Theoretical study of forbidden non-unique beta transitions

X. Mougeot

Radionuclide metrology

LNHB (National Laboratory Henri Becquerel) is the French Designated Institute for primary standards in ionizing radiation metrology.

→ **Definition of activity (Bq) and dose (Sv, Gy) units.**

The diversity of radioactive processes makes necessary a certain pre-knowledge to establish primary and secondary standards: decay schemes, atomic and nuclear data.

→ **Evaluation of atomic and nuclear decay data.**

✓ Coordination of the DDEP (Decay Data Evaluation Project) international collaboration.

✓ Decay data officially recommended by the BIPM.

Extension of SIR to almost pure beta emitters

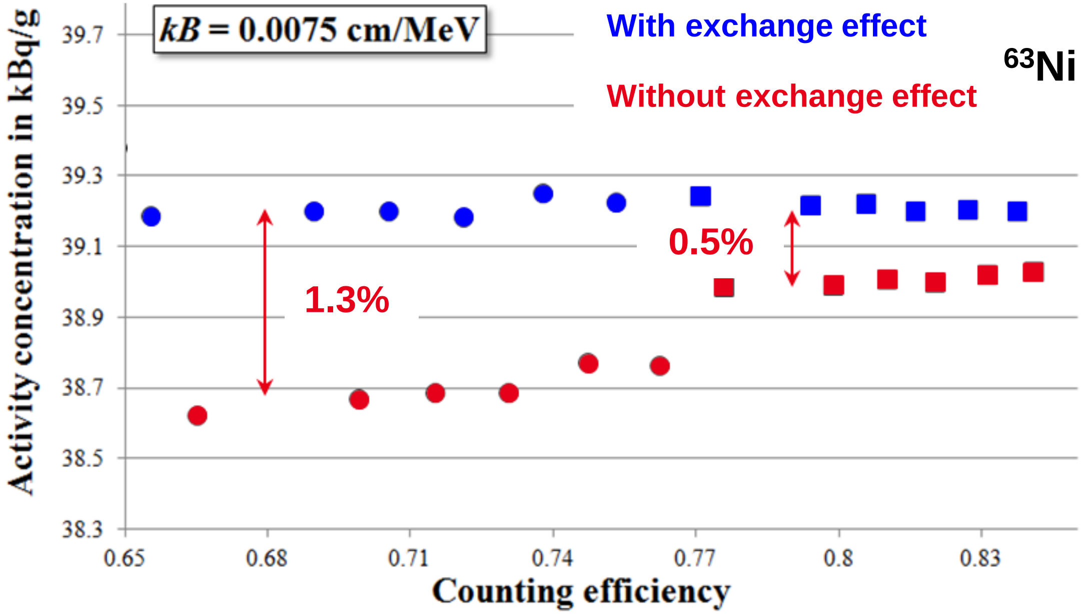
→ Primary activity by liquid scintillation counting.

→ Strong non-linear efficiency at low energy.

→ **Sensitivity to beta spectrum shape**.

Estimate of deposited dose in patient's cells

→ **Impact at DNA level**.

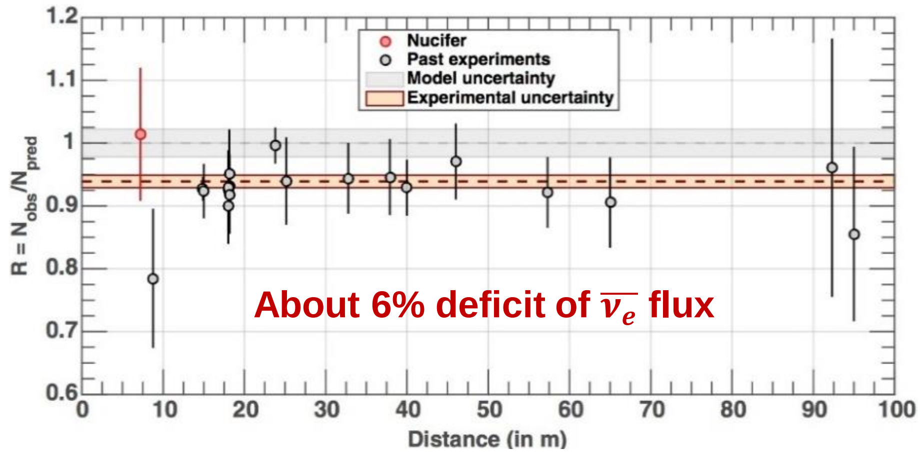


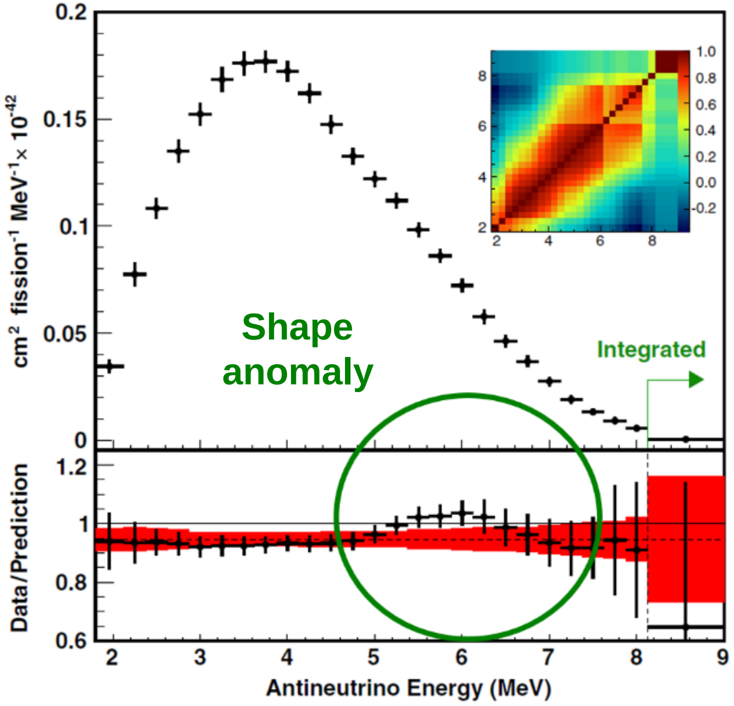
^{63}Ni: K. Kossert, X. Mougeot, Appl. Radiat. Isot. 101, 40 (2015)

^{60}Co: K. Kossert et al., Appl. Radiat. Isot. 134, 212 (2018)

^{90}Sr/^{90}Y: K. Kossert, X. Mougeot, Appl. Radiat. Isot. 168, 109478 (2021)

Sterile neutrino?





First evidence of divergence between measured and predicted antineutrino flux/spectra from CEA.

G. Mention et al., Phys. Rev. D 83, 073006 (2011)

CEA PhD thesis of L. Périssé (2018-2021)

- ✓ DRF (M. Vivier, supervisor) – DRT (X. Mougeot, director)
- ✓ Revision of nuclear databases and of BESTIOLE code.
- ✓ **Nuclear structure included for dominant forbidden non-unique beta transitions**.
- ✓ **First complete, robust and detailed uncertainty budget**.

L. Périssé et al., submitted to Phys. Rev. C (2023)

CEA PhD thesis of V. Satu (2018-2021)

- ✓ Phenomenological Gamow-Teller decay strength to model missing transitions in databases.

→ **Bias on reference electron spectra**.

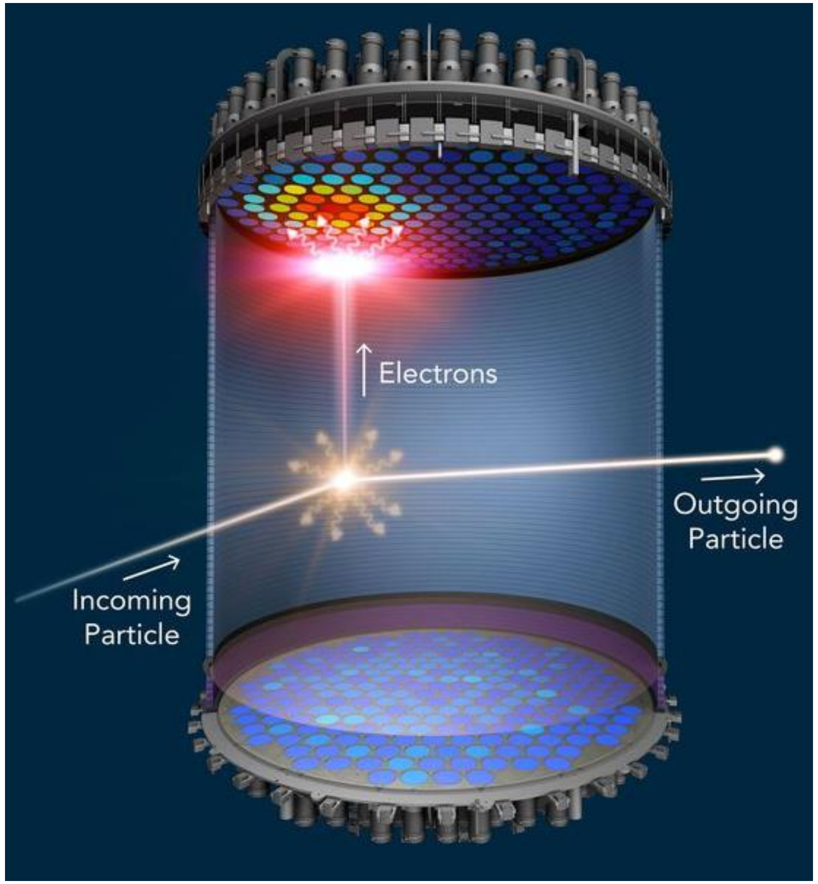
A. Letourneau et al., Phys. Rev. Lett. 130, 021801 (2023)

Dark matter

XENON collaboration

First contact with dark matter community: accurate description of low energy radioactive background is essential.

Excess of electron events as a mono-energetic peak at low energy (~ 2 keV).



Accurate calculations of beta spectra for this community

XENON

E. Aprile et al., Phys. Rev. D 102, 072004 (2020)

Spin-off

S.J. Haselschwardt et al., Phys. Rev. C 102, 065501 (2020)

DarkSide

P. Agnes et al., Phys. Rev. D 104, 082005 (2021)

P. Agnes et al., Phys. Rev. D 107, 063001 (2023)

P. Agnes et al., Phys. Rev. Lett. 130, 101001 (2023)

P. Agnes et al., Phys. Rev. Lett. 130, 101002 (2023)

Current situation in ENSDF database

Weak interaction properties in nuclear decay data are incomplete and come from calculation: mean energies, capture probabilities, log-*ft* values.

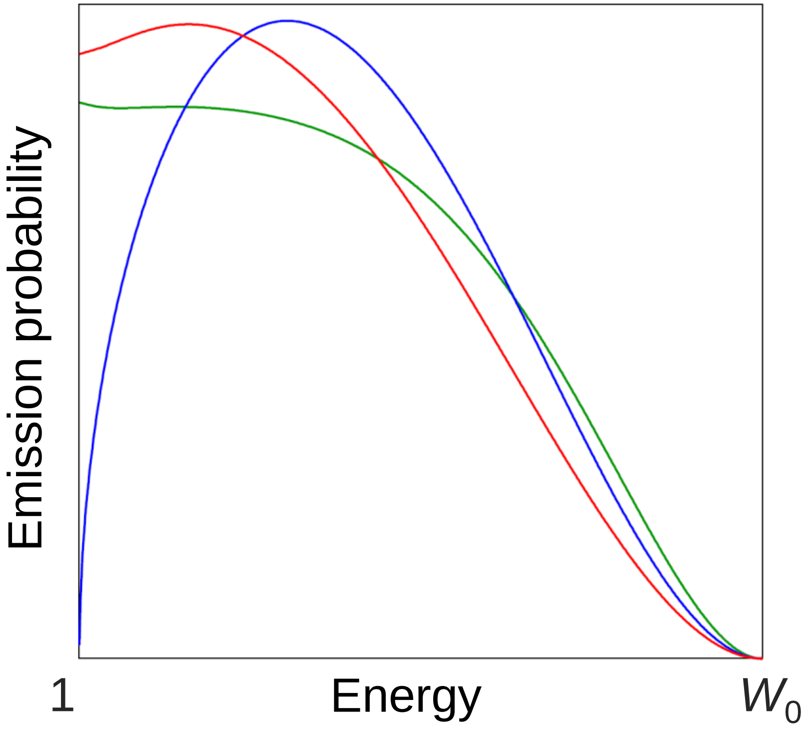
They have been determined for the last 50 years with the LogFT code, developed in the late 1960's. It is still the case in 2023.

Quite simple analytical models of beta transitions and electron captures. Limited in forbiddenness degree.

Beta spectrum shape

Phase space Fermi function Shape factor

$$\frac{dP}{dW} \propto \boxed{pWq^2} \cdot \boxed{F(Z,W)} \cdot \boxed{C(W)}$$



W electron energy, W_0 transition energy

p electron momentum, q neutrino momentum

Allowed	$C(W) = 1$
First forbidden unique	$C(W) = q^2 + \lambda_2 p^2$
Second forbidden unique	$C(W) = q^4 + \lambda_2 q^2 p^2 + \lambda_3 p^4$
Third forbidden unique	$C(W) = q^6 + \lambda_2 q^4 p^2 + \lambda_3 q^2 p^4 + \lambda_4 p^6$
	Etc.

$F(Z,W)$ and λ_k parameters are determined from the Coulomb amplitudes of the relativistic electron wave functions, determined by numerical solving of the Dirac equation.

→ The BetaShape program has been developed by CEA-LNHB and its next version (06/2023) will replace the LogFT code for the future ENSDF evaluations.

→ BetaShape also treats electron captures with an improved modelling.

For forbidden non-unique transitions, coupling with nuclear structure is needed.

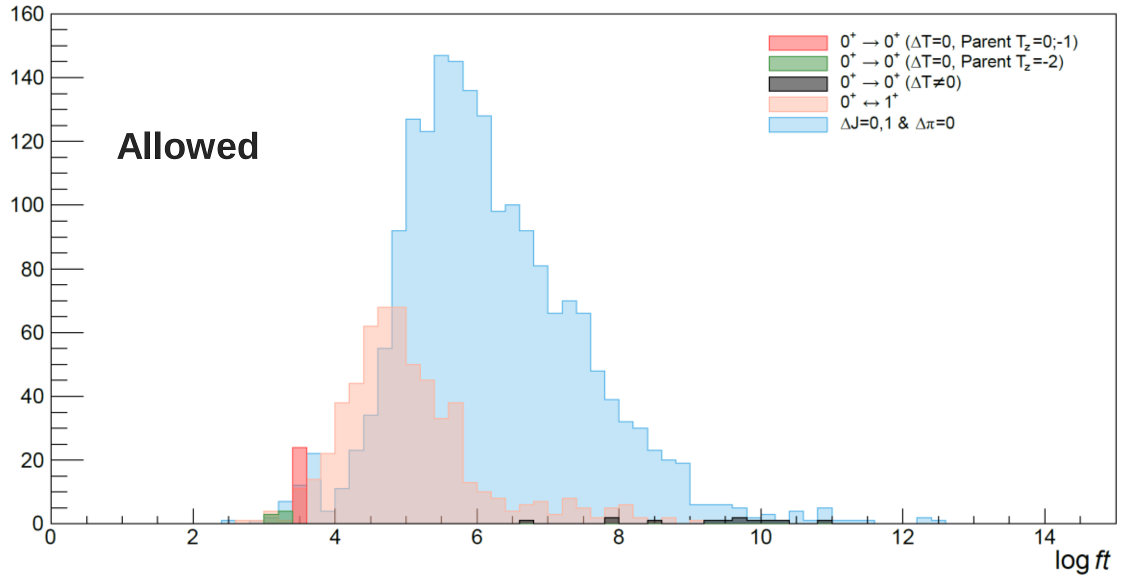
New review of log-*ft* values

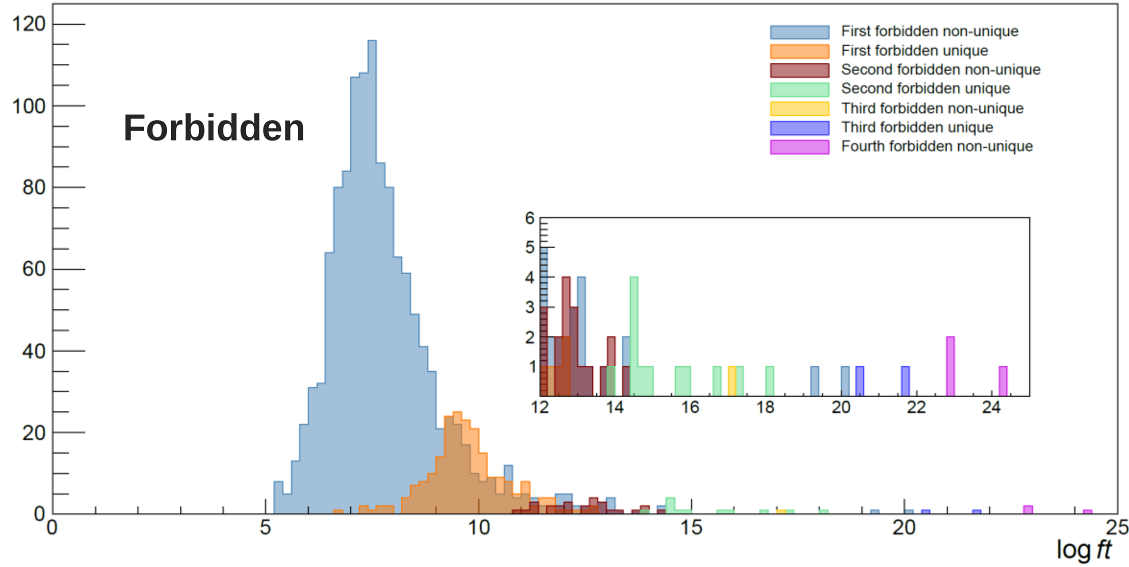
Former review of log-*ft* values, calculated with the LogFT code: B. Singh et al., Nuclear Data Sheets 84, 487 (1998)

These values are used in nuclear structure studies, e.g. to assign spin and parity to a level.

Collaborative work: B. Singh (McMaster University), S. Turkat and K. Zuber (TU Dresden), X. Mougeot (CEA-LNHB)

- ✓ Update of B and EC decays present in ENSDF database (as of 15 April 2023).
- ✓ Use of BetaShape to calculate the log-*ft* values (including the developments for the upcoming version 2.3).
- ✓ In total, 26 318 transitions calculated. Selection of well-defined transitions. Possible pandemonium nuclei flagged.
- ✓ 4 038 transitions survived this filtering. All distributions re-established. Specific transitions are discussed.
- ✓ Atomic Data and Nuclear Data Tables 101584 (2023) in press.



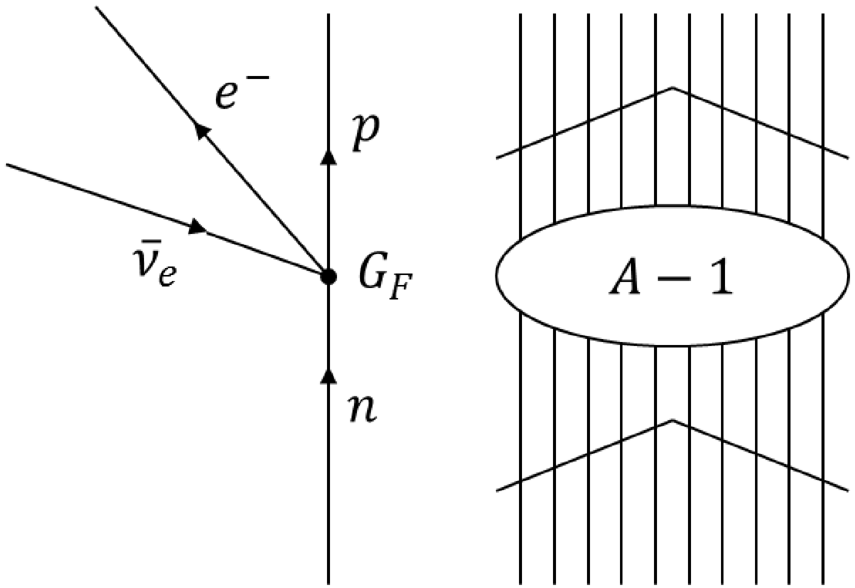


Theoretical shape factor

$$C(W_e) = \sum_{Kk_ek_\nu} \lambda_{k_e} \left[M_K^2(k_e,k_\nu) + m_K^2(k_e,k_\nu) - \frac{2\mu_{k_e}\gamma_{k_e}}{k_e W_e} M_K(k_e,k_\nu) m_K(k_e,k_\nu) \right]$$

H. Behrens, W. Bühring, *Electron Radial Wave functions and Nuclear Beta Decay*, Oxford Science Publications (1982)

Multipole expansion of lepton and nuclear currents. Calculation of shape factors, half-lives, branching ratios, log-*ft* values.



Fermi theory

- Vertex of the weak interaction is assumed to be point-like. No propagation of $W^\pm$ boson.
- Effective coupling constant G_F.
- Relativistic lepton and nuclear wave functions.
- Non-relativistic and relativistic vector or axial-vector matrix elements.

Impulse approximation

- The nucleon is assumed to feel only the weak interaction.
- Other nucleons are spectators.

Realistic nuclear structure

Nuclear state described as a **superposition of nucleon states**

$$\langle \xi_f J_f || T_\lambda || \xi_i J_i \rangle = \hat{\lambda}^{-1} \sum_{a,b} \langle a || T_\lambda || b \rangle \left\langle \xi_f J_f \left\| \left[c_a^\dagger \tilde{c}_b \right]_\lambda \right\| \xi_i J_i \right\rangle$$

transition matrix element

tensor rank

single particle matrix element

one-body transition density

One-body transition densities must be given by a nuclear structure model.

→ **NushellX@MSU**: spherical shell model, effective Hamiltonians fitted on nuclear data, can be used by non-experts.

Nuclear structure models are non-relativistic

The small component of the nucleon wave function can be estimated from the large (non-relativistic) component.

→ Such an approximation has been demonstrated for decades not to be sufficiently accurate for beta transitions.

Forbidden non-unique transitions

Relativistic matrix element

Non-relativistic matrix elements

$$M_K(k_e,k_\nu) = C_K (pR)^{k_e-1}(qR)^{k_\nu-1}\left\{-\sqrt{\frac{2K+1}{K}}\,{}^VF^{(0)}_{K,K-1,1} - \frac{\alpha Z}{2k_e+1}\,{}^VF^{(0)}_{K,K,0}(k_e,1,1,1)\right.$$
$$\left. - \left[\frac{WR}{2k_e+1}+\frac{qR}{2k_\nu+1}\right]{}^VF^{(0)}_{K,K,0} - \frac{\alpha Z}{2k_e+1}\sqrt{\frac{K+1}{K}}\,{}^AF^{(0)}_{K,K,1}(k_e,1,1,1) - \left[\frac{WR}{2k_e+1}-\frac{qR}{2k_\nu+1}\right]\sqrt{\frac{K+1}{K}}\,{}^AF^{(0)}_{K,K,1}\right\}$$

Nuclear structure models are non-relativistic but forbidden non-unique transitions are sensitive to $^{V}F_{K,K-1,1}$

→ **Conserved Vector Current (CVC) hypothesis**

- Derived from the gauge invariance of the weak interaction.
- Provides relationships between non-relativistic and relativistic vector matrix elements.
- Depends on the Coulomb displacement energy ΔE_C.

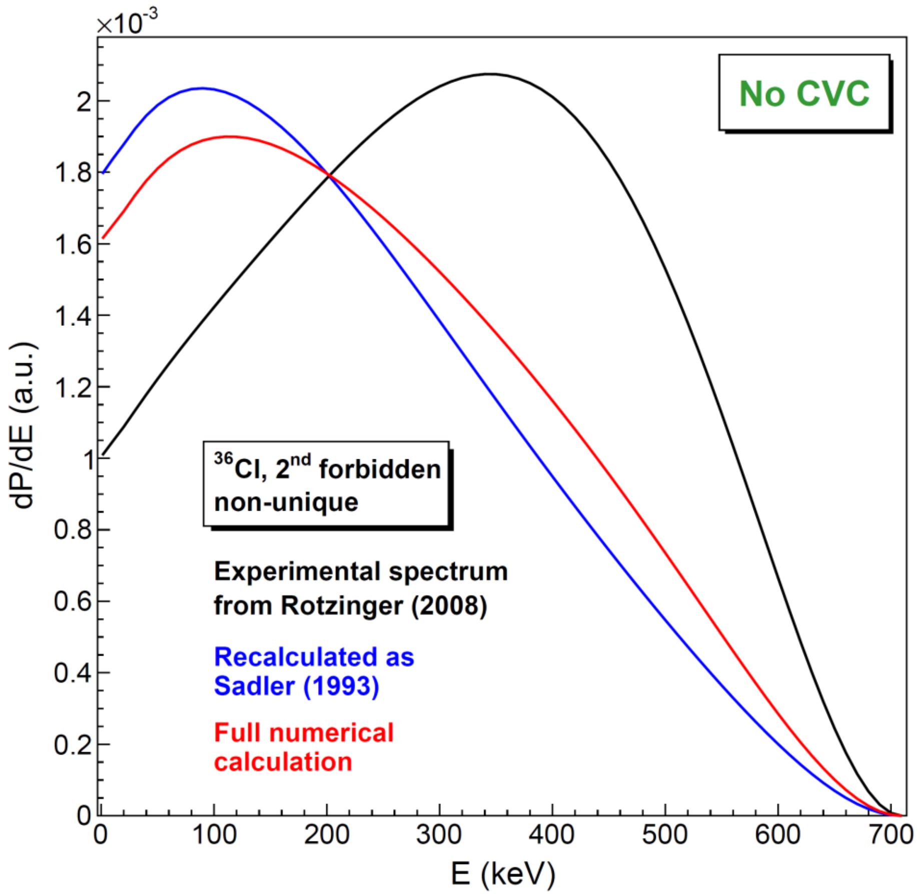
Influence of lepton current treatment: usually approximated only to dominant terms (formula here).

→ A modelling with full numerical treatment has also been implemented.

^{36}Cl decay

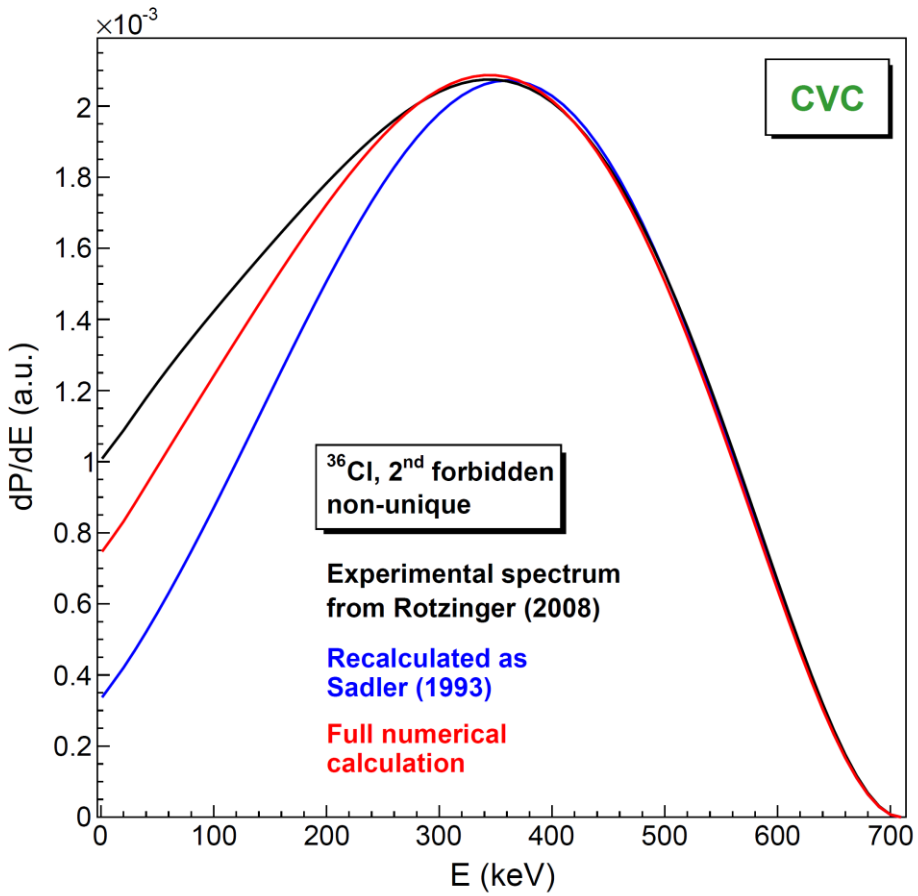
Precise measurement exists

Rotzinger et al., J. Low Temp. Phys. 151, 1087 (2008)



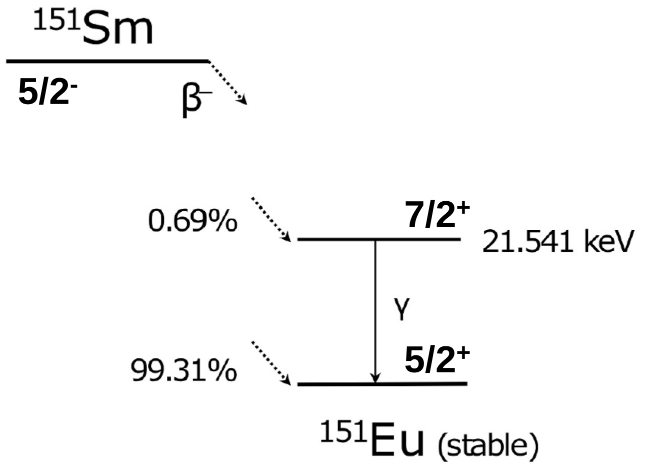
Detailed theoretical study (simplified lepton current)

Sadler, Behrens, Z. Phys. A 346, 25 (1993)



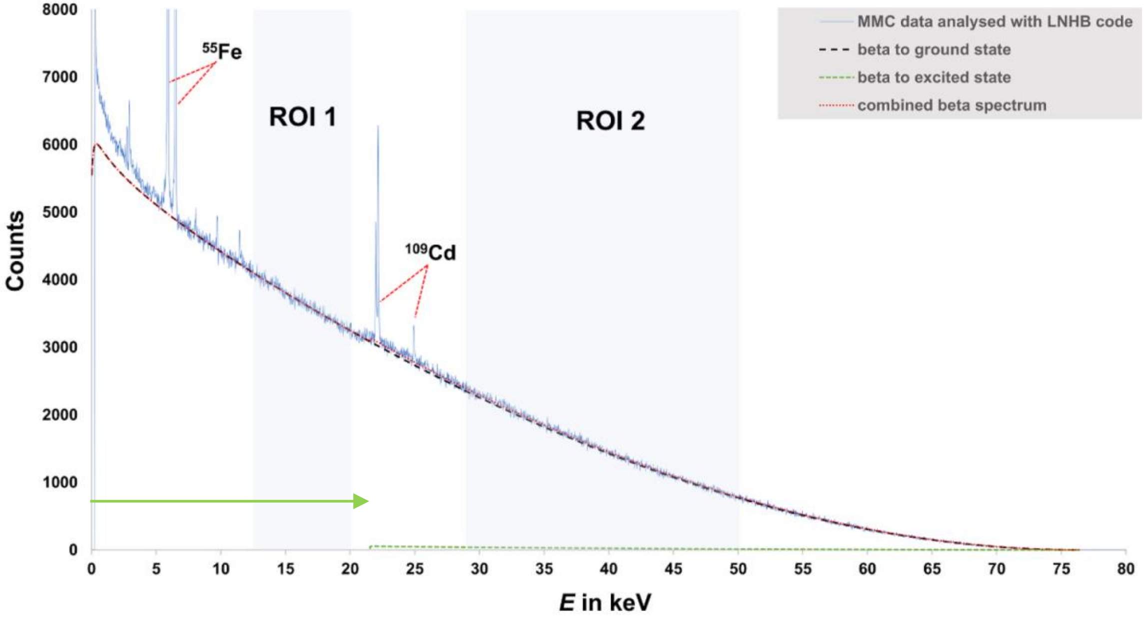
→ CVC hypothesis mandatory + Influence of lepton current treatment

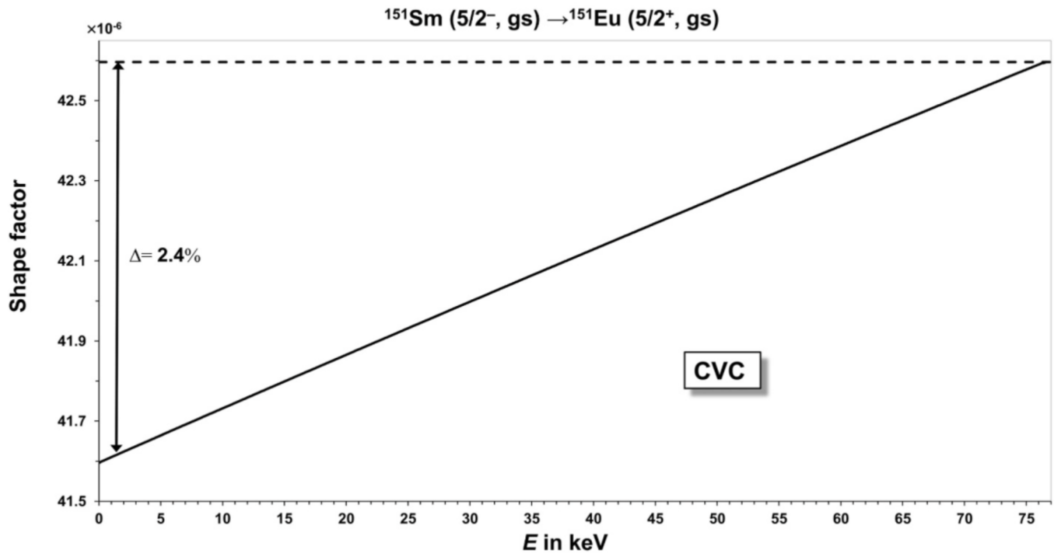
^{151}Sm decay



First forbidden non-unique transitions

- ✓ High-precision measurement of ^{151}Sm spectrum with Metallic Magnetic Calorimeters (MMC) at LNHB.
- ✓ **New Q-value = 76.430 (68) keV** more precise than AME2020 value of 76.5 (5) keV.
- ✓ New determination of branching ratios: 99.31 (11)% and 0.69 (11)%.
- ✓ Kossert et al., Appl. Radiat. Isot. 185, 110237 (2022)

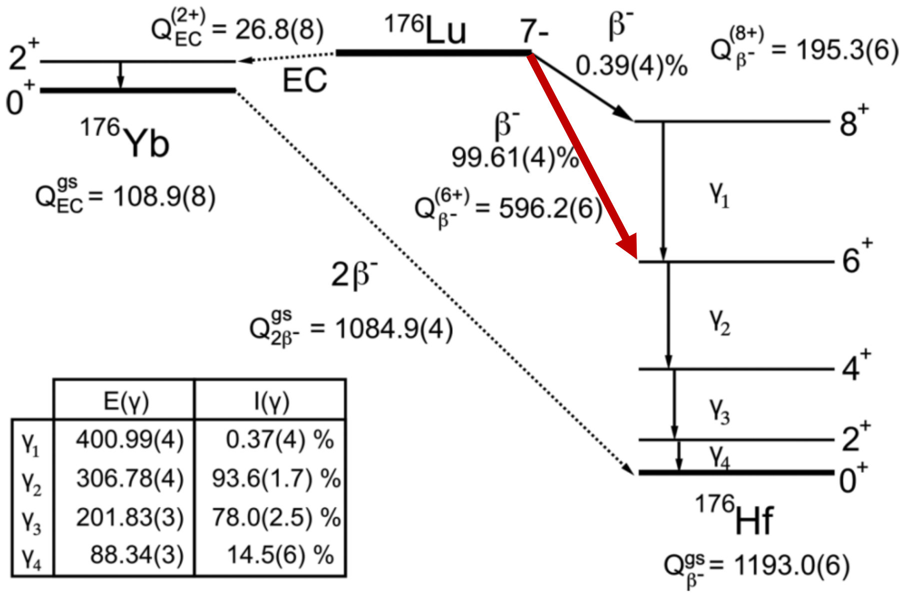




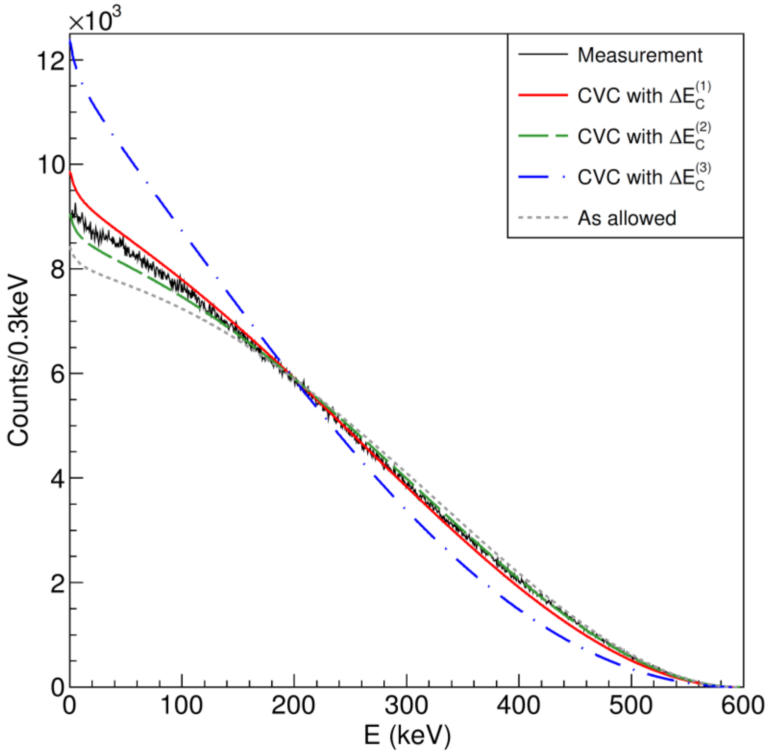
Detailed calculations with nuclear structure

→ Testing the accuracy of the ξ-approximation

^{176}Lu decay



	E(γ)	I(γ)
γ1	400.99(4)	0.37(4) %
γ2	306.78(4)	93.6(1.7) %
γ3	201.83(3)	78.0(2.5) %
γ4	88.34(3)	14.5(6) %

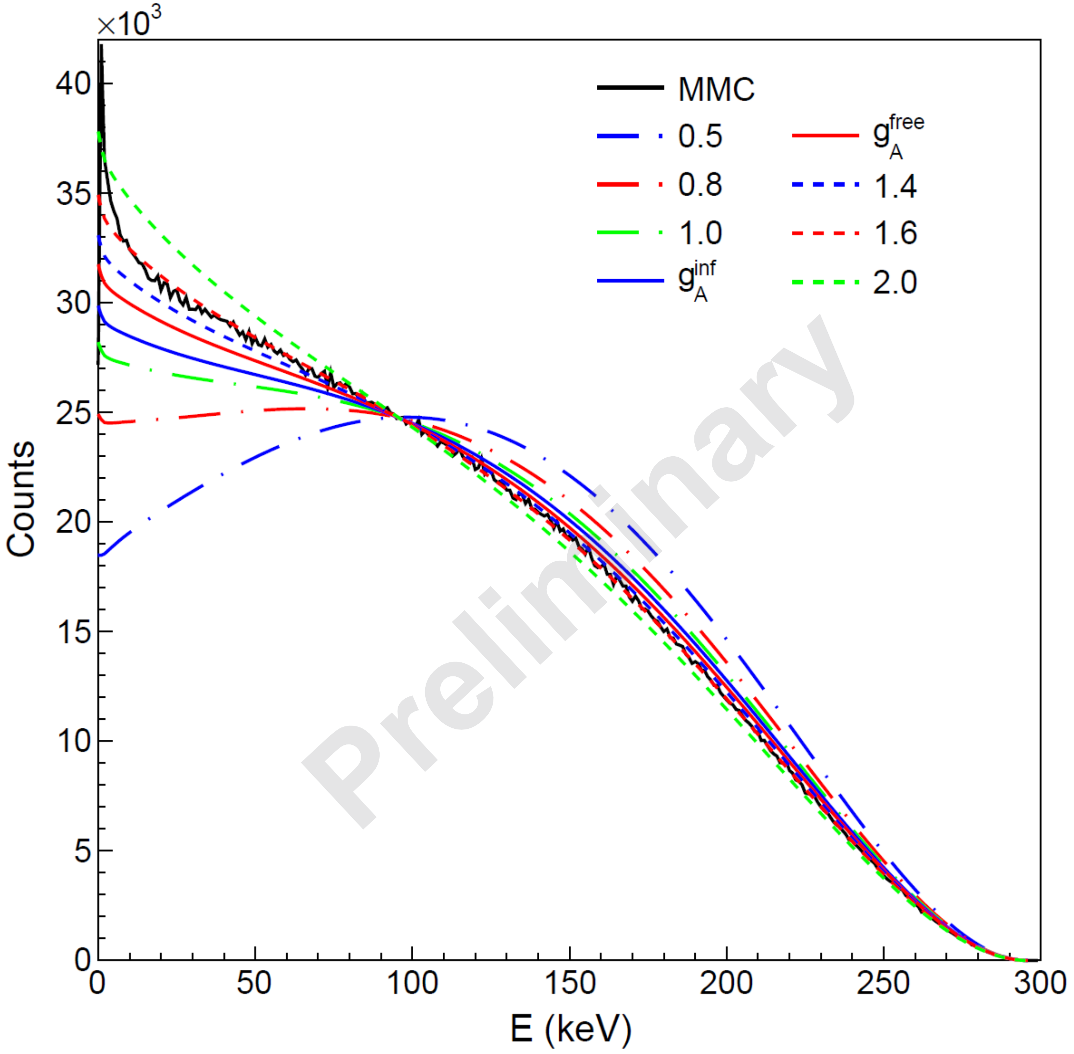


First forbidden non-unique transition

- ✓ First high-precision spectrum measurement from self-scintillation of a LuAG:Pr crystal at TU Delft.
- ✓ New Q-values: Q_β = 1193.0 (6) keV and Q_ε = 108.9 (8) keV. From AME2020: Q_β = 1194.1 (9) keV and Q_ε = 109.0 (12) keV.
- ✓ Spectrum shape retrieved adjusting the Coulomb displacement energy ΔE_C or the axial-vector coupling constant g_A.
- ✗ Calculated half-life shorter by 13 orders of magnitude!

  → **Detailed analysis would require accurate modelling with nuclear deformation: hindered transition ($\Delta K = 7$).**
- ✓ F.G.A. Quarati et al., Phys. Rev. C 107, 024313 (2023).

^{99}Tc decay

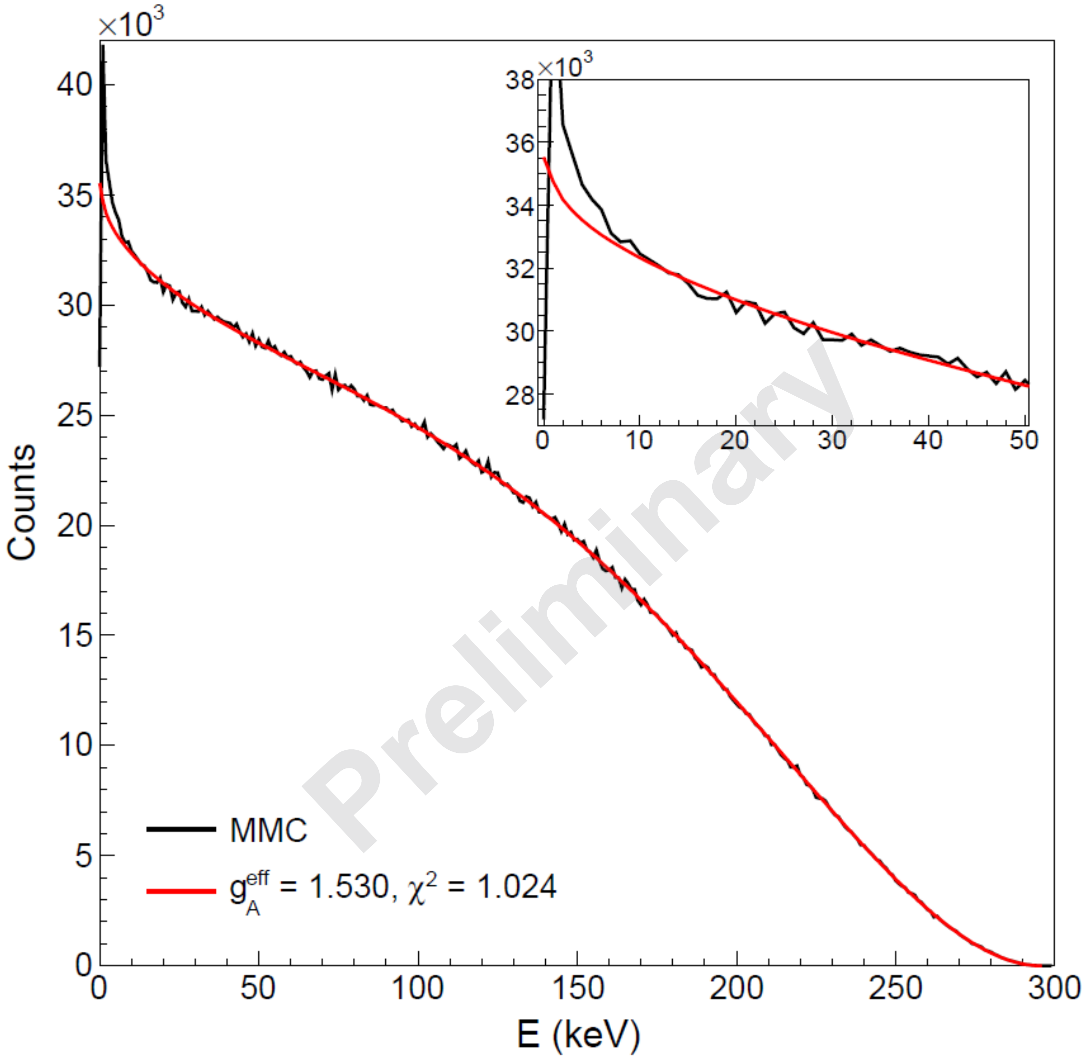


The value of g_A can be affected by several nuclear effects due to the necessary assumptions in nuclear models to deal with the many-body problem.

- ✓ High-precision measurements of ^{99}Tc spectrum with MMC at CEA-LNHB and PTB, and with Silicon detectors at CEA-LNHB.
- ✓ Excellent agreement of all the three spectra.
- ✓ **New Q-value = 295.82 (16) keV** not consistent with AME2020 value of 297.5 (9) keV.
- ✓ ^{99}Tc beta spectrum, second forbidden non-unique, has been predicted to be very sensitive to g_A. Free-nucleon value [PDG 2020] g_A^{free} = 1.2754 (13).

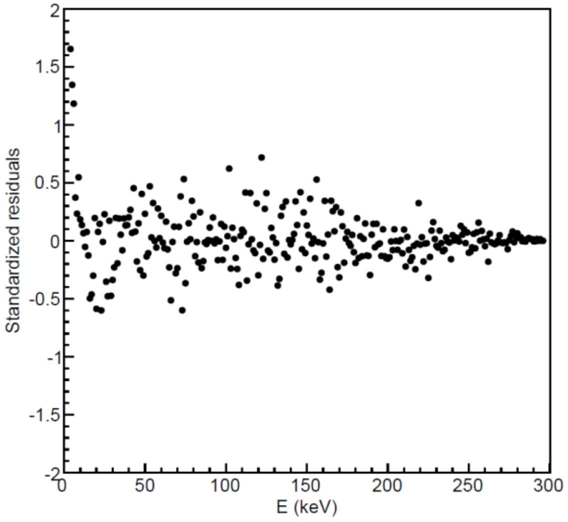
  → **Confirmed.**

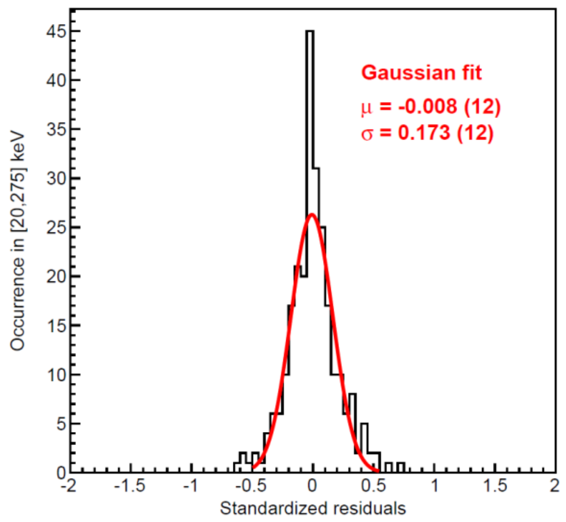
^{99}Tc decay



The value of g_A can be affected by several nuclear effects due to the necessary assumptions in nuclear models to deal with the many-body problem.

- ✓ Theoretical calculations with nuclear structure, CVC and complete lepton current.
- ✓ Best adjustment gives an effective axial-vector coupling constant g_A^{eff} = 1.53 (8).
- ✓ Excellent residuals, without any trend down to 6 keV.





Conclusion

Beta decays are of importance in many scientific areas: radionuclide metrology, nuclear medicine, nuclear reactors, neutrino physics, dark matter studies, etc.

Our aim is to improve nuclear decay data by providing theoretical predictions **for every possible beta transition** or electron capture validated against precise measurements.

- ✓ When no nuclear structure is required, the BetaShape code already improves the situation.
- ✓ We started the study of beta transitions that are sensitive to nuclear structure.

Ongoing developments and challenges

- ➢ **High-precision measurements**, which can be used as a probe of nuclear models.
- ➢ Treatment of the many-body problem → Effective axial-vector coupling constant.
- ➢ **Medium and heavy nuclei, nuclear deformation**
  - → Ongoing collaboration with S. Péru (CEA/DAM) and M. Martini (IPSA) in the framework of the European metrology project PrimA-LTD.

Technical adaptions are necessary of both nuclear and beta decay modelling.

Thank you for attention