News on Analytical Emittance Estimates

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Recap Analytical Formulas

- Formulas well documented in various papers by A Wolski and T Raubenheimer (e.g. SLAC-PUB-4937)

$$\frac{\epsilon_y}{< y_{sext}^2 >} \approx$$

$$\frac{J_x\left(1-\cos(2\pi\nu_x)\cos(2\pi\nu_y)\right)\epsilon_x}{J_y\left(\cos(2\pi\nu_x)-\cos(2\pi\nu_y)\right)^2}\sum_{sext}\beta_x\beta_y\left(\frac{k_2 L}{2}\right)^2$$

← Coupling contribution

$$+\frac{J_z\sigma_\delta^2}{\sin^2(\pi\nu_y)}\sum_{sext}\beta_y\eta_x^2\left(\frac{k_2 L}{2}\right)^2$$

← Dispersion contribution

$\frac{k_2 L}{2} \rightarrow k_1 L$ and $< y_{sext}^2 > \rightarrow < \theta_{quad}^2 >$ for quads

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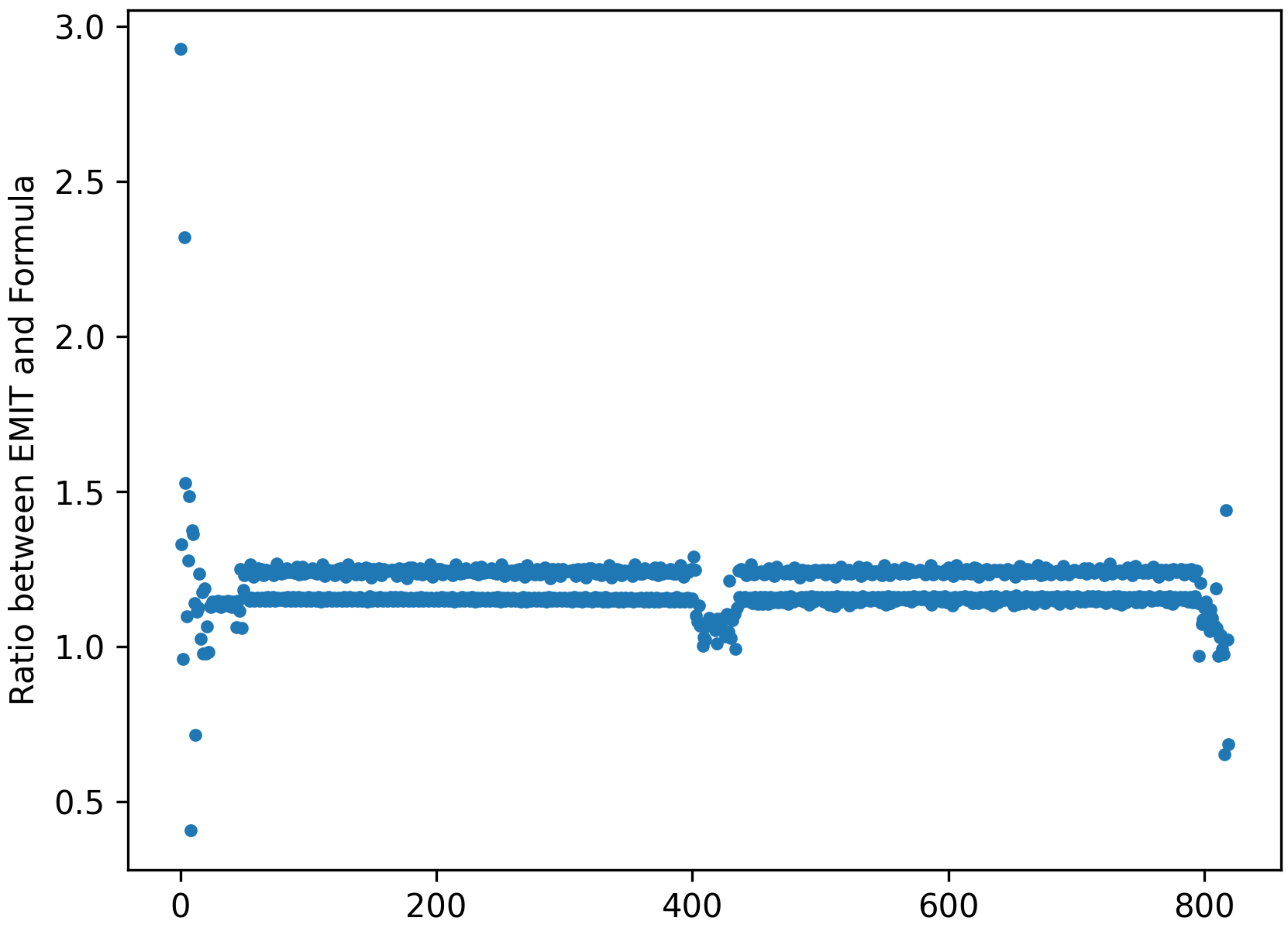
Recap Simulations vs Equations

- Good agreement between equations and simulations for sextupoles
  - Both for arc and IR sextupoles separately
- Results did not seem to agree well for arc quadrupoles
  - Off by an order of magnitude

Testing Quadrupole Equations

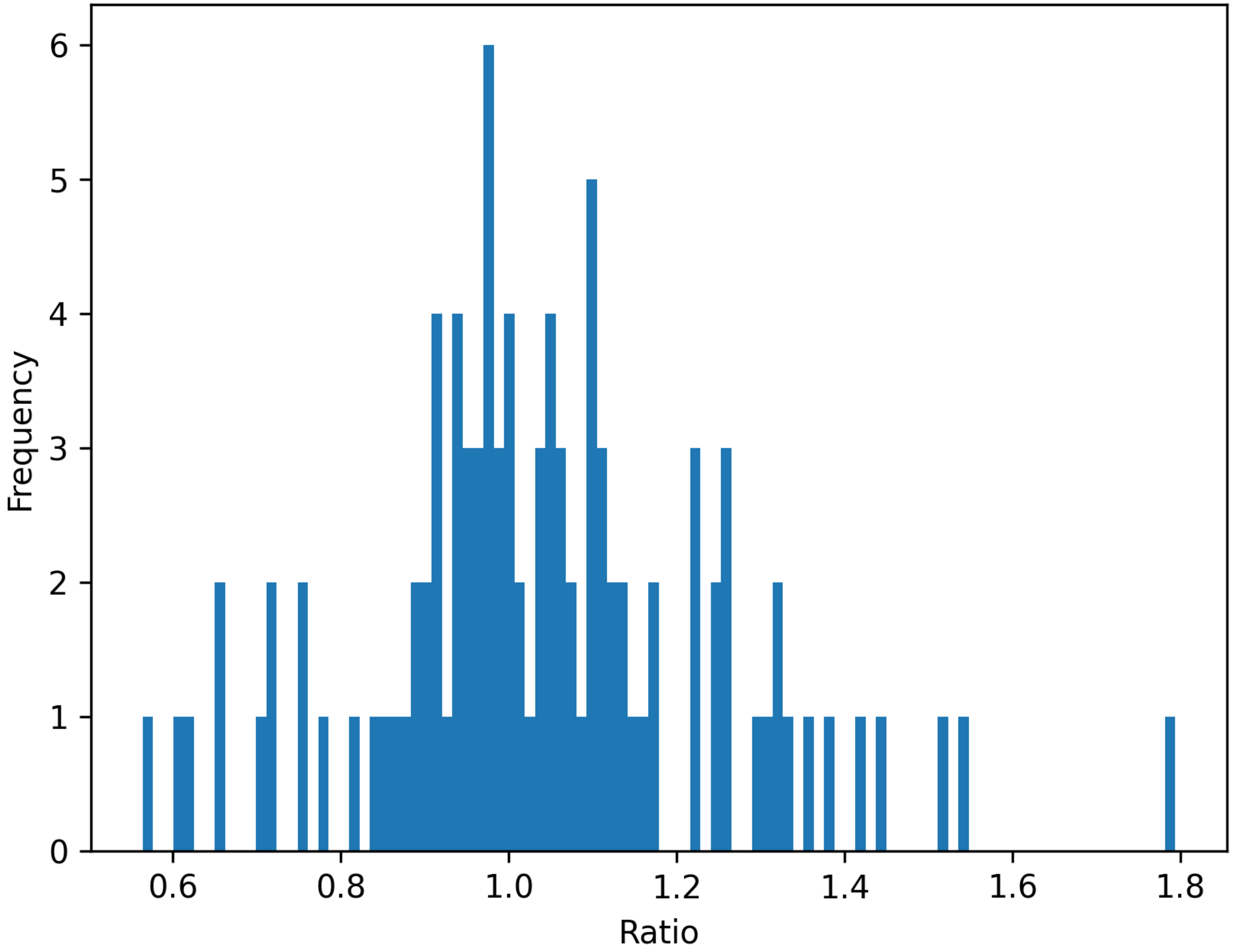
- Testing single quadrupole effect
  - Turn one quadrupole by $10\ \mu\text{rad}$
  - Evaluate emittance with MADX
  - Compare to emittance from analytical formula
  - Repeat for the next quadrupole
- Shows consistently good agreement


Ratio between EMIT and Formula
3.0
2.5
2.0
1.5
1.0
0.5
0
200
400
600
800

Testing Equations for Quadrupole Groups

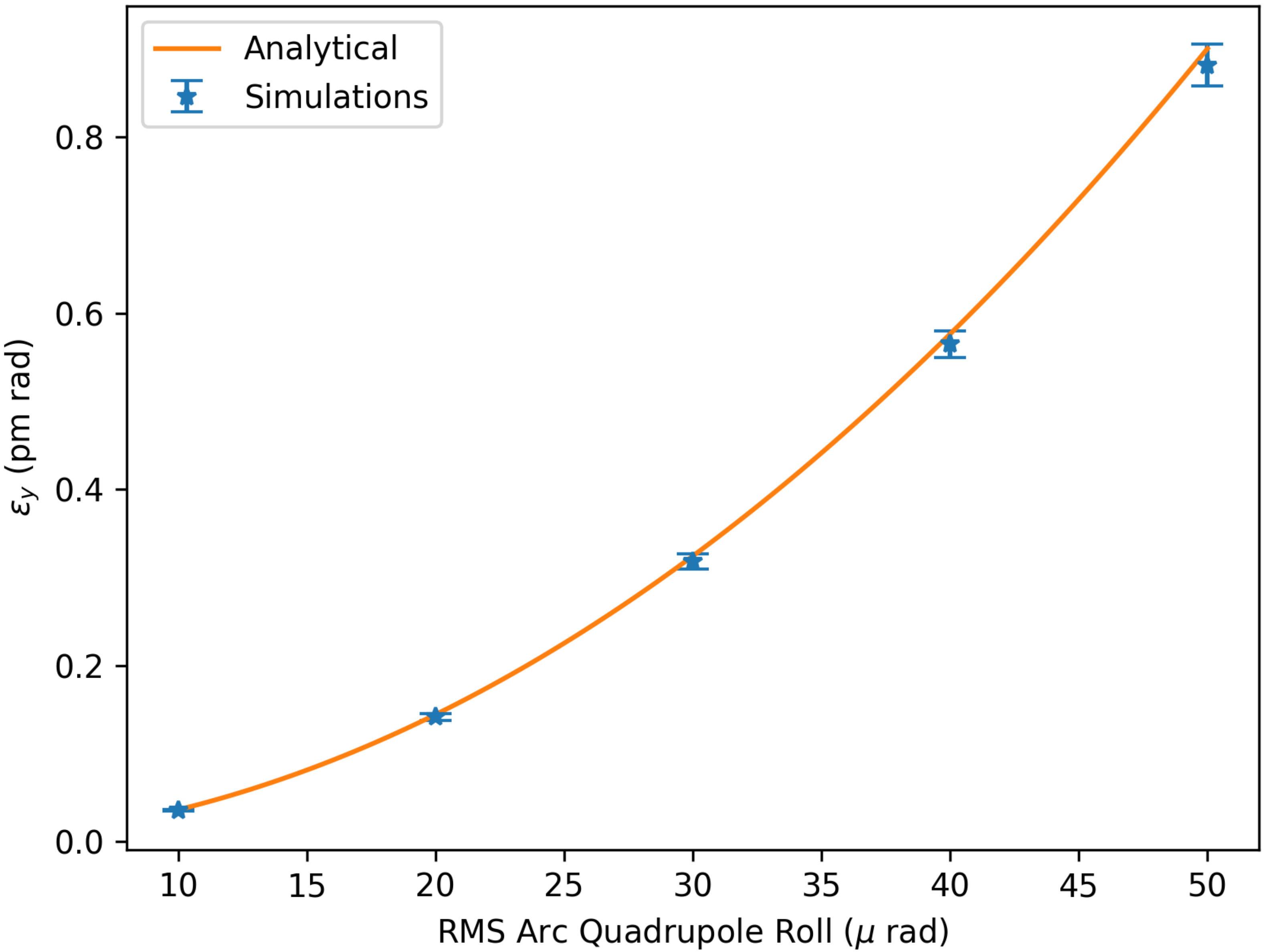
- Possible by using ABP’s optics server
  - Scheduling and parallelisation using OMC’s PyLHC submitter (J Dilly, M Hofer)
  - Using 30 cores allows 10,000 MADX scripts to run in about 15 min
- Python Scripts:
  - Create 100 groups of 41 randomly chosen quads (out of c.a. 3200)
  - Compute analytical emittance for all 100 groups
  - For each group generate and run 100 MADX scripts that
    - Apply random errors to the 41 quads (Based on error application scripts by T Charles)
    - Compute and save emittance in MADX
  - Read out MADX emittance and take averages
  - Compute average Ratio of MADX emittance / Analytical Prediction

Very Good Agreement



Arc Quadrupoles Revisited

- Repeated studies with roll errors on arc quadrupoles
- Double checking definitions of what is counted as arc quadrupole
- Double checking how errors are applied
- Simulations with 1000 seeds for each error size
- Determine mean emittance and standard error of the mean


Analytical
Simulations
0.8
0.6
0.4
0.2
0.0
ε_y (pm rad)
10
15
20
25
30
35
40
45
50
RMS Arc Quadrupole Roll (μ rad)

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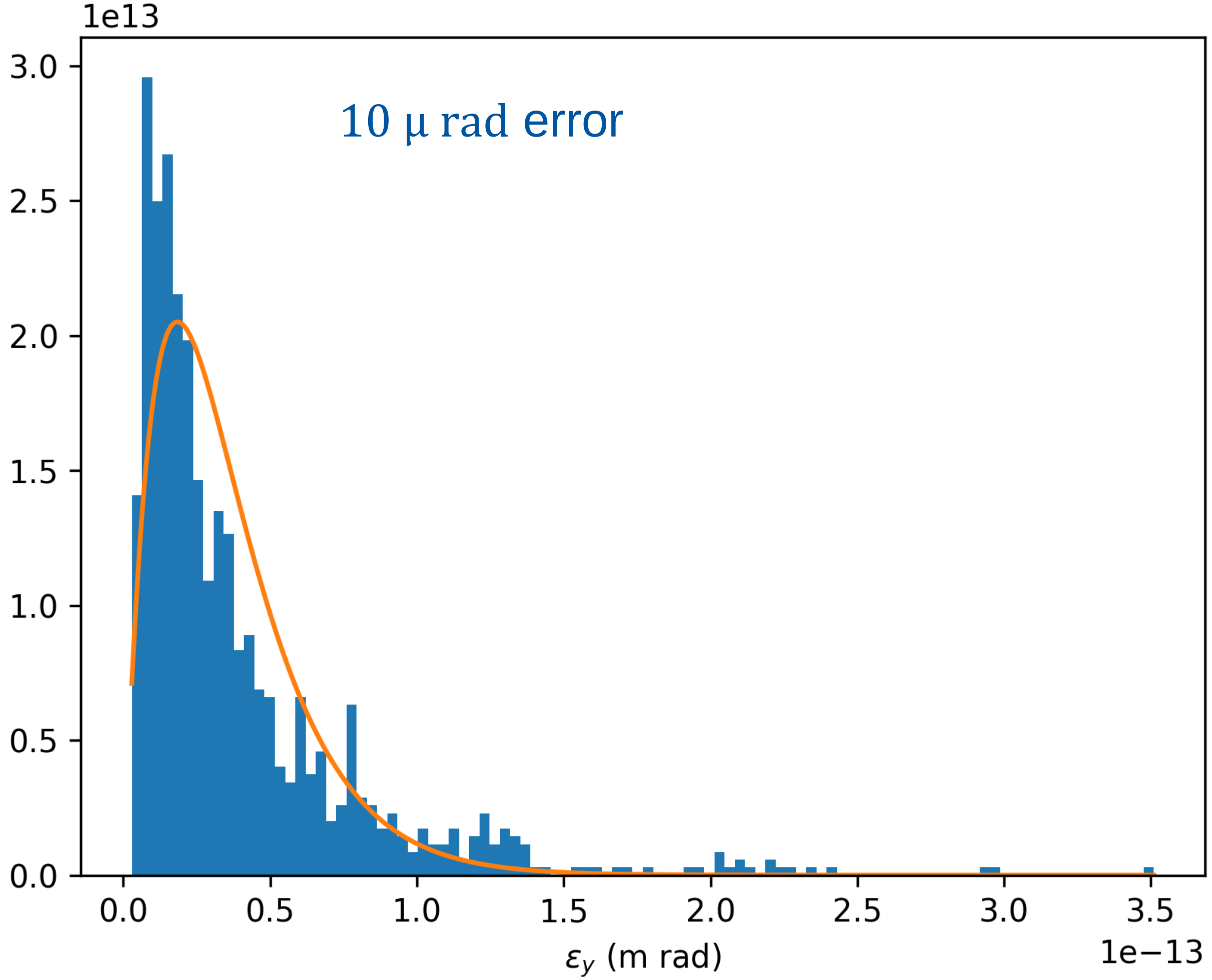
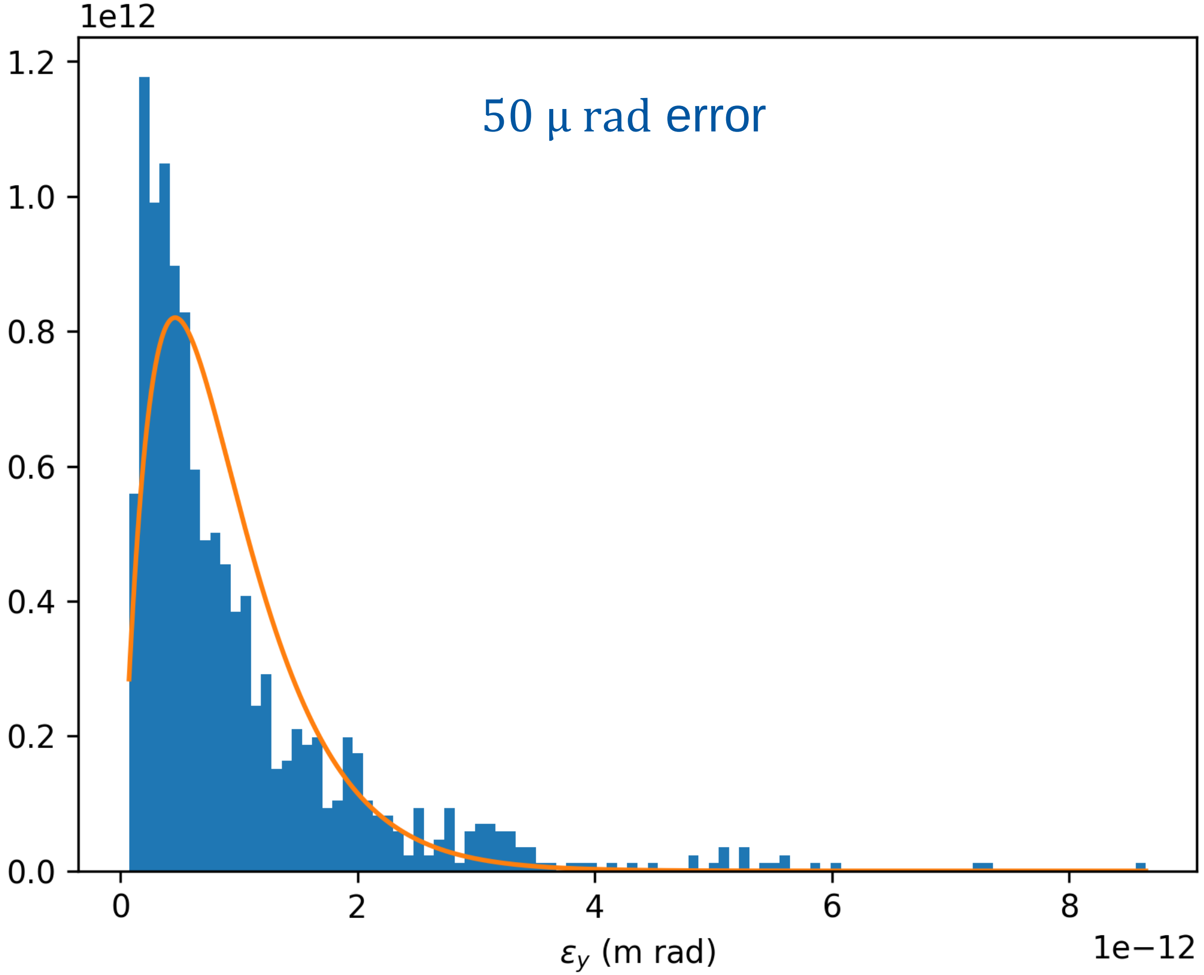
Emittance Distribution

- Very good agreement between mean emittance from simulations and analytical formulas
- Good agreement for any group of quadrupoles
- Can we also predict emittance distribution?
  - Emittances for the 1000 seeds non-gaussian
  - Need to know “worst-case” for feasibility
- Emittances shown to follow a gamma distribution
  - Shape defined by $< \epsilon_y >$ and Q_y

Emittance Distribution PDF

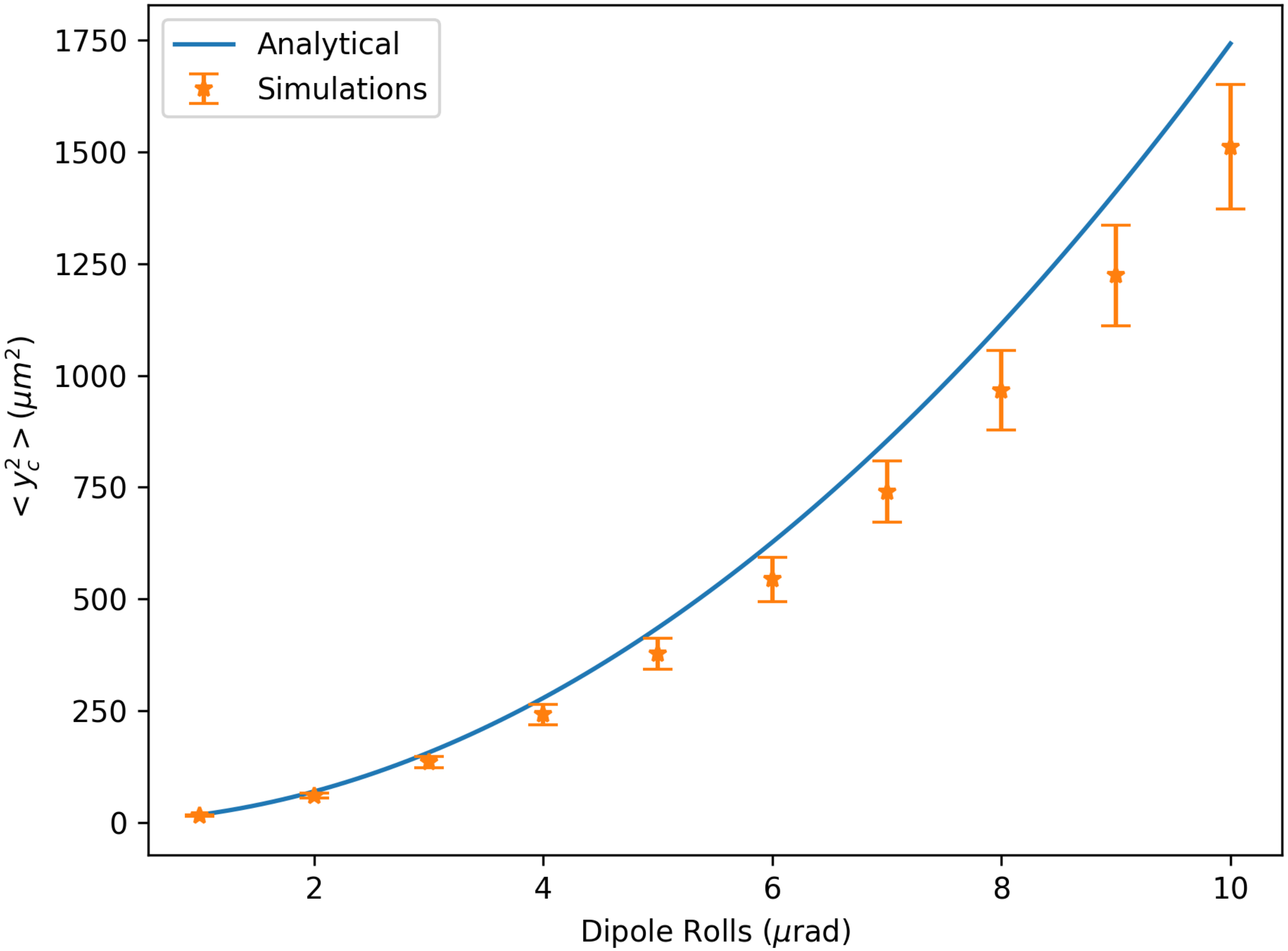




Closed Orbit

- Sextupole misalignments and quadrupole rolls drive emittance through coupling of optics
- Emittance growth also caused by non-zero closed orbit
  - Quadrupole misalignments
  - Dipole rolls
- There is also a well established formula for closed orbit
  - $< y_c(s) > = \frac{\beta_y(s)}{8\sin^2(\pi Q_y)} \sum < G^2 L^2 > \beta_y$
  - $G = \frac{1}{\rho_{bend}} \theta_{roll} + k_1 y_{error}$
- Test for dipole rolls with simulations with 100 seeds
  - **Preliminary results**

Closed Orbit



Outlook and Next Steps

- Closed orbit from quadrupole misalignments
- Emittance from closed orbit
  - Dispersion
  - Higher order chromatic effects
  - Coupling from orbit through sextupoles
- Estimate emittance with correctors
  - Formulas exist for such estimates
  - Rely on many assumptions about the nature of the corrections