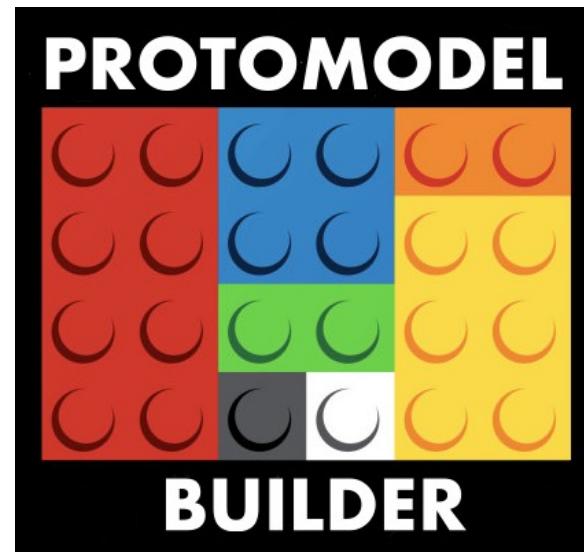


# ARTIFICIAL PROTO-MODELLING:

## BUILDING PRECURSORS OF A NEXT STANDARD MODEL FROM SIMPLIFIED MODELS RESULTS

arXiv:2012.12246



Reinterpretation  
forum workshop,  
18 feb 2021

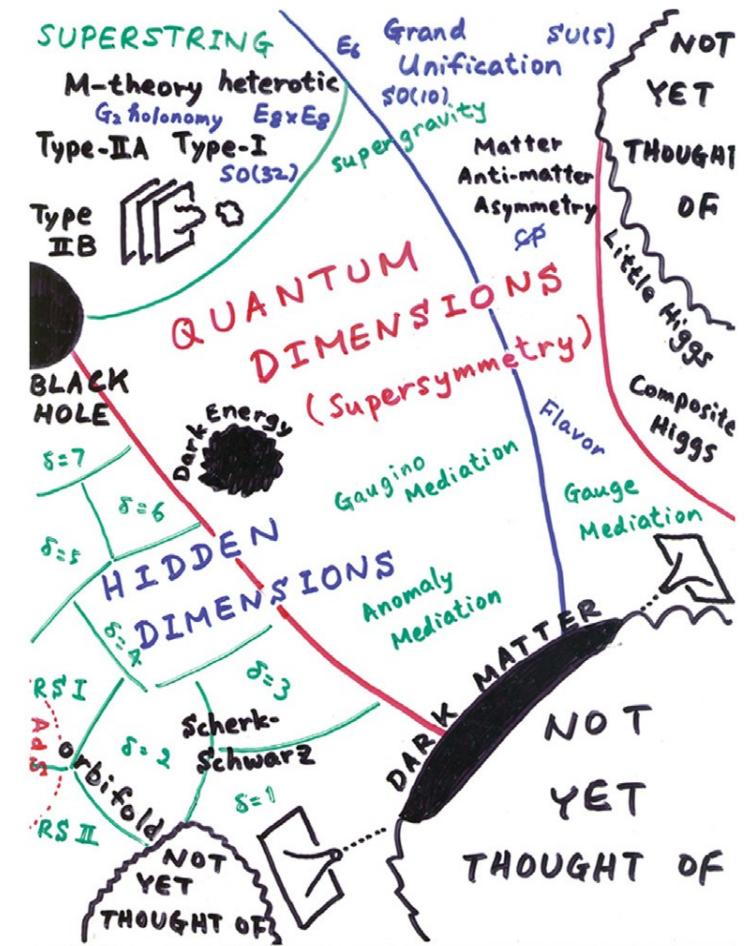
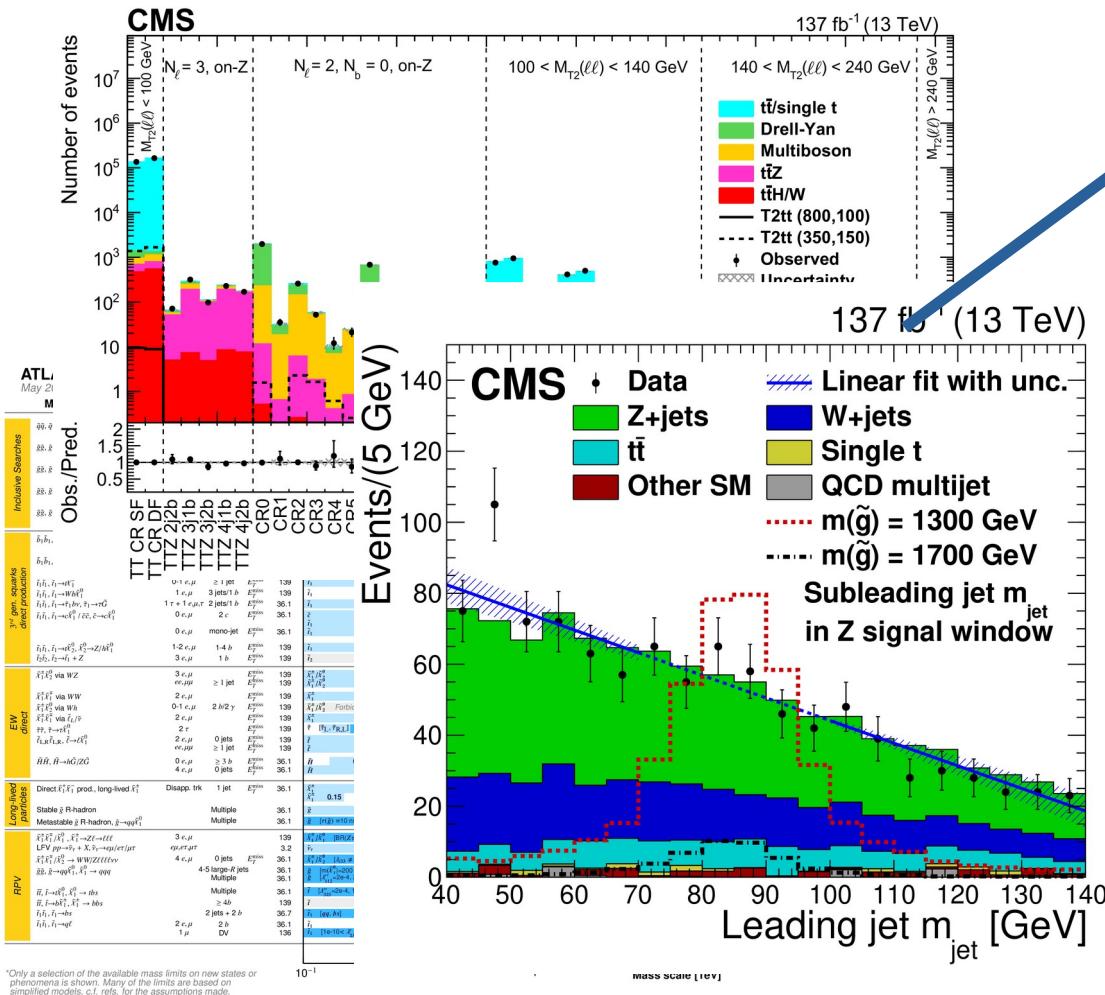


Wolfgang Waltenberger (ÖAW and Uni Vienna),  
Andre Lessa (UFABC São Paulo), Sabine Kraml (LPSC Grenoble)

<https://smodels.github.io/protomodels>

# PROBLEM STATEMENT

How will we infer the right hypothetical Next Standard Model (NSM) from the deluge of experimental results? Classical hypothesis testing might not anymore do the trick.



→ The Inverse Problem of  
Particle Physics

# OUR APPROACH

# OUR APPROACH

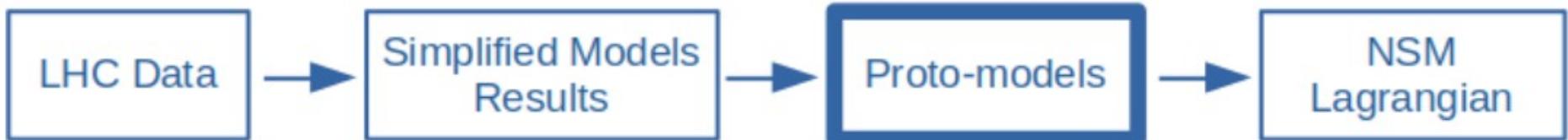


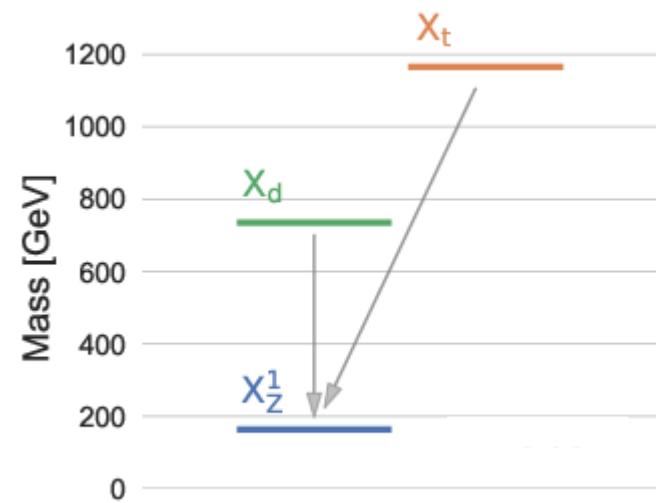
Fig. 1

Instead of testing BSM scenarios one-by-one against the experimental data:

- identify dispersed signals in the slew of published LHC analyses
- build candidate “proto-models” from them.

MCMC random walk through  
“proto-model” space of:

- particle content
- masses
- signal strengths [!]
- branching ratios

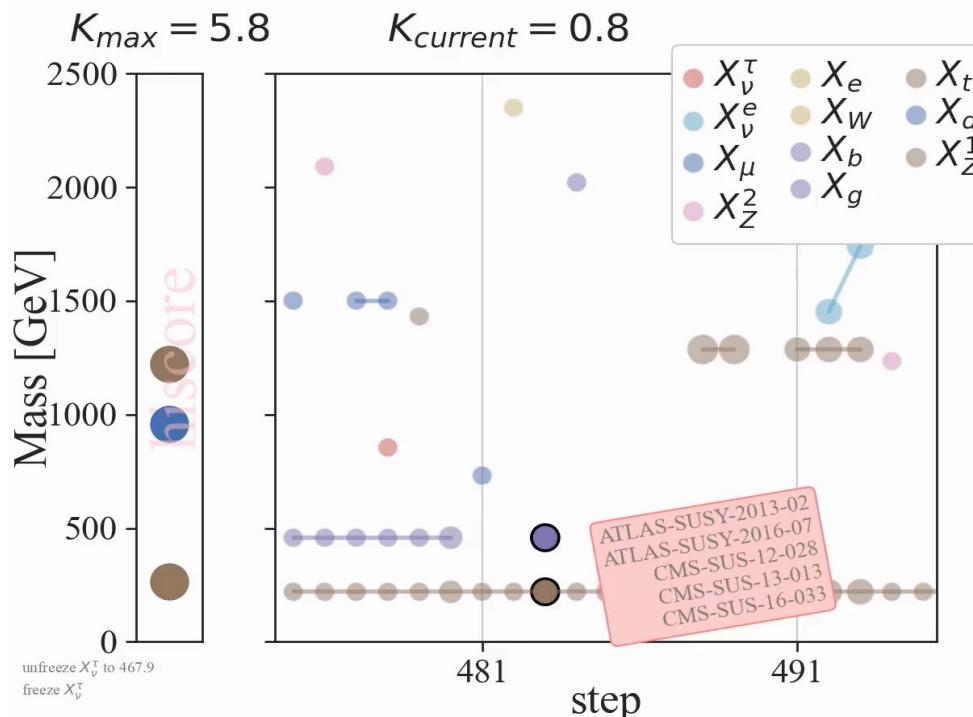


*a test statistic*

# OUR APPROACH

Particle spectra

A hiscore protomodel



Random modifications

an MCMC walk

<https://smmodels.github.io/protomodels/videos>

See also Sec. 4.2 “Toy Walk”  
for an illustration based on a  
very limited database

# INPUT DATA

The test statistic is based on likelihoods

- likelihood computation based on simplified models results in SModelS database
- vast number of efficiency and upper limit maps from 47 CMS and 48 ATLAS publications.

#	ID	Short Description	Type	$\mathcal{L} [\text{fb}^{-1}]$
1	CMS-PAS-EXO-16-036	hscp search	ul, eff	12.9
2	CMS-PAS-SUS-16-052	soft l, $<= 2$ jets	ul, eff	35.9
3	CMS-SUS-16-009	multijets + $\cancel{E}_T$ , top tagging	ul	2.3
4	CMS-SUS-16-032	Sbottom and compressed stop	ul	35.9
5	CMS-SUS-16-033	$0\ell + \text{jets} + \cancel{E}_T$	ul, eff	35.9
6	CMS-SUS-16-034	2 OSSF l's	ul	35.9
7	CMS-SUS-16-035	2 SS l's	ul	35.9
8	CMS-SUS-16-036	$0\ell + \text{jets} + \cancel{E}_T$	ul	35.9
9	CMS-SUS-16-037	$1\ell + \text{jets} + \cancel{E}_T$ with MJ	ul	35.9
10	CMS-SUS-16-039	multi-l EWK searches	ul	35.9
11	CMS-SUS-16-041	multi-ls + jets + $\cancel{E}_T$	ul	35.9
12	CMS-SUS-16-042	$1\ell + \text{jets} + \cancel{E}_T$	ul	35.9
13	CMS-SUS-16-043	EWK WH	ul	35.9
14	CMS-SUS-16-045	Sbottom to bHbH and $H \rightarrow \gamma\gamma$	ul	35.9
15	CMS-SUS-16-046	$\gamma + \cancel{E}_T$	ul	35.9
16	CMS-SUS-16-047	$\gamma + \text{HT}$	ul	35.9
17	CMS-SUS-16-049	All hadronic stop	ul	35.9
18	CMS-SUS-16-050	$0\ell + \text{top tag}$	ul	35.9
19	CMS-SUS-16-051	$1\ell$ stop	ul	35.9
20	CMS-SUS-17-001	Stop search in dil + jets + $\cancel{E}_T$	ul	35.9
21	CMS-SUS-17-003	2 taus + $\cancel{E}_T$	ul	35.9
22	CMS-SUS-17-004	EW-ino combination	ul	35.9
23	CMS-SUS-17-005	$1\ell + \text{multijets} + \cancel{E}_T$ , top tagging	ul	35.9
24	CMS-SUS-17-006	jets + boosted H(bb) + $\cancel{E}_T$	ul	35.9
25	CMS-SUS-17-009	SFOS l's + $\cancel{E}_T$	ul	35.9
26	CMS-SUS-17-010	2L stop	ul	35.9
27	CMS-SUS-18-002	$\gamma$ , jets, b-jets + $\cancel{E}_T$ , top tagging	ul	35.9
28	CMS-SUS-19-006	$0\ell + \text{jets}, \text{MHT}$	ul	137.0

14	CMS-SUS-13-012	$n_{\text{jets}} + \text{HTmiss}$	ul, eff	19.5
15	CMS-SUS-13-013	2 SS l's + (b-)jets + $\cancel{E}_T$	ul, eff	19.5
16	CMS-SUS-13-019	$>= 2$ jets + $\cancel{E}_T$ , MT2	ul	19.5
17	CMS-SUS-14-010	b-jets + 4 Ws	ul	19.5
18	CMS-SUS-14-021	soft l's, low $n_{\text{jets}}$ , high $\cancel{E}_T$	ul	19.7

#	ID	Short Description	Type	$\mathcal{L} [\text{fb}^{-1}]$
1	ATLAS-SUSY-2015-01	2 b-jets + $\cancel{E}_T$	ul	3.2
2	ATLAS-SUSY-2015-02	single l stop	ul, eff	3.2
3	ATLAS-SUSY-2015-06	$0\ell$ 's + 2-6 jets + $\cancel{E}_T$	eff	3.2
4	ATLAS-SUSY-2015-09	jets + 2 SS l's or $>= 3$ l's	ul	3.2
5	ATLAS-SUSY-2016-07	$0\ell + \text{jets} + \cancel{E}_T$	ul, eff	36.1
6	ATLAS-SUSY-2016-14	2 SS or 3 l's + jets + $\cancel{E}_T$	ul	36.1
7	ATLAS-SUSY-2016-15	$0\ell$ stop	ul	36.1
8	ATLAS-SUSY-2016-16	$1\ell$ stop	ul, eff	36.1
9	ATLAS-SUSY-2016-17	2 opposite sign l's + $\cancel{E}_T$	ul	36.1
10	ATLAS-SUSY-2016-19	stops to staus	ul	36.1
11	ATLAS-SUSY-2016-24	2-3 l's + $\cancel{E}_T$ , EWino	ul, eff	36.1
12	ATLAS-SUSY-2016-26	$>= 2$ c jets + $\cancel{E}_T$	ul	36.1
13	ATLAS-SUSY-2016-27	jets + $\gamma + \cancel{E}_T$	ul, eff	36.1
14	ATLAS-SUSY-2016-28	2 b-jets + $\cancel{E}_T$	ul	36.1
15	ATLAS-SUSY-2016-33	2 OSSF l's + $\cancel{E}_T$	ul	36.1
16	ATLAS-SUSY-2017-01	EWK WH(bb) + $\cancel{E}_T$	ul	36.1
17	ATLAS-SUSY-2017-02	$0\ell + \text{jets} + \cancel{E}_T$	ul	36.1
18	ATLAS-SUSY-2017-03	multi-l EWK searches	ul	36.1
19	ATLAS-SUSY-2018-04	2 hadronic taus	ul, eff	139.0
20	ATLAS-SUSY-2018-06	3 l's EW-ino	ul	139.0
21	ATLAS-SUSY-2018-31	2b + 2H(bb) + $\cancel{E}_T$	ul, eff	139.0
22	ATLAS-SUSY-2018-32	2 OS l's + $\cancel{E}_T$	ul	139.0
23	ATLAS-SUSY-2019-08	$1\ell + \text{higgs} + \cancel{E}_T$	ul, eff	139.0
14	ATLAS-SUSY-2013-19	2 OS l's + (b-)jets + $\cancel{E}_T$	ul	20.3
15	ATLAS-SUSY-2013-21	monojet or c-jet + $\cancel{E}_T$	eff	20.3
16	ATLAS-SUSY-2013-23	$1\ell + 2$ b-jets (or 2 $\gamma$ s) + $\cancel{E}_T$	ul	20.3
17	ATLAS-SUSY-2014-03	$>= 2$ (c-)jets + $\cancel{E}_T$	eff	20.3

# THE COMBINER

As we are chasing dispersed signals, we need to allow the machine to combine likelihoods.

ATLAS, 13 TeV

green:  
approximately  
uncorrelated  
→ combinable

red: correlated,  
not combinable

White: cannot  
construct a  
likelihood

Signal regions  
within each  
analysis:  
correlated

Les Houches effort:

<https://arxiv.org/abs/2002.12220>

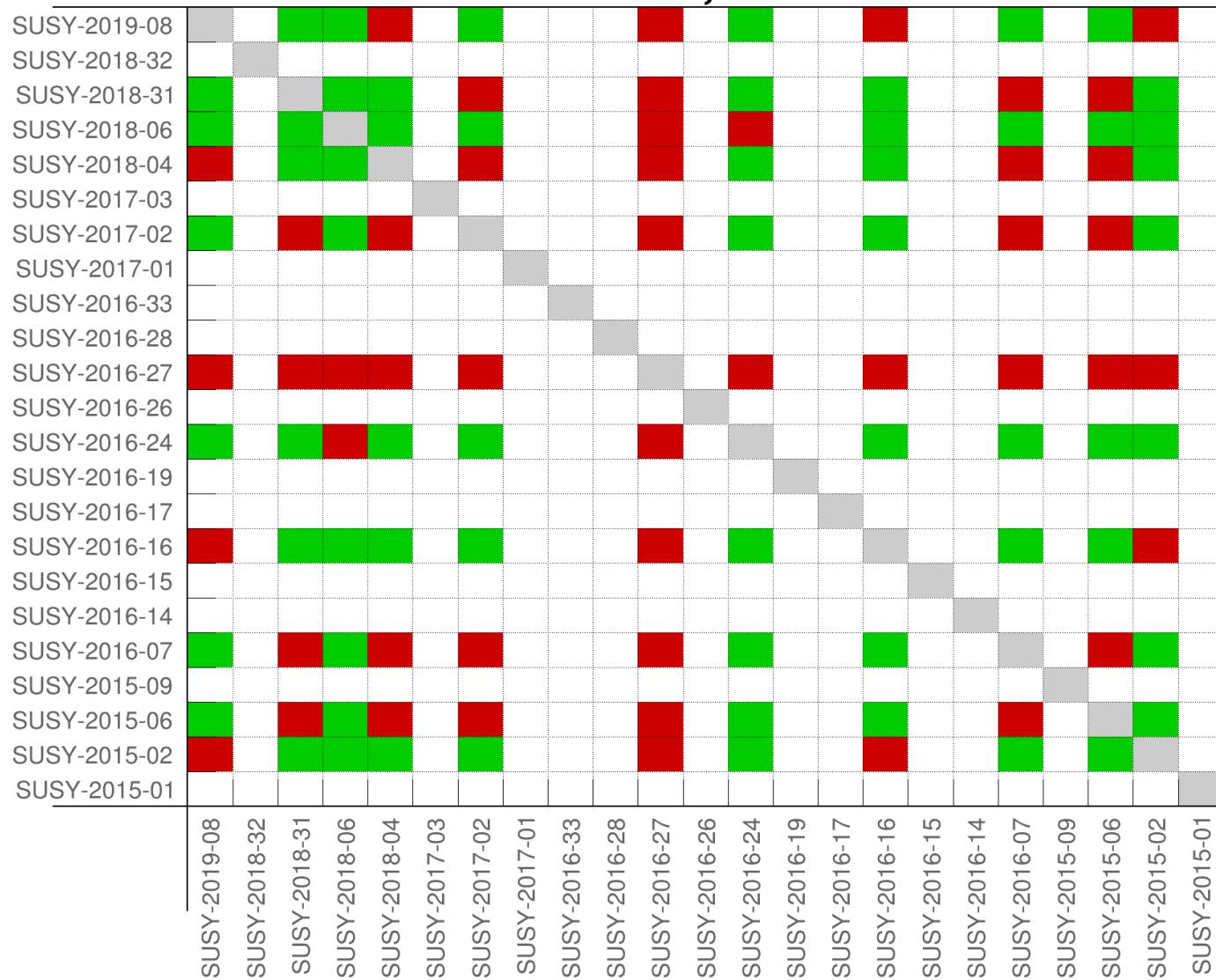
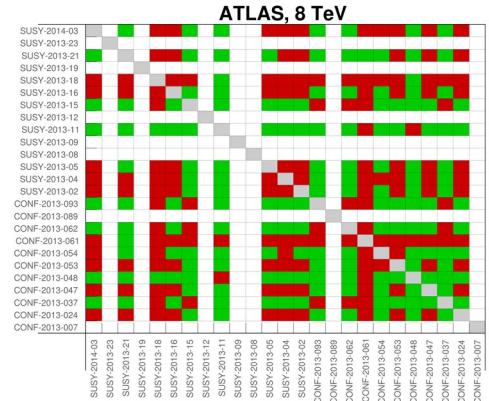
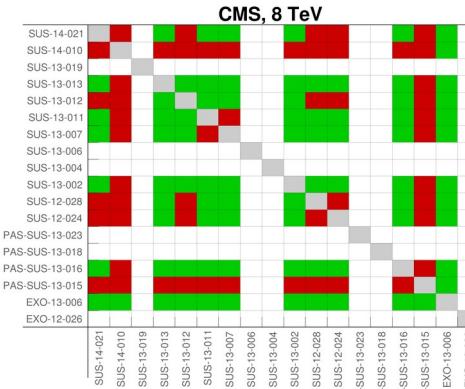


Fig. 2



# THE COMBINER

we allow the machine  
to combine likelihoods.

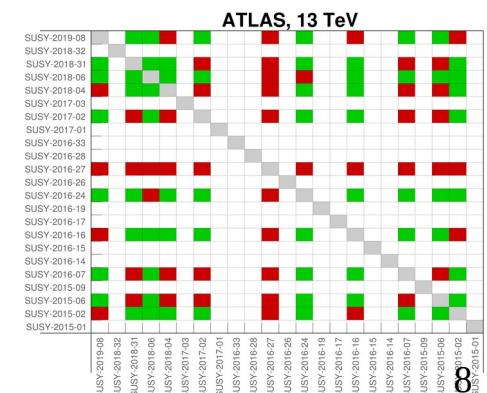
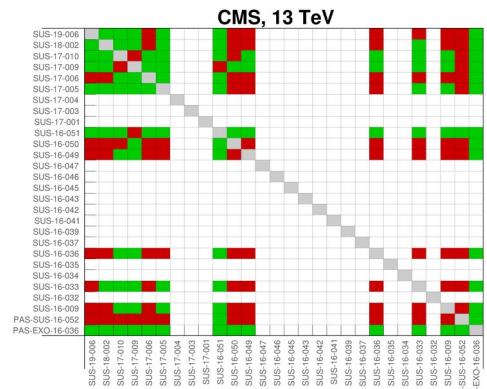
Fig. 2

Approximately uncorrelated are analyses that are:

- from different runs, and/or
  - from different experiments, and/or
  - looking for (clearly) different signatures

A combination “c” of analyses is “legal” if the following conditions are met:

- all results are mutually uncorrelated (= "combinable")
  - if a result can be added, it has to be added (any subset of a legal combination is not itself legal)
  - combined likelihood:  $L_c = \prod_{i \in c} L_i$



# THE TEST STATISTIC

For every legal combination, we define a test statistic  $K$

$$K^c := -2 \ln \frac{L_{\text{SM}}^c \cdot \pi(\text{SM})}{L_{\text{BSM}}^c(\hat{\mu}) \cdot \pi(\text{BSM})} \quad \text{Eq. 6}$$

(Remember, we have a database of results from  $\sim 100$  CMS+ATLAS searches. We want to find the most interesting combinations of these results, i.e. the ones that maximally violate the SM hypothesis)

Of all “legal” combinations of experimental results, the builder chooses the one combination “c” that maximizes  $K$ :

$$K := \max_{\forall c \in C} K^c \quad \text{Eq. 7}$$

$\mu$  denotes a global signal strength multiplier – the production cross sections are free parameters

$$\forall i, j : \sigma(pp \rightarrow X_i X_j) = \mu \bar{\sigma}(pp \rightarrow X_i X_j)$$

It is maximized in the denominator, but its support is confined such that no limits in the SModelS database are violated (the “critic”),

$$\hat{\mu} \in [0, \mu_{\max}]$$

# THE TEST STATISTIC

For every legal combination, we define a test statistic  $K$

$$K^c := -2 \ln \frac{L_{\text{SM}}^c \cdot \pi(\text{SM})}{L_{\text{BSM}}^c(\hat{\mu}) \cdot \pi(\text{BSM})} \quad \text{Eq. 6}$$

$\pi(\text{BSM})$  is the prior of the BSM model. We use it to “regularize” the model, i.e. impose the *law of parsimony*:

$$\pi(M) = \exp \left[ - \left( \frac{n_{\text{particles}}}{a_1} + \frac{n_{\text{BRs}}}{a_2} + \frac{n_{\text{production modes}}}{a_3} \right) \right] \quad \text{Eq. 9}$$

That way, one new particle with one non-trivial branching ratio and two production modes is similar to one degree of freedom in Akaike’s information criterion (the sign is however flipped, and it’s a likelihood ratio), i.e. the test statistic is roughly equivalent to

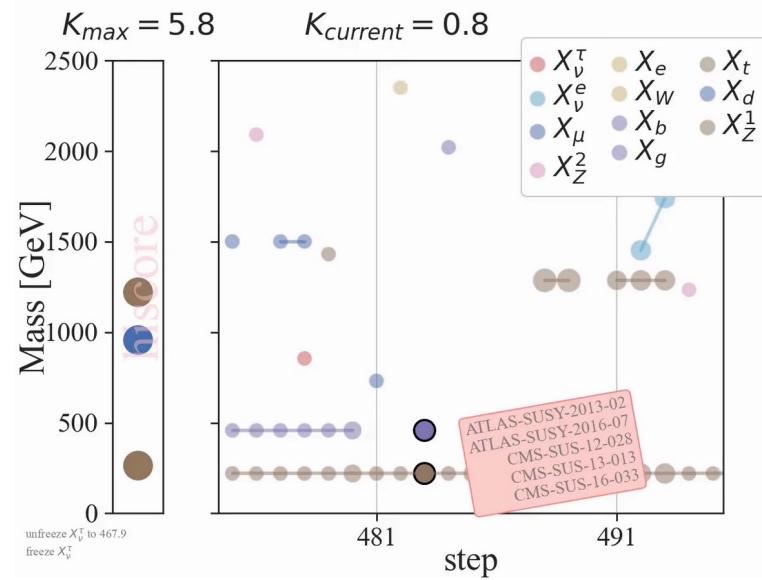
$$K \approx \Delta\chi^2 - 2n_{\text{particles}}$$

An additional particle will have to increase the “(delta-)chi-square” by approximately two units.

# THE WALKER

The Walker takes care of moving in the protomodel space with varying dimensionality by performing the following types of modifications to the protomodel:

- add or remove particles from the protomodel
- change the masses of particles
- change the signal strengths of production modes
- change decay channels and branching ratios



At each step the test statistic  $K$  is computed. An MCMC-like procedure[\*] is then applied in the sense that the step is reverted with a probability of

$$\exp \left[ \frac{1}{2} (K_i - K_{i-1}) \right]$$

if and only if  $K_i$  is smaller than  $K_{i-1}$

## Appendix A.1

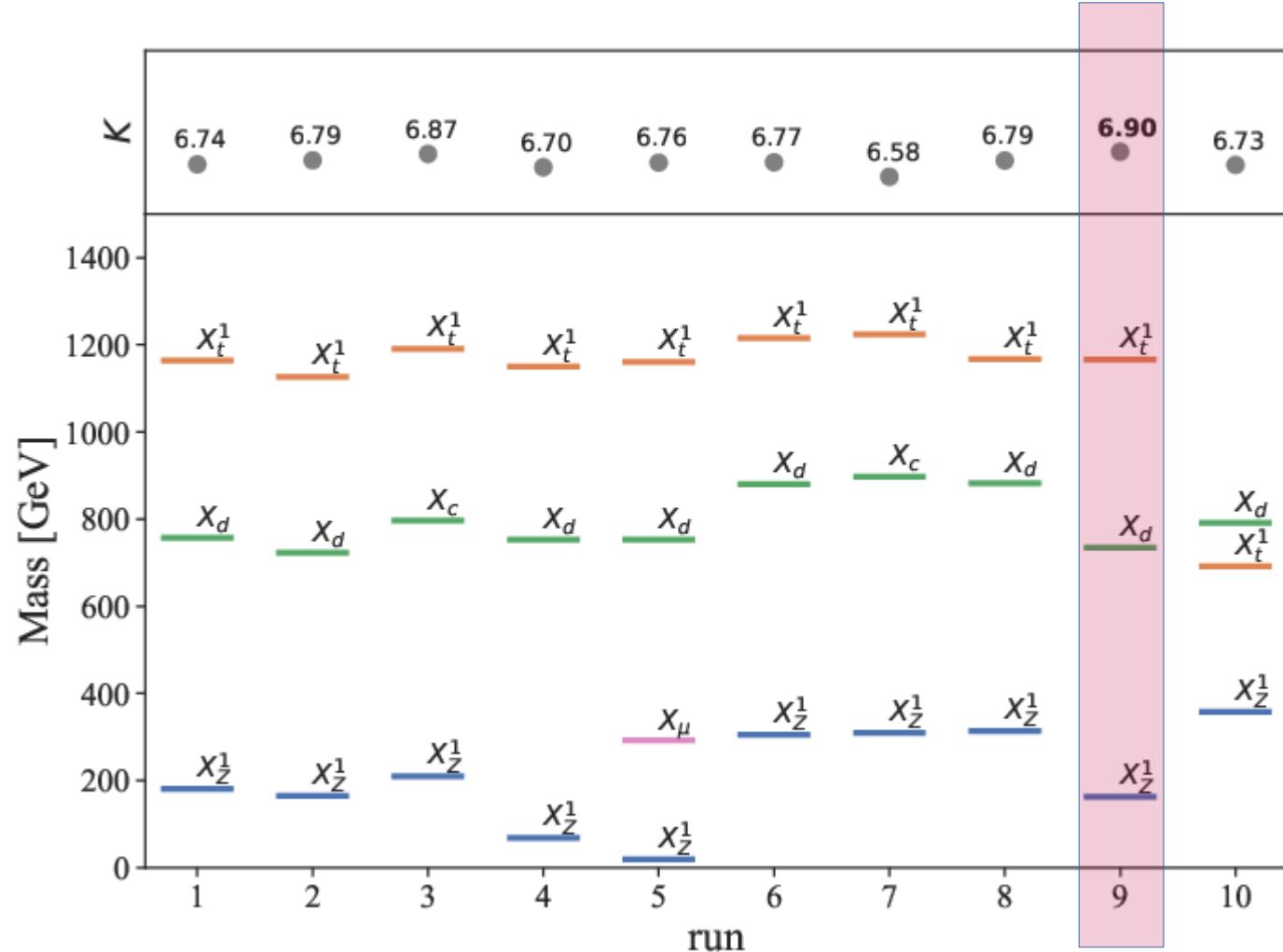
\* (note however, instead of ratios of unnormalized posteriors we have ratios of ratios of unnormalized posteriors)

# RESULTS

(will keep this brief – closure tests, discussions of a posteriori distributions, proofs of convergence, and more in the backup)

# WALKING OVER THE SMODELS DATABASE

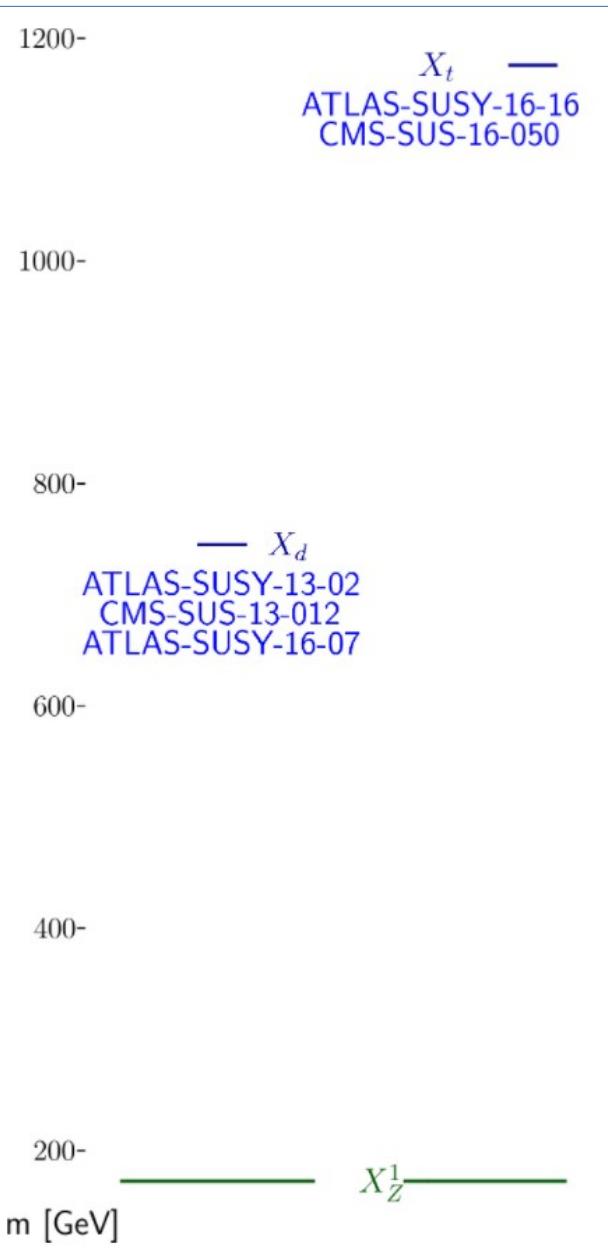
We defined a “run” as 50 parallel walkers, making 1,000 steps each.  
We performed 10 such runs on the SModelS database:



Sec 5.2  
Fig. 8

All 10 runs introduced a top partner as well as a light quark partner. The cross sections are compatible with values expected from the MSSM. The best test statistic was  $K=6.9$ .

# THE HISCORE PROTO-MODEL



Analysis	Dataset	Obs	Exp	Z	P	Signal
ATL multijet, 8 TeV [54]	SR6jtp	6	$4.9 \pm 1.6$	$0.4 \sigma$	$X_d$	0.25
ATL multijet, 13 TeV [55]	2j_Me ...	611	$526 \pm 31$	$2.2 \sigma$	$X_d$	44.18
ATL $1\ell$ stop, 13 TeV [48]	tN_high	8	$3.8 \pm 1$	$1.9 \sigma$	$X_t$	3.93
CMS multijet, 8 TeV [56]		30.8 fb	19.6 fb	$1.1 \sigma$	$X_d$	2.66 fb
CMS $0\ell$ stop, 13 TeV [49]		4.5 fb	2.5 fb	$1.6 \sigma$	$X_t$	2.62 fb

Table 3: Analyses contributing to the  $K$  value of the highest score proto-model

Table 3: the dispersed excess

Analysis (all CMS 13 TeV)	Prod	$\sigma_{XX}$ (fb)	$\sigma_{obs}^{UL}$ (fb)	$\sigma_{exp}^{UL}$ (fb)	$r_{obs}$
CMS multijet, $M_{H_T}$ , $137 \text{ fb}^{-1}$ [15]	$(\bar{X}_d, X_d)$	23.96	18.45	21.57	1.30
CMS multijet, $M_{H_T}$ , $137 \text{ fb}^{-1}$ [15]	$(\bar{X}_t, X_t)$	2.62	2.04	2.08	1.28
CMS multijet, $M_{H_T}$ , $36 \text{ fb}^{-1}$ [57]	$(\bar{X}_d, X_d)$	23.96	19.26	28.31	1.24
CMS multijet, $M_{T_2}$ , $36 \text{ fb}^{-1}$ [58]	$(\bar{X}_d, X_d)$	23.96	26.02	31.79	0.92
CMS $1\ell$ stop, $36 \text{ fb}^{-1}$ [59]	$(\bar{X}_t, X_t)$	2.62	2.91	4.44	0.90

Table 4: List of the most constraining results for the highest score proto-model. The

Table 4: what is driving the “critic”

Signal strength multipliers:  $(\bar{X}_t, X_t) = 1.2$ ;  $(\bar{X}_d, X_d), (X_d, X_Z^1), (\bar{X}_d, X_Z^1) = 0.49$

Contributions by particles:  $X_t : K_{\text{without}} = 2.59(59\%), X_d : K_{\text{without}} = 3.90(41\%)$   
Last updated: Mon Dec 14 20:08:06 2020

# GLOBAL P-VALUE

- running over “fake” databases with the fake observations obtained from sampling the background-only statistical model of the results
- from this a **global p-value** for the Standard Model hypothesis is computed as:

$$p_{\text{global}} := \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{i=1}^N 1_{[\bar{K}_{\text{obs}}, \infty)}(K_{\text{fake}}^i) \approx \int_{\bar{K}_{\text{obs}}}^{\infty} dK \rho(K) \approx 0.19 \quad \text{Eq. 12}$$

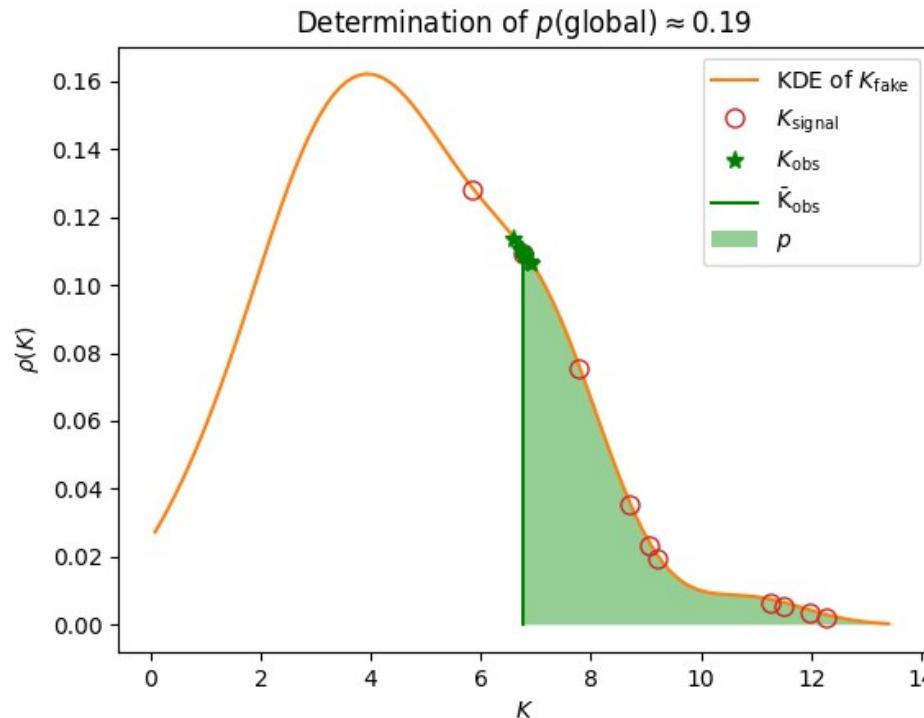


Fig. 9

**No look-elsewhere effect applies.** (Performing a meta-statistical analysis of the results in the SModelS database we are confident that the estimate is conservative – see slide 22 in backup)

# FUTURE IMPROVEMENTS

## Improvements of the SModelS database:

- add latest full run-2 CMS and ATLAS publications (Moriond!)
- produce efficiency maps for existing results
- enlarge mass range of older efficiency maps

## Improvements in speed:

- learn the SModelS database
- make everything differentiable

## Improvements in procedure:

- improve the “analyses correlation matrix”, automate the determination
- ponder relationship between proto-models and effective field theories
- connect proto-models with complete theories

<https://smodels.github.io/protomodels>

# BACKUP

# MUTUAL (IN-)COMPATIBILITY

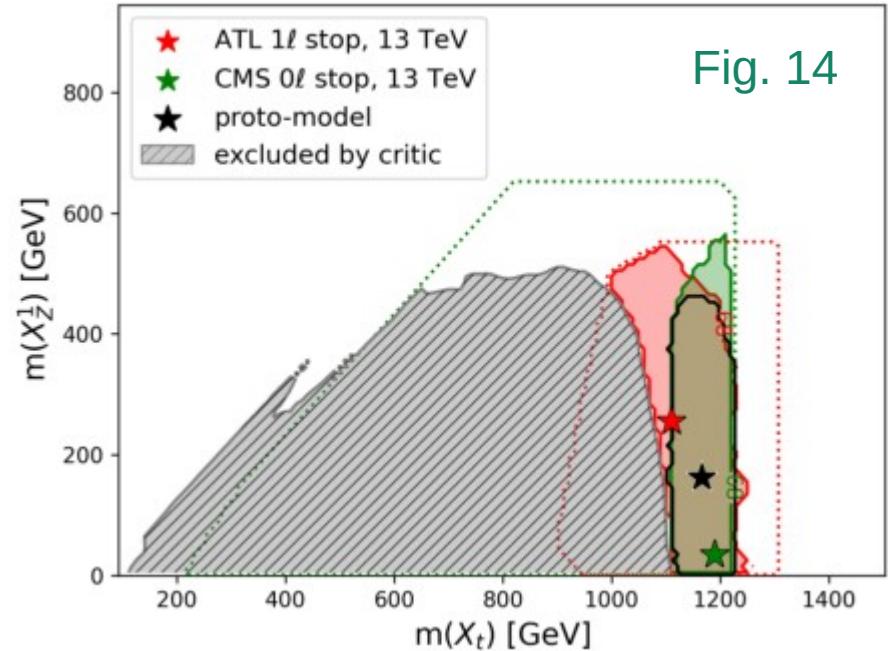
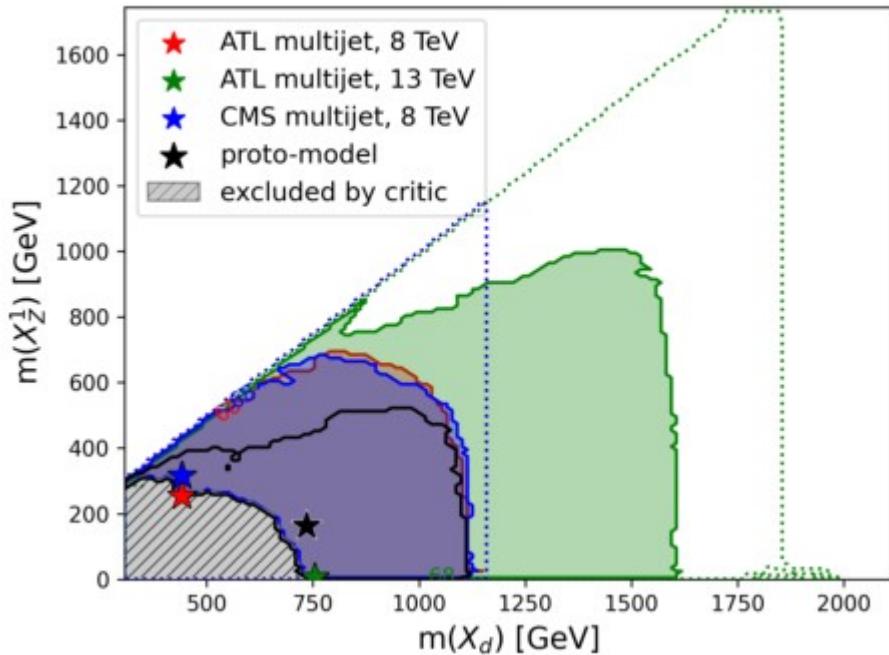


Fig. 14

**68% Bayesian credibility regions of the particle masses**, fixating all other parameters.

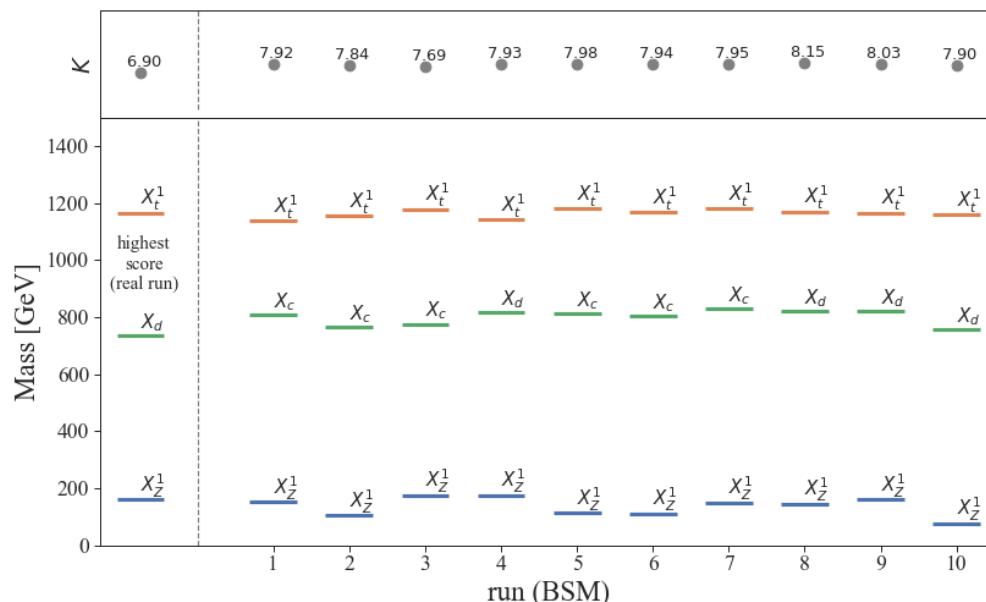
- very little handle on the masses
- results suffer from the fact that the efficiency and upper limit maps are limited in the mass ranges (the dashed lines are the limits of the maps). → try and fix in next iteration.
- tension between builder and critic – will understand this better with future, improved, efficiency maps
- Aim for full posteriors in next iteration of this effort

# WALKING OVER DATABASES WITH FAKE SIGNALS

To show closure of our method, we inject the winning protomodel as a signal in fake databases, and see if the algorithm can reconstruct the injected signal.

Sec 5.3

Technical closure test



Physics closure test

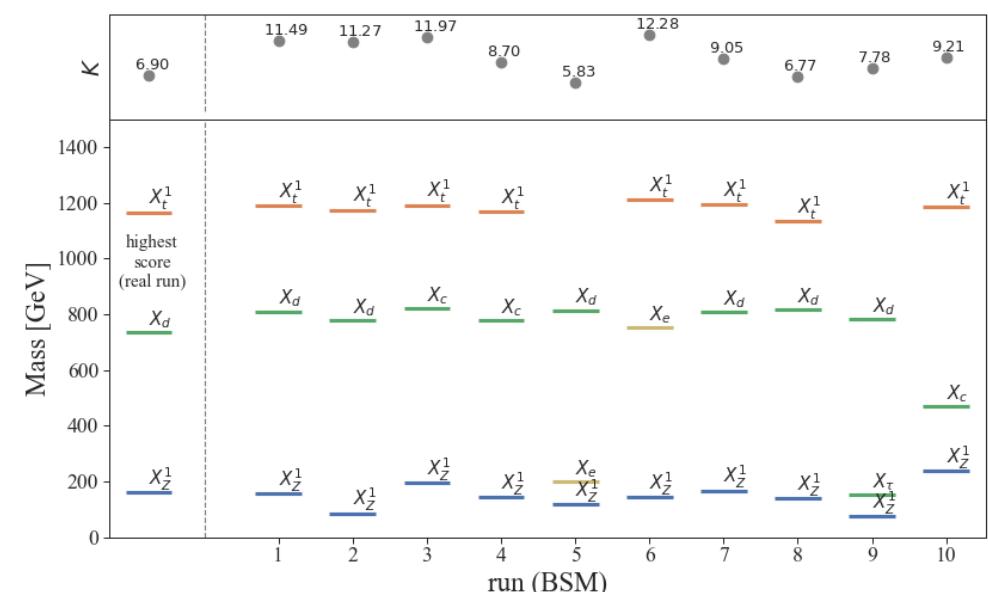


Fig. 11

No sampling of the models for the SRs, i.e.  
observed events := expected SM +  
expected signal events

Sampling turned on

Fig. 10

# THE WALKS

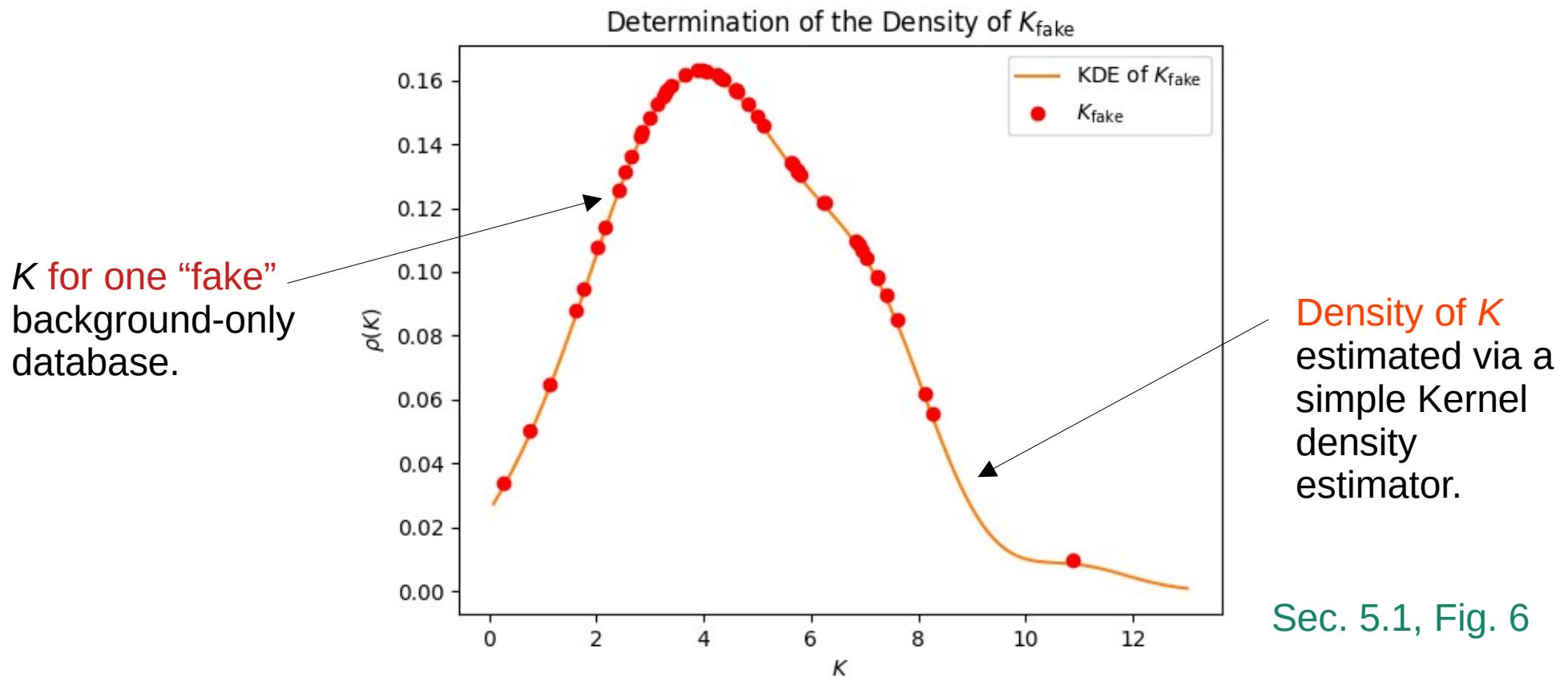
We define a “run” as 50 parallel walks, each taking 1000 steps.

We performed

- 10 runs on the SModelS database ([Sec. 5.2](#))
- 50 runs on fake “Standard Model-like” databases ([Sec 5.1](#))  
to be able to determine a global  $p$ -value under the SM hypothesis
- 2x10 runs on fake “Signal-like” databases ([Sec 5.3](#))  
to show closure of the method

# WALKING OVER FAKE STANDARD MODEL DATABASES

- Produced 50 “fake” SModelS databases by sampling background models
- Corresponds to typical LHC results if no new physics is in data
- Determine 50 “fake”  $K$  values by running 50 walkers on each of the 50 databases (50 x 50 walkers in total) → density of  $K$  under null SM-only hypothesis



# *P*-VALUES PER SIGNAL REGION

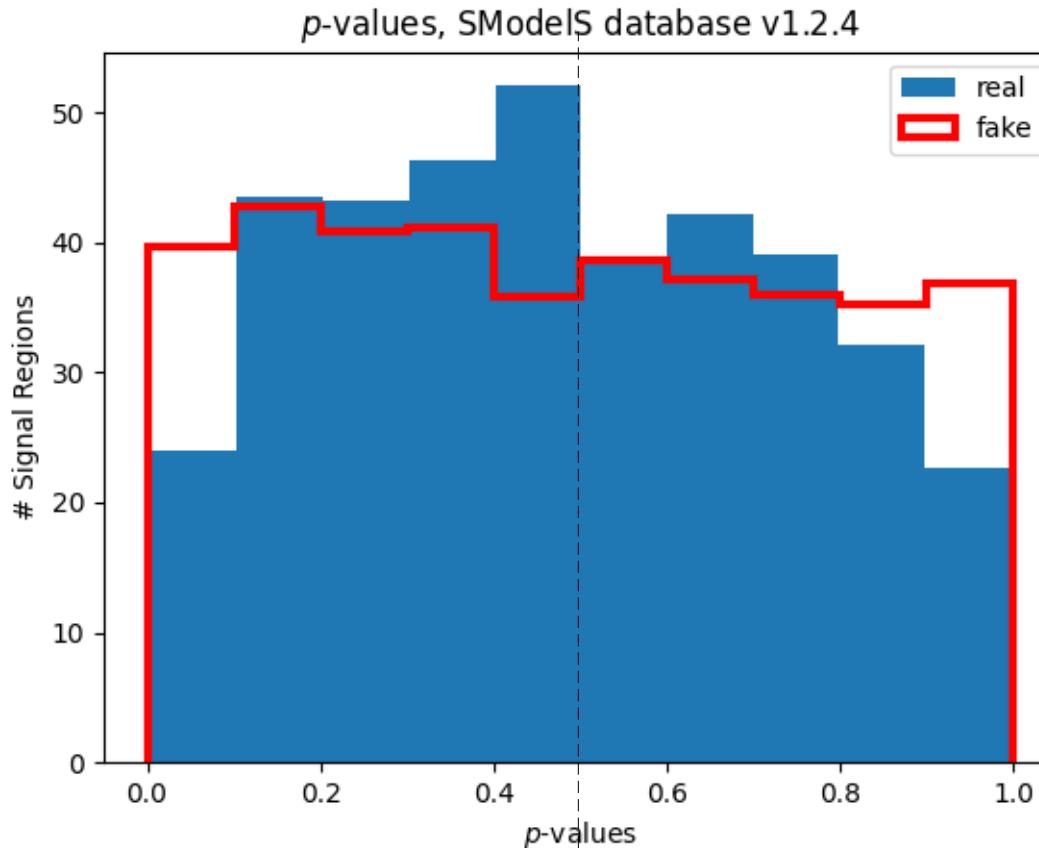


Fig. 7

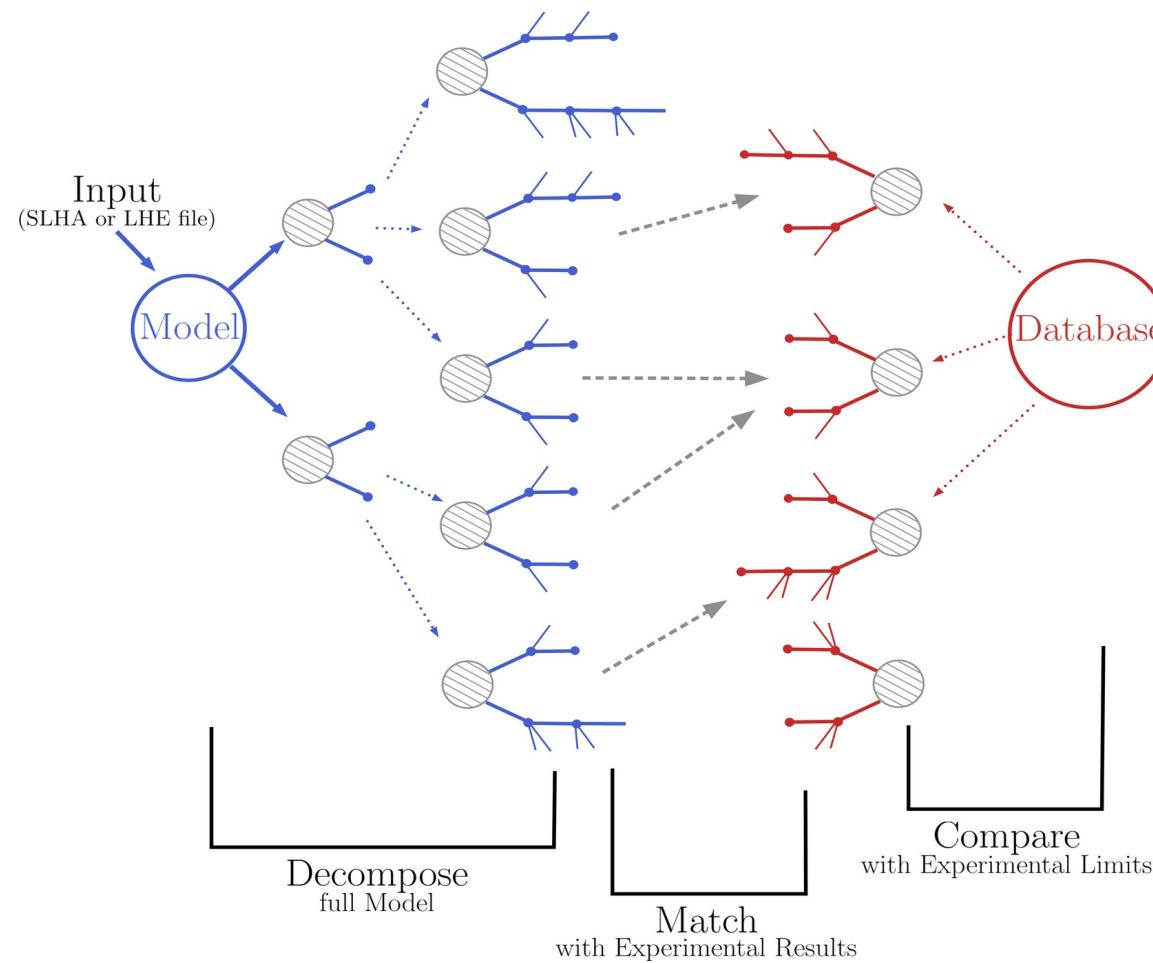
- *p*-values for signal regions in SModelS database
- errors on background estimated modelled as (“single enveloping”) Gaussian
- filtered out regions with expected number of the events < 3.5
- blue area is **real data**, red line is “fake” BG-only simulated databases
- results compatible with idea that **BG errors are conservative**, see also [arXiv:1410.2270](https://arxiv.org/abs/1410.2270)
- slightly **more excesses** ( $p \rightarrow 0$ ) than underfluctuations ( $p \rightarrow 1$ )

→ ***p*<sub>global</sub> is most likely conservative!**

# SModelS – a decomposer and a database



We decompose full theories into SMS topologies, and match them against our database. Depending on how much information we have access to, we can do different things.

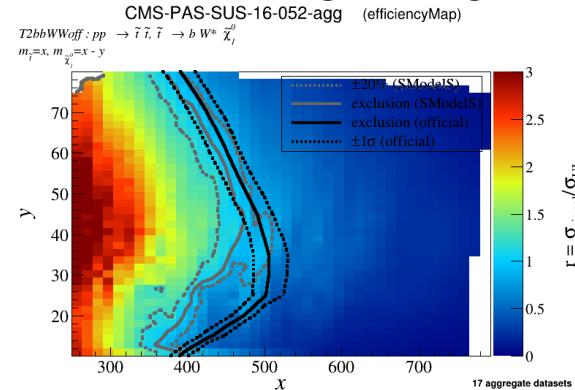


# SModelS – a decomposer and a database



## efficiency maps with simplified likelihoods

Around 2017/18, CMS started to publish simplified likelihoods for a handful of analyses, making it possible for outsiders to combine signal regions. Until then, SModelS has never been able to combine SRs.



**Simplified likelihood, v1:** All nuisances summarized in a single “all enveloping” multivariate Gaussian that “connects” all signal regions (which are Poissonian counting variables):

$$\mathcal{L}_S(\mu, \theta) = \prod_{i=1}^N \frac{(\mu \cdot s_i + b_i + \theta_i)^{n_i} e^{-(\mu \cdot s_i + b_i + \theta_i)}}{n_i!} \cdot \exp \left( -\frac{1}{2} \theta^T \mathbf{V}^{-1} \theta \right)$$

CMS-NOTE-2017-001

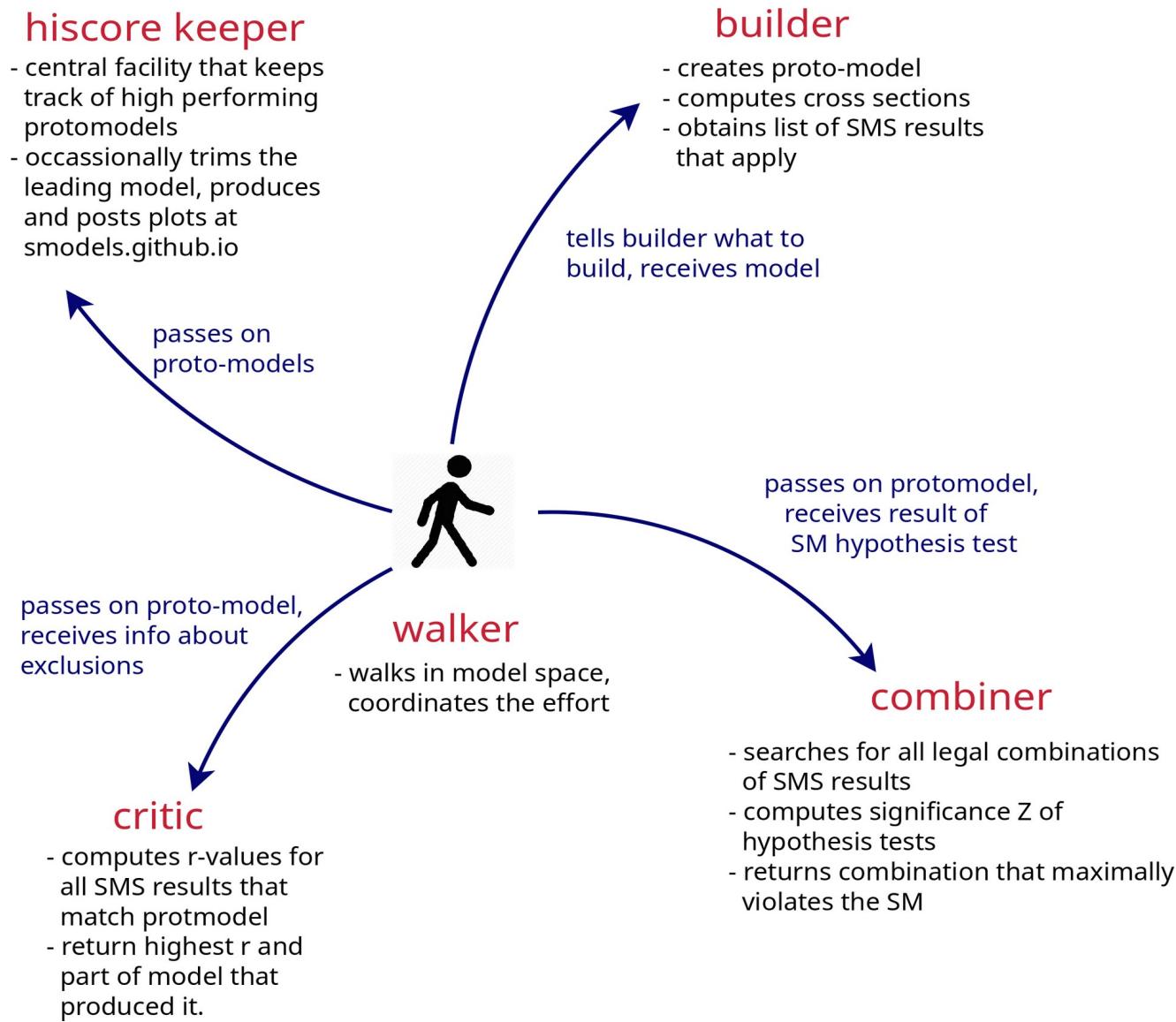
**Simplified likelihood, v2:** a skewness term is added to allow for asymmetrical distributions.

$$L_S(\alpha, \theta) = \prod_{I=1}^P \Pr\left(n_I^{\text{obs}} \mid n_{s,I}(\alpha) + a_I + b_I \theta_I + c_I \theta_I^2\right) \cdot \frac{e^{-\frac{1}{2}\theta^T \rho^{-1} \theta}}{\sqrt{(2\pi)^P}}$$

arXiv:1809.05548

JHEP 04 (2019) 064

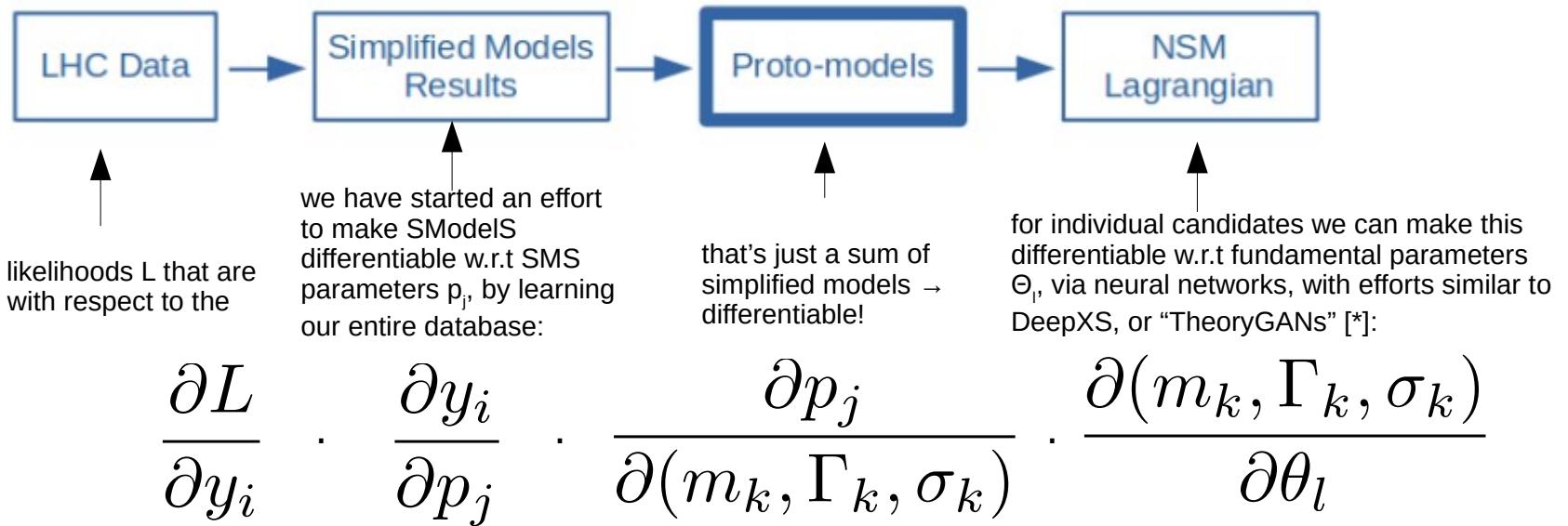
# proto-models



# WHY DIFFERENTIABLE?



If we had gradients we could perform gradient descent to find the best model, and we could use e.g. the Fisher information to infer the error on its parameters (or, alternatively we can then MCMC-sample).



Needless to say, the data pipeline sketched above is not the only feasible one. Differentiability however would be a helpful tool for all possible data pipelines. A similar rationale would apply also to EFTs, Wilson coefficients and data from measurements.

<https://arxiv.org/abs/1810.08312>

→ DIFFERENTIABLE INDUCTIVE REASONING!