Leptogenesis from $SU(5)$ GUT with $T_{13}$ Family Symmetry

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C.S. Fong, MHR, S. Saad, arXiv:2103.14691 [hep-ph]
Baryogenesis via Leptogenesis
- out of equilibrium decay of right-handed neutrinos explain baryon asymmetry of the universe

The Asymmetric Texture
- Yukawa texture based on $SU(5)$ GUT and $T_{13}$ family symmetry

Leptogenesis from the “asymmetric texture”
- thermal leptogenesis in both nonresonant and resonant regimes
Baryogenesis via Leptogenesis

**Baryon Asymmetry:**

Net baryon to photon ratio of the universe from CMB data:

$$\eta_B = (N_B - N_{\bar{B}})/N_\gamma \simeq (6.12 \pm 0.04) \times 10^{-10}$$

**Leptogenesis:**

$$\bar{N} \leftrightarrow \ell + H^*, \quad \bar{N} \leftrightarrow \bar{\ell} + H$$

out of equilibrium decays violate $L$, $C$ and $CP$; $B - L$ conserving sphaleron processes convert $L$ violation into $B$ violation

![Diagram showing leptogenesis processes](image)
Leptogenesis from the Asymmetric Texture

- $SU(5) \times T_{13}$ model describes the GUT-scale mass ratios and mixing angles of both quarks and leptons.

- Single $CP$ violating phase in the seesaw sector generates both Dirac and Majorana $CP$ violation.

- Requires four right-handed neutrinos to generate viable light neutrino mass spectrum.

- Can the decay of these right handed neutrinos translate the $CP$ violation into $CP$ asymmetry?
Asymmetric Texture and $\mathcal{T}_{13}$ Family Symmetry

- Lepton mixing is unlike quark mixing!

$$\theta_{12} = 33.65^\circ \pm 2.38^\circ, \quad \theta_{23} = 47.58^\circ \pm 3.66^\circ, \quad \theta_{13} = 8.49^\circ \pm 0.40^\circ$$

- Large angles from Tribimaximal mixing in the seesaw sector

$$U_{TBM} \equiv R(\theta_{12} = 35.3^\circ), \quad R(\theta_{23} = 45^\circ), \quad R(\theta_{13} = 0^\circ)$$

and the small reactor angle from EW sector: ‘Cabibbo haze’

- The “asymmetric texture”: minimal $SU(5)$ texture with a complex TBM mixing does the job! [MHR, Ramond, Xu PRD 98 (2018) 055030]

- The asymmetric term arises naturally from a $\mathcal{T}_{13}$ family symmetry [Pérez, MHR, Ramond, Stuart, Xu PRD 100 (2019) 075008]
Seesaw Parameters for Leptogenesis

- Seesaw matrix in terms of Dirac Yukawa and Majorana mass matrix:

\[ S = Y^{(0)} \mathcal{M}^{-1} Y^{(0)T} = U_{TBM}(\delta_{TBM}) \text{diag}(m_{\nu_1}, m_{\nu_2}, m_{\nu_3}) U_{TBM}(\delta_{TBM})^T \]

- Requires four right-handed neutrinos and predicts normal ordering of light neutrino masses: \(|m_{\nu_1}| = 27.6, |m_{\nu_2}| = 28.9, |m_{\nu_3}| = 57.8\) meV

- Neutrino masses and mixing angles are determined independently of the four undetermined parameters: \(b_1, b_2, b_3\) and \(M\)

- \(CP\) violation arises from the TBM phase, whose magnitude is constrained \(66^\circ \leq |\delta_{TBM}| \leq 85^\circ\), but sign is unresolved [Pérez, MHR, Ramond, Stuart, Xu PRD 101 (2020) 075018]

- If leptogenesis is viable, can the undetermined parameters \(b_1, b_2, b_3\) and \(M\) be constrained? Is the sign of \(\delta_{TBM}\) resolved by the sign of baryon asymmetry?
Classical Boltzmann Equations for three regimes:
(i) $T \gg 10^{12}$ GeV: one flavor, (ii) $10^9 \ll T \ll 10^{12}$ GeV: two flavors, (iii) $T \ll 10^9$ GeV: three flavors

One-flavor leptogenesis fails because of the flavor structure dictated by $T_{13}$

What about the transition regions? What if the mass scale is unknown? Density Matrix Equations

$3 \times 3$ $CP$ asymmetry matrix in flavor space with nonzero off-diagonal entries, accounts for flavor transitions

$B - L$ asymmetries are calculated by solving coupled differential equations

Final baryon asymmetry is proportional to the trace of the $B - L$ asymmetry matrix
The simplest case \((b_1, b_2, b_3) \equiv b(1, 1, 1)\) yields zero \(CP\) asymmetry.

Set \((b_1, b_2, b_3) \equiv b(1, f, 1)\), define \(M \equiv ab\).

For a fixed \(f \neq 1\), right-handed neutrino masses are determined as a function of \(a\) in the non-degenerate regions.

Required right-handed neutrino masses for successful leptogenesis are \(\mathcal{O}(10^{11-12})\) GeV.

Is something special going on near the \(M_3 - M_4\) degeneracy?

Figure: Right-handed neutrino mass spectrum required for successful leptogenesis setting \(f = 2\). Shaded regions represent non-degenerate masses.
The magnitude of $\delta_{TBM}$ is constrained, but the sign remained undetermined.

Final $B - L$ asymmetry is proportional to the diagonal $CP$ asymmetries, which are proportional to $\sin \delta_{TBM}$.

$-85^\circ \leq \delta_{TBM} \leq -66^\circ$ yields positive baryon asymmetry in all four regions and picks the correct sign of the Dirac $CP$ phase $1.27\pi \leq \delta_{CP} \leq 1.35\pi$, compared to the 2021 PDG estimate $1.37 \pm 0.17\pi$.

**Figure:** The asymmetric texture requires $66^\circ \leq |\delta_{TBM}| \leq 85^\circ$ to yield all PMNS angles within $3\sigma$ of their 2021 PDG estimate.
- **Exact degeneracy** yields zero $CP$ asymmetry, but **quasi-degeneracy** can enhance the $CP$ asymmetry to nearly $O(1)$

- $CP$ asymmetry is proportional to $\text{Re} \left[ (Y_\nu^\dagger Y_\nu)_{ij} \right]$, which is nonzero only for $N_3 - N_4$ quasi-degeneracy

- Resonance condition $|M_3 - M_4| \simeq \frac{\Gamma_{3,4}}{2}$ fixes $a$ in terms of $b$ and $f$

- There is a **minimum** $b$ below which the generated asymmetry is lower than the CMB value

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**Figure:** Minimum right-handed neutrino mass spectrum required for successful leptogenesis at $N_3 - N_4$ resonance.
- $M_1$ and $M_2$ are smaller than the resonant pair $M_3, M_4$

- There is a maximum $b$ above which the generated asymmetry is severely washed out by lighter right handed neutrinos

- The upper bound only occurs for $f \gtrsim 2$ below which the washout is not efficient

- There is a discontinuity at $f \approx 6.2$ due to change of sign of baryon asymmetry for nontrivial flavor interactions

Figure: Maximum right-handed neutrino mass spectrum required for successful leptogenesis at $N_3 - N_4$ resonance.
Experimental Constraints

Fong, MHR, Saad arXiv:2103.14691 [hep-ph]

(a) $f = 0.1$

(b) $f = 0.1$

(c) $f = 0.1$

(d) $f = 10$

(e) $f = 10$

(f) $f = 10$
Key Findings

Nonresonant Leptogenesis

- No unflavored leptogenesis, flavored leptogenesis requires right-handed neutrino masses $\mathcal{O}(10^{11-12})$ GeV
- TBM phase $\delta_{TBM}$ must be negative, yields positive baryon asymmetry and Dirac $CP$ phase consistent with global fits

Resonant Leptogenesis

- Nontrivial upper bound on right-handed neutrino mass because of washout by lighter neutrinos
- Minimum right-handed neutrino mass $\mathcal{O}(1)$ GeV, mixing parameters close to the sensitivity of DUNE
Backup Slides
Hunting the ‘Minimal’ Texture

What is the ‘minimal’ Yukawa texture that

- reproduces the GUT-scale mass relations, the CKM mixing angles, and
- generates enough ‘Cabibbo haze’ for the reactor angle?

The Asymmetric Texture arXiv:1805.10684

\[ Y^{(2)} \sim \text{diag} \ (\lambda^8, \lambda^4, 1), \]

\[ Y^{-1} \sim \begin{pmatrix} bd\lambda^4 & a\lambda^3 & b\lambda^3 \\ a\lambda^3 & c\lambda^2 & g\lambda^2 \\ d\lambda & g\lambda^2 & 1 \end{pmatrix}, \quad Y^{-1} \sim \begin{pmatrix} bd\lambda^4 & a\lambda^3 & d\lambda \\ a\lambda^3 & -3c\lambda^2 & g\lambda^2 \\ b\lambda^3 & g\lambda^2 & 1 \end{pmatrix} \]

\[ U_{\text{Seesaw}} = \text{diag} \ (1, 1, e^{i\delta}) \ U_{TBM} \]

\[ a = c = \frac{1}{3}, \quad g = A, \quad b = A\sqrt{\rho^2 + \eta^2}, \quad d = \frac{2a}{g} = \frac{2}{3A}, \quad \cos \delta = 0.2 \]
TBM Mixing with a Phase

\[ U_{PMNS} = U^{(-1)\dagger} U_{TBM}(\delta) \]

\[ U_{TBM}(\delta) = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{e^{i\delta}}{\sqrt{6}} & \frac{e^{i\delta}}{\sqrt{3}} & \frac{e^{i\delta}}{\sqrt{2}} \end{pmatrix} \]

With real TBM mixing \( \delta = 0 \),

\[ |\sin \theta_{13}| = \frac{1}{\sqrt{2}} \left| U_{21}^{(-1)} + U_{31}^{(-1)} \right| \]

\( \theta_{13} \approx 10.59^\circ \ (2.26^\circ \text{ above PDG}), \quad \theta_{12} \approx 39.81^\circ \ (6.16^\circ \text{ above PDG}), \quad \theta_{23} \approx 42.67^\circ \ (2.90^\circ \text{ below PDG}) \)

With complex TBM mixing,

\[ |\sin \theta_{13}| = \frac{1}{\sqrt{2}} \left| U_{21}^{(-1)} + e^{i\delta} U_{31}^{(-1)} \right| \]
One phase to rule them all: $66^\circ < |\delta| < 85^\circ$ brings all three angles within $3\sigma$ of PDG 2020 value.
Suppose $SU(5)$ matter fields $F \sim \bar{5}$ and $T \sim 10$ are triplets of some discrete family symmetry group, $G_f$.

**$F$ and $T$ must be different triplets of $G_f$**

- $Y^{-1/3}$ and $Y^{-1}$ comes from $F \otimes T = (\bar{5}, r) \otimes (10, s)$
- $3 \times 3 \Rightarrow$ either symmetric or antisymmetric
  
  We need a group with at least two different triplets!

**Candidates:**

- $S_4$ (order 24), $\Delta(27)$ (order 27), $T_{13}$ (order 39)

**$s \otimes s$ must distinguish diagonal from off-diagonal**

- $Y^{2/3}$ comes from $T \otimes T = (10, s) \otimes (10, s)$

Only $T_{13}$ survives!
Generating the Asymmetric Term

\[ Y^\left(-\frac{1}{3}\right) \leftarrow F \bar{T} H_5 \varphi^\left(-\frac{1}{3}\right) \]

\[
\begin{pmatrix} F_1 \\ F_2 \\ F_3 \end{pmatrix}_{3_1} \times \begin{pmatrix} T_1 \\ T_3 \\ T_2 \end{pmatrix}_{3_2} = \begin{pmatrix} F_3T_2 \\ F_1T_1 \\ F_2T_3 \end{pmatrix}_{3_1} \oplus \begin{pmatrix} F_3T_1 \\ F_1T_3 \\ F_2T_2 \end{pmatrix}_{3_2} \oplus \begin{pmatrix} F_3T_3 \\ F_1T_2 \\ F_2T_1 \end{pmatrix}_{\bar{3}_2}
\]

\( \mathcal{T}_{13} \) can dial individual matrix elements!

\[ Y^\left(\frac{2}{3}\right) \leftarrow T \bar{T} \bar{H}_5 \varphi^\left(\frac{2}{3}\right) \]

\[
\begin{pmatrix} T_1 \\ T_3 \\ T_2 \end{pmatrix}_{3_2} \times \begin{pmatrix} T_1 \\ T_3 \\ T_2 \end{pmatrix}_{3_2} = \begin{pmatrix} T_3T_3 \\ T_2T_2 \\ T_1T_1 \end{pmatrix}_{3_1} \oplus \begin{pmatrix} T_3T_2 \\ T_2T_1 \\ T_1T_3 \end{pmatrix}_{\bar{3}_1} \oplus \begin{pmatrix} T_2T_3 \\ T_1T_2 \\ T_3T_1 \end{pmatrix}_{\bar{3}_2}
\]

Diagonals are distinguished from off-diagonals!
Lepton Sector of the $SU(5) \times T_{13} \times Z_{12}$ Model


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Table: Transformation properties of matter, Higgs, messenger and familon fields in the seesaw sector. Here $\omega^{12} = 1$. The $Z_{12}$ 'shaping' symmetry prevents unwanted tree-level operators.

$$
\mathcal{L}_{ss} \supset y_A F \Lambda \bar{H}_5 + y_A' \bar{N} \bar{\Lambda} \varphi_A + y_B \bar{N} \bar{N} \varphi_B + M_\Lambda \bar{\Lambda} \Lambda + y'_v \bar{N}_4 \bar{\Lambda} \varphi_v + M \bar{N}_4 \bar{N}_4
$$

$$
\supset \frac{1}{M_\Lambda} y_A y'_A F \bar{N}_4 \bar{N} \bar{H}_5 \varphi_A + \frac{1}{M_\Lambda} y_A y'_v F \bar{N}_4 \bar{H}_5 \varphi_v + y_B \bar{N} \bar{N} \varphi_B + M \bar{N}_4 \bar{N}_4.
$$
Familon Structure

- **Vacuum Expectation Value (VEV) of the familons:**

  \[ y_A y'_A \langle \varphi_A \rangle = \frac{M_\Lambda}{v} \sqrt{m_v b_1 b_2 b_3} \left( -b_2^{-1} e^{i\delta}, b_1^{-1}, b_3^{-1} \right), \]

  \[ y_B \langle \varphi_B \rangle = (b_1, b_2, b_3), \]

  \[ y_A y'_v \langle \varphi_v \rangle = \frac{M_\Lambda}{v} \sqrt{M m'_v} \left( 2, -1, e^{i\delta} \right), \]

- **Majorana matrix from** \( y_B \bar{N} \bar{N} \varphi_B + M \bar{N}_4 \bar{N}_4 \): \[
\mathcal{M} \equiv \begin{pmatrix}
0 & b_2 & b_3 & 0 \\
b_2 & 0 & b_1 & 0 \\
b_3 & b_1 & 0 & 0 \\
0 & 0 & 0 & M
\end{pmatrix}
\]
Vacuum Expectation Value (VEV) of the familons:

\[ y_A y'_A \langle \varphi_A \rangle = \frac{M_{\Lambda}}{v} \sqrt{m_v b_1 b_2 b_3} \left( -b_2^{-1} e^{i\delta}, b_1^{-1}, b_3^{-1} \right), \]

\[ y_B \langle \varphi_B \rangle = (b_1, b_2, b_3), \]

\[ y_A y'_v \langle \varphi_v \rangle = \frac{M_{\Lambda}}{v} \sqrt{M m'_v} \left( 2, -1, e^{i\delta} \right), \]

Yukawa matrix from \( \frac{1}{M_{\Lambda}} y_A y'_A F \tilde{N} \tilde{H}_5 \varphi_A + \frac{1}{M_{\Lambda}} y_A y'_v F \tilde{N}_4 \tilde{H}_5 \varphi_v: \)

\[ Y^{(0)} \equiv \frac{\sqrt{b_1 b_2 b_3 m_v}}{v} \begin{pmatrix}
0 & b_3^{-1} & 0 & 2 \frac{M m'_v}{b_1 b_2 b_3 m_v} \\
b_1^{-1} & 0 & 0 & -\sqrt{\frac{M m'_v}{b_1 b_2 b_3 m_v}} \\
0 & 0 & -e^{i\delta} b_2^{-1} & e^{i\delta} \frac{M m'_v}{b_1 b_2 b_3 m_v}
\end{pmatrix} \]
- $\sum_i |m_{\nu_i}| = 114.3$ meV compared to Planck: $\sum_i |m_{\nu_i}| < 120$ meV [arXiv:1807.06209].

- Combining data from Euclid and LSST to DESI and WFIRST, the error bound on $\sum_i |m_{\nu_i}|$ will be constrained to $8 - 11$ meV.

- Normal ordering is preferred above $3\sigma$ by Super-K, T2K and NOvA [arXiv:1710.09126].

- DUNE and Hyper-K will resolve the correct mass ordering beyond $5\sigma$ in $5 - 7$ yrs [arXiv:1807.10334, 1805.04163].
Neutrinoless Double Beta Decay

- Dirac $\mathcal{CP}$ Jarlskog-Greenberg Invariant, $|\mathcal{J}| = 0.028$

- Majorana Invariants, $|\mathcal{I}_1| = 0.106$, $|\mathcal{I}_2| = 0.011$

**Prediction for $0\nu\beta\beta$**

$|m_{\beta\beta}| = 13.02$ or $25.21$ meV

compared to $|m_{\beta\beta}| \leq 61 - 165$ meV by KamLAND-Zen [arXiv:1605.02889]

- Our predictions are expected to be tested in several next generation experiments


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<th>Sensitivity (meV)</th>
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<td>CUPID</td>
<td>6 - 17</td>
<td>PandaX-III</td>
<td>20 - 55</td>
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Define $\beta = \sqrt{\frac{a}{11.5f}}$. Dirac and Majorana matrices:

\[
Y^{(0)} = \frac{\sqrt{bfm_\nu}}{v} \begin{pmatrix}
0 & 1 & 0 & 2\beta \\
1 & 0 & 0 & -\beta \\
0 & 0 & -f^{-1}e^{i\delta} & \beta e^{i\delta}
\end{pmatrix}, \quad M = b \begin{pmatrix}
0 & f & 1 & 0 \\
f & 0 & 1 & 0 \\
1 & 1 & 0 & 0 \\
0 & 0 & 0 & a
\end{pmatrix}
\]

Takagi factorization: $M = U_m D_m U_m^T$

$M_1 = bf$, $M_2 = \frac{b}{2} \left( \sqrt{f^2 + 8} - f \right)$, $M_3 = \frac{b}{2} \left( \sqrt{f^2 + 8} + f \right)$, $M_4 = ab$

\[
U_m = \begin{pmatrix}
-\frac{i}{\sqrt{2}} & -\frac{i}{2} \sqrt{1 - \frac{f}{\sqrt{f^2 + 8}}} & \frac{1}{2} \sqrt{1 + \frac{f}{\sqrt{f^2 + 8}}} & 0 \\
\frac{i}{\sqrt{2}} & -\frac{i}{2} \sqrt{1 - \frac{f}{\sqrt{f^2 + 8}}} & \frac{1}{2} \sqrt{1 + \frac{f}{\sqrt{f^2 + 8}}} & 0 \\
0 & \frac{i}{\sqrt{2}} \sqrt{1 + \frac{f}{\sqrt{f^2 + 8}}} & -\frac{1}{2} \sqrt{1 - \frac{f}{\sqrt{f^2 + 8}}} & 0 \\
0 & \frac{i}{\sqrt{2}} \sqrt{1 + \frac{f}{\sqrt{f^2 + 8}}} & -\frac{1}{2} \sqrt{1 - \frac{f}{\sqrt{f^2 + 8}}} & 0
\end{pmatrix}
\]
Viability of Leptogenesis

- Neutrino Yukawa matrix $Y_\nu = U^{(-1)} Y^{(0)} U^*_m$

$$Y_\nu^\dagger Y_\nu = \frac{bfm_\nu}{v^2}$$

$$\times \begin{pmatrix}
1 & 0 & 0 \\
\ast & \frac{1}{2} \left( 1 - \frac{f^3 - f - \sqrt{f^2 + 8}}{f^2 \sqrt{f^2 + 8}} \right) & 0 \\
\ast & * & 1 \left( 1 + \frac{f^3 - f + \sqrt{f^2 + 8}}{f^2 \sqrt{f^2 + 8}} \right) \\
\ast & * & *
\end{pmatrix}$$

- Unflavored leptogenesis is ruled out

$$\varepsilon_i \equiv \frac{\Gamma_i - \bar{\Gamma}_i}{\Gamma_i + \bar{\Gamma}_i} = \frac{1}{8\pi} \sum_{j \neq i} \text{Im} \left[ \left( (Y_\nu^\dagger Y_\nu)_{ij} \right)^2 \right] \frac{\xi \left( M_j^2 \right)}{(M_i^2)}$$
Unflavored Leptogenesis

- Structure of $Y^{(0)}$ and $\mathcal{M} = U_m \mathcal{D}_m U_m^T$ determined by $\mathcal{T}_{13}$

- Change basis to weak basis: $Y^{(0)} \rightarrow Y_\nu = U^{(-1)} Y^{(0)} U_m^*$

- Charged-lepton flavors can be neglected for $T \gg 10^{12}$ GeV. Unflavored $CP$ asymmetry depends on

\[
\text{Im} \left[ \left( Y_\nu^\dagger Y_\nu \right)^2 \right] = \text{Im} \left[ \left( U_m^T Y^{(0)} Y^{(0)} U_m^* \right)^2 \right]
\]

- $Y^{(0)} = \text{diag}(1, 1, e^{i\delta}) Y_{\text{real}}^{(0)}$ implies $Y^{(0)} Y^{(0)} \dagger$ is real

- Real, symmetric $\mathcal{M}$ implies $U_m = U_{m, \text{real}} \mathcal{P}$ where $\mathcal{P}$ is a diagonal phase matrix with entries $\pm 1$ or $\pm i$

- $\text{Im} \left[ \left( Y_\nu^\dagger Y_\nu \right)^2 \right] = \text{Im} \left[ \left( \mathcal{P}^T \text{ Real} \mathcal{P}^* \right)^2 \right]$ vanishes, unflavored leptogenesis fails!

[MHR PRD 103 (2021) 035011]
Flavored Leptogenesis

- **Density Matrix Equations**

\[
\frac{dN_{N_i}}{dz} = -(D_i + S_i)(N_{N_i} - N_{N_i}^{eq})
\]

\[
\frac{dN_{\alpha\beta}}{dz} = \sum_i \varepsilon^{(i)}_{\alpha\beta} D_i (N_{N_i} - N_{N_i}^{eq}) - \frac{1}{2} \sum_i W_i \{ P^{0(i)}, N \}_{\alpha\beta}
\]

\[
- \frac{\text{Im}(\Lambda_\tau)}{Hz} \left[ \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, N \right]_{\alpha\beta}
\]

\[
- \frac{\text{Im}(\Lambda_\mu)}{Hz} \left[ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, N \right]_{\alpha\beta}
\]

- **Final value of the $B - L$ asymmetry:**

\[
N_{B-L}^f \equiv \sum_{\alpha=e,\mu,\tau} N_{\alpha\alpha}^f
\]

- **Baryon asymmetry** $\eta_B \approx 1.28 \times 10^{-2} N_{B-L}^f$ to be compared with $\eta_{B_{CMB}}^C = (6.12 \pm 0.04) \times 10^{-10}$
The Simplest Case

- All seesaw parameters are described in terms of the familon VEV $\langle \varphi_B \rangle \equiv (b_1, b_2, b_3)$ and mass parameter $M$

- The simplest case $(b_1, b_2, b_3) \equiv b(1, 1, 1)$ fails

- **Degeneracy** in the mass spectrum
  \[
  M_1 = M_2 = b, \quad M_3 = 2b, \quad M_4 = M \equiv ab
  \]

- **$CP$ asymmetries**
  \[
  \varepsilon^{(1)} = -\varepsilon^{(2)}, \quad \varepsilon^{(3)} = \varepsilon^{(4)} = 0
  \]

- Final $B - L$ asymmetry
  \[
  N_{B-L}^f \propto \sum_i \varepsilon^{(i)} \times \text{Rate of Number density}
  \]
A Simpler Case

- Set $b_1, b_2, b_3 \equiv b(1, f, 1)$, define $M \equiv ab$

- $f \neq 1$ breaks degeneracy in the mass spectrum

$$M_1 = bf, M_2 = \frac{1}{2} \left( \sqrt{f^2 + 8} - f \right)$$

$$M_4 = ab, M_3 = \frac{1}{2} \left( \sqrt{f^2 + 8} + f \right)$$

- Non-hierarchical mass spectrum: asymmetry generated by heavier neutrinos are not entirely washed out

**Figure:** Right-handed neutrino mass spectrum setting $f = 2$. The four shaded regions represent...
Three undetermined parameters: $a$, $b$ and $f$

- $b$ is the overall mass scale, $f \neq 1$ lifts the degeneracy between $M_1$ and $M_2$, $a$ determines how heavy $M_4$ is w.r.t. others

Set $f$ to a fixed value \quad \rightarrow \quad$Vary $a$ to cover the four non-degenerate regions \quad \rightarrow \quad$Solve the density matrix equations for a trial value of $b$ \quad \rightarrow \quad$Determine $b$ requiring the calculated baryon asymmetry matches the CMB value
Upper Bound on Right-Handed Neutrino Mass

Figure: Maximum $B - L$ asymmetry at the resonance $M_3 \simeq M_4$ for (a) $f = 0.1$ and (b) $f = 0.1$ without considering $N_1$ washout. For large $b$, the $B - L$ asymmetry saturates at a value higher than the CMB value in case (a) and (b), thus indicating that there is no upper limit on $b$. $N_1$ washout decreases the final asymmetry by only a factor of 10, and is not very efficient.
Upper Bound on Right-Handed Neutrino Mass

Figure: Maximum $B - L$ asymmetry at the resonance $M_3 \simeq M_4$ for (a) $f = 10$, and (b) $f = 10$ without considering $N_2$ washout. For large $b$, the $B - L$ asymmetry saturates below the CMB value for large $b$, thus setting an upper limit above which successful resonant leptogenesis is not feasible. If the $N_2$ washout is disregarded, the final asymmetry is $\mathcal{O}(10^{12})$ times large, as shown in case (b), thus...
Figure: Decay parameter times branching ratio as a function of $f$ at resonance. The asymmetry generated by $N_3$ and $N_4$ at resonance is partially washed out by $N_1$ and $N_2$, and is proportional to $e^{-P_{1\alpha}K_i}$.