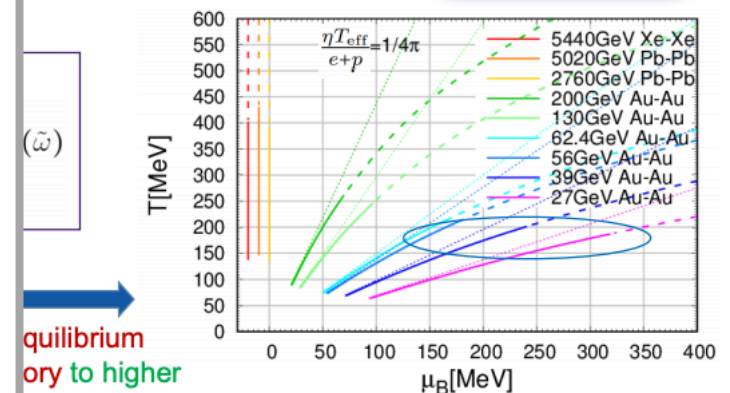
Charged particle multiplicity:

$$\frac{dN_{ch}}{d\eta} \simeq \frac{N_{ch}}{JS} (\tau s)_{eq} S_T \simeq 0.12 (\tau s)_{eq} S_T$$

Entropy per baryon:

$$\frac{S}{N_B} = \left(\frac{\tau s}{\tau \Delta n_B} \right)_{eq}$$

Xiaojuan Du's
talk



equilibrium
ory to higher
n density

To what extent does hydro occur at
lower-energy ?

BEC up to high multiplicity (pp @ 13 TeV)

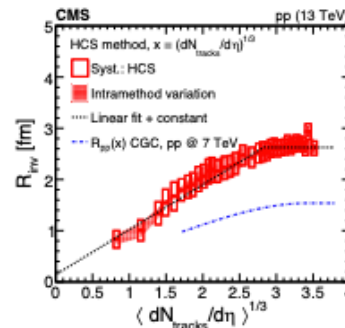
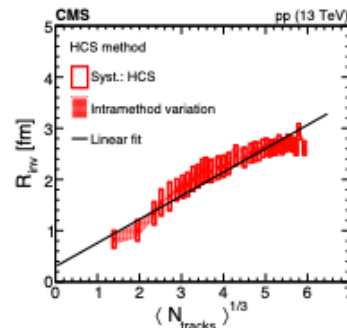
Data vs. CGC and vs. Hydro

- Color Glass Condensate
 - R_{inv} saturates at high multiplicity
- Above 1.7 : fit with same function obtained from CGC prediction (dashed black curve; stat. uncert. only)
 - [McLerran, Schenke, *NPA* **916** (2013) 210; P. T. A. Bzdak et al, *PRC* **87** (2013) 064906]

$$R_{pp}(x) = \begin{cases} (1 \text{ fm}) \times [a + b x + c x^2 + d x^3], & \text{for } x < 3.4 \\ e \text{ (fm)}, & \text{for } x \geq 3.4 \end{cases}$$

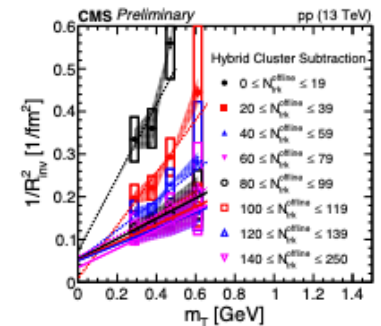
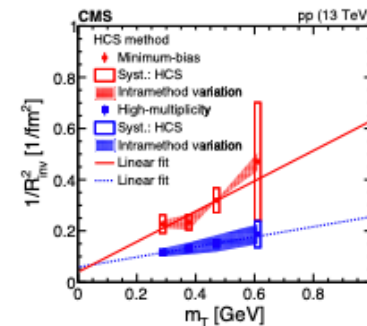
$$x = (dN_{\text{tracks}}/d\eta)^{1/3}$$

- Hydrodynamics
 - R_{inv} continuously grow w/ $\langle N_{\text{tracks}} \rangle$



$$1/R^2 \text{ vs. } m_T = \sqrt{m_\pi^2 + k_T^2}$$

- In hydrodynamic models [Sinyukov et al., *NPA* 946 (2016) 227]
 - Intercept \longleftrightarrow source geometrical size (at freeze-out)
 - Slope: reflects the flow component ...
 - Larger slope (faster expansion) \rightarrow MB (low N_{tracks}) (similar to peripheral AA collisions)
 - Lower slope \rightarrow higher multiplicities (similar to more central AA collisions)



Sandra Padula's talk

19

\rightarrow Results compatible with expectations from both, within uncertainties

JHEP 03 (2020) 014

sPHENIX – The Next Step

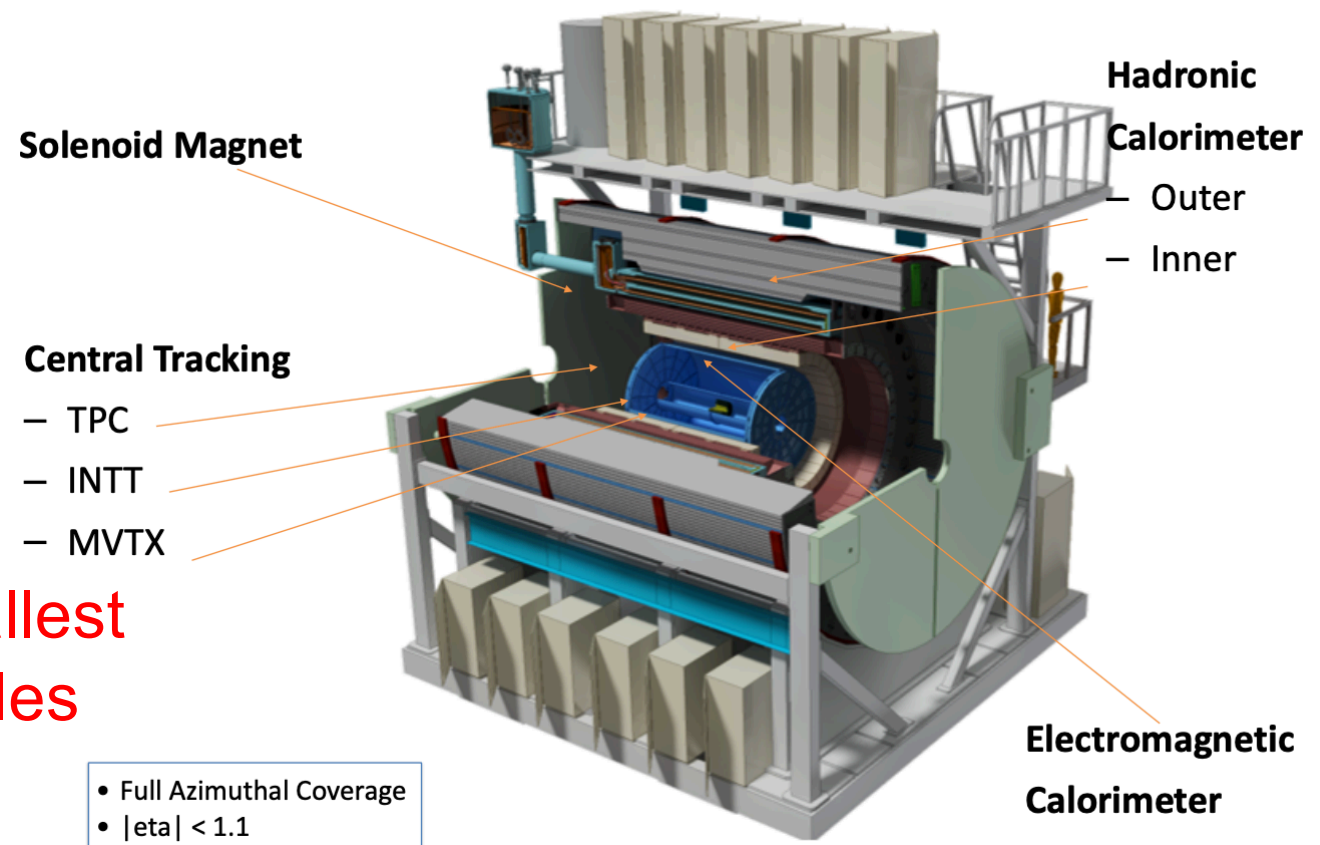
- Open question:

How do asymptotically free quarks and gluons conspire to form the world's most perfect fluid?

- To answer:

Probe it on the smallest possible length scales with jets

sPHENIX: State-of-the-Art Jet Detector at RHIC



The Papers

- *Quark gluon plasma and color glass condensate at RHIC? The Perspective from the BRAHMS experiment,*
Nucl.Phys. **A757** (2005) 1-27, [nucl-ex/0410020](#)
- *Formation of dense partonic matter in relativistic nucleus-nucleus collisions at RHIC: Experimental evaluation by the PHENIX collaboration,*
Nucl.Phys. **A757** (2005) 184-283, [nucl-ex/0410003](#)
- *The PHOBOS perspective on discoveries at RHIC,*
Nucl.Phys. **A757** (2005) 28-101, [nucl-ex/0410022](#)
- *Experimental and theoretical challenges in the search for the quark gluon plasma: The STAR Collaboration's critical assessment of the evidence from RHIC collisions,*
Nucl.Phys. **A757** (2005) 102-183, [nucl-ex/0501009](#)

Addressing the nature of QGP discovery

- From the PHENIX “White Paper”
- [nucl-ex/0410003](#)
- (2777 citations)

Q: What is the most relevant “experimentally observed property”?

A. *Viscosity*
(suitably normalized)

so that concepts such as temperature, chemical potential and flow velocity apply and the system can be characterized by an experimentally determined equation of state. Additionally, experiments eventually should be able to determine the physical characteristics of the transition, for example the critical temperature, the order of the phase transition, and the speed of sound along with the nature of the underlying quasi-particles. While at (currently unobtainable) very high temperatures $T \gg T_c$ the quark-gluon plasma may act as a weakly interacting gas of quarks and gluons, in the transition region near T_c the fundamental degrees of freedom may be considerably more complex. It is therefore appropriate to argue that the quark-gluon plasma must be defined in terms of its unique properties *at a given temperature*. To date the definition is provided by lattice QCD calculations. Ultimately we would expect to validate this by characterizing the quark-gluon plasma in terms of its experimentally observed properties. However, the real discoveries will be of the fascinating properties of high temperature nuclear matter, and not the naming of that matter.

1.2 *Experimental Program*

The theoretical discussion of the nature of hadronic matter at extreme densities has been greatly stimulated by the realization that such conditions could be studied via relativistic heavy ion collisions [32]. Early investigations at the Berkeley Bevalac (c. 1975–1985), the BNL AGS (c. 1987–1995) and the CERN SPS (c. 1987–present) have reached their culmination with the commissioning of BNL’s Relativistic Heavy Ion Collider (RHIC), a dedicated facility for the study of nuclear collisions at ultra-relativistic energies [33].

Small Viscosity Compared to What ?

- Various measures lead to

$$\left(\frac{\eta}{\text{s}} \right)_{RHIC} \sim 0.1 \hbar$$

- This is *small*.
- It implies damping time $\sim 1 / 0.1 = 10 \times$ longer than natural thermal time $\sim 1 / (\text{Temperature})$
 $\sim \hbar / (\text{Temperature})$

(Near)-Perfect Fluids

- Ideal aka “Perfect” fluid characterized by

- ▶ mean-free-path $\lambda = 0$
- viscosity $\eta \sim n \langle p \rangle \lambda = 0$

- Near-perfect fluid characterized by

- ▶ Small λ (relative to system size R)
- “small” viscosity $\eta \sim n \langle p \rangle \lambda \sim \langle p \rangle / \sigma$

- ▶ “Small” η implies large σ
- ▶ “Large” σ implies **strong coupling** !

(Jet quenching *also* implies **strong coupling** .)

Remember
that
 $\lambda = 1/(n \sigma)$

Alternative History

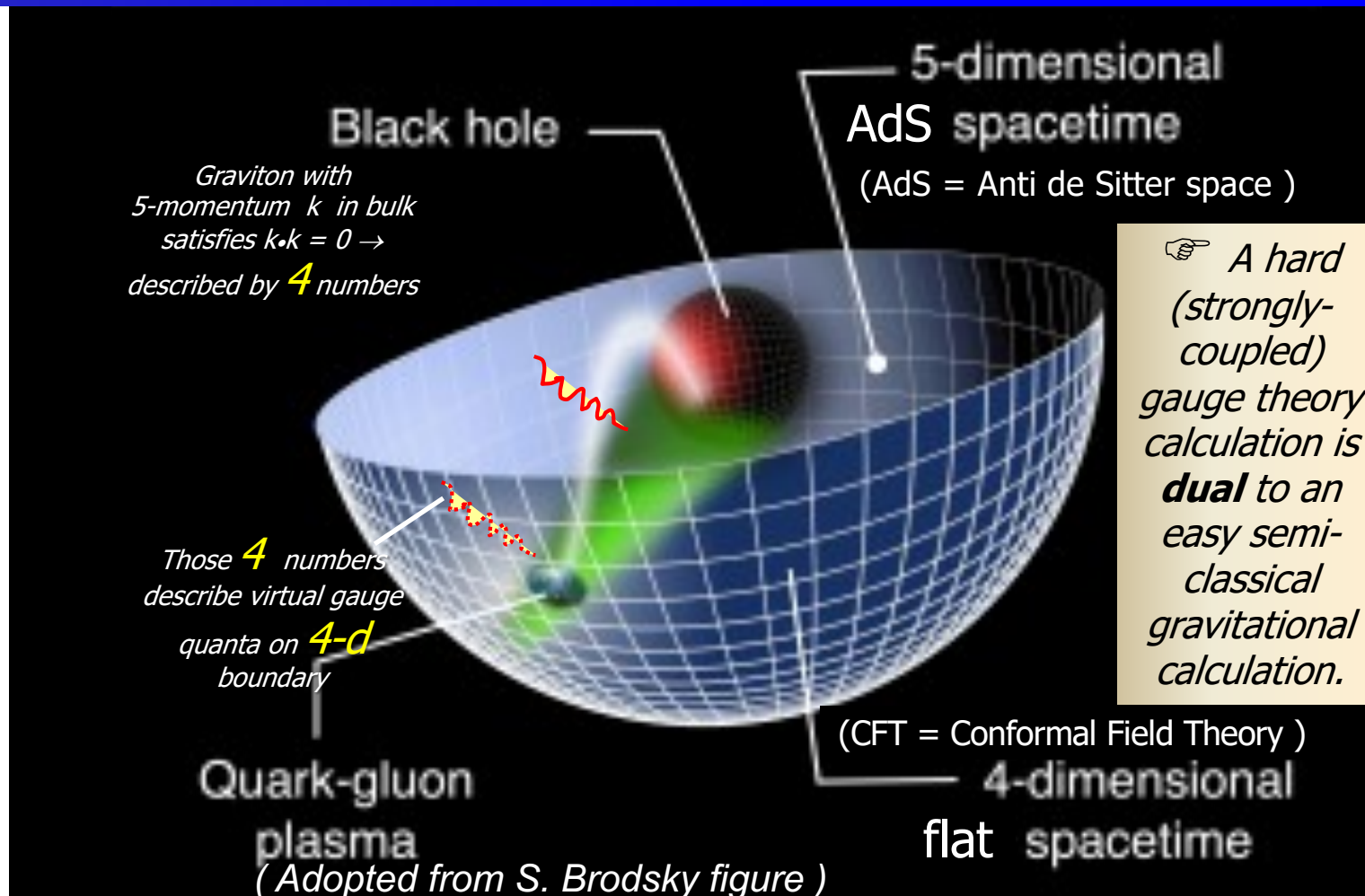
- So the “perfect fluid” observed at RHIC with

$$\left(\frac{\eta}{s} \right)_{RHIC} \sim 0.1 \hbar$$

was immediately recognized as confirming the 1985 uncertainty principle estimate of Danielewicz and Gyulassy

- Except that’s not what happened...

AdS / CFT in a Picture



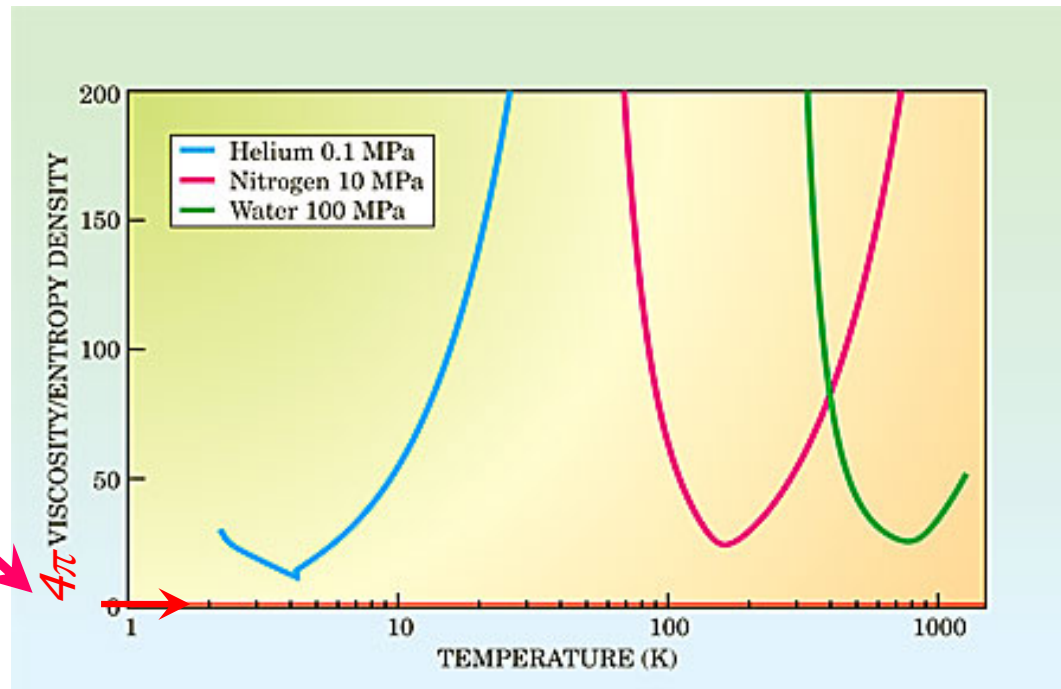
Is The Bound Respected ?

- All ordinary fluids exceed the KSS bound by factors of 10-1000

▶ “A Viscosity Bound Conjecture”,
 P. Kovtun,
 D.T. Son,
 A.O. Starinets, hep-th/0405231

$$\frac{\eta}{s} \geq \frac{\hbar}{4\pi}$$

- RHIC
 (and now LHC)
 sQGP fluids
 are at
 ~1-3
 on this
 scale (!)



Paradigm Shift

RHIC Scientists Serve Up 'Perfect' Liquid ²⁰

RHIC Scientists Serve Up 'Perfect' Liquid
New state of matter more remarkable than predicted — raising many new questions

Monday, April 18, 2005

TAMPA, Fla. — The four detector groups conducting research at the **Relativistic Heavy Ion Collider (RHIC)** — a giant atom "smasher" located at the U.S. Department of Energy's Brookhaven National Laboratory — say they've created a new state of hot, dense matter out of the quarks and gluons that are the basic particles of atomic nuclei, but it is a state quite different and even more remarkable than had been predicted. In **peer-reviewed papers** summarizing the first three years of RHIC findings, the scientists say that instead of behaving like a gas of free quarks and gluons, as was expected, the matter created in RHIC's heavy ion collisions appears to be more like a liquid.

+

A Long Time Ago (1985) ²⁴

Miklos Gyulassy and Pawel Danielewicz:

► **Dissipative Phenomena In Quark-Gluon Plasmas**
P. Danielewicz, M. Gyulassy
[Phys. Rev. D31, 53, 1985.](#)

noted restrictions on smallest allowed η :

- Most restrictive:
- $\lambda > \hbar / c p \Rightarrow \eta > \sim n / 3$
- But recall $s = 3.6 n$ for the quanta they were considering
- $\Rightarrow \eta / s > 1 / (3.6 \times 3) \sim \hbar / (4 \pi)$
 $\sim 0.1 \hbar$

Before estimating λ , via Eq. (3.2) we state several physical constraints on λ . First, the uncertainty principle implies that quanta transporting typical momenta (p) cannot be localized to distances smaller than $(p)^{-1}$. Hence, it is meaningless to speak about mean free paths smaller than $(p)^{-1}$. Requiring $\lambda \geq (p)^{-1}$ leads to the lower bound

$$\eta \geq \frac{\hbar}{3.6 n} \quad (3.3)$$

where $n = \sum_i n_i$ is the total density of quanta. What seems amazing about (3.3) is that it is independent of dynamical details. There is a finite viscosity regardless of how large is the free-space cross section between the quanta. See Refs. 21 and 22 for examples illustrating how the thermalization rate of many-body systems is limited by the uncertainty principle.

+

AdS / CFT in a Picture ²¹

Click to add text

Black hole

5-dimensional AdS spacetime (AdS = Anti de Sitter space)

Graviton with 5-momentum k in bulk satisfies $k^2 = 0 \rightarrow$ described by 4 numbers

Those 4 numbers describe virtual gauge quanta on 4-d boundary

Quark-gluon plasma

(Adopted from S. Brodsky figure)

A hard (strongly-coupled) gauge theory calculation is dual to an easy semi-classical gravitational calculation.

(CFT = Conformal Field Theory)

4-dimensional flat spacetime

- ➡ The realization that the key property of the quark-gluon plasma is its “perfect liquidity”, as quantified by η/s being at or near the quantum bound

And the AdS/CFT Approach Is “Routinely” Deployed

ITP-UU-13/22

A fully dynamical simulation of central nuclear collisions

Wilke van der Schee,¹ Paul Romatschke,² and Scott Pratt³

¹*Institute for Theoretical Physics and Institute for Subatomic Physics, Utrecht University, Leuvenlaan 4, 3584 CE Utrecht, The Netherlands*

²*Department of Physics, 390 UCB, University of Colorado, Boulder, CO 80309-0390, USA*

³*Department of Physics and Astronomy and National Superconducting Cyclotron Laboratory, Michigan State University, East Lansing, Michigan 48824, USA*

We present a fully dynamical simulation of central nuclear collisions around mid-rapidity at LHC energies. Unlike previous treatments, we simulate all phases of the collision, including the equilibration of the system. For the simulation, we use numerical relativity solutions to AdS/CFT for the pre-equilibrium stage, viscous hydrodynamics for the plasma equilibrium stage and kinetic theory for the low density hadronic stage. Our pre-equilibrium stage provides initial conditions for hydrodynamics, resulting in sizable radial flow. The resulting light particle spectra reproduce the measurements from the ALICE experiment at all transverse momenta.

1. Introduction. High precision experimental data from nucleus-nucleus collisions at the Relativistic Heavy Ion Collider (RHIC) and the Large Hadron Collider

is itself matched onto a standard hadronic cascade code, thereby achieving a fully dynamical simulation of a boost-invariant heavy-ion collision.

For the simulation, we use numerical relativity solutions to AdS/CFT for the pre-equilibrium stage, viscous hydrodynamics for the plasma equilibrium stage and kinetic theory for the low density hadronic stage.

ulations, however, involve several unknown parameters and functions, such as the initial conditions for hydrodynamics (starting time τ_{hydro} , energy density, velocity, shear tensor). Understanding these unknown parameters can therefore lead to more precise extraction of QCD properties, and consequently to a better understanding of strongly coupled theories like QCD.

In particular, very little is known about the far-from-equilibrium evolution towards hydrodynamics, such that until now reasonable initial conditions at τ_{hydro} had to be guessed. In this Letter we attempt to reduce this ambiguity by including the far-from-equilibrium stage obtained from colliding shock waves in AdS, which through the AdS/CFT correspondence are thought to be similar to heavy ion collisions. After the matter has equilibrated, we match the AdS/CFT results onto a standard viscous hydrodynamics code which, once the matter has cooled below the QCD phase transition temperature T_c ,

is the dominant force at high energies, but generalizations are straightforward. For a head-on (central) collision this shockwave collision has been written down and solved near the boundary of AdS in Ref. [5], resulting in the stress-energy tensor at early times. In polar Milne coordinates τ, ξ, ρ, θ with $t = \tau \cosh \xi$, $z = \tau \sinh \xi$, $\rho^2 = x^2 + y^2$, $\tan \theta = y/x$, $T^{\mu\nu}$ can be decomposed into

$$T^{\mu\nu} = (\epsilon + P(\epsilon))u^\mu u^\nu + P(\epsilon)g^{\mu\nu} + \pi^{\mu\nu}, \quad (2)$$

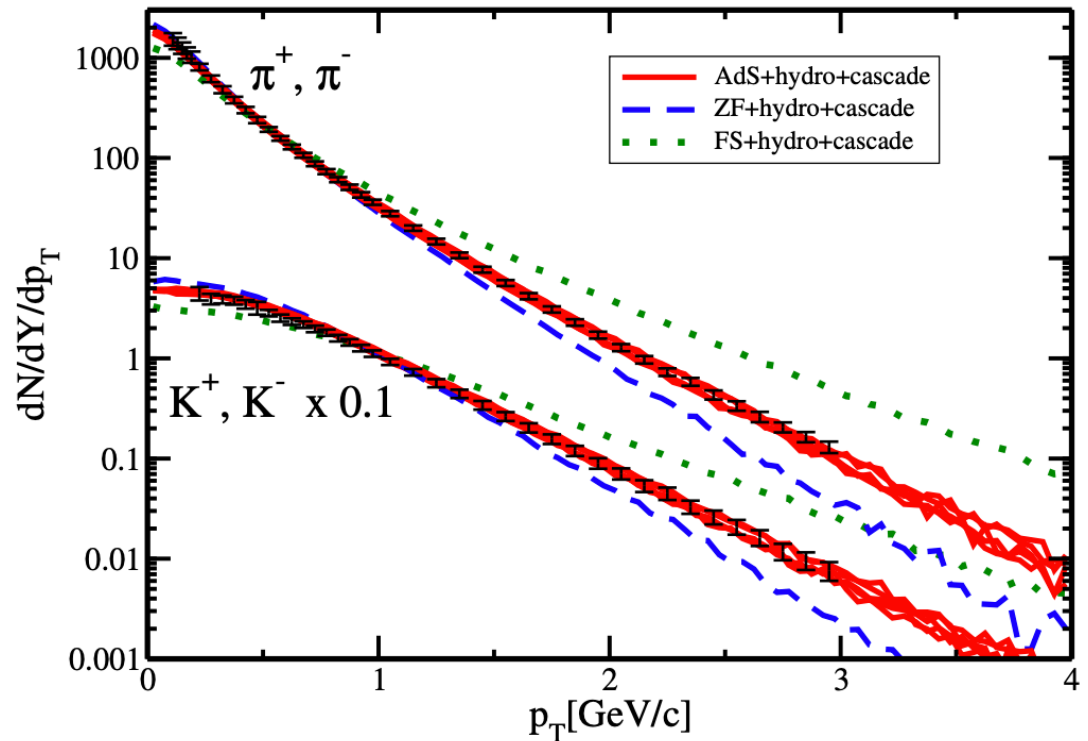
with ϵ , $P(\epsilon)$, u^μ , $\pi^{\mu\nu}$ the local energy density, equation of state (EoS), velocity and shear tensor. In Ref. [5], it was found to leading order in t that

$$\epsilon = 2T_\lambda^2(\rho)\tau^2, \quad u^\rho = -\frac{T_\lambda(\rho)}{3T_\lambda(\rho)}\tau, \quad \frac{P_L}{P_T} = -\frac{3}{2}, \quad (3)$$

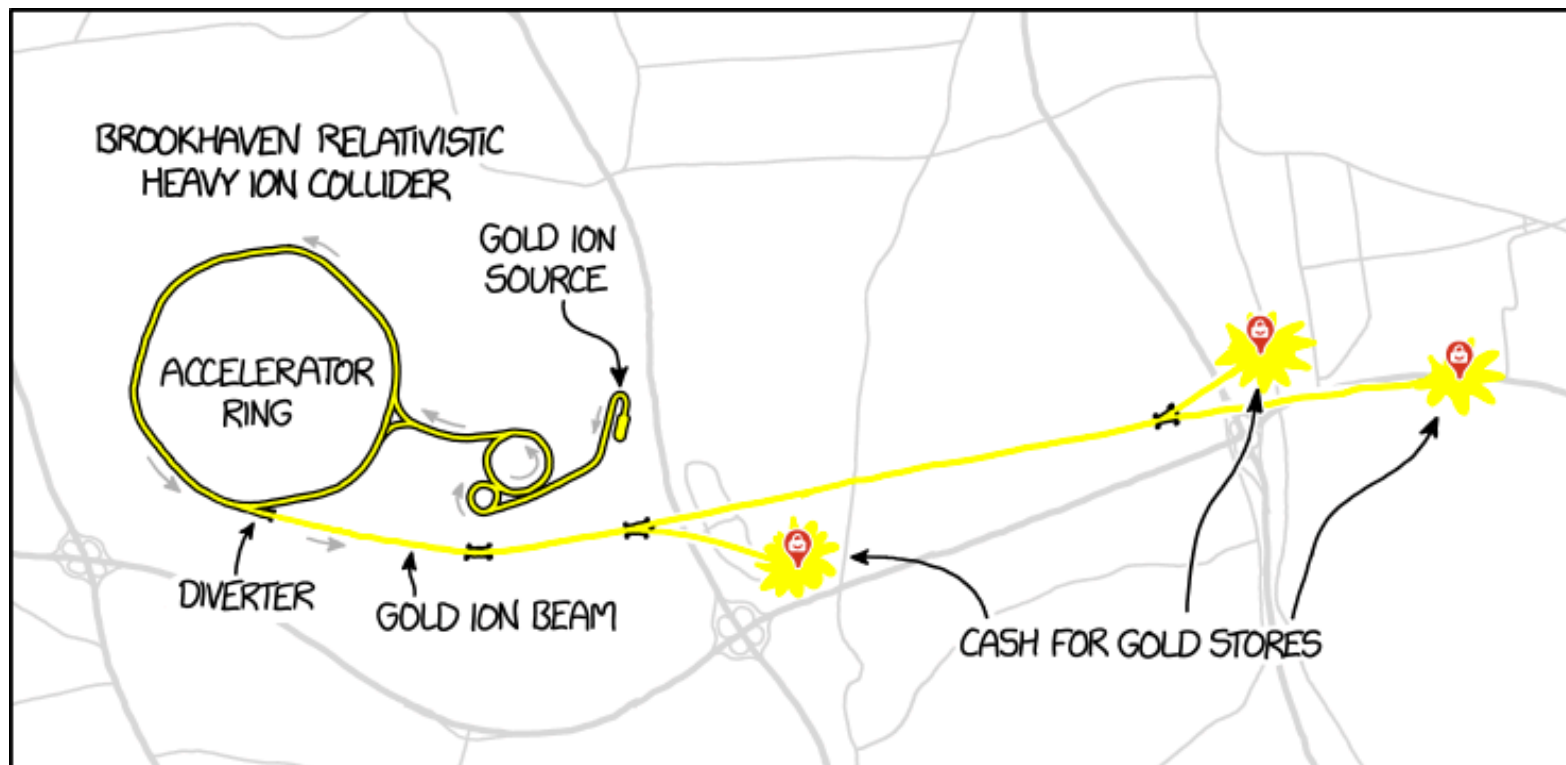
where in the local rest frame $T_\mu^\mu = \text{diag}(-\epsilon, P_T, P_T, P_L)$. The velocity dependence and pressure anisotropy are consistent with previous results [6-9]. To leading order in

Light Particle Spectra

Pb+Pb @ $\sqrt{s} = 2.76$ TeV



- https://imgs.xkcd.com/comics/brookhaven_rhic.png



SADLY, BROOKHAVEN REJECTED MY PROPOSED EXPERIMENT.

Perfect Primordial Fluid?

- Naming has been pre-empted by Gamow over 70 years ago:
 - ▶ R. Alpher, H. Bethe, G. Gamow, "*Origin of Chemical Elements*," *Phys. Rev.* 73, 803, (1948)

• y·lem A form of matter hypothesized by proponents of the big bang theory to have existed before the formation of the chemical elements.

[Middle English, universal matter, from Old French, from Medieval Latin, accusative of, matter, from Greek]



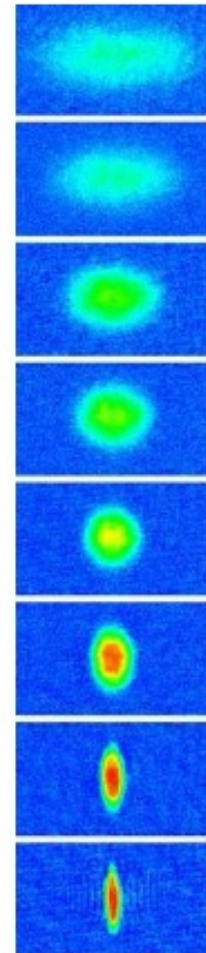
Connections to Other Fields

- The “cartoon” I used to illustrate elliptic flow ...

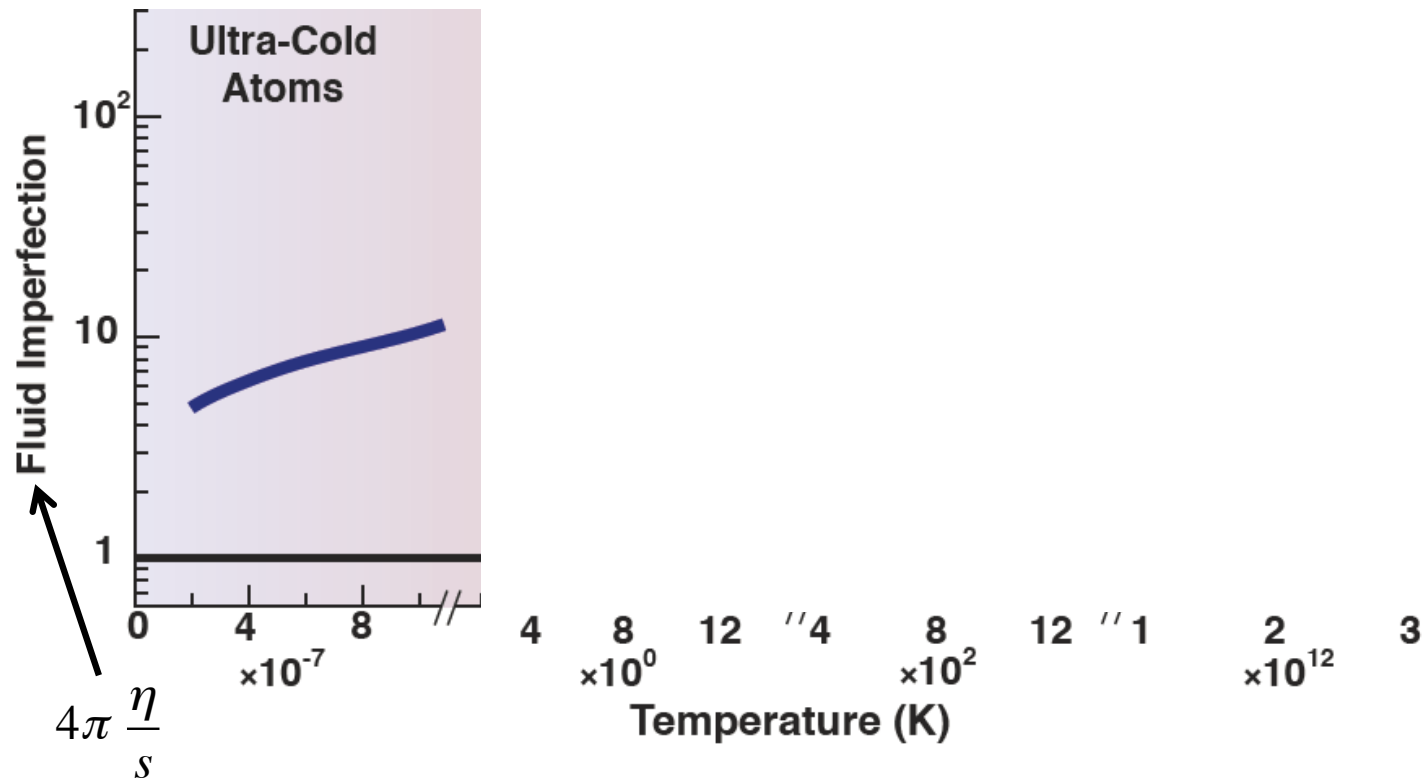
is actually a *real* picture of cold atoms expanding as a nearly perfect fluid with $\eta/s \sim (4-5)/4\pi$.

- John Thomas and collaborators

- *Observation of a Strongly-Interacting Degenerate Fermi Gas of Atoms*,
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Quark-Gluon Plasma Remains the Winner...



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Small System Collectivity in Relativistic Hadronic and Nuclear Collisions

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Abstract

The bulk motion of nuclear matter at the ultrahigh temperatures created in heavy ion collisions at the Relativistic Heavy Ion Collider and the Large Hadron Collider is well described in terms of nearly inviscid hydrodynamics, thereby establishing this system of quarks and gluons as the most perfect fluid in nature. A revolution in the field is under way, spearheaded by the discovery of similar collective, fluid-like phenomena in much smaller systems including p - p , p - A , d - Au , and $^3\text{He}+Au$ collisions. We review these exciting new observations and their profound implications for hydrodynamic descriptions of small and/or out-of-equilibrium systems.

Keywords

QCD, RHIC, LHC, heavy ion collisions, quark-gluon plasma, QGP, relativistic hydrodynamics, perfect liquid, shear viscosity, relativistic fluid dynamics, gauge/gravity duality

1. INTRODUCTION