

Multi-lepton anomalies at the LHC and implications

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Based Sciences

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The Simplified Model and 2HDM+S

The 2HDM+S

Eur. Phys. J. C (2016) 76:580

See also M.Muhlleitner et al.

arXiv:1612.01309

arXiv:1708.01578

Introduce singlet real scalar, S .

2HDM potential, $\mathcal{V}(\Phi_1, \Phi_2)$

$$\begin{aligned} &= m_1^2 \Phi_1^\dagger \Phi_1 + m_2^2 \Phi_2^\dagger \Phi_2 - m_{12}^2 (\Phi_1^\dagger \Phi_2 + \text{h.c.}) \\ &+ \frac{1}{2} \lambda_1 (\Phi_1^\dagger \Phi_1)^2 + \frac{1}{2} \lambda_2 (\Phi_2^\dagger \Phi_2)^2 \\ &+ \lambda_3 (\Phi_1^\dagger \Phi_1) (\Phi_2^\dagger \Phi_2) + \lambda_4 |\Phi_1^\dagger \Phi_2|^2 \\ &+ \frac{1}{2} \lambda_5 \left[(\Phi_1^\dagger \Phi_2)^2 + \text{h.c.} \right] \\ &+ \left\{ \left[\lambda_6 (\Phi_1^\dagger \Phi_1) + \lambda_7 (\Phi_2^\dagger \Phi_2) \right] \Phi_1^\dagger \Phi_2 + \text{h.c.} \right\} \end{aligned}$$

2HDM+S potential

$$\begin{aligned} &\mathcal{V}(\Phi_1, \Phi_2) + \frac{1}{2} m_{S_0}^2 S^2 + \frac{\lambda_{S_1}}{2} \Phi_1^\dagger \Phi_1 S^2 \\ &+ \frac{\lambda_{S_2}}{2} \Phi_2^\dagger \Phi_2 S^2 + \frac{\lambda_{S_3}}{4} (\Phi_1^\dagger \Phi_2 + \text{h.c.}) S^2 \\ &+ \frac{\lambda_{S_4}}{4!} S^4 + \mu_1 \Phi_1^\dagger \Phi_1 S + \mu_2 \Phi_2^\dagger \Phi_2 S \\ &+ \mu_3 \left[\Phi_1^\dagger \Phi_2 + \text{h.c.} \right] S + \mu_S S^3. \end{aligned}$$

Out of considerations of simplicity, assume S to be Higgs-like, which is not too far fetched.

The Decays of H

- In the general case, H can have couplings as those displayed by a Higgs boson in addition to decays involving the intermediate scalar and Dark Matter

$$H \rightarrow WW, ZZ, q\bar{q}, gg, Z\gamma, \gamma\gamma, \chi\chi$$
$$+ H \rightarrow SS, Sh, hh$$

Dominant decays

Diboson decay

$$H \rightarrow h(+X), S(+X)$$

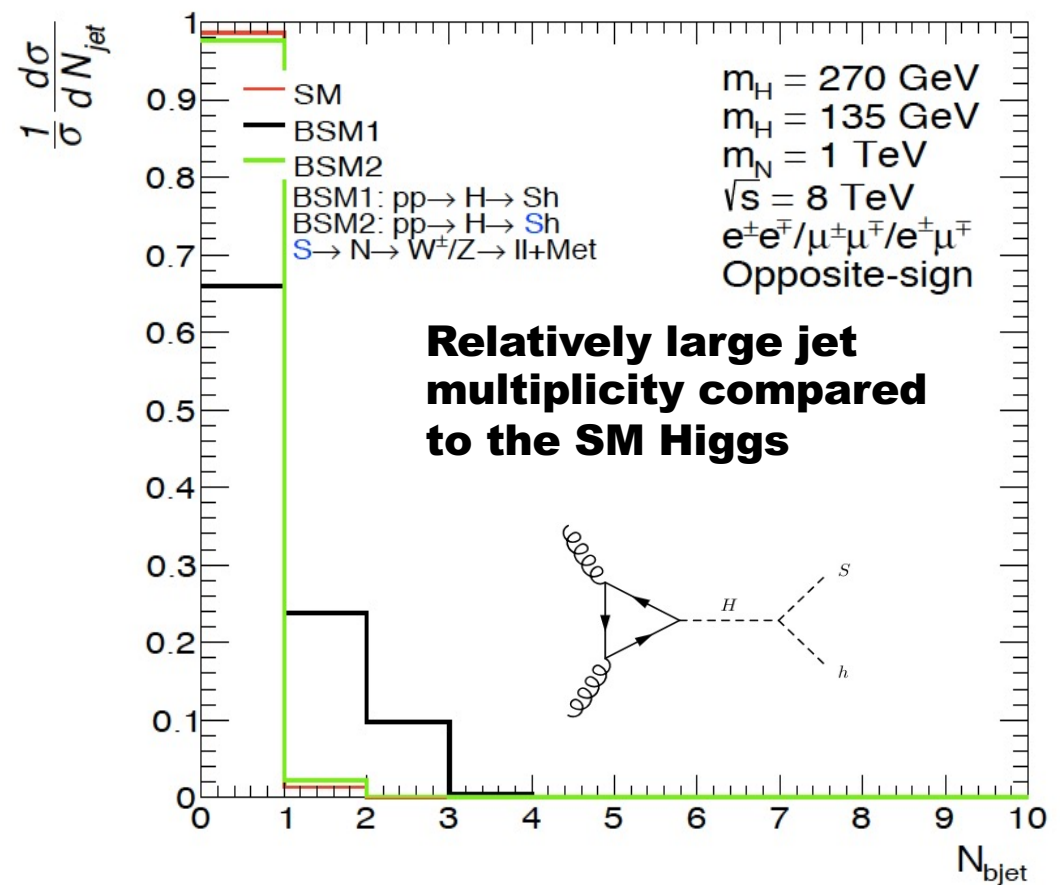
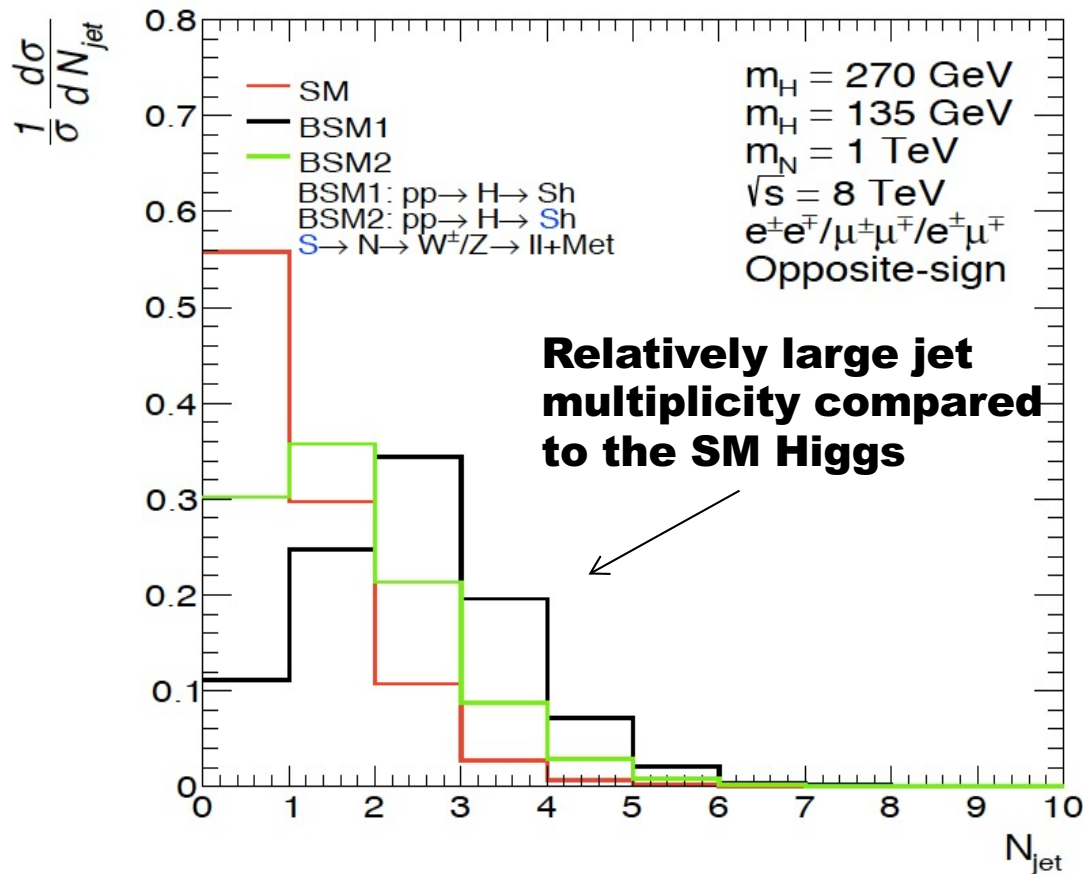
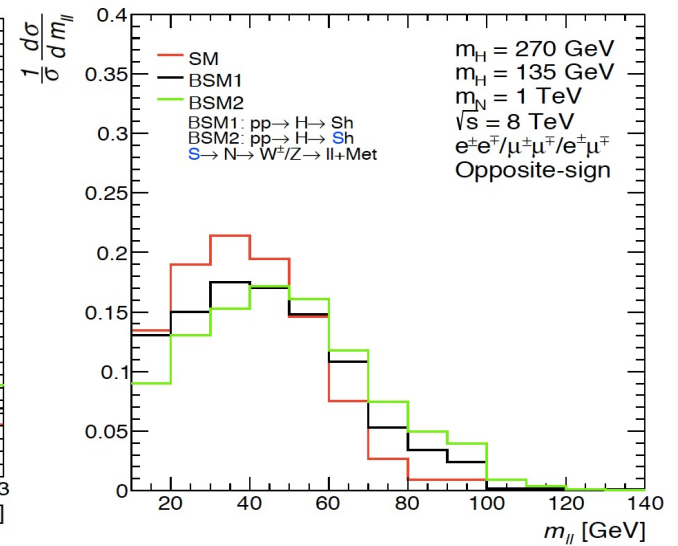
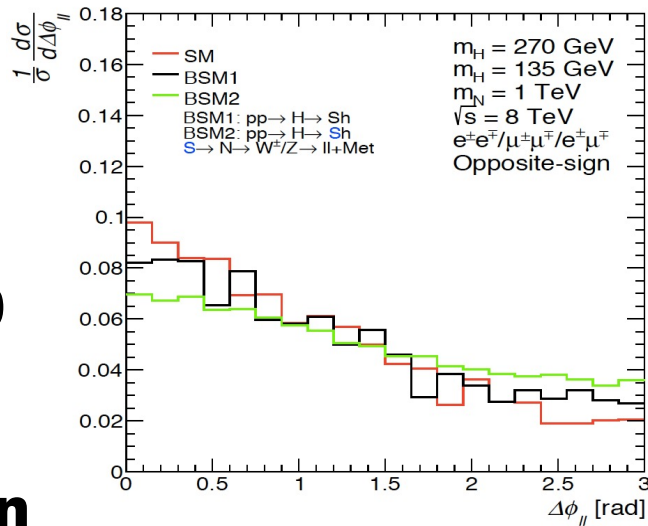
Multi-lepton final states

It is paramount to remark that the excesses are seen in final states that were predicted 2015/2016 on the basis of a simplified model and not the result of scan of the available phase-space. Additionally, the parameters of the model were fixed then leaving only one degree of freedom: normalization. Thus, no look-elsewhere effects in parameter or phase-space

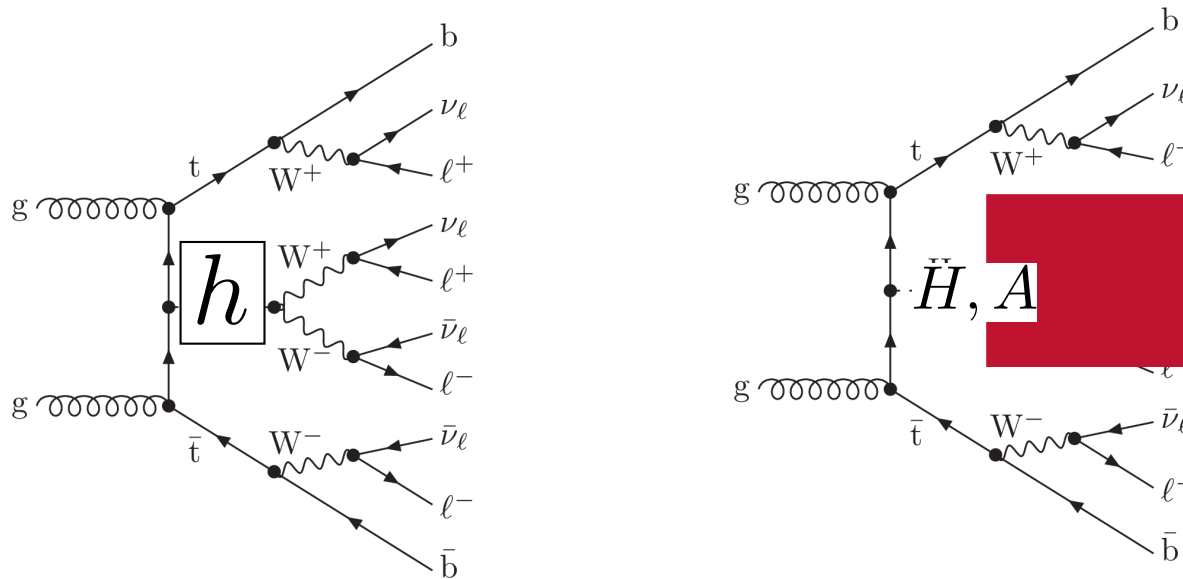
$$pp \rightarrow H \rightarrow Sh$$

$$\rightarrow l^+ l^- + X$$

Expect di-leptons ($m_{ll} < 100$ GeV) with jets and b-jets with rates comparable to that of the SM Higgs boson

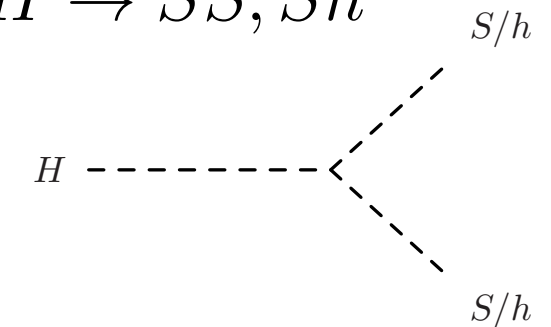


Top associated Higgs production (Multi-lepton final states)



$$A \rightarrow t\bar{t}, ZH$$

$$H \rightarrow SS, Sh$$



Reduced cross-section of $t\bar{t}H+tH$ is compensated by di-boson, (SS, Sh) decay and large $\text{Br}(S \rightarrow WW)$. Production of same sign leptons, three leptons is enhanced. Enhanced tH cross-section

Produces SS 2l, 3l with b-jets, including 3 b-jets

Explains anomalously large $t\bar{t}W+t\bar{t}h+4t$ cross-sections seen by ATLAS and CMS

Methodology

(to avoid biases and look-else-where effects)

Based Higgs p_T , hh, tth, VV in Run 1
Eur. Phys. J. C (2016) 76:580

Model defined and predictions made for
multilepton excesses

Multi-lepton excesses in Run 1 and few
Run 2 results available in 2017

J.Phys.G 45 (2018) 11, 115003

Model parameters fixed in 2017 with
 $m_H=270$ GeV, $m_S=150$ GeV,
S treated as SM Higgs-like,
dominance of $H \rightarrow Sh, SS$

Fixed final states and phase-space
defined by fixed model parameters.
NO tuning, NO scanning

Update same final states with
more data in Run 2

Study new final states where
excesses predicted and data
available in Run 1 and Run 2
(e.g., SS0b, 3l0b, ZW0b)

J.Phys. G46 (2019) no.11, 115001

JHEP 1910 (2019) 157

Chin.Phys.C 44 (2020) 6, 063103

Physics Letters B 811 (2020) 135964

Eur.Phys.J.C 81 (2021) 365

Combination of fit results (2019)

- **Simultaneous fit for all measurements:**
- **To the right: (-2 log) profile likelihood ratio for each individual result and the combination of them all**
- **The significance for each fit is calculated as**

$$\sqrt{-2 \log \lambda(0)}$$

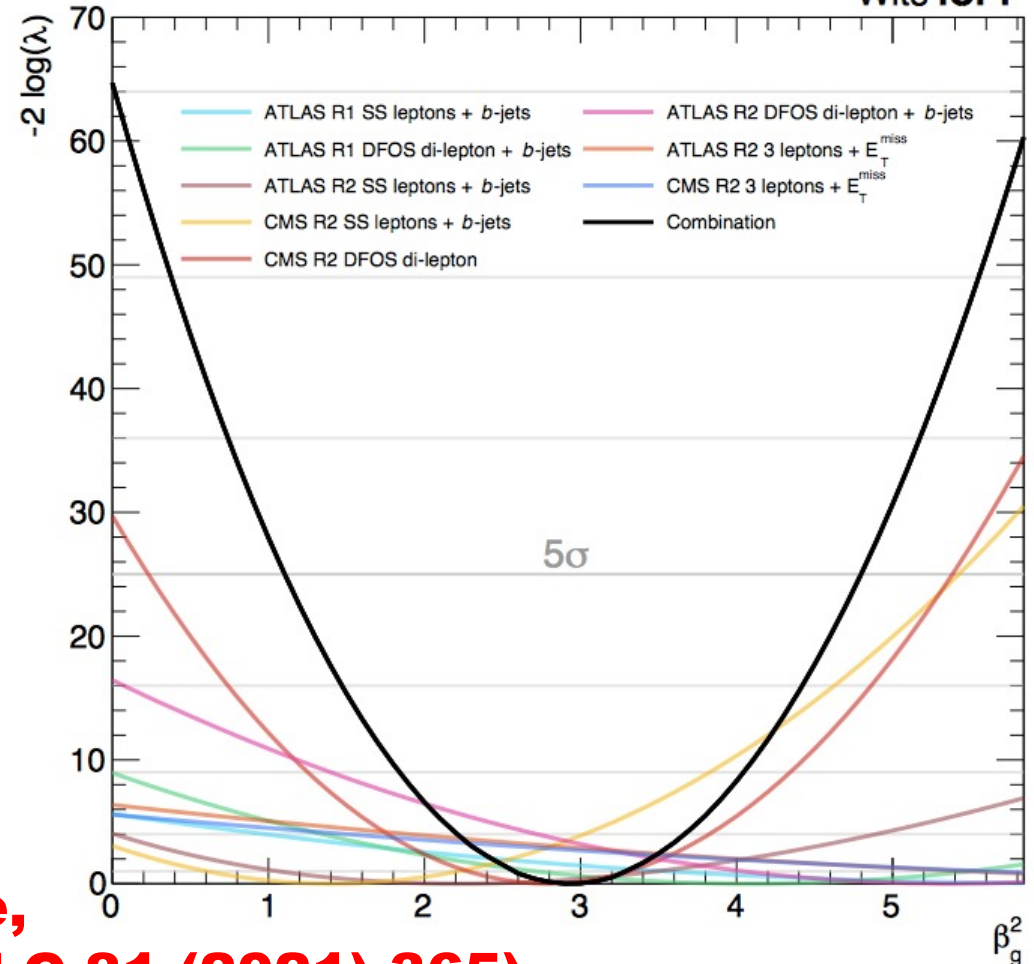
- **Best-fit: $\beta_g^2 = 2.92 \pm 0.35$**
- **Corresponds to 8.04σ**

Excesses have been growing since, and new have emerged (Eur.Phys.J.C 81 (2021) 365)

Interpretation: Measure of the inability of current MC tools to describe multiple-lepton data and how a simplified model with $H \rightarrow Sh$ is able to capture the effect with one parameter

JHEP 1910 (2019) 157

Wits ICPP



Anatomy of the multi-lepton anomalies

Final state	Characteristic	Dominant SM process	Significance
l^+l^- + jets, b-jets	$m_{ll} < 100$ GeV, dominated by 0b-jet and 1b-jet	tt+Wt	$>5\sigma$
l^+l^- + full-jet veto	$m_{ll} < 100$ GeV	WW	$\sim 3\sigma$
$l^\pm l^\pm$ & $l^\pm l^\pm l$ + b-jets	Moderate H_T	ttW, 4t	$>3\sigma$
$l^\pm l^\pm$ & $l^\pm l^\pm l$ et al., no b-jets	In association with h	Wh	3.8σ
Z($\rightarrow l^+l^-$)+l	$p_{TZ} < 100$ GeV	ZW	$\sim 3\sigma$

Anomalies cannot be explained by mismodelling of a particular process, e.g. ttbar production alone.

We do not think that the Toponium will explain away any of the multi-lepton anomalies described there, as excess in l^+l^-+b -jets appears in low b-jet multiplicity and inconsistent with tt events. See seminar in [Korea](#)

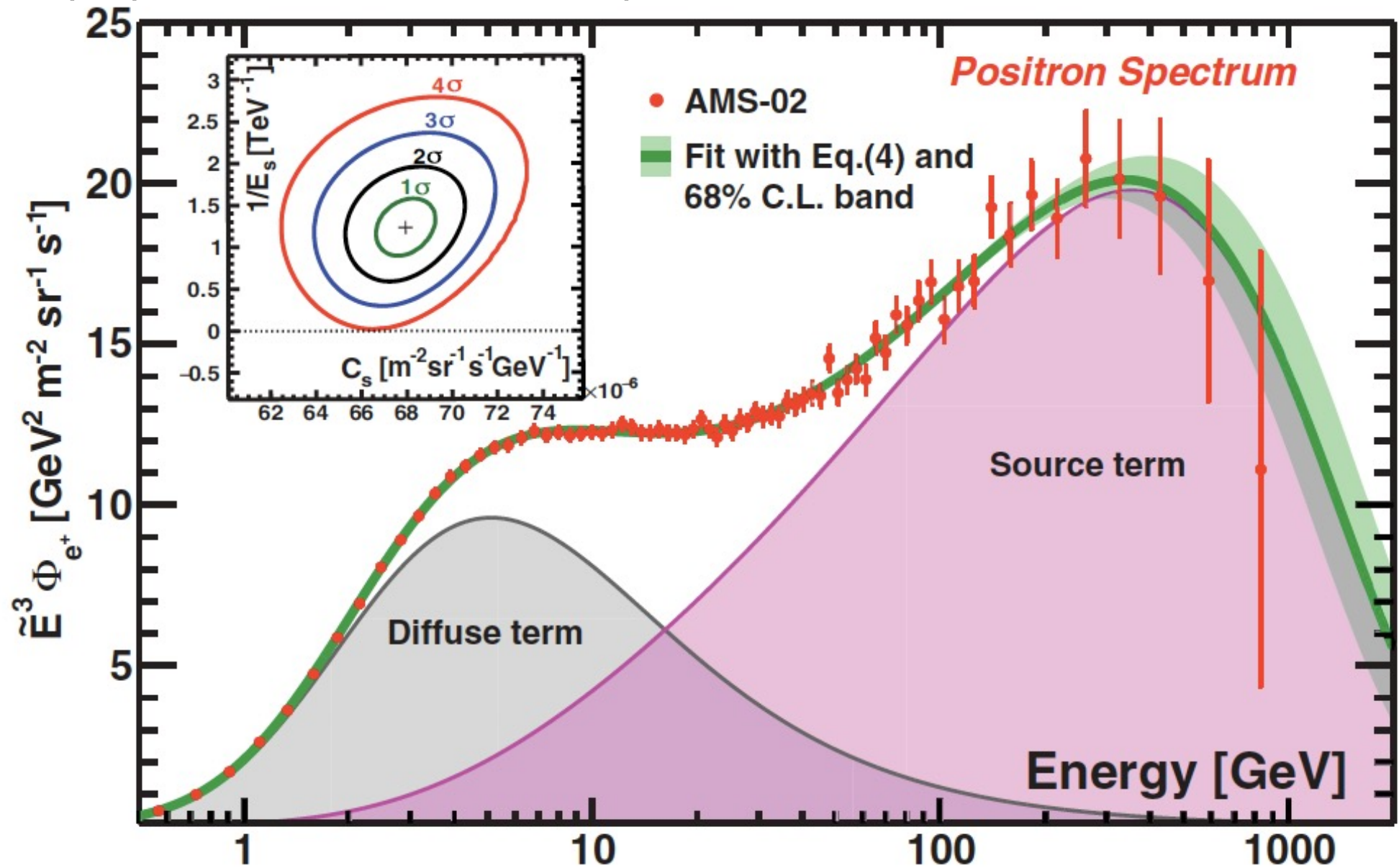
However, we believe it is important that the experiments need to incorporate the Toponium into the MCs since it will improve the description of ttbar final states.

Want to thank authors of Toponium paper because they have been among the first to realise that the excess in l^+l^-+b -jets may not be due to MC mismodelling, but due to something missing in the description of the data.

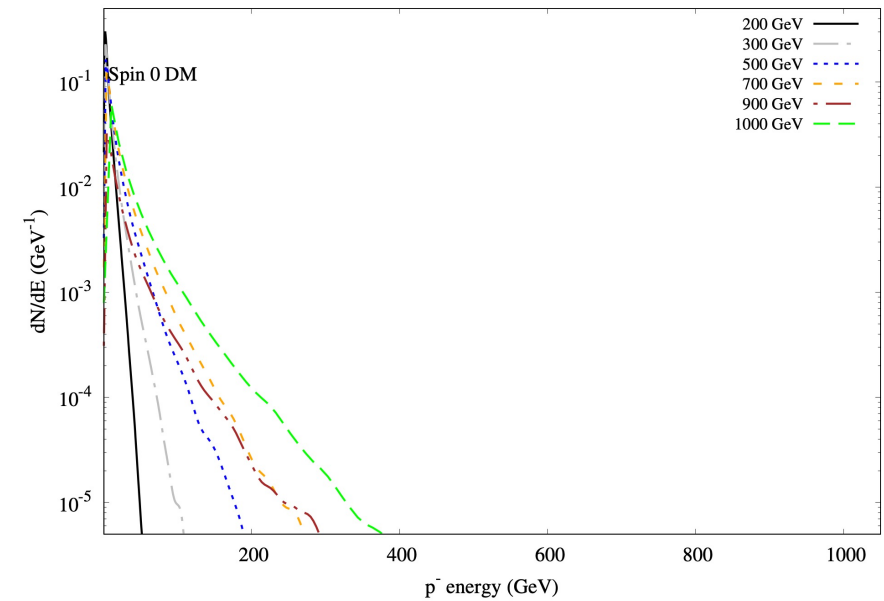
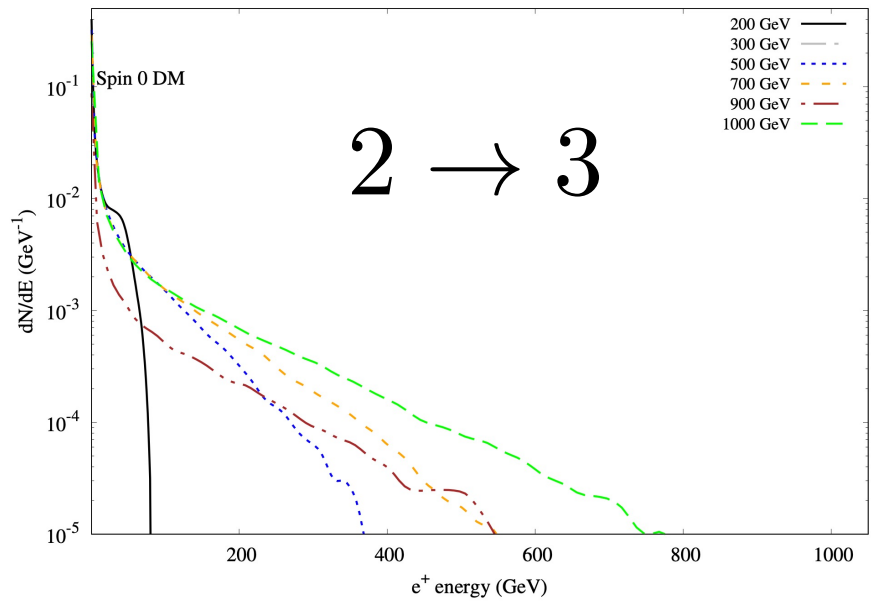
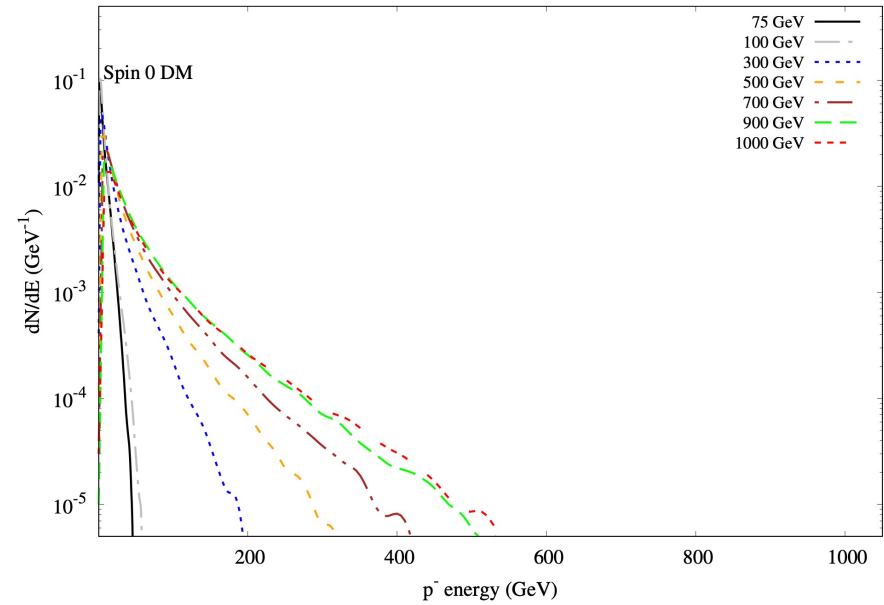
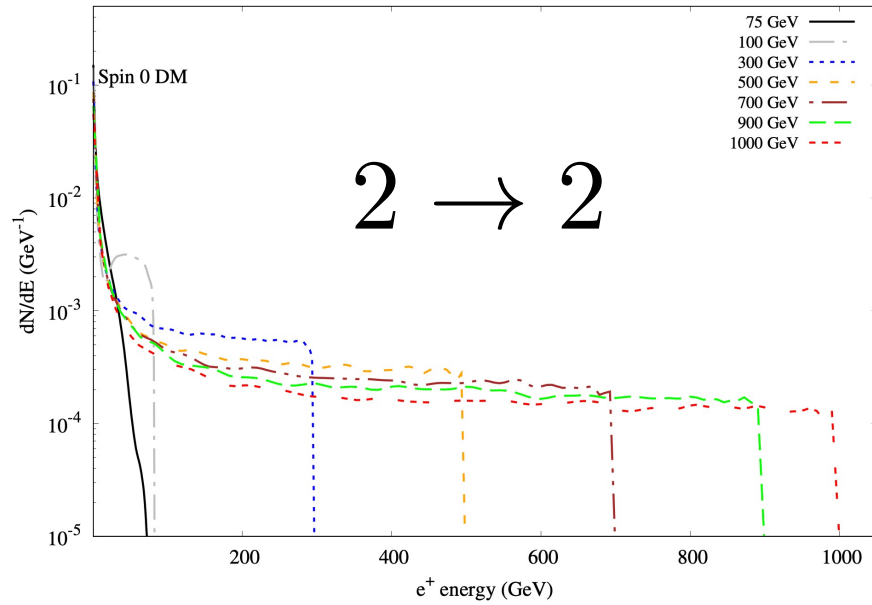
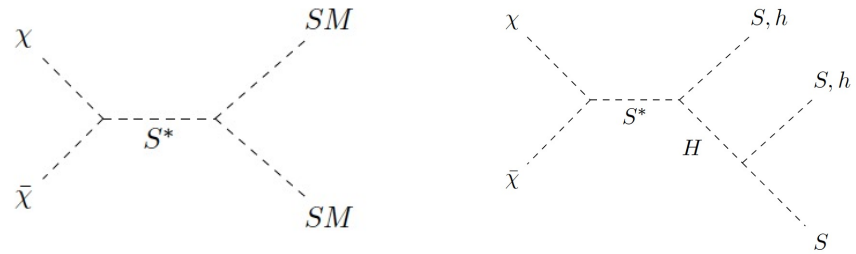
The multi-lepton anomalies and excesses in astrophysics

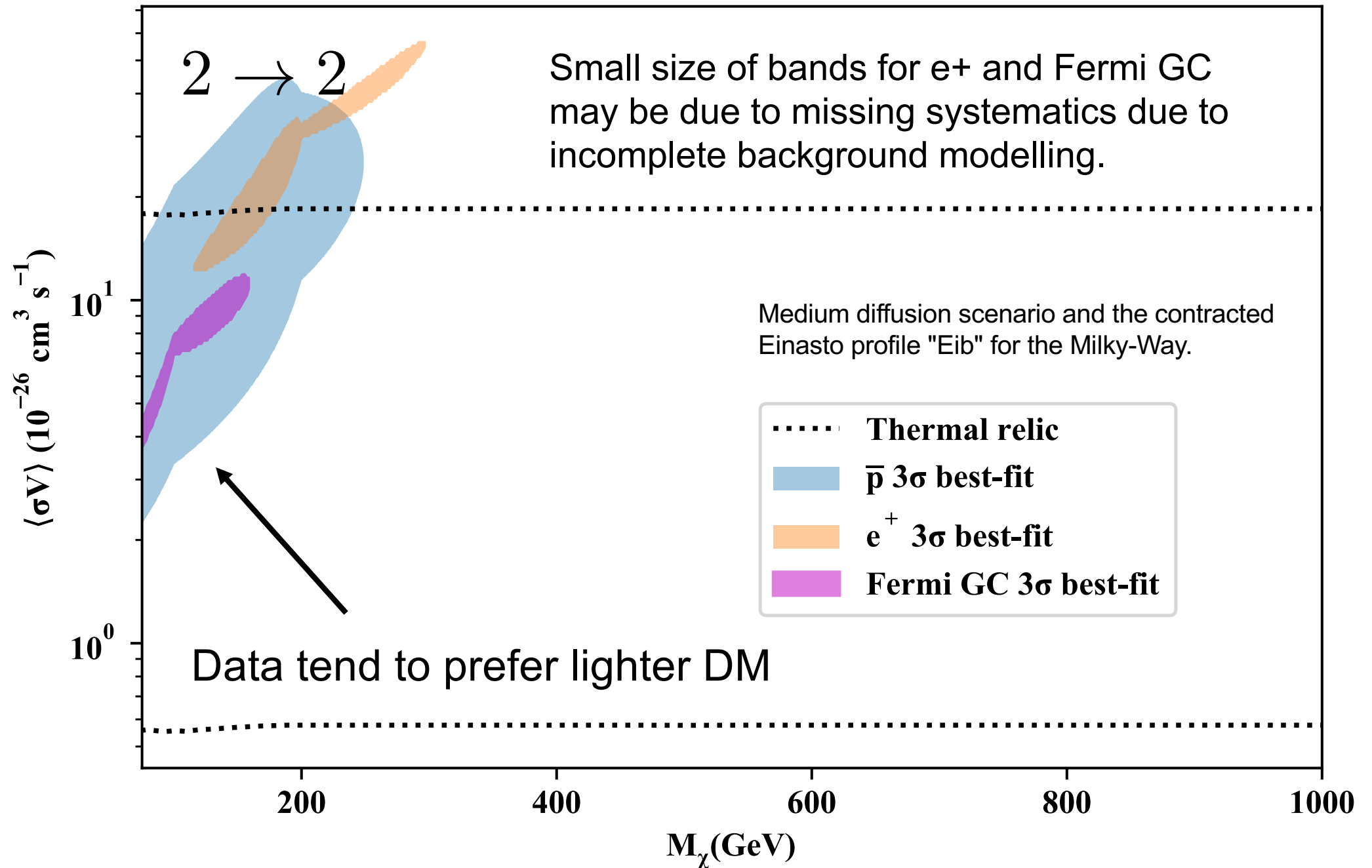


Leptophilic excesses, such as positron rise in PAMELA/AMS02

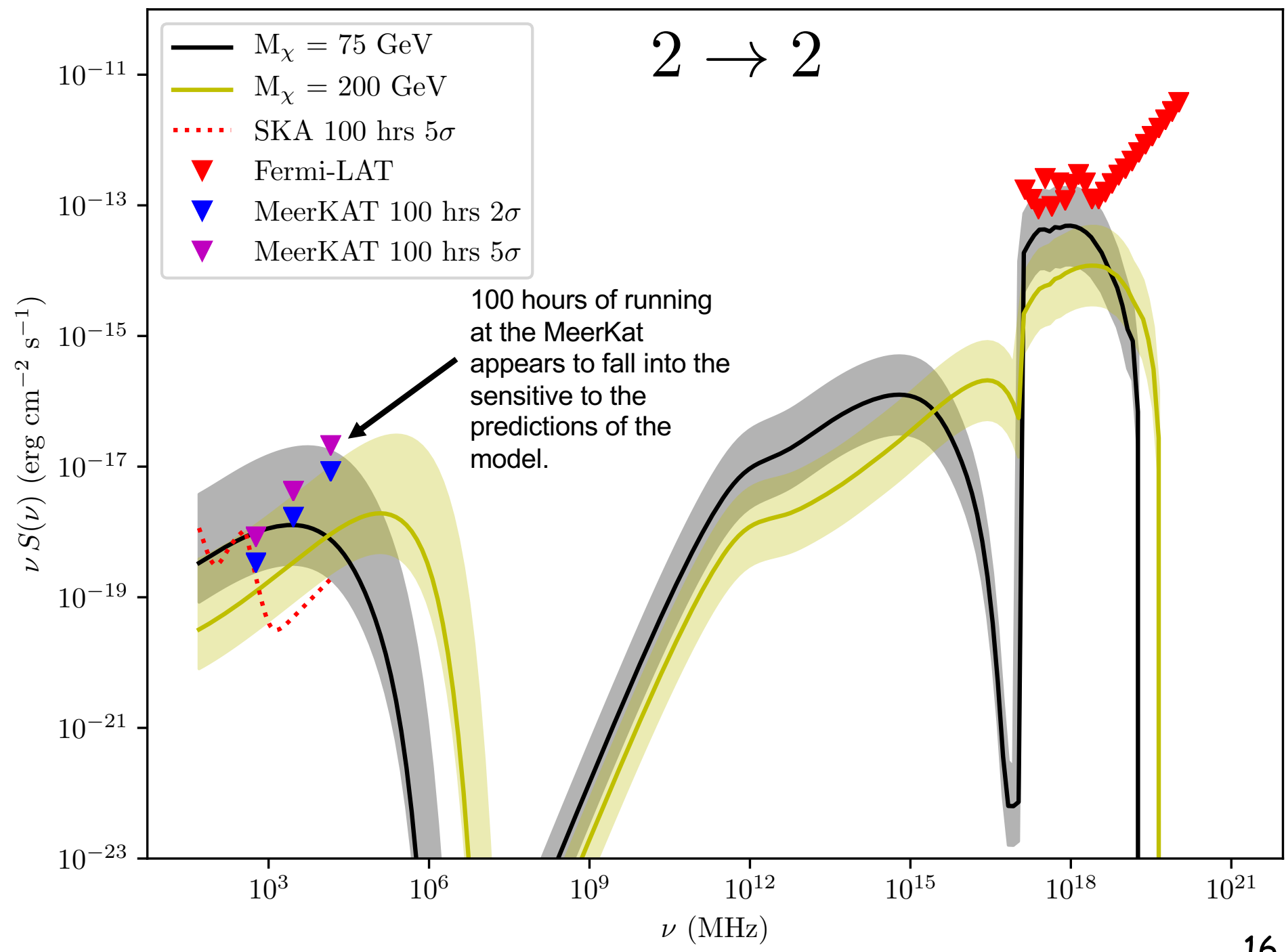


Dark matter annihilation.
Leptons, photons and protons
from the decays of S .



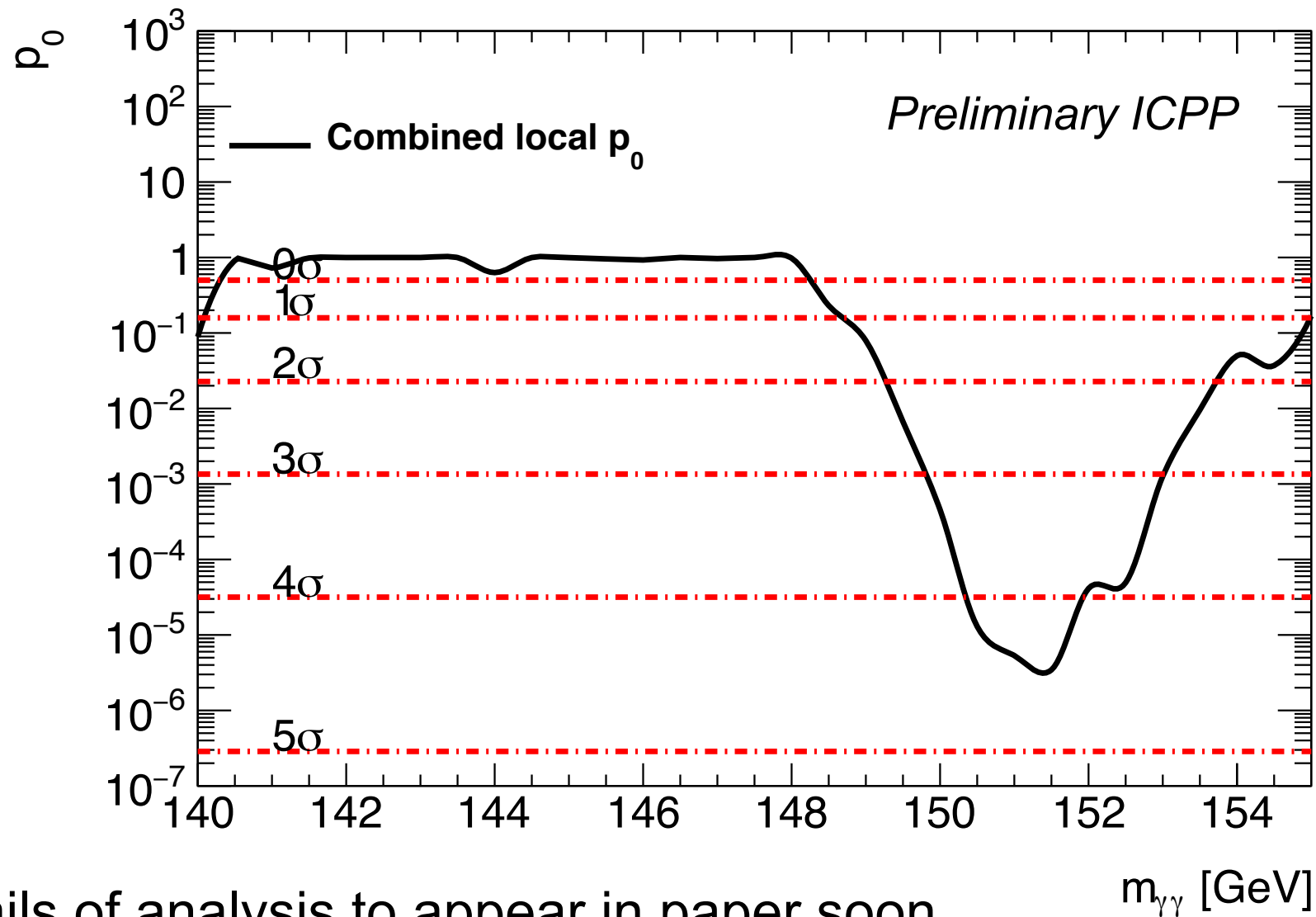


$2 \rightarrow 2$



**A possible
candidate of S**

Analysis of publicly available di-photon spectra in associated production gives $>4\sigma$ excess around 151 GeV. Fiducial yields consistent with $H \rightarrow SS^*$ hypothesis with $m_H=270$ GeV (see above)



Details of analysis to appear in paper soon

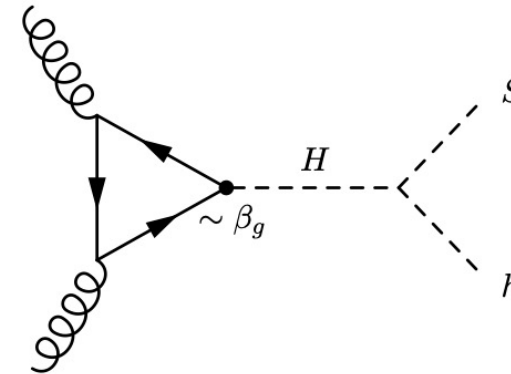
Outlook and Conclusions

- ❑ **Discrepancies in multi-lepton final states at LHC w.r.t. current MCs are not statistical fluctuations**
 - ❑ **They appear in corners of the phase-space dominated by different processes ($Wt/tt/4t$, VV , ttV , Vh)**
 - **Hard to explain with MC mismodelling**
 - ❑ **Discrepancies interpreted with simplified model where $H \rightarrow SS$, S_h is treated as SM Higgs-like and one parameter is floated**
- ❑ **Features of the Higgs data from LHC agree with predictions the simplified model used here**
- ❑ **Connection made with excesses in astro-physics, where MeerKat has sensitivity to probe**
- ❑ **Analysis of di-photon spectra in associated production gives $>4\sigma$ excess around 151 GeV**

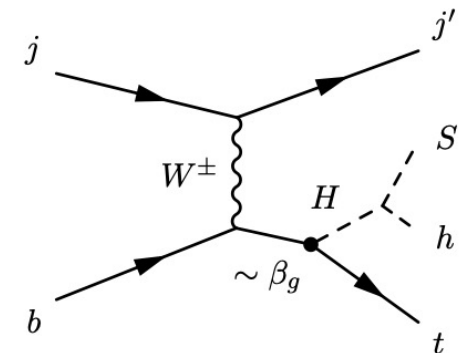
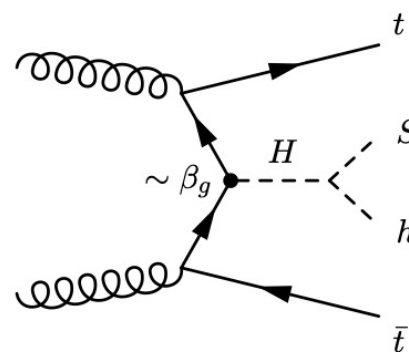
Additional Slides

The simplified Model (from Run I)

- 1. The starting point of the hypothesis is the existence of a boson, H, that contains Higgs-like interactions, with a mass in the range 250-280 GeV**
- 2. In order to avoid large quartic couplings, incorporate a mediator scalar, S, that interacts with the SM and Dark Matter.**
- 3. Dominance of $H \rightarrow Sh, SS$ decay over other decays**



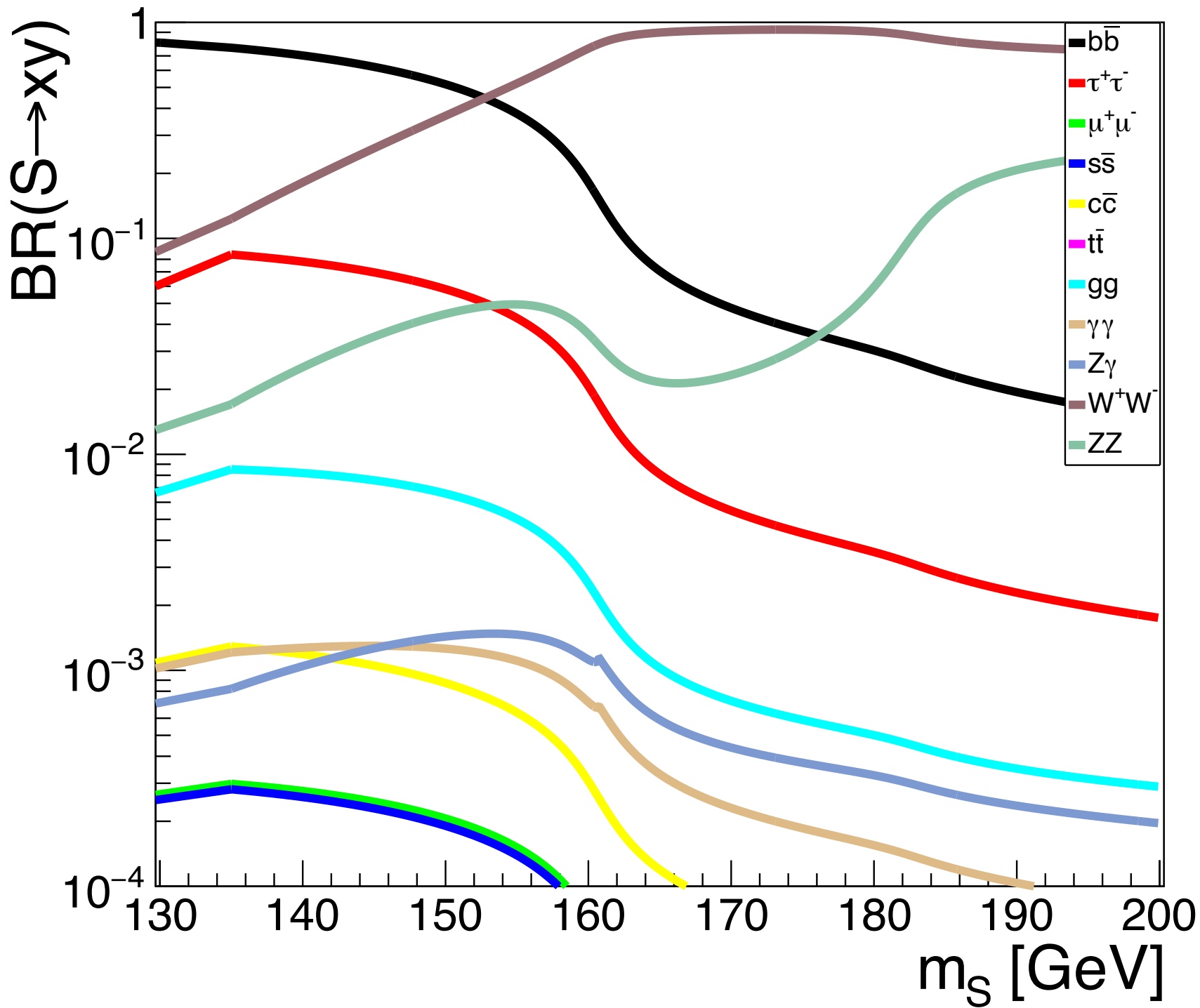
(a) Gluon fusion (ggF).

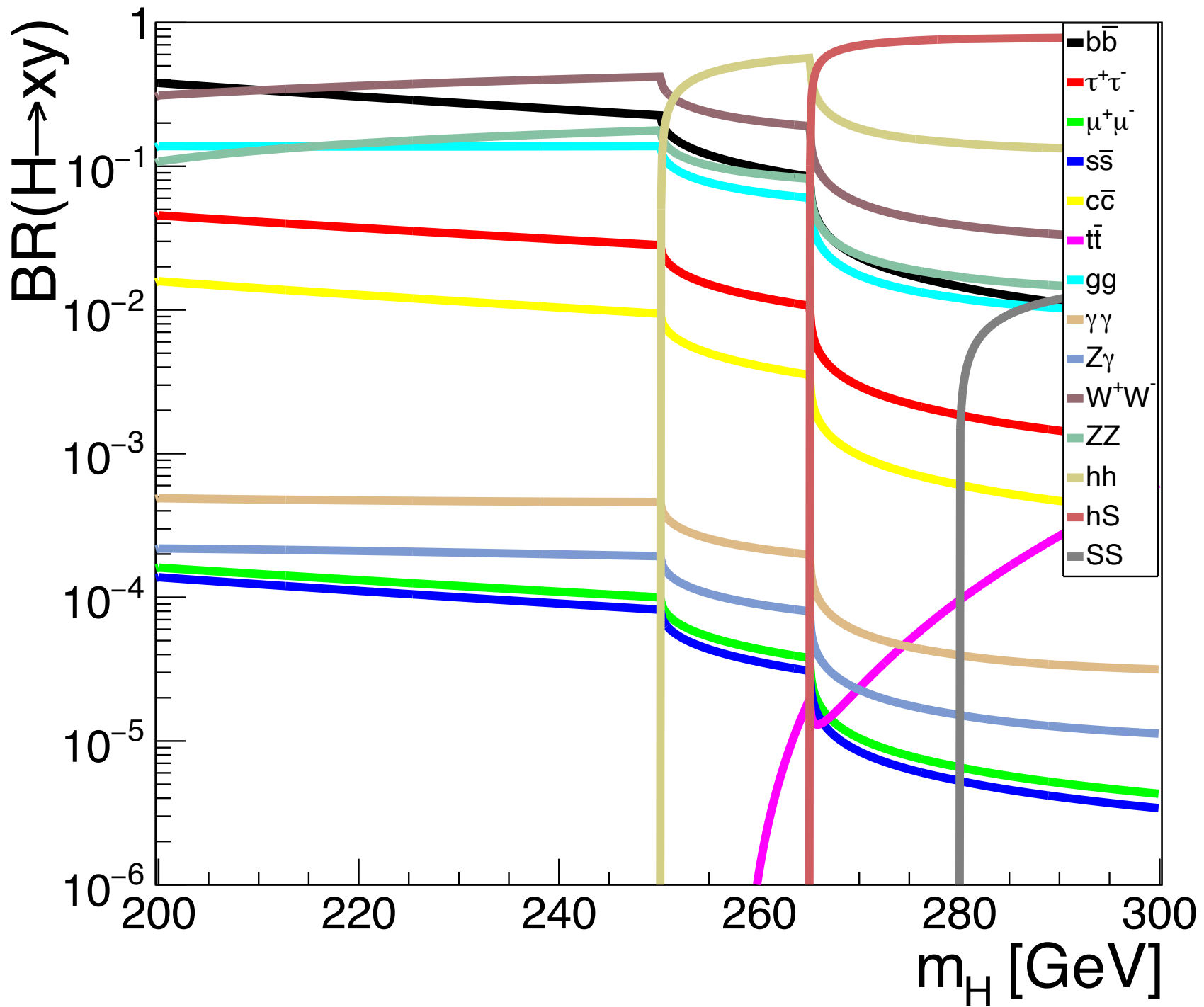


$$\mathcal{L}_{\text{int}} \supset -\beta_g \frac{m_t}{v} t\bar{t}H + \beta_V \frac{m_V^2}{v} g_{\mu\nu} V^\mu V^\nu H$$

$$\mathcal{L}_{HhS} = -\frac{1}{2} v \left[\lambda_{hhS} hhS + \lambda_{hSS} hSS + \lambda_{HHS} HHS + \lambda_{HSS} HSS + \lambda_{HhS} HhS \right],$$

For simplicity we will assume that the S decays like the SM Higgs boson





The model leads to rich phenomenology. Of particular interest are multilepton signatures

S. No.	Scalars	Decay modes
D.1	h	$b\bar{b}, \tau^+\tau^-, \mu^+\mu^-, s\bar{s}, c\bar{c}, gg, \gamma\gamma, Z\gamma, W^+W^-, ZZ$
D.2	H	D.1, hh, SS, Sh
D.3	A	D.1, $t\bar{t}, Zh, ZH, ZS, W^\pm H^\mp$
D.4	H^\pm	$W^\pm h, W^\pm H, W^\pm S$
D.5	S	D.1, $\chi\chi$

Scalar	Production mode	Search channels
H	$gg \rightarrow H, Hjj$ (ggF and VBF)	Direct SM decays as in Table 1 $\rightarrow SS/Sh \rightarrow 4W \rightarrow 4\ell + E_T^{\text{miss}}$ $\rightarrow hh \rightarrow \gamma b\bar{b}, b\bar{b}\tau\tau, 4b, \gamma\gamma WW$ etc. $\rightarrow Sh$ where $S \rightarrow \chi\chi \implies \gamma\gamma, b\bar{b}, 4\ell + E_T^{\text{miss}}$
	$pp \rightarrow Z(W^\pm)H$ ($H \rightarrow SS/Sh$)	$\rightarrow 6(5)l + E_T^{\text{miss}}$ $\rightarrow 4(3)l + 2j + E_T^{\text{miss}}$ $\rightarrow 2(1)l + 4j + E_T^{\text{miss}}$
	$pp \rightarrow t\bar{t}H, (t + \bar{t})H$ ($H \rightarrow SS/Sh$)	$\rightarrow 2W + 2Z + E_T^{\text{miss}}$ and b -jets $\rightarrow 6W \rightarrow 3$ same sign leptons + jets and E_T^{miss}
H^\pm	$pp \rightarrow tH^\pm$ ($H^\pm \rightarrow W^\pm H$)	$\rightarrow 6W \rightarrow 3$ same sign leptons + jets and E_T^{miss}
	$pp \rightarrow tbH^\pm$ ($H^\pm \rightarrow W^\pm H$)	Same as above with extra b -jet
	$pp \rightarrow H^\pm H^\mp$ ($H^\pm \rightarrow HW^\pm$)	$\rightarrow 6W \rightarrow 3$ same sign leptons + jets and E_T^{miss}
	$pp \rightarrow H^\pm W^\pm$ ($H^\pm \rightarrow HW^\pm$)	$\rightarrow 6W \rightarrow 3$ same sign leptons + jets and E_T^{miss}
A	$gg \rightarrow A$ (ggF)	$\rightarrow t\bar{t}$ $\rightarrow \gamma\gamma$
	$gg \rightarrow A \rightarrow ZH$ ($H \rightarrow SS/Sh$)	Same as $pp \rightarrow ZH$ above, but with resonance structure over final state objects
	$gg \rightarrow A \rightarrow W^\pm H^\mp$ ($H^\mp \rightarrow W^\mp H$)	6W signature with resonance structure over final state objects

Masses in the 2HDM+S

$$\begin{pmatrix} H_1 \\ H_2 \\ H_3 \end{pmatrix} = \mathbb{R} \begin{pmatrix} \rho_1 \\ \rho_2 \\ \rho_S \end{pmatrix},$$

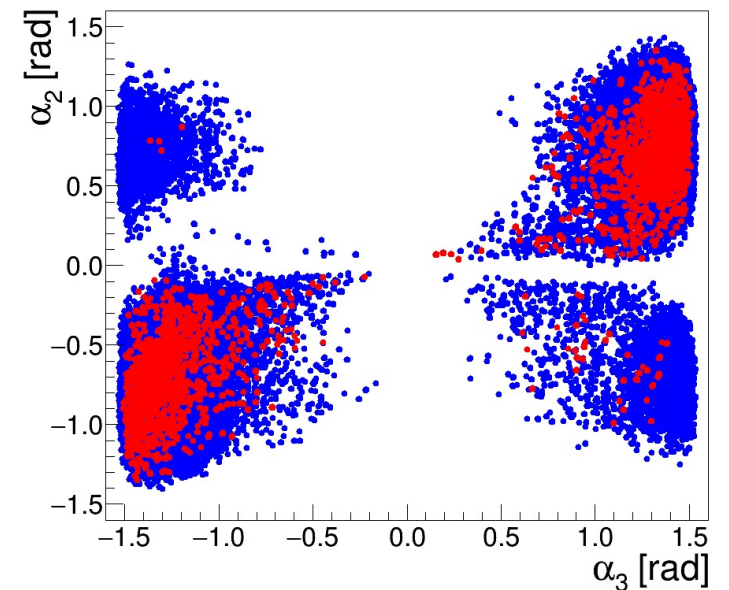
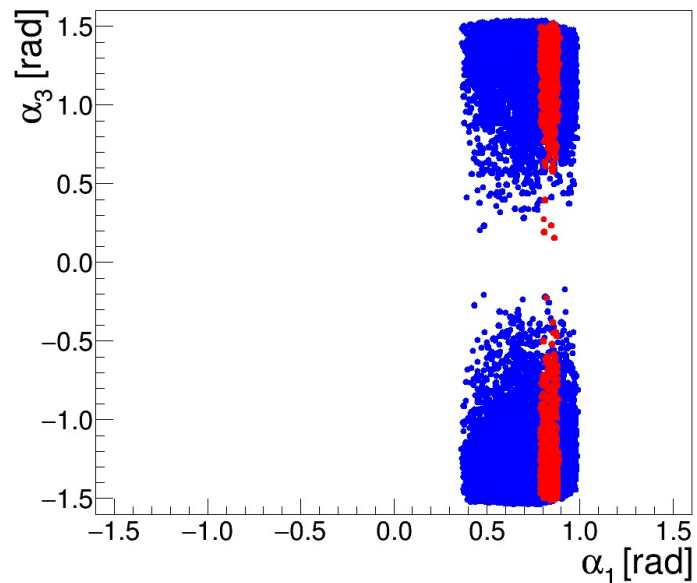
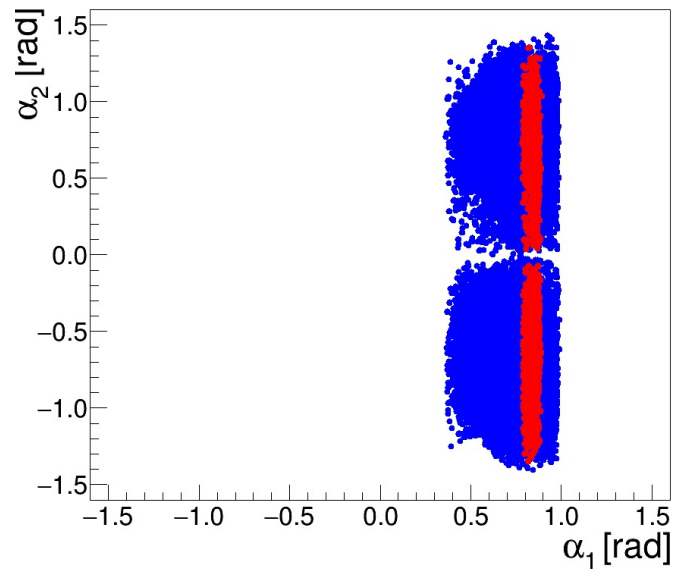
Mass-matrix for the CP-even scalar sector will be modified with respect to 2HDM and that needs a 3 x 3 matrix (three mixing angles). Couplings are modified.

$$\mathbb{R} = \begin{pmatrix} c_{\alpha_1} c_{\alpha_2} & s_{\alpha_1} c_{\alpha_2} & s_{\alpha_2} \\ - (c_{\alpha_1} s_{\alpha_2} s_{\alpha_3} + s_{\alpha_1} c_{\alpha_3}) & c_{\alpha_1} c_{\alpha_3} - s_{\alpha_1} s_{\alpha_2} s_{\alpha_3} & c_{\alpha_2} s_{\alpha_3} \\ -c_{\alpha_1} s_{\alpha_2} s_{\alpha_3} + s_{\alpha_1} s_{\alpha_3} & - (c_{\alpha_1} s_{\alpha_3} + s_{\alpha_1} s_{\alpha_2} c_{\alpha_3}) & c_{\alpha_2} c_{\alpha_3} \end{pmatrix}$$

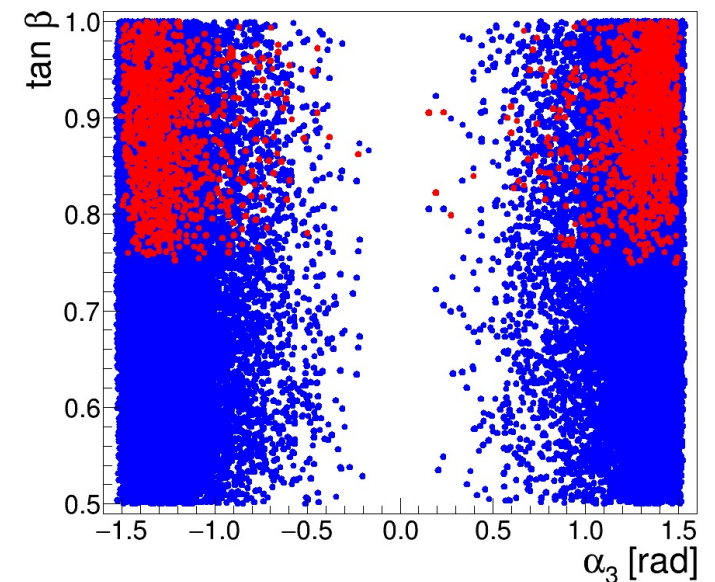
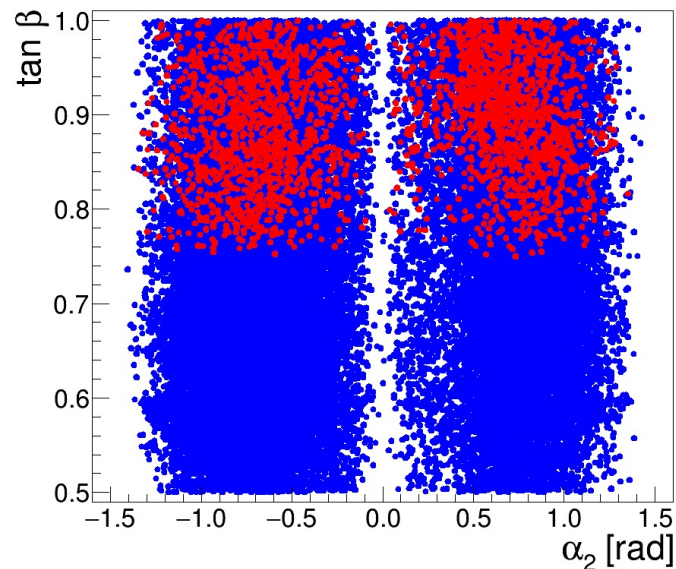
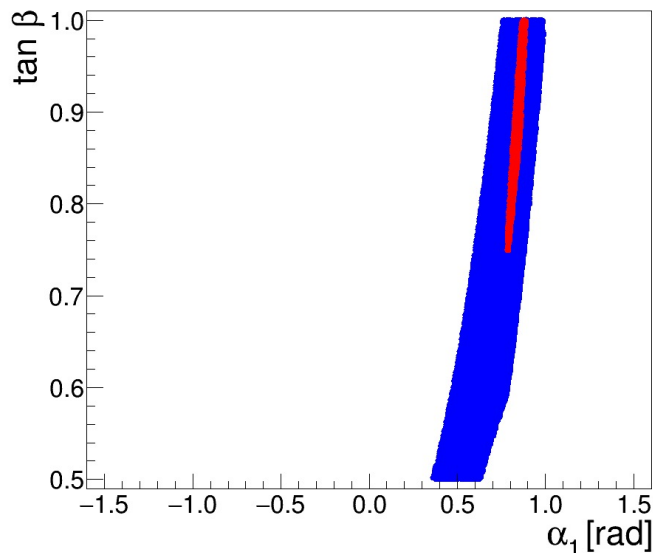
$$M_{\text{CP-even}}^2 = \begin{pmatrix} 2\lambda_1 v_1^2 - m_{12} \frac{v_2}{v_1} & m_{12} + \lambda_{345} v_1 v_2 & 2\kappa_1 v_1 v_S \\ m_{12} + \lambda_{345} v_1 v_2 & -m_{12} \frac{v_2}{v_1} + 2\lambda_2 v_2^2 & 2\kappa_2 v_2 v_S \\ 2\kappa_1 v_1 v_S & 2\kappa_2 v_2 v_S & \frac{1}{3} \lambda_S v_S^2 \end{pmatrix}$$

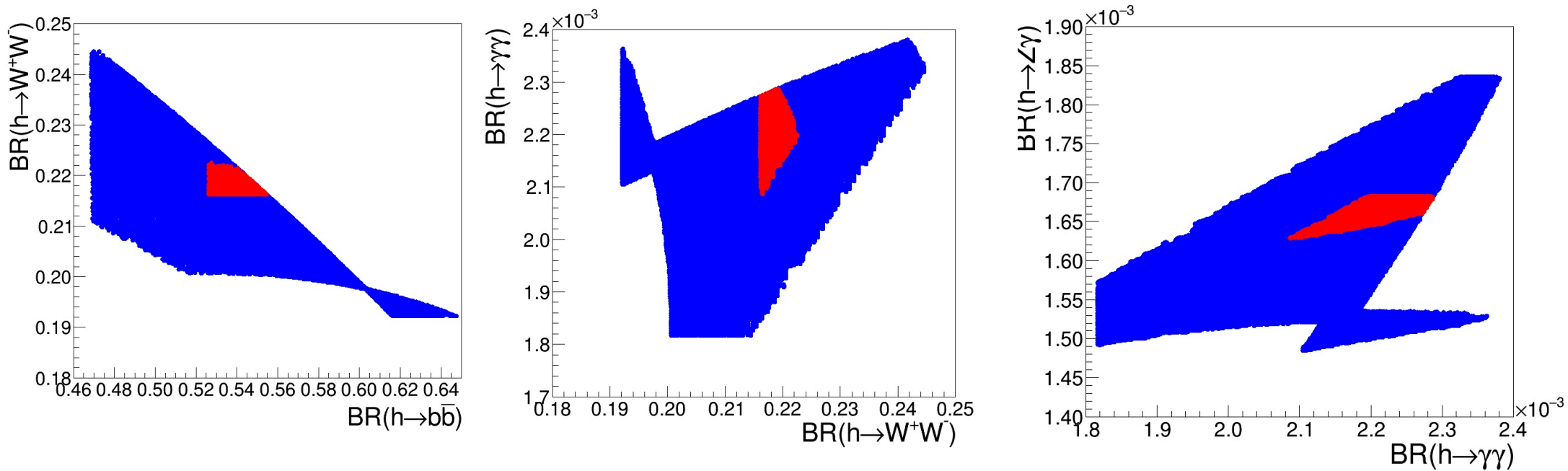
$$\begin{aligned}
m_{H_1}^2 &= v_S \sin \alpha_2 [\lambda_7 v \cos \alpha_1 \cos \alpha_2 \cos \beta + \lambda_8 v \sin \alpha_1 \cos \alpha_2 \sin \beta + \lambda_6 v_S \sin \alpha_2], \\
m_{H_2}^2 &= (\cos \alpha_1 \cos \alpha_3 - \sin \alpha_1 \sin \alpha_2 \sin \alpha_3) \left[\cos \alpha_1 \cos \alpha_2 (\lambda_{345} v^2 \sin \beta \cos \beta - m_{12}^2) \right. \\
&\quad \left. + \sin \alpha_1 \cos \alpha_2 (m_{12}^2 \cot \beta + \lambda_2 v^2 \sin^2 \beta) + \lambda_8 v v_S \sin \alpha_2 \sin \beta \right], \\
m_{H_3}^2 &= (\sin \alpha_1 \sin \alpha_3 - \sin \alpha_2 \cos \alpha_1 \cos \alpha_3) \left[\cos \alpha_1 \cos \alpha_2 (m_{12}^2 \tan \beta + \lambda_1 v^2 \cos^2 \beta) \right. \\
&\quad \left. + \sin \alpha_1 \cos \alpha_2 (\lambda_{345} v^2 \sin \beta \cos \beta - m_{12}^2) + \lambda_7 v v_S \sin \alpha_2 \cos \beta \right]. \tag{2.17}
\end{aligned}$$

Perform scans after fixing masses of physical bosons ($m_{h_1}=125$ GeV, $m_{h_2}=140$, $m_{h_3}=270$ GeV, $m_A=600$ GeV, $m_{H^\pm}=600$ GeV) in addition to the constraints described in arXiv:1711.07874, including the signal Yukawa coupling strength of $\beta_g^2=1.38 \pm 0.22$ (translated into $\tan^2 \beta$)

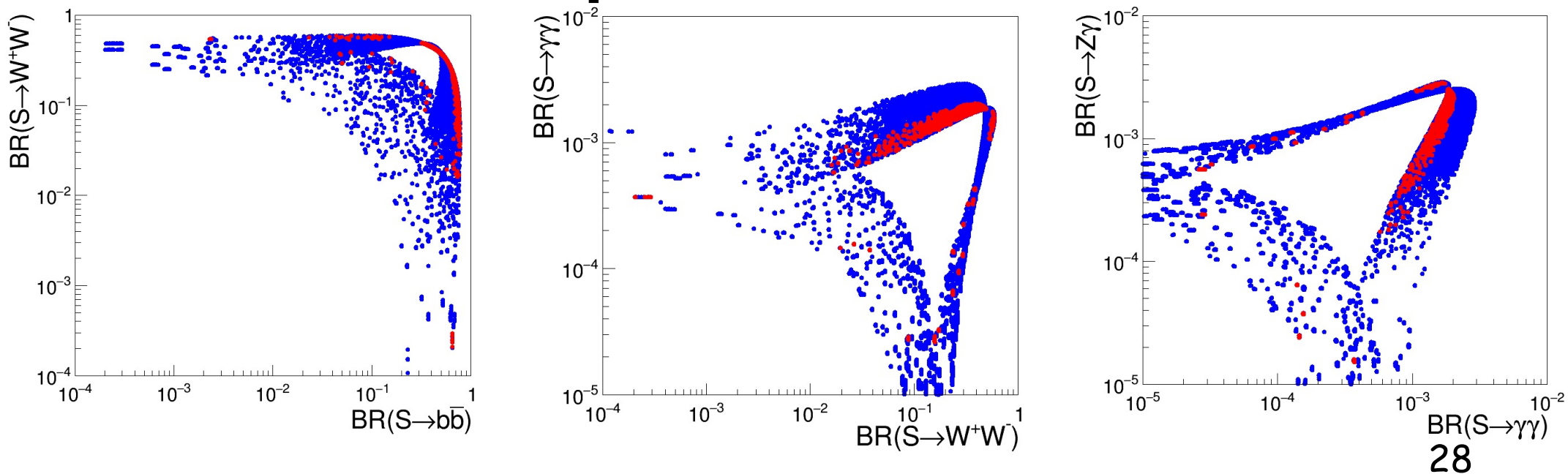


Correlation plots for the three mixing angles and $\tan\beta$. Blue (red) points correspond to $\text{Br}(h \rightarrow \text{SM})$ within 10% (20%) of the SM h values (J.Phys. G46 (2019) no.11, 115001)





Results using N2HDECAY (arXiv:1612.01309) for one benchmark point



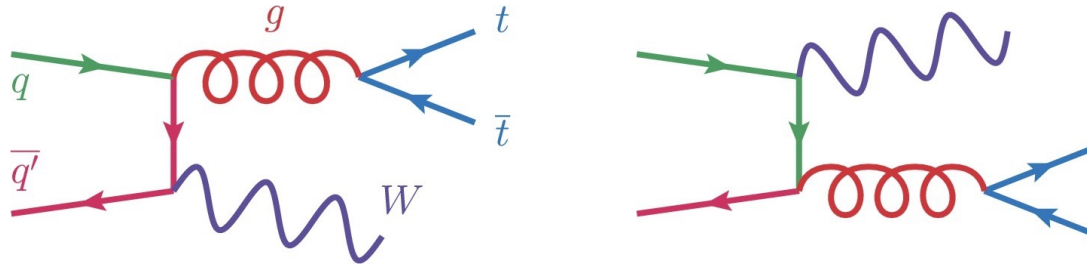
The fitting procedure

- **The RooStats workspace is made by HistFactory**
- **From the workspace, a profile likelihood ratio is calculated,**

$$\lambda(\beta_g^2) = \frac{L(\beta_g^2 | \hat{\theta})}{L(\hat{\beta}_g^2 | \hat{\theta})} \quad (\text{here } \theta \text{ denotes the nuisance parameters})$$

- **The best-fit value of β_g^2 is then calculated as the minimum of $-2\log(\lambda)$, with an error corresponding to a unit of deviation in this quantity from the best-fit point**
- **The significance is calculated as $\sqrt{-2 \log \lambda(0)}$, since $\beta_g^2 = 0$ corresponds to the SM-only hypothesis**

The anatomy of inclusive ttW at the LHC



S.Buddenbrock, R.Ruiz
and B.M.
Physics Letters B 811
(2020) 135964

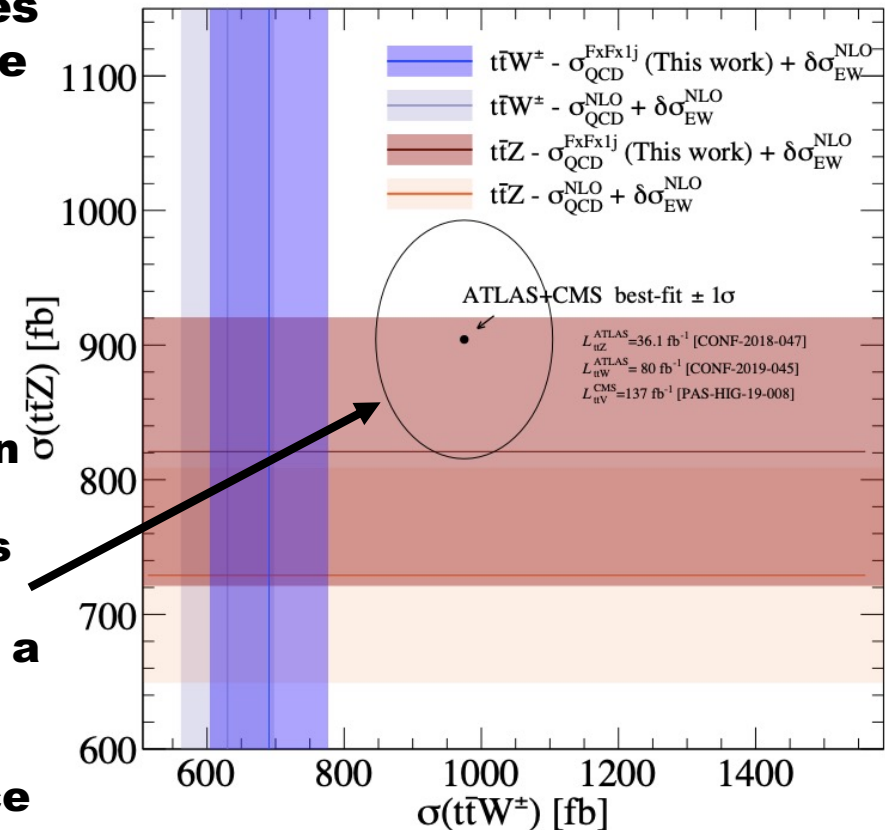
Using fixed order computations at $O(\alpha_s^4\alpha)$ and NLO multi-jet matching yielding similar (10%-14%) corrections to the inclusive rate

		$ij \rightarrow t\bar{t}W^\pm kl$				
(i, j)	(k, l)	$p_T^{j1} \text{ min}$	$p_T^{j2} \text{ min}$	σ [fb]	$\pm\delta_{\mu_f, \mu_r}$	$\pm\delta_{\text{PDF}}$
All	All	75 GeV	75 GeV	34.7 (100%)	+57%	+1.1%
(g, Q)	(g, Q)			23.7 (68%)	-34%	-1.1%
(Q, Q)	(Q, Q)			6.99 (20%)		
(Q, Q)	(g, g)			3.63 (10%)		
(g, g)	(q, \bar{q})			0.437 (1.3%)		
All	All	100 GeV	75 GeV	33.1 (100%)	+57%	+1.0%
(g, Q)	(g, Q)			22.6 (68%)	-34%	-1.0%
(Q, Q)	(Q, Q)			6.78 (20%)		
(Q, Q)	(g, g)			3.28 (9.9%)		
(g, g)	(q, \bar{q})			0.409 (1.2%)		
All	All	100 GeV	100 GeV	21.2 (100%)	+57%	+1.1%
(g, Q)	(g, Q)			14.3 (67%)	-34%	-1.1%
(Q, Q)	(Q, Q)			4.91 (23%)		
(Q, Q)	(g, g)			1.75 (8%)		
(g, g)	(q, \bar{q})			2.58 (1%)		
(g, q_V)	(g, q_V)	75 GeV	75 GeV	20.1 (58%)	+58%	+2.3%
(g, q_V)	(g, q_V)	100 GeV	75 GeV	19.3 (58%)	-35%	-2.3%
(g, q_V)	(g, q_V)	100 GeV	100 GeV	12.2 (58%)	+58%	+2.3%
(g, q_V)	(g, q_V)	75 GeV	75 GeV	20.1 (58%)	-35%	-2.3%
(g, q_V)	(g, q_V)	100 GeV	100 GeV	12.2 (58%)	+59%	+2.4%
(g, q_V)	(g, q_V)	100 GeV	100 GeV	12.2 (58%)	-35%	-2.4%

Table 2: Total cross sections [fb] at $\sqrt{s} = 13$ TeV for the $pp \rightarrow t\bar{t}W^\pm jj$ process at LO, with scale and PDF uncertainties [%], for representative $p_T^{jk} \text{ min}$ with $|\eta^j| < 4.0$. Also shown is the decomposition according to partonic channel, for $q_V \in \{u, d\}$, $q \in \{u, d, c, s\}$, and $Q \in \{q, \bar{q}\}$.

Detailed studies that include the decomposition in partonic channels and differential distributions

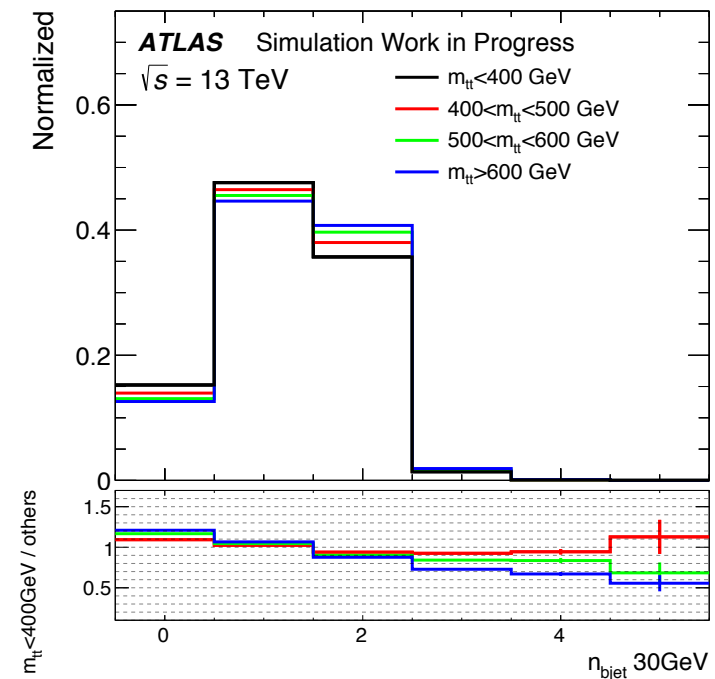
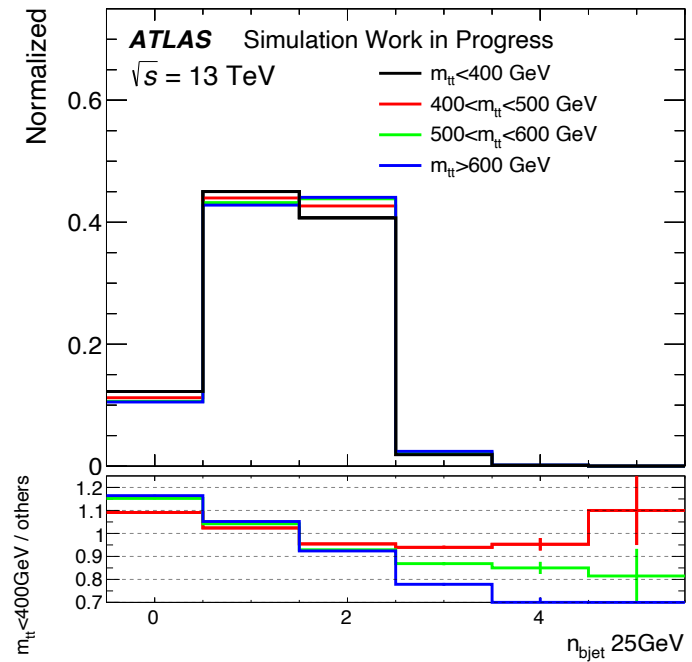
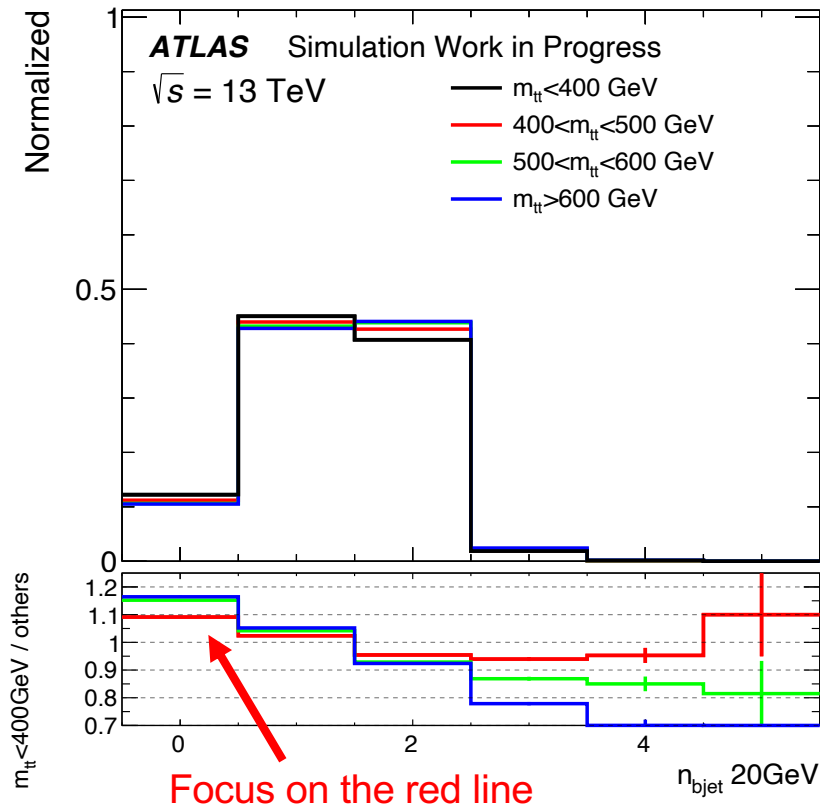
Tension between data and predictions does not wane. For this process a complete NNLO computation is needed to reduce theory uncertainty



Can Toponium explain the anomalies

Final state	Characteristic	Dominant SM process	Toponium
$l^+l^- + \text{jets, b-jets}$	$m_{ll} < 100 \text{ GeV}$, dominated by 0b-jet and 1b-jet	$tt+Wt$	Let's take a closer look
$l^+l^- + \text{full-jet veto}$	$m_{ll} < 100 \text{ GeV}$	WW	No
$l^\pm l^\pm$ & $l^\pm l^\pm + \text{b-jets}$	Moderate H_T	$ttW, 4t$	No
$l^\pm l^\pm$ & $l^\pm l^\pm$ et al., no b-jets	In association with h	Wh	No
$Z(\rightarrow l^+l^-)+l$	$p_{TZ} < 100 \text{ GeV}$	ZW	No

Toponium leads to a tt final state and contributes to $l^+l^- + \text{b-jets}$ with large b-jet multiplicity. BUT the excess prefers low b-jet multiplicity.



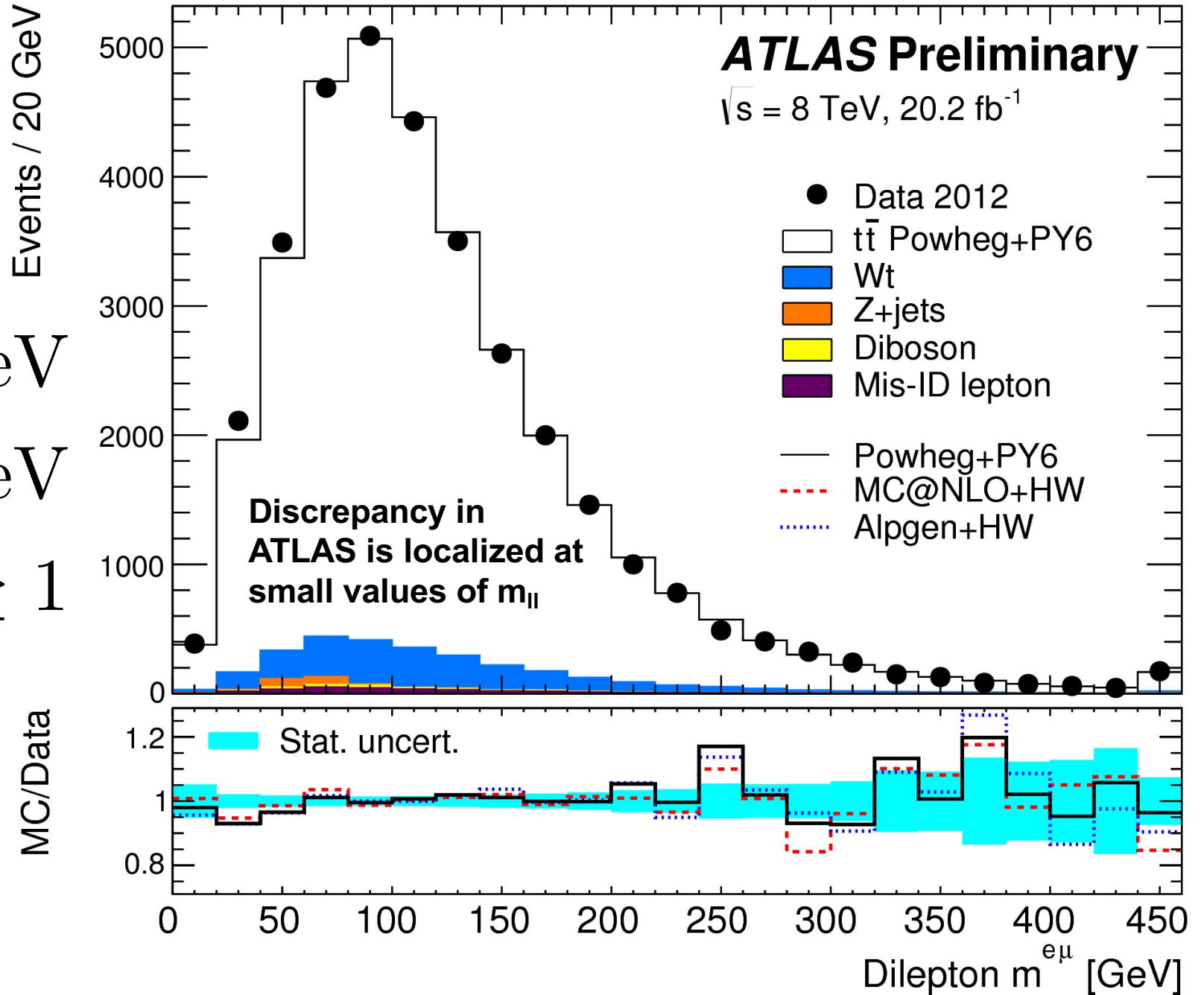
Toponium will produce $t\bar{t}$ at threshold. MC studies show that the b-jet multiplicity changes little with the $t\bar{t}$ invariant mass. Note small difference in b-jet multiplicity for $m_{t\bar{t}} < 400$ vs $400 < m_{t\bar{t}} < 500$ GeV. The effect would need to be an order of magnitude higher to explain the excess with OS di-leptons with b-jets.

Is the discrepancy due to $t\bar{t}$ events?

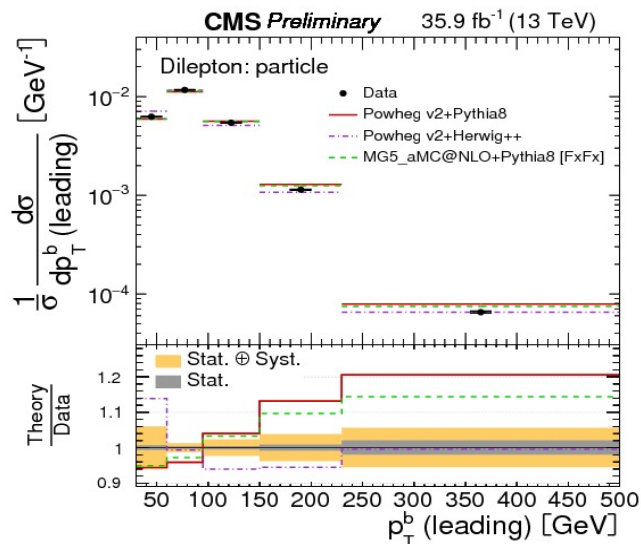
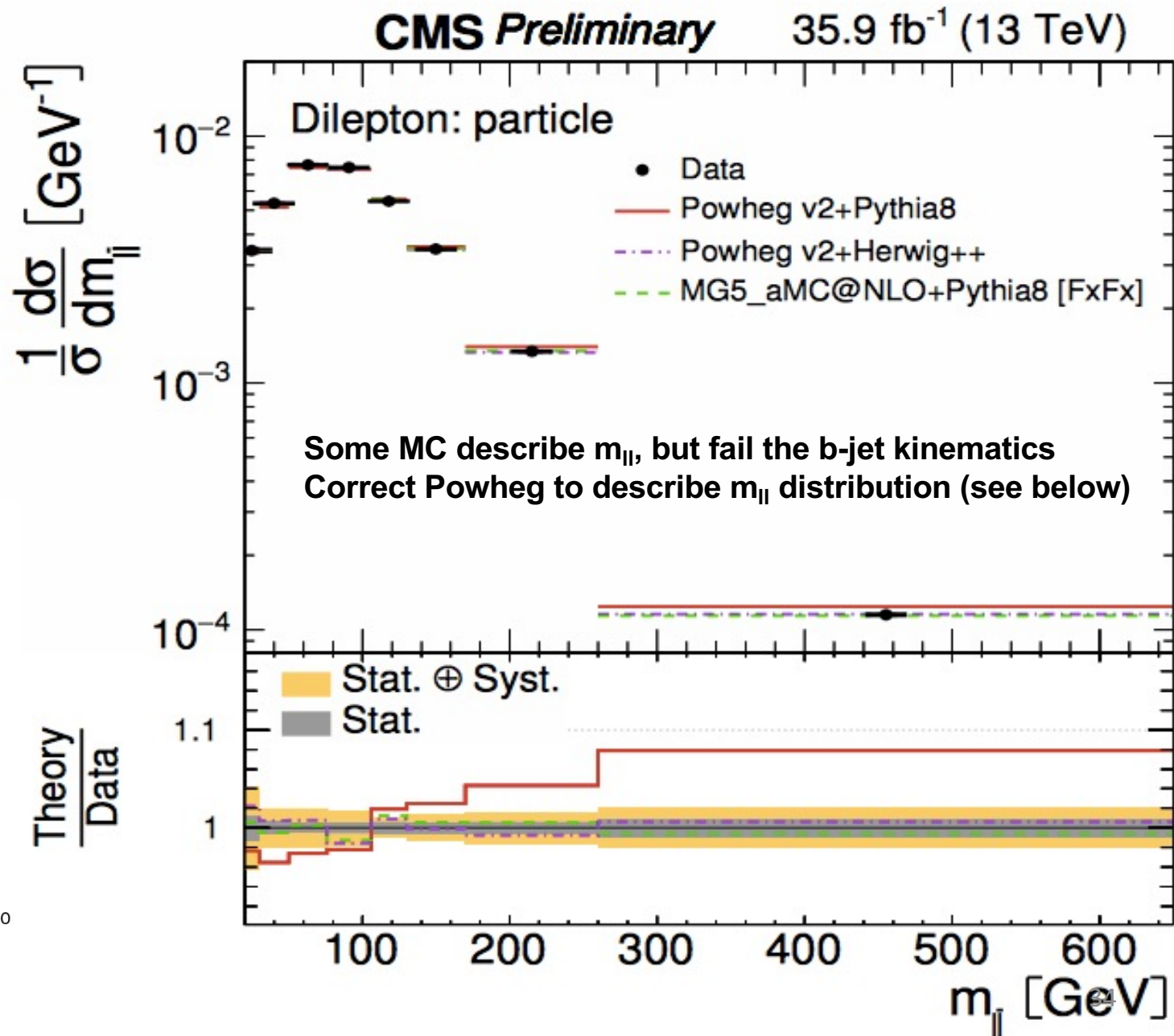
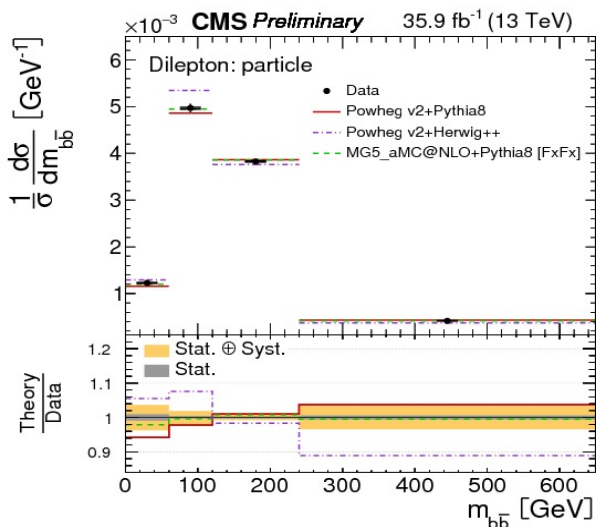
$$p_{T\ell} > 25 \text{ GeV}$$

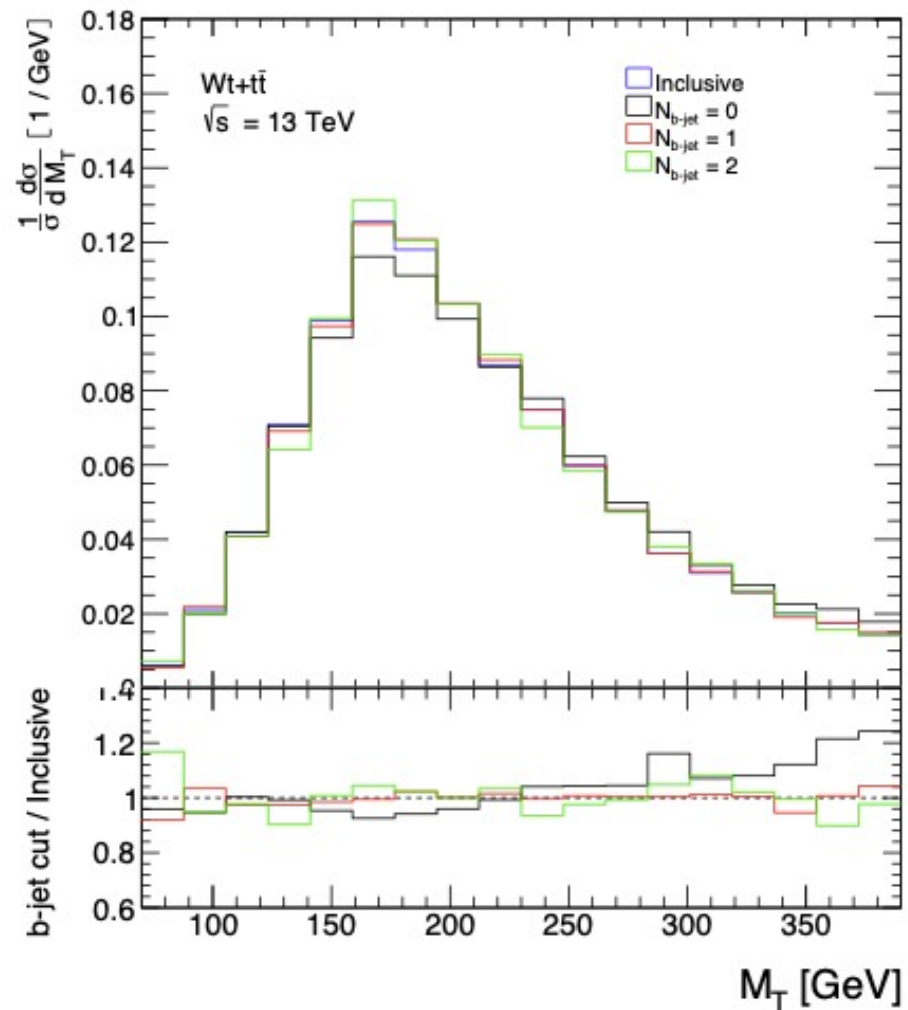
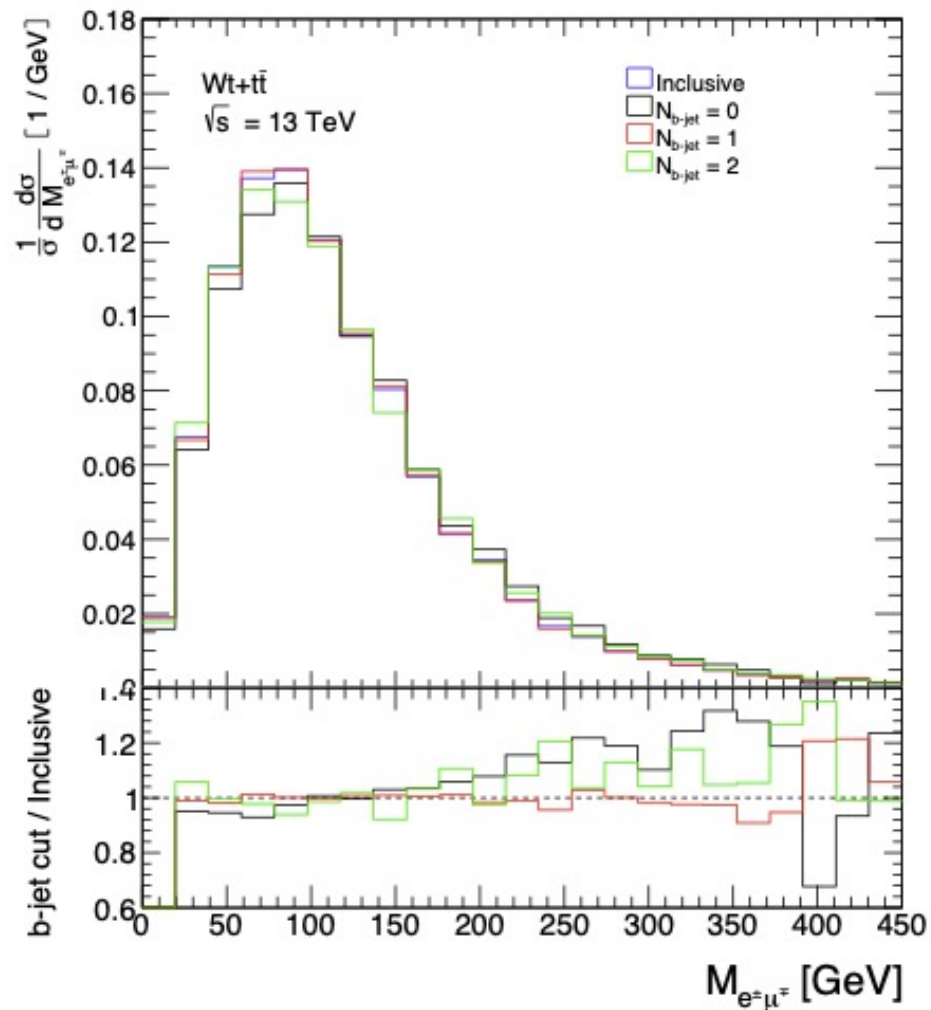
$$p_{Tb} > 25 \text{ GeV}$$

$$N_{bjet} \geq 1$$



**Event selection with exactly two leptons (e,μ),
m_{ll}>20 GeV and at least 2b-jets**



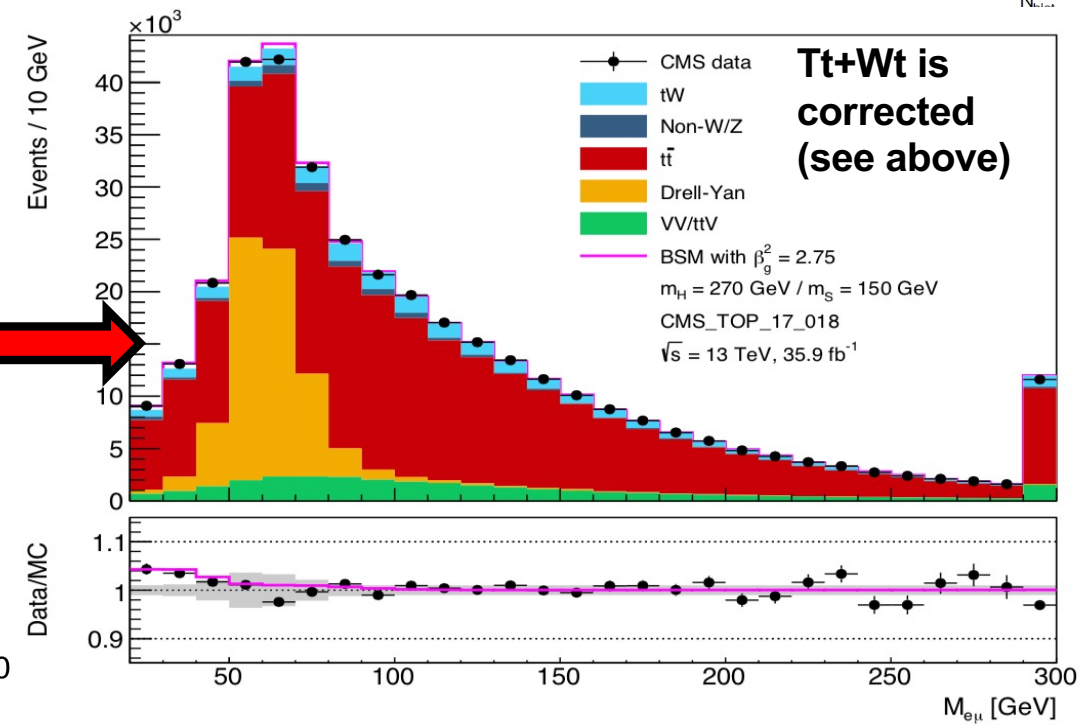
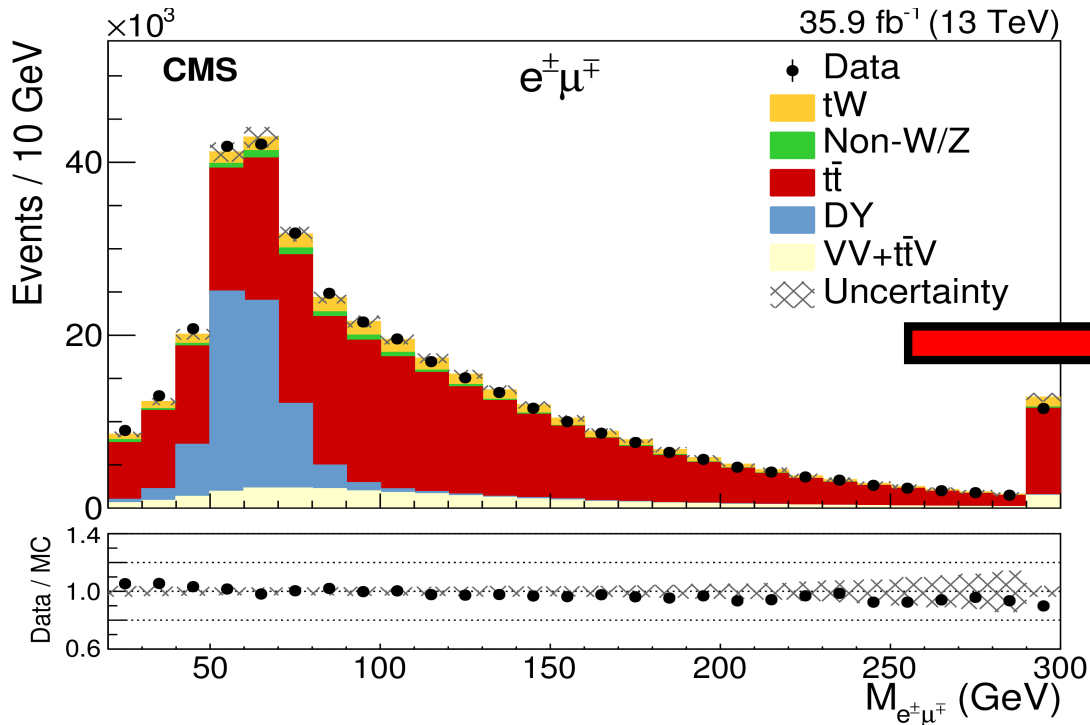
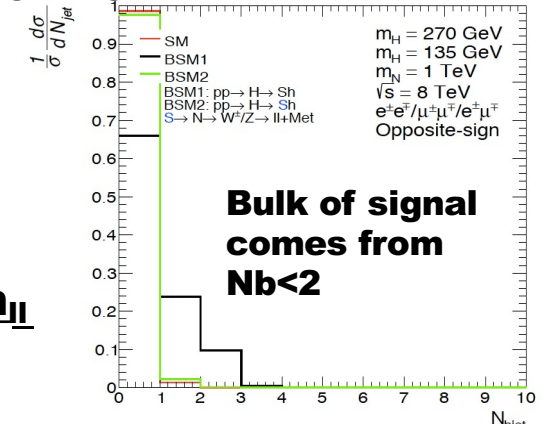


b-jet multiplicity is robust theoretically

Figure 9: Leptonic distributions produced by $t\bar{t}$ and tW processes (see text) as a function of the b -tagged jet multiplicity. The di-lepton invariant mass (left) and the transverse mass of the di-lepton and missing transverse energy system are displayed. Distributions are normalised to unity. The insert shows the ratio of the distributions with exclusive b -tagged jet bins relative to that obtained inclusively.

- Poor modeling of POWHEG + Pythia8 distribution is improved through reweighting
- We fix the normalisation of the SM by scaling it to the data in the region $m_{ll} > 110$ GeV
 - A normalisation systematic of 3% is applied to all but DY
 - DY systematic = 6.8%. 3% systematic on m_{ll} shape in top
 - The fit is done to the region below 110 GeV
- Fit results:
 - $\beta_g^2 = 2.79 \pm 0.52$
 - Fit is extremely well constrained

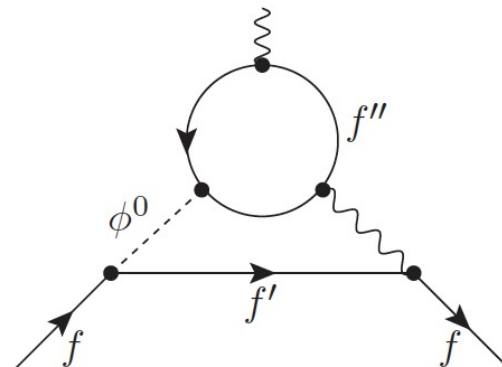
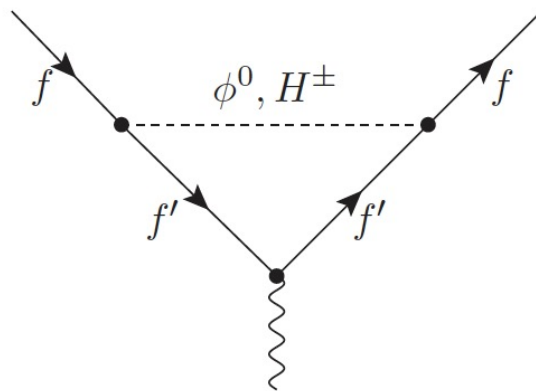
Negligible MC dependence, as m_{ll} shape comes from data



Used conservative assumption that $l+l+2b$ -jet final state is perfectly described by the SM. The discrepancy comes from events with $N_b < 2$. Excess unlikely due to tt

$$\Delta a_\mu = a_\mu^{\text{Exp}} - a_\mu^{\text{SM}} = 2.87(80) \times 10^{-9}$$

The Muon $g-2$ and the 2HDM+S

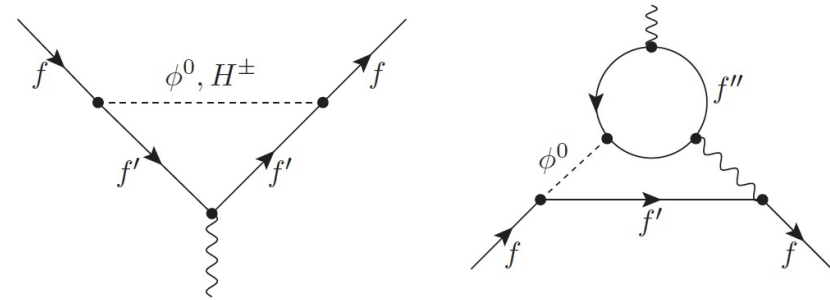


2HDM+S potential with fixed parameters from multi-lepton anomalies at the LHC

$$\begin{aligned}
 V(\Phi_1, \Phi_2, \Phi_S) &= m_{11}^2 |\Phi_1|^2 + m_{22}^2 |\Phi_2|^2 - m_{12}^2 (\Phi_1^\dagger \Phi_2 + \text{h.c.}) \\
 &+ \frac{\lambda_1}{2} (\Phi_1^\dagger \Phi_1)^2 + \frac{\lambda_2}{2} (\Phi_2^\dagger \Phi_2)^2 + \lambda_3 (\Phi_1^\dagger \Phi_1) (\Phi_2^\dagger \Phi_2) \\
 &+ \lambda_4 (\Phi_1^\dagger \Phi_2) (\Phi_2^\dagger \Phi_1) + \frac{\lambda_5}{2} \left[(\Phi_1^\dagger \Phi_2)^2 + \text{h.c.} \right] \\
 &+ \frac{1}{2} m_S^2 \Phi_S^2 + \frac{\lambda_6}{8} \Phi_S^4 + \frac{\lambda_7}{2} (\Phi_1^\dagger \Phi_1) \Phi_S^2 + \frac{\lambda_8}{2} (\Phi_2^\dagger \Phi_2) \Phi_S^2
 \end{aligned}$$

Consider extra degrees of freedom in the form of SM singlet vector-like fermions

$$\mathcal{L} \supset -y_f^S \bar{l}_R \Phi_S f'_L - \sum_{i=1}^2 y_f^i \bar{L}_l \Phi_i f'_R + \text{h.c.},$$



Allowed fermion masses with different choices of Yukawa couplings

