The Weak Eightfold Way: $SU(3)_W$ unification of the electroweak interactions

P. Q. Hung

UNIVERSITY OF VIRGINIA

PHENO 2021
Once upon a time there was a star \( \sin^2 \theta_W \). Everybody was parading the star under the banner gauge unification and quark-lepton unification, leaving behind a big desert. All led to proton decay.

The proton hasn’t decayed and slowly the star was forgotten. Recently it as resurfaced with a new banner: No GUT, no quark-lepton unification!. I am a TeV-star!
\[ \sin^2 \theta_W \text{ without GUT} \]

- \( \sin^2 \theta_W = \frac{g'^2}{(g'^2 + g^2)} \) with \( g' \in U(1)_Y \) and \( g \in SU(2) \).
- Prediction for \( \sin^2 \theta_W \Rightarrow g' = ??g \). Depends only on gauge unification of \( SU(2) \) and \( U(1)_Y \).
- The form of unification depends on an independent prediction for \( \sin^2 \theta_W \). Could this prediction come from TeV-scale physics?
- Unexpected twist: Dirac Quantization Condition on an electroweak monopole \( \Rightarrow \sin^2 \theta_W = 1/4 \) at the mass scale of the monopole (2-3 TeV).
- Renormalization of \( \sin^2 \theta_W = 1/4 \) down to \( M_Z \) gives \( \sin^2 \theta_W(M_Z) \approx 0.231 - 0.232 \).
- Electroweak monopole?
Electroweak monopole and $\sin^2 \theta_W$

- For EW $SU(2)$, the existence of an electroweak monopole depends on the existence of a real Higgs triplet (topological argument).

- Which model contains a real Higgs triplet and for what reasons?


- 3 steps: 1) $\nu_R$ Majorana mass from coupling to a complex Higgs triplet $\tilde{\chi}$: $g_{MR} \nu_M$ ($\langle \tilde{\chi} \rangle = v_M$); 2) Z boson width: $M_R > M_Z/2$; 3) Value of $v_M$ badly violates custodial symmetry which guarantees $M_W = M_Z \cos \theta_W$ at tree level!

- Cure: Introduce, in addition, a real Higgs triplet $\xi \Rightarrow$ Custodial symmetry is restored!
Electroweak monopole and $\sin^2 \theta_W$

- Magnetic charge given by Topological Quantization Condition: $g_M = (1/g)n$. Magnetic field at large distances: $B_i = \frac{g_M}{r^2} \hat{r}_i = \frac{\sin \theta_W}{er^2} \hat{r}_i$
- Dirac Quantization Condition for an electron going around the monopole: $eg_M = m/2$.
The Weak Eightfold Way

- \( \sin^2 \theta_W = 1/4 \) implies that \( g'^2 = g^2/3 \) \( \Rightarrow \) Unification of \( SU(2) \) and \( U(1)_Y \) at some scale \( M_U \sim O(\text{TeV}) \).
- Simplest possibility: \( SU(3)_W \rightarrow SU(2) \times U(1)_Y \).
- Comparing \( D_\mu = \partial_\mu + igU(\lambda^a/2)A^a_\mu \) of \( SU(3)_W \) with \( D_\mu = \partial_\mu + ig(\tau^i/2)W^i_\mu + ig'(Y/2)B_\mu \) of \( SU(2) \times U(1)_Y \), and identifying \( A^8_\mu \) with \( B_\mu \), one obtains \( gU(\lambda^8/2) = g'(Y/2) \). Explicitly \( \pm \frac{Y}{2} = \text{diag}(\pm \frac{1}{2}, \pm \frac{1}{2}, \mp 1) \).
- With \( \lambda^8/2 = \text{diag}(1, 1, -2)/2\sqrt{3} \) \( \Rightarrow \) \( g = -\sqrt{3}g' = g_U \) \( \Rightarrow \) \( \sin^2 \theta_W = 1/4 \).
- Requirements: 1) Fermion representations should contain all SM degrees of freedom; 2) \( SU(3)_W \) is anomaly-free.
The Weak Eightfold Way


Gauge bosons: \[
\left( \frac{\lambda^a}{2} \right) A^a_\mu = \left( \begin{array}{c}
\frac{W_{3\mu}}{2} + \frac{B_\mu}{2\sqrt{3}}, \ W^+_\mu, \ \frac{X^+_\mu}{\sqrt{2}} \\
-W^-_\mu, -\frac{W_{3\mu}}{2} + \frac{B_\mu}{2\sqrt{3}}, \ \frac{X^-_\mu}{\sqrt{2}} \\
\frac{X^-_\mu}{\sqrt{2}}, \ \frac{X^0_\mu}{\sqrt{2}}, \ -\frac{B_\mu}{\sqrt{3}}
\end{array} \right).
\]

Representations $(m, n)$ with dimension $(m + 1)(n + 1)(m + n + 2)/2$ are classified under $T_{3W} = -T_W, \ldots, T_W$ and $Y/2$ with $T_W = (p + q)/2, \frac{Y_q}{2} = \frac{p}{2} - \frac{q}{2} + \frac{1}{3}(n - m)$ for quarks, and $\frac{Y_l}{2} = \frac{3p}{2} - \frac{3q}{2} + (n - m)$ for leptons, with $0 \leq p \leq m; 0 \leq q \leq n$.

All SM fermions are written as left-handed fields (just as in GUT).

In analogy with the old flavor $SU(3)$: $T^\pm_W = \frac{\lambda_1 \pm i\lambda_2}{2}; U^\pm_W = \frac{\lambda_6 \pm i\lambda_7}{2}; V^\pm_W = \frac{\lambda_4 \pm i\lambda_5}{2}; W^\pm_\mu = \frac{W^1_\mu \mp iW^2_\mu}{\sqrt{2}}; X^\pm_\mu = \frac{A^4_\mu \mp iA^5_\mu}{\sqrt{2}}; X^0_\mu = \frac{A^6_\mu \mp iA^7_\mu}{\sqrt{2}}$.

Couplings: $\frac{1}{\sqrt{2}}(T^\pm_W W^\pm_\mu + V^\pm_W X^\pm_\mu + U^\pm_W X^0_\mu)$
The Weak Eightfold Way

lepton: $\bar{3}_L = (0,1)$. $p = q = 0$ and $n = 1$, $m = 0$.

![Diagram of lepton structure]

down quark: $\bar{3}_L^d = (0,1)$. $1/3$ ($p = q = 0$) and $-1/6$ ($p = 0$, $q = 1$).

![Diagram of down quark structure]
The Weak Eightfold Way

up quark: \( 6_L = (2,0) \). \( m = 2 \) \( (p = 0) \) and \( n = 0 \) \( (q = 0) \).

- Anomaly coefficients: \( Tr \left[ \{ T^R_a, T^R_b \} T^R_c \right] = d_{abc} A(R) \)
- \( A(\bar{\mathbf{3}}_L^d) = A(\mathbf{3}_d^L) = -1 \) and \( A(\mathbf{6}_L) = 7 \). With color factors: \( A_{tot} = -1 - 3 + 21 = 17 \). NOT ANOMALY-FREE!
- Simplest option: representations with the same dimensionality but with opposite anomaly coefficients \( \Rightarrow \) Right-handed multiplets \( \Rightarrow \) Mirror fermions of the EW-\( \nu_R \) model!
The Weak Eightfold Way

SU(3)\(_W\) unification of the electroweak interactions
Conclusion

- Unification of $SU(2)$ and $U(1)_Y$ leads to the predictions: 1) $\sin^2 \theta_W = 1/4$; 2) Existence of mirror fermions and, hence non-sterile, electroweak-scale right-handed neutrinos; 3) Vector-like quarks with unconventional electric charges $V = (V_{L,R}(-2/3), V_{L,R}(1/3), V_{L,R}(4/3))$ and $v = (\nu_{L,R}(-2/3), \nu_{L,R}(1/3))$.

- A rich phenomenology involving vector-like quarks, X-gauge bosons and BSM scalars (not shown here).