Neutrino Decoherence in Simple Open Quantum Systems

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Content

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Neutrino Oscillation

The neutrino oscillation is a quantum mechanical consequence which can be described by the Schrödinger equation

$$i\frac{\partial}{\partial t}|\nu\rangle = H_\nu|\nu\rangle, \quad H_\nu = \frac{\delta m^2}{4E}\sigma_3$$

or the Liouville–von Neumann equation

$$\dot{\rho}_\nu(t) = -i[H_\nu, \rho_\nu(t)]$$

Initial state

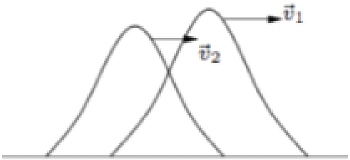
$$|\nu_e\rangle = \cos\theta|\nu_1\rangle + \sin\theta|\nu_2\rangle, \quad \rho_\nu = |\nu_e\rangle\langle\nu_e|$$

Survival probability

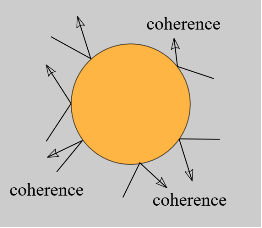
$$P_{ee}(t) = |\langle\nu_e|\nu_e(t)\rangle|^2 = Tr[\rho_e\rho_e(t)] = 1 - \frac{1}{2}\sin^2(2\theta)(1-\cos(\frac{\delta m^2}{2E}t))$$

Decoherence

- Kinematic decoherence - Wave packet separation



- Dynamic decoherence - Interactions with environment



Lindblad equation

A popular way to account for the environment interactions is the Gorini–Kossakowski–Sudarshan–Lindblad (GKSL) equation

$$\dot{\rho}_\nu(t) = -i[H_\nu, \rho_\nu] + D[\rho_\nu]$$

The dynamical decoherence affects the evolution of the density matrix by adding the Lindblad decohering term

$$D[\rho_\nu] = \sum_m (L_m \rho_\nu L_m^\dagger - \frac{L_m^\dagger L_m \rho_\nu + \rho_\nu L_m^\dagger L_m}{2})$$

Constraints on Lindblad operators

- Entropy increasing $L_m^\dagger = L_m$
- Energy conservation $[H_\nu, L_m] = 0$

Solution to the Lindblad equation

For a two-state system $L = \begin{pmatrix} l_1 & 0 \\ 0 & l_2 \end{pmatrix}$, the resulting solution is

$$\rho_\nu(t) = \begin{pmatrix} \rho_{11} & \rho_{12} e^{i\frac{\delta m^2}{2E}t - \Gamma t} \\ \rho_{21} e^{-i\frac{\delta m^2}{2E}t - \Gamma t} & \rho_{22} \end{pmatrix}$$

Decoherence rate

$$\Gamma = \frac{1}{2}(l_1 - l_2)^2$$

Survival probability

$$P_{ee}(t) = 1 - \frac{1}{2}\sin^2(2\theta)[1 - e^{-\Gamma t}\cos(\frac{\delta m^2}{2E}t)].$$

What is Γ? Energy dependence?

The QM approach

Total Hamiltonian $H = H_\nu + H_{Ev} + H_I$
Energy conservation $[H_\nu, H_I] = 0$

$$H_I = |\nu_1\rangle\langle\nu_1| \otimes H_1 + |\nu_2\rangle\langle\nu_2| \otimes H_2$$

Initial state $\rho(0) = \rho_\nu(0) \otimes \rho_{Ev}(0)$

$$\begin{aligned}\rho(t) &= e^{-i(H_\nu + H_{Ev} + H_I)t} \rho_\nu \otimes \rho_{Ev} e^{i(H_\nu + H_{Ev} + H_I)t} \\ &= \sum_{ij} \rho_{ij} e^{-iE_i t} |\nu_i\rangle\langle\nu_j| e^{iE_j t} \otimes e^{-i(H_{Ev} + H_i)t} \rho_{Ev} e^{i(H_{Ev} + H_j)t}\end{aligned}$$

The QM approach

To get the reduced density matrix of neutrinos, we take the partial trace

$$\rho_\nu(t) = Tr_{Ev}[\rho(t)] = \begin{pmatrix} \rho_{11} & \rho_{12} e^{i\frac{\delta m_{ij}^2}{2E}t} F(t) \\ \rho_{21} e^{-i\frac{\delta m_{ij}^2}{2E}t} F^*(t) & \rho_{22} \end{pmatrix}$$

Define the form factor $F(t) = Tr_{Ev}[e^{i(H_{Ev}+H_j)t} e^{-i(H_{Ev}+H_i)t} \rho_{Ev}]$
Decoherence happens when $F(t) \to 0$, the state becomes an incoherence sum $\rho_\nu = \rho_{11}|\nu_1\rangle\langle\nu_1| + \rho_{22}|\nu_2\rangle\langle\nu_2|$

Environment as forced harmonic oscillators

We model the environment by N identical harmonic oscillators

$$H_{Ev} = \sum_{s=1}^{N} \omega a_s^\dagger a_s.$$

The interaction with neutrinos is modeled as a linear coupling

$$H_I = (\lambda_1 |\nu_1\rangle\langle\nu_1| + \lambda_2 |\nu_2\rangle\langle\nu_2|) \otimes \sum_{s=1}^{N} f_s(t)(a_s^\dagger + a_s).$$

For a "white noise" power spectrum, the autocorrelation function gives

$$C_f(\tau) \equiv \langle f_s(t) f_s(t+\tau)\rangle = g\delta(\tau),$$

which corresponds to the vacuum fluctuations of the background field.

Environment as forced harmonic oscillators

This Hamiltonian evolves the s'th harmonic oscillator from the ground state to coherent states

$$e^{\omega a_s^\dagger a_s + \lambda_i f_s(t)(a_s^\dagger + a_s)}|0\rangle = |z_{is}\rangle.$$

The form factor reads

$$F(t) = \prod_s \langle z_{1s}|z_{2s}\rangle = \exp\{-\frac{1}{2}(\lambda_1 - \lambda_2)^2 Ngt\}.$$

Corresponding to the Lindblad operator

$$L = \sqrt{Ng}\begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix}.$$

The QFT approach

A realistic interaction of neutrinos with the environment should be described by local quantum field operators.
Suppose the interaction term is given by (ignore the spins)

$$\mathcal{H}_I = (\lambda_1 \nu_1 \nu_1 + \lambda_2 \nu_2 \nu_2)\phi\phi$$

Particles are initially at their momentum eigenstates

$$|i\rangle = \mathcal{N}|\nu_e(p)\rangle|\phi(q)\rangle$$

where $\mathcal{N}^{-1} = 2V\sqrt{E_\nu E_\phi}$, $V = (2\pi)^3\delta^{(3)}(0)$.

The QFT approach

Consider the elastic scattering , the final state is given by $|f\rangle = S|i\rangle$, with

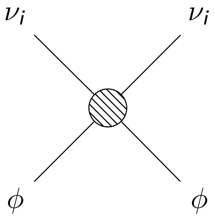
$$S = e^{-iHt} = 1 + i\mathcal{T}.$$

The S-matrix elements are given by

$$\langle \nu_i(p')\phi(q')|\mathcal{T}|\nu_i(p)\phi(q)\rangle = (2\pi)^4\delta^{(4)}(p+q-p'-q')\mathcal{M}_i(p,q,p',q'),$$

where $\mathcal{M}_i$ is given by Feynman diagrams.

$$i\mathcal{M}_i =$$



The QFT approach

The final state can be written as

$$|f\rangle = \mathcal{N}(\cos\theta|f_1\rangle + \sin\theta|f_2\rangle),$$

where

$$|f_1\rangle = |\nu_1(p)\phi(q)\rangle + i\int \frac{d^3p'}{(2\pi)^3 2E_j(p')}\frac{d^3q'}{(2\pi)^3 2E_j(q')} (2\pi)^4\delta^{(4)}(p+q-p'-q')\mathcal{M}_1(p,q,p',q')|\nu_1(p')\phi(q')\rangle,$$

$$|f_2\rangle = |\nu_2(p)\phi(q)\rangle + i\int \frac{d^3p'}{(2\pi)^3 2E_j(p')}\frac{d^3q'}{(2\pi)^3 2E_j(q')} (2\pi)^4\delta^{(4)}(p+q-p'-q')\mathcal{M}_2(p,q,p',q')|\nu_2(p')\phi(q')\rangle.$$

The QFT approach

Tracing out the environmental degrees of freedom and integrate over the momentum space, we get the reduced density matrix of neutrinos after one scattering event

$$\rho_{\nu,f} = \begin{pmatrix} \rho_{11} & \rho_{12}(1+\Delta) \\ \rho_{21}(1+\Delta^*) & \rho_{22} \end{pmatrix}$$

The diagonal terms of ρ_ν stay unchanged due to unitarity.
The off-diagonal term is altered by

$$\begin{aligned} \Delta =& i\mathcal{N}^2 VT\mathcal{M}_1(p,q,p,q) - i\mathcal{N}^2 VT\mathcal{M}_2^*(p,q,p,q) \\ &+ \mathcal{N}^2 VT \int \frac{d^3p'}{(2\pi)^3 2E_j(p')} \frac{d^3q'}{(2\pi)^3 2E_j(q')} (2\pi)^4 \\ &\delta^{(4)}(p+q-p'-q')\mathcal{M}_1(p,q,p',q')\mathcal{M}_2^*(p,q,p',q'). \end{aligned}$$

where $VT = (2\pi)^4\delta^{(4)}(0)$.

The QFT approach

Using the optical theorem

$$\text{Im}[\mathcal{M}_i(p,q,p,q)] = 2E_\nu E_\phi v \sigma_i$$

where v is the relative speed between neutrinos and environment, and σ_i is the total cross section between ν_1 and ϕ.
Consider the flux of environmental particles with number density N_ϕ and integrate over time, we finally get the decoherence rate

$$\Gamma = -\frac{N_\phi v}{2}(\sigma_1 + \sigma_2 - 2\sqrt{\sigma_1 \sigma_2})$$

with the form factor

$$F(t) = e^{-\Gamma t}$$

Corresponding to the Lindblad operator

$$L = \begin{pmatrix} \sqrt{N_\phi v \sigma_1} & 0 \\ 0 & \sqrt{N_\phi v \sigma_2} \end{pmatrix}$$

Conclusion

- The exponential decay $F(t) = e^{-\Gamma t}$ is a common feature for environment induced decoherence, as long as the interactions are weak and stochastic (Born-Markov approximation).
- The Lindblad operators are determined by the coupling strength (or scattering cross sections) of neutrinos with environment.
- The standard model contribution $\Gamma \sim N_e G_F^2 m_e E_\nu \sim 10^{-30} GeV$
- Experimental bounds: $\Gamma_{12} < 1.2 \times 10^{-23} GeV$ and $\Gamma_{23} < 7.7 \times 10^{-25} GeV$ at 90% C.L. (1805.09818)
- This result could be helpful for looking for interactions beyond standard model, dark matter detection, testing quantum gravity, etc.

Thank You!