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Distinguishing different neutrinoless double beta mechanisms (+ a 0νββ tool)

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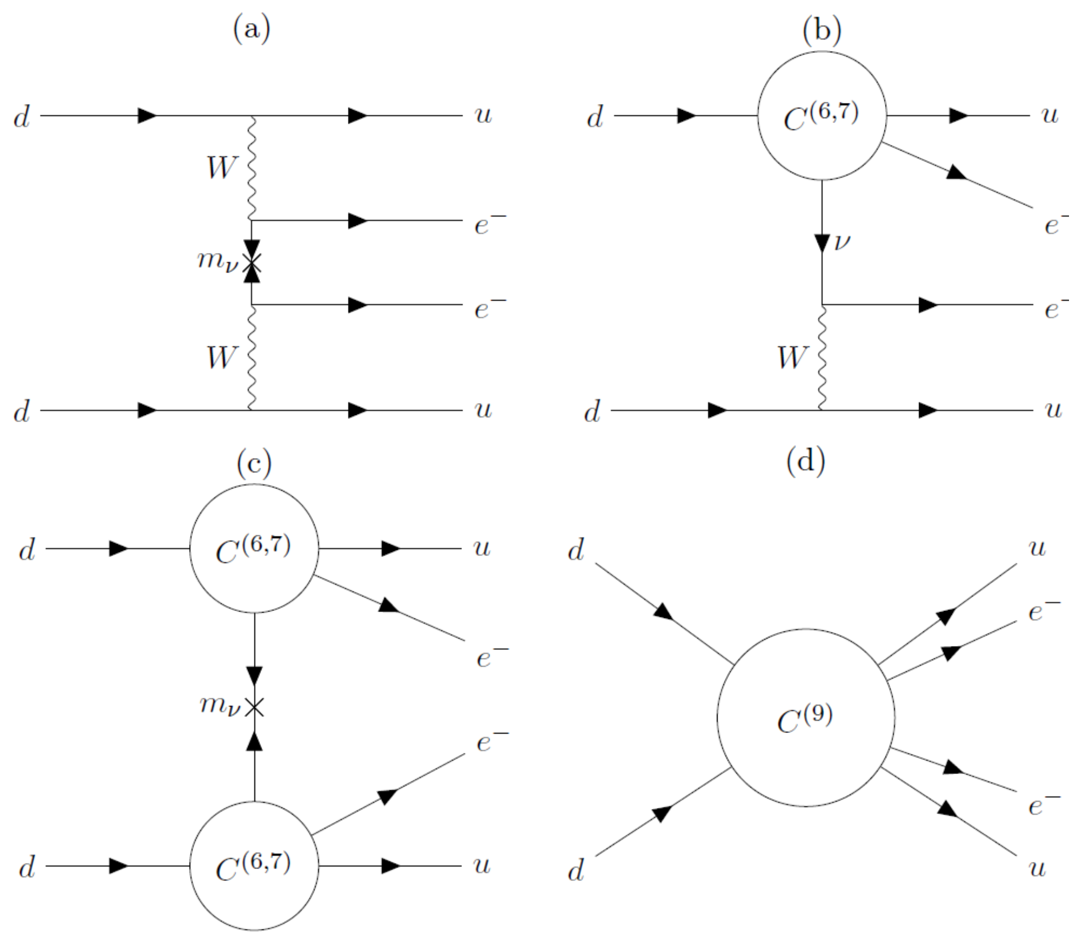
0νββ mechanisms - Standard Classification

a) mass mechanism

b) long-range

c) ignore

d) short-range



EFT Approach

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Goal: Describe 0vββ in terms of a limited set of EFT operators

Low-Energy decay process → use LEFT (no light new physics!)

→ "Master-Formula" framework [Cirigliano et al. ArXiv:1806.02780]

$$\left(T_{1/2}^{0\nu}\right)^{-1} = g_A^4 \Big[G_{01} \left(|\mathcal{A}_\nu|^2 + |\mathcal{A}_R|^2 \right) - 2\left(G_{01} - G_{04}\right) \mathrm{Re}\left[\mathcal{A}_\nu^* \mathcal{A}_R\right] + 4 G_{02} |\mathcal{A}_E|^2 + 2 G_{04} \left(|\mathcal{A}_{m_e}|^2 + \mathrm{Re}\left[\mathcal{A}_{m_e}^* \left(\mathcal{A}_\nu + \mathcal{A}_R\right)\right] \right) - 2 G_{03} \mathrm{Re}\left[\left(\mathcal{A}_\nu + \mathcal{A}_R\right)\mathcal{A}_E^* + 2\mathcal{A}_{m_e}\mathcal{A}_E^*\right] + G_{09} |\mathcal{A}_M|^2 + G_{06} \mathrm{Re}\left[\left(\mathcal{A}_\nu - \mathcal{A}_R\right)\mathcal{A}_M^*\right] \Big]$$

Wilson Coefficients + 3 numerical inputs (NMEs, PSFs, LECs)

EFT Approach

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32 different operators

1 mass mechanism (dim 3)

7 long-range (dim 6,7)

24 short-range (dim 9)

→ Goal: experimentally distinguish among these 32 operators

Class	Op. Name	Op. Structure
Dim 3		
Ψ^2	$\mathcal{O}_{m_{\beta\beta}}$	$-\frac{1}{2}m_{ee}\overline{\nu_{L,e}^C}\nu_{L,e}$
Dim 6		
Ψ^4	$\mathcal{O}_{SL}^{(6)}$	$[\overline{u_R}d_L][\overline{e_L}\nu_L^C]$
	$\mathcal{O}_{SR}^{(6)}$	$[\overline{u_L}d_R][\overline{e_L}\nu_L^C]$
	$\mathcal{O}_{VL}^{(6)}$	$[\overline{u_L}\gamma^\mu d_L][\overline{e_R}\gamma_\mu\nu_L^C]$
	$\mathcal{O}_{VR}^{(6)}$	$[\overline{u_R}\gamma^\mu d_R][\overline{e_R}\gamma_\mu\nu_L^C]$
	$\mathcal{O}_{T}^{(6)}$	$[\overline{u_L}\sigma^{\mu\nu}d_R][\overline{e_L}\sigma_{\mu\nu}\nu_L^C]$
Dim 7		
$\Psi^4\partial$	$\mathcal{O}_{VL}^{(7)}$	$[\overline{u_L}\gamma^\mu d_L][\overline{e_L}\overleftrightarrow{\partial}_\mu\nu_L^C]$
	$\mathcal{O}_{VR}^{(7)}$	$[\overline{u_R}\gamma^\mu d_R][\overline{e_L}\overleftrightarrow{\partial}_\mu\nu_L^C]$
Dim 9		
Ψ^6	$\mathcal{O}_{1L,R}^{(9)}$	$[\overline{u_L}\gamma_\mu d_L][\overline{u_L}\gamma^\mu d_L][\overline{e_{L,R}}e_{L,R}^C]$
	$\mathcal{O}_{1L,R}^{(9)\prime}$	$[\overline{u_R}\gamma_\mu d_R][\overline{u_R}\gamma^\mu d_R][\overline{e_{L,R}}e_{L,R}^C]$
	$\mathcal{O}_{2L,R}^{(9)}$	$[\overline{u_R}d_L][\overline{u_R}d_L][\overline{e_{L,R}}e_{L,R}^C]$
	$\mathcal{O}_{2L,R}^{(9)\prime}$	$[\overline{u_L}d_R][\overline{u_L}d_R][\overline{e_{L,R}}e_{L,R}^C]$
	$\mathcal{O}_{3L,R}^{(9)}$	$[\overline{u_R}^\alpha d_L^\beta][\overline{u_R}^\beta d_L^\alpha][\overline{e_{L,R}}e_{L,R}^C]$
	$\mathcal{O}_{3L,R}^{(9)\prime}$	$[\overline{u_L}^\alpha d_R^\beta][\overline{u_L}^\beta d_R^\alpha][\overline{e_{L,R}}e_{L,R}^C]$
	$\mathcal{O}_{4L,R}^{(9)}$	$[\overline{u_L}\gamma^\mu d_L][\overline{u_R}\gamma_\mu d_R][\overline{e_{L,R}}e_{L,R}^C]$
	$\mathcal{O}_{5L,R}^{(9)}$	$[\overline{u_L}^\alpha\gamma^\mu d_L^\beta][\overline{u_R}^\beta\gamma_\mu d_R^\alpha][\overline{e_{L,R}}e_{L,R}^C]$
	$\mathcal{O}_{6}^{(9)}$	$[\overline{u_L}\gamma_\mu d_L][\overline{u_L}d_R][\bar{e}\gamma^\mu\gamma_5 e^C]$
	$\mathcal{O}_{6}^{(9)\prime}$	$[\overline{u_R}\gamma_\mu d_R][\overline{u_R}d_L][\bar{e}\gamma^\mu\gamma_5 e^C]$
	$\mathcal{O}_{7}^{(9)}$	$[\overline{u_L}t^A\gamma_\mu d_L][\overline{u_L}t^A d_R][\bar{e}\gamma^\mu\gamma_5 e^C]$
	$\mathcal{O}_{7}^{(9)\prime}$	$[\overline{u_R}t^A\gamma_\mu d_R][\overline{u_R}t^A d_L][\bar{e}\gamma^\mu\gamma_5 e^C]$
	$\mathcal{O}_{8}^{(9)}$	$[\overline{u_L}\gamma_\mu d_L][\overline{u_R}d_L][\bar{e}\gamma^\mu\gamma_5 e^C]$
	$\mathcal{O}_{8}^{(9)\prime}$	$[\overline{u_R}\gamma_\mu d_R][\overline{u_L}d_R][\bar{e}\gamma^\mu\gamma_5 e^C]$
	$\mathcal{O}_{9}^{(9)}$	$[\overline{u_L}t^A\gamma_\mu d_L][\overline{u_R}t^A d_L][\bar{e}\gamma^\mu\gamma_5 e^C]$
	$\mathcal{O}_{9}^{(9)\prime}$	$[\overline{u_R}t^A\gamma_\mu d_R][\overline{u_L}t^A d_R][\bar{e}\gamma^\mu\gamma_5 e^C]$

How to distinguish?

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Assume only 1 operator at a time

3 different observables in decay experiments

1) Half-Life

2) Single Electron Spectra

3) Angular Correlation

1) Half-Life → Ratios

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$$\left(T_{1/2}^{\mathcal{O}_i}(^AX)\right)^{-1} = |C^{\mathcal{O}_i}|^2 |\mathcal{M}^{\mathcal{O}_i}(^AX)|^2 G^{\mathcal{O}_i}(^AX)$$

Study Ratios of Half-Lifes [see Deppisch and Päs arXiv:hep-ph/0612165]

$$R^{\mathcal{O}_i}(^{\mathrm{A}}\mathrm{X}) \equiv \frac{T_{1/2}^{\mathcal{O}_i}(^{\mathrm{A}}\mathrm{X})}{T_{1/2}^{\mathcal{O}_i}(^{76}\mathrm{Ge})} = \frac{|\mathcal{M}^{\mathcal{O}_i}(^{76}\mathrm{Ge})|^2 G^{\mathcal{O}_i}(^{76}\mathrm{Ge})}{|\mathcal{M}^{\mathcal{O}_i}(^{\mathrm{A}}\mathrm{X})|^2 G^{\mathcal{O}_i}(^{\mathrm{A}}\mathrm{X})}$$

Distinguish 2 operators i, j by

study different lepton currents

$$R_{ij}(^AX) = \frac{R^{\mathcal{O}_i}(^AX)}{R^{\mathcal{O}_j}(^AX)}$$

Study different quark currents

Distinguish from mass mechanism (i.e. identify existence of non-standard mechanisms)

$\rightarrow R_{im_{\beta\beta}}$

1) Half-Life → Ratios

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12 Groups of distinguishable operators (depends on unknown LECs!)

$m_{\beta\beta}$	$C_{VL}^{(6)}$	$C_{VR}^{(6)}$	$C_{T}^{(6)}$	$C_{6,7}$	$C_{S1}^{(9)}$	$C_{S2}^{(9)}$	$C_{S3}^{(9)}$	$C_{S4}^{(9)}$	$C_{S5}^{(9)}$	$C_{V}^{(9)}$	$\tilde{C}_{V}^{(9)}$
$m_{\beta\beta}$	$C_{VL}^{(6)}$	$C_{VR}^{(6)}$	$C_{T}^{(6)}$	$C_{SL}^{(6)}$	$C_{1L}^{(9)}$	$C_{2L}^{(9)}$	$C_{3L}^{(9)}$	$C_{4L}^{(9)}$	$C_{5L}^{(9)}$	$C_{6}^{(9)}$	$C_{7}^{(9)}$
-	-	-	-	$C_{SR}^{(6)}$	$C_{1R}^{(9)}$	$C_{2R}^{(9)}$	$C_{3R}^{(9)}$	$C_{4R}^{(9)}$	$C_{5R}^{(9)}$	$C_{6}^{(9)\prime}$	$C_{7}^{(9)\prime}$
-	-	-	-	$C_{VL}^{(7)}$	$C_{1L}^{(9)\prime}$	$C_{2L}^{(9)\prime}$	$C_{3L}^{(9)\prime}$	-	-	$C_{8}^{(9)}$	$C_{9}^{(9)}$
-	-	-	-	$C_{VR}^{(7)}$	$C_{1R}^{(9)\prime}$	$C_{2R}^{(9)\prime}$	$C_{3R}^{(9)\prime}$	-	-	$C_{8}^{(9)\prime}$	$C_{9}^{(9)\prime}$

1) Half-Life → Ratios

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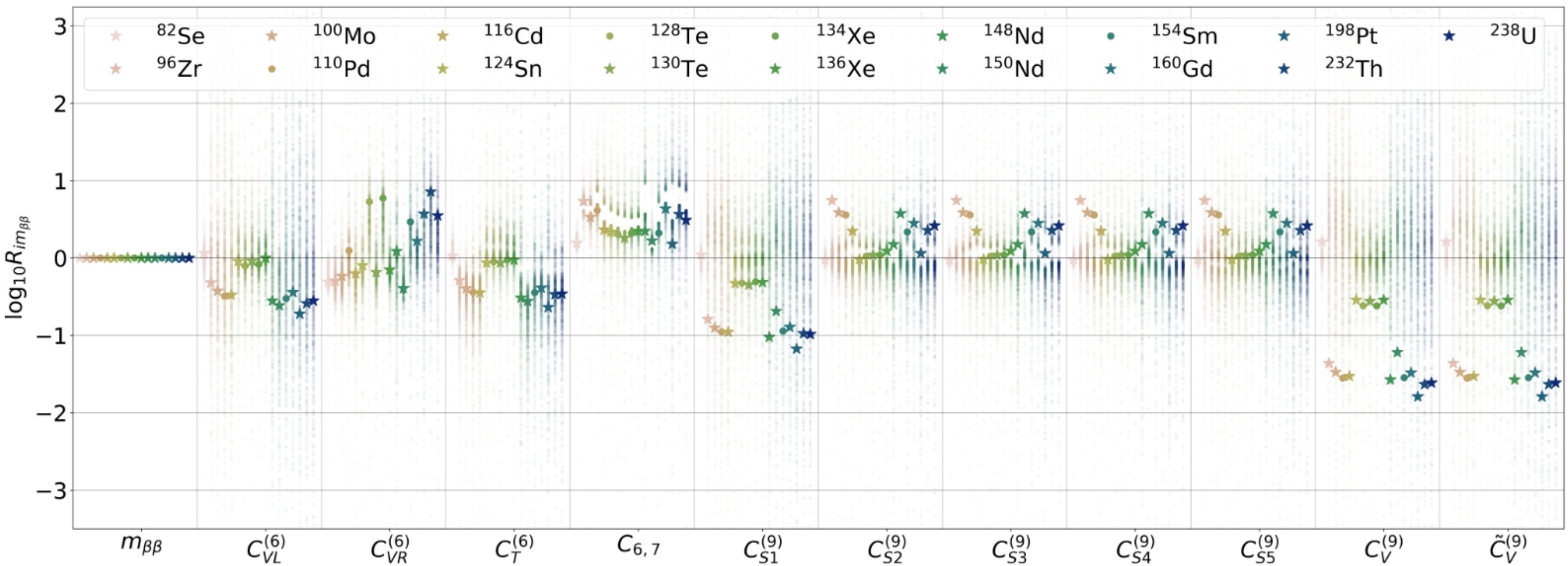
12 Groups of distinguishable operators (depends on unknown LECs!)



Preliminary results, soon to be published...

2) + 3) Phase Space Observables

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Angular Correlation and Single Electron Spectra result from the leptonic phase space → distinguish different lepton currents

Again order operators into groups:

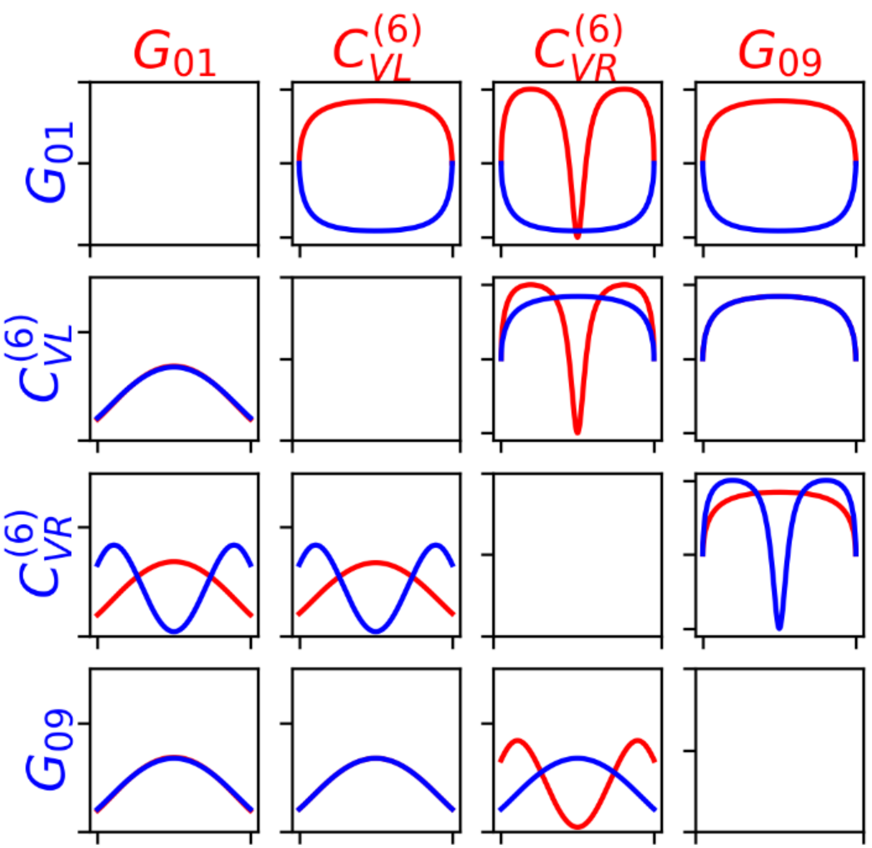
→ 4 different groups of operators

G_{01}	G_{02}	G_{03}	G_{04}	G_{06}	G_{09}
$m_{\beta\beta}$	$C_{VL}^{(6)}, C_{VR}^{(6)}$	$C_{VL}^{(6)}, C_{VR}^{(6)}$	$C_{VL}^{(6)}, C_{VR}^{(6)}$	-	$C_{VL}^{(6)}$
$C_{SL}^{(6)}, C_{SR}^{(6)}$	-	-	-	-	$C_6^{(9)}, C_6^{(9)\prime}$
$C_T^{(6)}$	-	-	-	-	$C_7^{(9)}, C_7^{(9)\prime}$
$C_{VL}^{(7)}, C_{VR}^{(7)}$	-	-	-	-	$C_8^{(9)}, C_8^{(9)\prime}$
$C_{1L}^{(9)}, C_{1R}^{(9)}$	-	-	-	-	$C_9^{(9)}, C_9^{(9)\prime}$
$C_{1L}^{(9)\prime}, C_{1R}^{(9)\prime}$	-	-	-	-	$C_9^{(9)}, C_9^{(9)\prime}$
$C_{2L}^{(9)}, C_{2R}^{(9)}$	-	-	-	-	-
$C_{2L}^{(9)\prime}, C_{2R}^{(9)\prime}$	-	-	-	-	-
$C_{3L}^{(9)}, C_{3R}^{(9)}$	-	-	-	-	-
$C_{3L}^{(9)\prime}, C_{3R}^{(9)\prime}$	-	-	-	-	-
$C_{4L}^{(9)}, C_{4R}^{(9)}$	-	-	-	-	-
$C_{5L}^{(9)}, C_{5R}^{(9)}$	-	-	-	-	-

2) + 3) Phase Space Observables

→ 4 different groups (plot for 136Xe)

Angular correlation



Normalized Spectra

Conclusion

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- The 32 LEFT operators can be put into 12 different distinguishable groups
- Currently unknown LECs need to be fixed!
- Long formula and calculation

$$
\begin{aligned}
\left(T_{1/2}^{0\nu}\right)^{-1} = g_A^4 \Big[& G_{01}\left(|\mathcal{A}_\nu|^2 + |\mathcal{A}_R|^2\right) - 2\left(G_{01} - G_{04}\right)\mathrm{Re}\left[\mathcal{A}_\nu^* \mathcal{A}_R\right] \\
& + 4G_{02}\,|\mathcal{A}_E|^2 + 2G_{04}\left(|\mathcal{A}_{m_e}|^2 + \mathrm{Re}\left[\mathcal{A}_{m_e}^*\left(\mathcal{A}_\nu + \mathcal{A}_R\right)\right]\right) \\
& - 2G_{03}\,\mathrm{Re}\left[\left(\mathcal{A}_\nu + \mathcal{A}_R\right)\mathcal{A}_E^* + 2\mathcal{A}_{m_e}\mathcal{A}_E^*\right] \\
& + G_{09}\,|\mathcal{A}_M|^2 + G_{06}\,\mathrm{Re}\left[\left(\mathcal{A}_\nu - \mathcal{A}_R\right)\mathcal{A}_M^*\right] \Big]
\end{aligned}
$$

Conclusion

- The 32 LEFT operators can be put into 12 different distinguishable groups
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$$\left(T_{1/2}^{0\nu}\right)^{-1} = g_A^4 \Big[G_{01} \left(|\mathcal{A}_\nu|^2 + |\mathcal{A}_R|^2 \right) - 2\left(G_{01} - G_{04}\right) \mathrm{Re}\left[\mathcal{A}_\nu^* \mathcal{A}_R\right] + 4 G_{02} |\mathcal{A}_E|^2 + 2 G_{04} \left(|\mathcal{A}_{m_e}|^2 + \mathrm{Re}\left[\mathcal{A}_{m_e}^* \left(\mathcal{A}_\nu + \mathcal{A}_R\right)\right]\right) - 2 G_{03}\, \mathrm{Re}\left[\left(\mathcal{A}_\nu + \mathcal{A}_R\right)\mathcal{A}_E^* + 2\mathcal{A}_{m_e}\mathcal{A}_E^*\right] + G_{09} |\mathcal{A}_M|^2 + G_{06}\, \mathrm{Re}\left[\left(\mathcal{A}_\nu - \mathcal{A}_R\right)\mathcal{A}_M^*\right] \Big]$$

$$\mathcal{A}_\nu = \frac{m_{\beta\beta}}{m_e} \mathcal{M}_\nu^{(3)} + \frac{m_N}{m_e} \mathcal{M}_\nu^{(6)} \left(C_{\mathrm{SL}}^{(6)}, C_{\mathrm{SR}}^{(6)}, C_{\mathrm{T}}^{(6)}, C_{\mathrm{VL}}^{(7)}, C_{\mathrm{VR}}^{(7)} \right) + \frac{m_N^2}{m_e v} \mathcal{M}_\nu^{(9)} \left(C_{1\mathrm{L}}^{(9)}, C_{1\mathrm{L}}^{(9)\prime}, C_{2\mathrm{L}}^{(9)}, C_{2\mathrm{L}}^{(9)\prime}, C_{3\mathrm{L}}^{(9)}, C_{3\mathrm{L}}^{(9)\prime}, C_{4\mathrm{L}}^{(9)}, C_{5\mathrm{L}}^{(9)} \right)$$

Conclusion

- The 32 LEFT operators can be put into 12 different distinguishable groups
- Currently unknown LECs need to be fixed!
- Long formula and calculation

$$\left(T^{0\nu}_{1/2}\right)^{-1} = g_A^4 \Big[G_{01}\left(|\mathcal{A}_\nu|^2 + |\mathcal{A}_R|^2\right) - 2\left(G_{01} - G_{04}\right) \mathrm{Re}\left[\mathcal{A}_\nu^* \mathcal{A}_R\right]$$
$$+ 4G_{02}\left|\mathcal{A}_E\right|^2 + 2G_{04}\left(|\mathcal{A}_{m_e}|^2 + \mathrm{Re}\left[\mathcal{A}_{m_e}^*\left(\mathcal{A}_\nu + \mathcal{A}_R\right)\right]\right)$$
$$- 2G_{03}\,\mathrm{Re}\left[\left(\mathcal{A}_\nu + \mathcal{A}_R\right)\mathcal{A}_E^* + 2\mathcal{A}_{m_e}\mathcal{A}_E^*\right]$$
$$\ldots\ \mathrm{Re}\left[\left(\mathcal{A}_\nu - \mathcal{A}_R\right)\mathcal{A}_M^*\right] \Big]$$

$$\mathcal{A}_R = \frac{m_N^2}{m_e v}\mathcal{M}_R^{(9)}\left(C_{1R}^{(9)}, C_{1R}^{(9)\prime}, C_{2R}^{(9)}, C_{2R}^{(9)\prime}, C_{3R}^{(9)}, C_{3R}^{(9)\prime}, C_{4R}^{(9)}, C_{5R}^{(9)}\right),$$
$$\mathcal{A}_E = \mathcal{M}_{E,L}^{(6)}\left(C_{VL}^{(6)}\right) + \mathcal{M}_{E,R}^{(6)}\left(C_{VR}^{(6)}\right),$$
$$\mathcal{A}_{m_e} = \mathcal{M}_{m_e,L}^{(6)}\left(C_{VL}^{(6)}\right) + \mathcal{M}_{m_e,R}^{(6)}\left(C_{VR}^{(6)}\right),$$
$$\mathcal{A}_M = \frac{m_N}{m_e}\mathcal{M}_M^{(6)}\left(C_{VL}^{(6)}\right) + \frac{m_N^2}{m_e v}\mathcal{M}_M^{(9)}\left(C_6^{(9)}, C_6^{(9)\prime}, C_7^{(9)}, C_7^{(9)\prime}, C_8^{(9)}, C_8^{(9)\prime}, C_9^{(9)}, C_9^{(9)\prime}\right)$$

$$\ldots \frac{m_N}{m_e}\mathcal{M}_\nu^{(6)}\left(C_{SL}^{(6)}, C_{SR}^{(6)}, C_T^{(6)}, C_{VL}^{(7)}, C_{VR}^{(7)}\right)$$
$$\ldots \mathcal{M}_\nu^{(9)}\left(C_{1L}^{(9)}, C_{1L}^{(9)\prime}, C_{2L}^{(9)}, C_{2L}^{(9)\prime}, C_{3L}^{(9)}, C_{3L}^{(9)\prime}, C_{4L}^{(9)}, C_{5L}^{(9)}\right)$$

Conclusion

- The 32 LEFT operators can be ... 12 different distinguishable groups
- Currently unknown LECs ne...
- Long formula and calculation

$$\mathcal{M}_\nu^{(3)} = -V_{ud}^2\left(-\frac{1}{g_A^2}M_F + \mathcal{M}_{GT} + \mathcal{M}_T + 2\frac{m_\pi^2 \mathbf{g}_\nu^{\mathrm{NN}}}{g_A^2}M_{F,sd}\right),$$

$$\mathcal{M}_\nu^{(6)} = V_{ud}\left(\frac{B}{m_N}\left(C_{SL}^{(6)} - C_{SR}^{(6)}\right) + \frac{m_\pi^2}{m_N v}\left(C_{VL}^{(7)} - C_{VR}^{(7)}\right)\right)\mathcal{M}_{PS} + V_{ud}C_{\mathrm{T}}^{(6)}\mathcal{M}_{T6},$$

$$\mathcal{M}_\nu^{(9)} = -\frac{1}{2m_N^2}C_{\pi\pi L}^{(9)}\left(M_{GT,sd}^{AP} + M_{T,sd}^{AP}\right) - \frac{2m_\pi^2}{g_A^2 m_N^2}C_{NNL}^{(9)}M_{F,sd},$$

$$\begin{aligned}\left(T_{1/2}^{0\nu}\right)^{-1} = g_A^4\Big[&G_{01}\left(|\mathcal{A}_\nu|^2 + |\mathcal{A}_R|^2\right) - 2\left(G_{01} - G_{04}\right)\mathrm{Re}\left[\mathcal{A}_\nu^*\mathcal{A}_R\right] \\ &+ 4G_{02}|\mathcal{A}_E|^2 + 2G_{04}\left(|\mathcal{A}_{m_e}|^2 + \mathrm{Re}\left[\mathcal{A}_{m_e}^*\left(\mathcal{A}_\nu + \mathcal{A}_R\right)\right]\right) \\ &- 2G_{03}\,\mathrm{Re}\left[\left(\mathcal{A}_\nu + \mathcal{A}_R\right)\mathcal{A}_E^* + 2\mathcal{A}_{m_e}\mathcal{A}_E^*\right] \\ &\cdots\,\mathrm{Re}\left[\left(\mathcal{A}_\nu - \mathcal{A}_R\right)\mathcal{A}_M^*\right]\Big]\end{aligned}$$

$$\mathcal{A}_R = \frac{m_N^2}{m_e v}\mathcal{M}_R^{(9)}\left(C_{1R}^{(9)}, C_{1R}^{(9)\prime}, C_{2R}^{(9)}, C_{2R}^{(9)\prime}, C_{3R}^{(9)}, C_{3R}^{(9)\prime}, C_{4R}^{(9)}, C_{5R}^{(9)}\right),$$

$$\mathcal{A}_E = \mathcal{M}_{E,L}^{(6)}\left(C_{\mathrm{VL}}^{(6)}\right) + \mathcal{M}_{E,R}^{(6)}\left(C_{\mathrm{VR}}^{(6)}\right),$$

$$\mathcal{A}_{m_e} = \mathcal{M}_{m_e,L}^{(6)}\left(C_{\mathrm{VL}}^{(6)}\right) + \mathcal{M}_{m_e,R}^{(6)}\left(C_{\mathrm{VR}}^{(6)}\right),$$

$$\mathcal{A}_M = \frac{m_N}{m_e}\mathcal{M}_M^{(6)}\left(C_{\mathrm{VL}}^{(6)}\right) + \frac{m_N^2}{m_e v}\mathcal{M}_M^{(9)}\left(C_6^{(9)}, C_6^{(9)\prime}, C_7^{(9)}, C_7^{(9)\prime}, C_8^{(9)}, C_8^{(9)\prime}, C_9^{(9)}, C_9^{(9)\prime}\right)$$

$$\cdots\frac{m_N}{m_e}\mathcal{M}_\nu^{(6)}\left(C_{\mathrm{SL}}^{(6)}, C_{\mathrm{SR}}^{(6)}, C_{\mathrm{T}}^{(6)}, C_{\mathrm{VL}}^{(7)}, C_{\mathrm{VR}}^{(7)}\right) \cdots \mathcal{M}_\nu^{(9)}\left(C_{1\mathrm{L}}^{(9)}, C_{1\mathrm{L}}^{(9)\prime}, C_{2\mathrm{L}}^{(9)}, C_{2\mathrm{L}}^{(9)\prime}, C_{3\mathrm{L}}^{(9)}, C_{3\mathrm{L}}^{(9)\prime}, C_{4\mathrm{L}}^{(9)}, C_{5\mathrm{L}}^{(9)}\right)$$

Conclusion

- The 32 … can be put into 12 different distinguishable …
- … unknown LECs need …
- … ong formula and calculation

$$\mathcal{M}_{E,L}^{(6)} = -\frac{V_{ud} C_{VL}^{(6)}}{3}\left(\frac{g_V^2}{g_A^2} M_F + \frac{1}{3}\left(2M_{GT}^{AA} + M_T^{AA}\right) + \frac{6 g_{VL}^E}{g_A^2} M_{F,sd}\right),$$

$$\mathcal{M}_{E,R}^{(6)} = -\frac{V_{ud} C_{VR}^{(6)}}{3}\left(\frac{g_V^2}{g_A^2} M_F - \frac{1}{3}\left(2M_{GT}^{AA} + M_T^{AA}\right) + \frac{6 g_{VL}^E}{g_A^2} M_{F,sd}\right),$$

$$\mathcal{M}_\nu^{(3)} = -V_{ud}^2\left(-\frac{1}{g_A^2} M_F + \mathcal{M}_{GT} + \mathcal{M}_T + 2\frac{m_\pi^2 g_\nu^{NN}}{g_A^2} M_{F,sd}\right),$$

$$\mathcal{M}_\nu^{(6)} = V_{ud}\left(\frac{B}{m_N}\left(C_{SL}^{(6)} - C_{SR}^{(6)}\right) + \frac{m_\pi^2}{m_N v}\left(C_{VL}^{(7)} - C_{VR}^{(7)}\right)\right)\mathcal{M}_{PS} + V_{ud} C_T^{(6)} \mathcal{M}_{T6},$$

$$\mathcal{M}_\nu^{(9)} = -\frac{1}{2m_N^2} C_{\pi\pi L}^{(9)}\left(M_{GT,sd}^{AP} + M_{T,sd}^{AP}\right) - \frac{2m_\pi^2}{g_A^2 m_N^2} C_{NNL}^{(9)} M_{F,sd},$$

$$\left(T_{1/2}^{0\nu}\right)^{-1} = g_A^4\Big[G_{01}\left(|\mathcal{A}_\nu|^2 + |\mathcal{A}_R|^2\right) - 2\left(G_{01} - G_{04}\right)\mathrm{Re}\left[\mathcal{A}_\nu^* \mathcal{A}_R\right]$$
$$+ 4G_{02}|\mathcal{A}_E|^2 + 2G_{04}\left(|\mathcal{A}_{m_e}|^2 + \mathrm{Re}\left[\mathcal{A}_{m_e}^*\left(\mathcal{A}_\nu + \mathcal{A}_R\right)\right]\right)$$
$$- 2G_{03}\,\mathrm{Re}\left[\left(\mathcal{A}_\nu + \mathcal{A}_R\right)\mathcal{A}_E^* + 2\mathcal{A}_{m_e}\mathcal{A}_E^*\right]$$
$$\ldots\,\mathrm{Re}\left[\left(\mathcal{A}_\nu - \mathcal{A}_R\right)\mathcal{A}_M^*\right]\Big]$$

$$\mathcal{A}_R = \frac{m_N^2}{m_e v}\mathcal{M}_R^{(9)}\left(C_{1R}^{(9)}, C_{1R}^{(9)\prime}, C_{2R}^{(9)}, C_{2R}^{(9)\prime}, C_{3R}^{(9)}, C_{3R}^{(9)\prime}, C_{4R}^{(9)}, C_{5R}^{(9)}\right),$$

$$\mathcal{A}_E = \mathcal{M}_{E,L}^{(6)}\left(C_{VL}^{(6)}\right) + \mathcal{M}_{E,R}^{(6)}\left(C_{VR}^{(6)}\right),$$

$$\mathcal{A}_{m_e} = \mathcal{M}_{m_e,L}^{(6)}\left(C_{VL}^{(6)}\right) + \mathcal{M}_{m_e,R}^{(6)}\left(C_{VR}^{(6)}\right),$$

$$\mathcal{A}_M = \frac{m_N}{m_e}\mathcal{M}_M^{(6)}\left(C_{VL}^{(6)}\right) + \frac{m_N^2}{m_e v}\mathcal{M}_M^{(9)}\left(C_6^{(9)}, C_6^{(9)\prime}, C_7^{(9)}, C_7^{(9)\prime}, C_8^{(9)}, C_8^{(9)\prime}, C_9^{(9)}, C_9^{(9)\prime}\right)$$

$$\ldots \frac{m_N}{m_e}\mathcal{M}_\nu^{(6)}\left(C_{SL}^{(6)}, C_{SR}^{(6)}, C_T^{(6)}, C_{VL}^{(7)}, C_{VR}^{(7)}\right)$$
$$\ldots \mathcal{M}_\nu^{(9)}\left(C_{1L}^{(9)}, C_{1L}^{(9)\prime}, C_{2L}^{(9)}, C_{2L}^{(9)\prime}, C_{3L}^{(9)}, C_{3L}^{(9)\prime}, C_{4L}^{(9)}, C_{5L}^{(9)}\right)$$

Conclusion

- The 32 … can be put into 12 different distinguishable …

$$\mathcal{M}_{E,L}^{(6)} = \ldots \left(\frac{g_V^2}{g_A^2}M_F + \frac{1}{3}\left(2M_{GT}^{AA} + M_T^{AA}\right) + \frac{6\mathbf{g}_{\mathbf{V\,L}}^{\mathbf{E}}}{g_A^2}M_{F,sd}\right),$$

$$\mathcal{M}_{E,R}^{(6)} = \ldots \left(\ldots \frac{1}{3}\left(2M_{GT}^{AA} + M_T^{AA}\right) + \frac{6\mathbf{g}_{\mathbf{V\,L}}^{\mathbf{E}}}{g_A^2}M_{F,sd}\right),$$

$$\mathcal{M}_{\nu}^{(3)} = -V_{ud}^2\left(-\frac{1}{g_A^2}M_F + \mathcal{M}_{GT} + \mathcal{M}_T + 2\frac{m_\pi^2\mathbf{g}_\nu^{\mathbf{NN}}}{g_A^2}M_{F,sd}\right),$$

$$\mathcal{M}_{m_e,L}^{(6)} = \frac{V_{ud}C_{VL}^{(6)}}{6}\left(\frac{g_V^2}{g_A^2}M_F - \frac{1}{3}\left(M_{GT}^{AA} - 4M_T^{AA}\right) - 3\left(M_{GT}^{AP} + M_{GT}^{PP} + M_T^{AP} + M_T^{PP}\right) - \frac{12\mathbf{g}_{\mathbf{V\,L}}^{\mathbf{m_e}}}{g_A^2}M_{F,sd}\right),$$

$$\ldots\right) + \frac{m_\pi^2}{m_N v}\left(C_{VL}^{(7)} - C_{VR}^{(7)}\right)\Big)\mathcal{M}_{PS} + V_{ud}C_T^{(6)}\mathcal{M}_{T6},$$

$$\ldots + M_{T,sd}^{AP}\right) - \frac{2m_\pi^2}{g_A^2 m_N^2}C_{NNL}^{(9)}M_{F,sd},$$

$$\mathcal{M}_{m_e,R}^{(6)} = \frac{V_{ud}C_{VR}^{(6)}}{6}\left(\frac{g_V^2}{g_A^2}M_F + \frac{1}{3}\left(M_{GT}^{AA} - 4M_T^{AA}\right) + 3\left(M_{GT}^{AP} + M_{GT}^{PP} + M_T^{AP} + M_T^{PP}\right) - \frac{12\mathbf{g}_{\mathbf{V\,L}}^{\mathbf{m_e}}}{g_A^2}M_{F,sd}\right),$$

$$\ldots \mathcal{A}_R]$$

$$+ 4G_{02}\left|\mathcal{A}_E\right|^2 + 2G_{04}\left(\left|\mathcal{A}_{m_e}\right|^2 + \mathrm{Re}\left[\mathcal{A}_{m_e}^*\left(\mathcal{A}_\nu + \mathcal{A}_R\right)\right]\right)$$

$$- 2G_{03}\,\mathrm{Re}\left[\left(\mathcal{A}_\nu + \mathcal{A}_R\right)\mathcal{A}_E^* + 2\mathcal{A}_{m_e}\mathcal{A}_E^*\right]$$

$$\ldots \mathrm{Re}\left[\left(\mathcal{A}_\nu - \mathcal{A}_R\right)\mathcal{A}_M^*\right]\Big]$$

$$\mathcal{A}_R = \frac{m_N^2}{m_e v}\mathcal{M}_R^{(9)}\left(C_{1R}^{(9)}, C_{1R}^{(9)\prime}, C_{2R}^{(9)}, C_{2R}^{(9)\prime}, C_{3R}^{(9)}, C_{3R}^{(9)\prime}, C_{4R}^{(9)}, C_{5R}^{(9)}\right),$$

$$\mathcal{A}_E = \mathcal{M}_{E,L}^{(6)}\left(C_{VL}^{(6)}\right) + \mathcal{M}_{E,R}^{(6)}\left(C_{VR}^{(6)}\right),$$

$$\mathcal{A}_{m_e} = \mathcal{M}_{m_e,L}^{(6)}\left(C_{VL}^{(6)}\right) + \mathcal{M}_{m_e,R}^{(6)}\left(C_{VR}^{(6)}\right),$$

$$\mathcal{A}_M = \frac{m_N}{m_e}\mathcal{M}_M^{(6)}\left(C_{VL}^{(6)}\right) + \frac{m_N^2}{m_e v}\mathcal{M}_M^{(9)}\left(C_6^{(9)}, C_6^{(9)\prime}, C_7^{(9)}, C_7^{(9)\prime}, C_8^{(9)}, C_8^{(9)\prime}, C_9^{(9)}, C_9^{(9)\prime}\right)$$

$$\ldots \frac{m_N}{m_e}\mathcal{M}_\nu^{(6)}\left(C_{SL}^{(6)}, C_{SR}^{(6)}, C_T^{(6)}, C_{VL}^{(7)}, C_{VR}^{(7)}\right)$$

$$\ldots \mathcal{M}_\nu^{(9)}\left(C_{1L}^{(9)}, C_{1L}^{(9)\prime}, C_{2L}^{(9)}, C_{2L}^{(9)\prime}, C_{3L}^{(9)}, C_{3L}^{(9)\prime}, C_{4L}^{(9)}, C_{5L}^{(9)}\right)$$

Conclusion

- The 32 ... can be ...

$$\mathcal{M}_{m_e,L}^{(6)} = \frac{V_{ud}C_{VL}^{(6)}}{6}\left(\frac{g_V^2}{g_A^2}M_F - \frac{1}{3}\left(M_{GT}^{AA} - 4M_T^{AA}\right) - 3\left(M_{GT}^{AP} + M_{GT}^{PP} + M_T^{AP} + M_T^{PP}\right) - \frac{12\mathbf{g_{V\,L}^{m_e}}}{g_A^2}M_{F,sd}\right),$$

$$\mathcal{M}_{m_e,R}^{(6)} = \frac{V_{ud}C_{VR}^{(6)}}{6}\left(\frac{g_V^2}{g_A^2}M_F + \frac{1}{3}\left(M_{GT}^{AA} - 4M_T^{AA}\right) + 3\left(M_{GT}^{AP} + M_{GT}^{PP} + M_T^{AP} + M_T^{PP}\right) - \frac{12\mathbf{g_{V\,L}^{m_e}}}{g_A^2}M_{F,sd}\right),$$

$$\mathcal{M}_{E,L}^{(6)} = \ldots \left(\frac{g_V^2}{g_A^2}M_F + \frac{1}{3}\left(2M_{GT}^{AA} + M_T^{AA}\right) + \frac{6\mathbf{g_{V\,L}^{E}}}{g_A^2}M_{F,sd}\right),$$

$$\mathcal{M}_{E,R}^{(6)} = \ldots \frac{1}{3}\left(2M_{GT}^{AA} + M_T^{AA}\right) + \frac{6\mathbf{g_{V\,L}^{E}}}{g_A^2}M \ldots$$

$$\mathcal{M}_M^{(6)} = V_{ud}C_{VL}^{(6)}\left[2\frac{g_A}{g_M}\left(M_{GT}^{MM} + M_T^{MM}\right) + \frac{m_\pi^2}{m_N^2}\left(-\frac{2}{g_A^2}\mathbf{g_{VL}^{NN}}M_{F,sd} + \frac{1}{2}\mathbf{g_{VL}^{\pi N}}\left(M_{GT,sd}^{AP} + M_{T,sd}^{AP}\right)\right)\right],$$

$$\mathcal{M}_M^{(9)} = \frac{m_\pi^2}{m_N^2}\left[-\frac{2}{g_A^2}\left(\mathbf{g_6^{NN}}C_V^{(9)} + \mathbf{g_7^{NN}}\tilde{C}_V^{(9)}\right)M_{F,sd} + \frac{1}{2}\left(\mathbf{g_V^{\pi N}}C_V^{(9)} + \mathbf{\tilde{g}_V^{\pi N}}\tilde{C}_V^{(9)}\right)\left(M_{GT,sd}^{AP} + M_{T,sd}^{AP}\right)\right].$$

$$\mathcal{M}_\nu^{(3)} = -V_{ud}^2\left(-\frac{\ldots}{g_A^2} \ldots\right) \ldots + \frac{m_\pi^2}{m_N v}\left(C_{VL}^{(6)} - C_{VR}^{(6)}\right) \ldots + M_{T,sd}^{AP}) - \frac{2m_\pi^2}{g_A^2 m_N^2}C_{NNL}^{(9)}M_{F,sd}, \quad \ldots t_{T6},$$

$$\ldots \mathcal{A}_\nu^* \mathcal{A}_R]$$

$$+ 4G_{02}\left|\mathcal{A}_E\right|^2 + 2G_{04}\left(\left|\mathcal{A}_{m_e}\right|^2 + \mathrm{Re}\left[\mathcal{A}_{m_e}^*\left(\mathcal{A}_\nu + \mathcal{A}_R\right)\right]\right)$$

$$- 2G_{03}\,\mathrm{Re}\left[\left(\mathcal{A}_\nu + \mathcal{A}_R\right)\mathcal{A}_E^* + 2\mathcal{A}_{m_e}\mathcal{A}_E^*\right]$$

$$\ldots \mathrm{Re}\left[\left(\mathcal{A}_\nu - \mathcal{A}_R\right)\mathcal{A}_M^*\right]\Big]$$

$$\mathcal{A}_R = \frac{m_N^2}{m_e v}\mathcal{M}_R^{(9)}\left(C_{1R}^{(9)}, C_{1R}^{(9)\prime}, C_{2R}^{(9)}, C_{2R}^{(9)\prime}, C_{3R}^{(9)}, C_{3R}^{(9)\prime}, C_{4R}^{(9)}, C_{5R}^{(9)}\right),$$

$$\mathcal{A}_E = \mathcal{M}_{E,L}^{(6)}\left(C_{VL}^{(6)}\right) + \mathcal{M}_{E,R}^{(6)}\left(C_{VR}^{(6)}\right),$$

$$\mathcal{A}_{m_e} = \mathcal{M}_{m_e,L}^{(6)}\left(C_{VL}^{(6)}\right) + \mathcal{M}_{m_e,R}^{(6)}\left(C_{VR}^{(6)}\right),$$

$$\mathcal{A}_M = \frac{m_N}{m_e}\mathcal{M}_M^{(6)}\left(C_{VL}^{(6)}\right) + \frac{m_N^2}{m_e v}\mathcal{M}_M^{(9)}\left(C_6^{(9)}, C_6^{(9)\prime}, C_7^{(9)}, C_7^{(9)\prime}, C_8^{(9)}, C_8^{(9)\prime}, C_9^{(9)}, C_9^{(9)\prime}\right)$$

$$\frac{m_N}{m_e}\mathcal{M}_\nu^{(6)}\left(C_{SL}^{(6)}, C_{SR}^{(6)}, C_T^{(6)}, C_{VL}^{(7)}, C_{VR}^{(7)}\right)$$

$$\mathcal{M}_\nu^{(9)}\left(C_{1L}^{(9)}, C_{1L}^{(9)\prime}, C_{2L}^{(9)}, C_{2L}^{(9)\prime}, C_{3L}^{(9)}, C_{3L}^{(9)\prime}, C_{4L}^{(9)}, C_{5L}^{(9)}\right)$$

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$$\mathcal{M}_{GT} = M_{GT}^{AA} + M_{GT}^{AP} + M_{GT}^{PP} + M_{GT}^{MM},$$

$$\mathcal{M}_{T} = M_{T}^{AP} + M_{T}^{PP} + M_{T}^{MM},$$

$$\mathcal{M}_{PS} = \frac{1}{2}M_{GT}^{AP} + M_{GT}^{PP} + \frac{1}{2}M_{T}^{AP} + M_{T}^{PP},$$

$$\mathcal{M}_{T6} = 2\frac{\mathbf{g'_T} - \mathbf{g_T^{NN}}}{g_A^2}\frac{m_\pi^2}{m_N^2}M_{F,sd} - \frac{8g_T}{g_M}\left(M_{GT}^{MM} + M_T^{MM}\right) + \mathbf{g_T^{\pi\pi}}\frac{m_\pi^2}{4m_N^2}\left(M_{GT,sd}^{PP} + M_{T,sd}^{PP}\right) + \mathbf{g_T^{\pi N}}\frac{m_\pi^2}{4m_N^2}\left(M_{GT,sd}^{AP} + M_{T,sd}^{AP}\right)$$

$$\mathcal{M}^{(6)}_{M} = V_{ud}C_{VL}^{(6)}\left[2\frac{g_A}{g_M}\left(M_{GT}^{MM} + M_T^{MM}\right) + \frac{m_\pi^2}{m_N^2}\left(-\frac{2}{g_A^2}\mathbf{g_{VL}^{NN}}M_{F,sd} + \frac{1}{2}\mathbf{g_{VL}^{\pi N}}\left(M_{GT,sd}^{AP} + M_{T,sd}^{AP}\right)\right)\right],$$

$$\left[-\frac{2}{g_A^2}\left(\mathbf{g_6^{NN}}C_V^{(9)} + \mathbf{g_7^{NN}}\tilde{C}_V^{(9)}\right)M_{F,sd} + \frac{1}{2}\left(\mathbf{g_V^{\pi N}}C_V^{(9)} + \mathbf{\tilde{g}_V^{\pi N}}\tilde{C}_V^{(9)}\right)\left(M_{GT,sd}^{AP} + M_{T,sd}^{AP}\right)\right].$$

$$\mathcal{M}_\nu^{(3)} = -V_{ud}^2\left(-\frac{[illegible]}{g_A^2}\ldots\right)$$

$$+ M_{T,sd}^{AP}) - \frac{2m_\pi^2}{g_A^2 m_N^2}C_{NNL}^{(9)}M_{F,sd},$$

$$\mathcal{M}_{E,L}^{(6)} = \ldots \quad \mathcal{M}_{m_e,L}^{(6)} = \frac{V_{ud}C_{VL}^{(6)}}{6}\left(\frac{g_V^2}{g_A^2}M_F - \frac{1}{3}\left(M_{GT}^{AA} - 4M_T^{AA}\right) - 3\left(M_{GT}^{AP} + M_{GT}^{PP} + M_T^{AP} + M_T^{PP}\right) - \frac{12\mathbf{g_{VL}^{m_e}}}{g_A^2}M_{F,sd}\right),$$

$$\ldots + \frac{m_\pi^2}{m_N v}\left(C_{VL}^{(6)} - C_{VR}^{(6)}\right)$$

$$\mathcal{M}_{E,R}^{(6)} = \ldots \quad \mathcal{M}_{m_e,R}^{(6)} = \frac{V_{ud}C_{VR}^{(6)}}{6}\left(\frac{g_V^2}{g_A^2}M_F + \frac{1}{3}\left(M_{GT}^{AA} - 4M_T^{AA}\right) + 3\left(M_{GT}^{AP} + M_{GT}^{PP} + M_T^{AP} + M_T^{PP}\right) - \frac{12\mathbf{g_{VL}^{m_e}}}{g_A^2}M_{F,sd}\right),$$

$$\ldots \mathcal{A}_\nu^* \mathcal{A}_R]$$

$$+ 4G_{02}|\mathcal{A}_E|^2 + 2G_{04}\left(|\mathcal{A}_{m_e}|^2 + \mathrm{Re}\left[\mathcal{A}_{m_e}^*\left(\mathcal{A}_\nu + \mathcal{A}_R\right)\right]\right)$$

$$- 2G_{03}\,\mathrm{Re}\left[\left(\mathcal{A}_\nu + \mathcal{A}_R\right)\mathcal{A}_E^* + 2\mathcal{A}_{m_e}\mathcal{A}_E^*\right]$$

$$\ldots\,\mathrm{Re}\left[\left(\mathcal{A}_\nu - \mathcal{A}_R\right)\mathcal{A}_M^*\right]\Big]$$

$$\mathcal{A}_R = \frac{m_N^2}{m_e v}\mathcal{M}_R^{(9)}\left(C_{1R}^{(9)}, C_{1R}^{(9)\prime}, C_{2R}^{(9)}, C_{2R}^{(9)\prime}, C_{3R}^{(9)}, C_{3R}^{(9)\prime}, C_{4R}^{(9)}, C_{5R}^{(9)}\right),$$

$$\mathcal{A}_E = \mathcal{M}_{E,L}^{(6)}\left(C_{VL}^{(6)}\right) + \mathcal{M}_{E,R}^{(6)}\left(C_{VR}^{(6)}\right),$$

$$\mathcal{A}_{m_e} = \mathcal{M}_{m_e,L}^{(6)}\left(C_{VL}^{(6)}\right) + \mathcal{M}_{m_e,R}^{(6)}\left(C_{VR}^{(6)}\right),$$

$$\mathcal{A}_M = \frac{m_N}{m_e}\mathcal{M}_M^{(6)}\left(C_{VL}^{(6)}\right) + \frac{m_N^2}{m_e v}\mathcal{M}_M^{(9)}\left(C_6^{(9)}, C_6^{(9)\prime}, C_7^{(9)}, C_7^{(9)\prime}, C_8^{(9)}, C_8^{(9)\prime}, C_9^{(9)}, C_9^{(9)\prime}\right)$$

$$\ldots\frac{m_N}{m_e}\mathcal{M}_\nu^{(6)}\left(C_{SL}^{(6)}, C_{SR}^{(6)}, C_T^{(6)}, C_{VL}^{(7)}, C_{VR}^{(7)}\right)$$

$$\ldots\mathcal{M}_\nu^{(9)}\left(C_{1L}^{(9)}, C_{1L}^{(9)\prime}, C_{2L}^{(9)}, C_{2L}^{(9)\prime}, C_{3L}^{(9)}, C_{3L}^{(9)\prime}, C_{4L}^{(9)}, C_{5L}^{(9)}\right)$$

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$$\mathcal{M}_{T6} = 2\frac{\mathbf{g_T'} - \mathbf{g_T^{NN}}}{g_A^2}\frac{m_\pi^2}{m_N^2}M_{F,sd} - \frac{8g_T}{g_M}\left(M_{GT}^{MM} + M_T^{MM}\right) + \mathbf{g_T^{\pi N}}\frac{m_\pi^2}{4m_N^2}\left(M_{GT,sd}^{AP} + M_{T,sd}^{AP}\right) + \mathbf{g_T^{\pi\pi}}\frac{m_\pi^2}{4m_N^2}\left(M_{GT,sd}^{PP} + M_{T,sd}^{PP}\right)$$

$$\mathcal{M}_M^{(6)} = V_{ud}C_{VL}^{(6)}\Big[2\frac{g_A}{g_M}\left(M_{GT}^{MM} + M_T^{MM}\right) + \frac{m_\pi^2}{m_N^2}\Big(-\frac{2}{g_A^2}\mathbf{g_{VL}^{NN}}M_{F,sd} + \frac{1}{2}\mathbf{g_{VL}^{\pi N}}\left(M_{GT,sd}^{AP} + M_{T,sd}^{AP}\right)\Big)\Big],$$

$$\frac{m_\pi^2}{m_N^2}\Big[-\frac{2}{g_A^2}\left(\mathbf{g_6^{NN}}C_V^{(9)} + \mathbf{g_7^{NN}}\tilde{C}_V^{(9)}\right)M_{F,sd} + \frac{1}{2}\left(\mathbf{g_V^{\pi N}}C_V^{(9)} + \mathbf{\tilde{g}_V^{\pi N}}\tilde{C}_V^{(9)}\right)\left(M_{GT,sd}^{AP} + M_{T,sd}^{AP}\right)\Big].$$

$$\mathcal{M}_\nu^{(3)} = -V_{ud}^2 \left(-\frac{\ldots}{g_A^2}\ldots\right)$$

$$\mathcal{M}_{E,L}^{(6)} = \ldots \quad \mathcal{M}_{m_e,L}^{(6)} = \ldots \quad \frac{1}{\ldots}\left(M_{GT}^{AA} - 4M_T^{AA}\right) - 3\left(M_{GT}^{AP} + M_{GT}^{PP} + M_T^{AP} + M_T^{PP}\right) \ldots + \frac{m_\pi^2}{m_N v}\left(C_{VL}^{(6)} - \ldots\right)$$

$$C_V^{(9)} = C_6^{(9)} + C_6^{(9)\prime} + C_8^{(9)} + C_8^{(9)\prime}, \quad \tilde{C}_V^{(9)} = C_7^{(9)} + C_7^{(9)\prime} + C_9^{(9)} + C_9^{(9)\prime},$$

$$\mathcal{M}_{E,R}^{(6)} = \quad \mathcal{M}_{m_e,R}^{(6)} = \ldots + M_{T,sd}^{AP}) - \frac{2m_\pi^2}{g_A^2 m_N^2}C_{NNL}^{(9)}M_{F,sd},$$

$$C_{\pi\pi L}^{(9)} = g_2^{\pi\pi}\left(C_{2L}^{(9)} + C_{2L}^{(9)\prime}\right) + g_3^{\pi\pi}\left(C_{3L}^{(9)} + C_{3L}^{(9)\prime}\right) - g_4^{\pi\pi}C_{4L}^{(9)} - g_5^{\pi\pi}C_{5L}^{(9)} - \frac{5}{3}g_1^{\pi\pi}m_\pi^2\left(C_{1L}^{(9)} + C_{1L}^{(9)\prime}\right)$$

$$C_{\pi NL}^{(9)} = \left(\mathbf{g_1^{\pi N}} - \frac{5}{6}g_1^{\pi\pi}\right)\left(C_{1L}^{(9)} + C_{1L}^{(9)\prime}\right)$$

$$C_{NNL}^{(9)} = \mathbf{g_1^{NN}}\left(C_{1L}^{(9)} + C_{1L}^{(9)\prime}\right) + \mathbf{g_2^{NN}}\left(C_{2L}^{(9)} + C_{2L}^{(9)\prime}\right) + \mathbf{g_3^{NN}}\left(C_{3L}^{(9)} + C_{3L}^{(9)\prime}\right) + \mathbf{g_4^{NN}}C_{4L}^{(9)} + \mathbf{g_5^{NN}}C_{5L}^{(9)}$$

$$C_{\{\pi\pi,\pi N,NN\}R} = C_{\{\pi\pi,\pi N,NN\}L}|_{L\to R}.$$

$$+ 4G_{02}\left|\mathcal{A}_E\right| \ldots$$

$$- 2G_{03}\,\mathrm{Re}\left[\left(\mathcal{A}_\nu + \mathcal{A}_R\right)\mathcal{A}_E^* + \ldots\right]$$

$$\ldots\,\mathrm{Re}\left[\left(\mathcal{A}_\nu - \mathcal{A}_R\right)\mathcal{A}_M^*\right]\Big]$$

$$\mathcal{A}_R = \frac{m_N^2}{m_e v}\mathcal{M}_R^{(9)}\left(C_{1R}^{(9)}, C_{1R}^{(9)\prime}, C_{2R}^{(9)}, C_{2R}^{(9)\prime}, C_{3R}^{(9)}, C_{3R}^{(9)\prime}, C_{4R}^{(9)}, C_{5R}^{(9)}\right),$$

$$\mathcal{A}_E = \mathcal{M}_{E,L}^{(6)}\left(C_{VL}^{(6)}\right) + \mathcal{M}_{E,R}^{(6)}\left(C_{VR}^{(6)}\right),$$

$$\mathcal{A}_{m_e} = \mathcal{M}_{m_e,L}^{(6)}\left(C_{VL}^{(6)}\right) + \mathcal{M}_{m_e,R}^{(6)}\left(C_{VR}^{(6)}\right),$$

$$\mathcal{A}_M = \frac{m_N}{m_e}\mathcal{M}_M^{(6)}\left(C_{VL}^{(6)}\right) + \frac{m_N^2}{m_e v}\mathcal{M}_M^{(9)}\left(C_6^{(9)}, C_6^{(9)\prime}, C_7^{(9)}, C_7^{(9)\prime}, C_8^{(9)}, C_8^{(9)\prime}, C_9^{(9)}, C_9^{(9)\prime}\right)$$

$$\ldots C_{VL}^{(7)}, C_{VR}^{(7)}\Big)$$

$$\frac{m_N}{m_e}\mathcal{M}_\nu^{(6)}\left(C_{SL}^{(6)}, C_{SR}^{(6)}, \ldots\right)$$

$$\mathcal{M}_\nu^{(9)}\left(C_{1L}^{(9)}, C_{1L}^{(9)\prime}, C_{2L}^{(9)}, C_{2L}^{(9)\prime}, C_{3L}^{(9)}, C_{3L}^{(9)\prime}, C_{4L}^{(9)}, C_{5L}^{(9)}\right)$$

Conclusion

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- The 32 LEFT operators can be put into 12 different distinguishable groups
- Currently unknown LECs need to be fixed!
- Long formula and calculation

=>Automated generalized $0\nu\beta\beta$ tool

Conclusion

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- The 32 LEFT operators can be put into 12 different distinguishable groups
- Currently unknown LECs need to be fixed!
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Thank you for your attention!