Higgs Mechanism From On-shell Massive Amplitudes

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DL, Zhewei Yin: work in progress

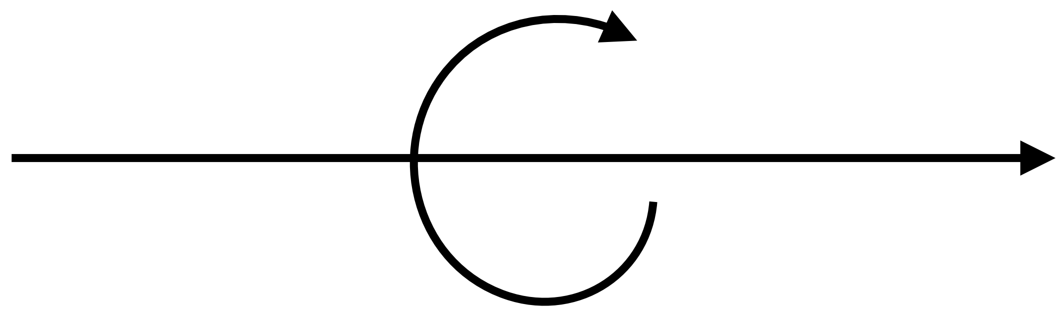
Outline

- Little Group
- On-shell Massless and Massive Amplitude
- Higgs Mechanism from Tree-level Unitarity

Little Group

- A subgroup of Lorentz group
- Its transformation leaves the momentum invariant
- Particles can be defined as irreducible representations

Little Group: Massless particles



$$k^\mu = E(1,0,0,1)$$

$$W(\Lambda, p; k) \in ISO(2)$$

To avoid continuum of states, only U(1) is relevant: helicity

Massless Spinor-Helicity Variables

$SO(3,1) \simeq SL(2,C)$

$p_\mu \qquad p_{\alpha\dot{\alpha}} = p_\mu \sigma^\mu_{\alpha\dot{\alpha}}$

$p^2 \longleftrightarrow \det p_{\alpha\dot{\alpha}}$

$p^2 = 0 \longrightarrow p_{\alpha\dot{\alpha}} = \lambda_\alpha \tilde{\lambda}_{\dot{\alpha}}$

Massless Spinor-Helicity Variables

$$p_{\alpha\dot{\alpha}} = \lambda_\alpha \tilde{\lambda}_{\dot{\alpha}} \equiv |\lambda\rangle[\tilde{\lambda}|$$

Little group scaling

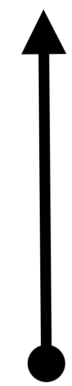
$$\lambda_\alpha \to w^{-1}\lambda, \qquad \tilde{\lambda} \to w\tilde{\lambda}$$

Helicity amplitudes

$$\mathcal{M}(w^{-1}\lambda, w\tilde{\lambda}) = w^{2h}\mathcal{M}(\lambda, \tilde{\lambda})$$

Completely fixes the 3-particle on-shell amplitudes

Little Group: Massive particles



$$k^\mu = (M, 0, 0, 0)$$

$$W(\Lambda, p; k) \in SO(3)$$

Spin degrees of freedom

Massive Spinor-Helicity Variables

$$SO(3,1) \simeq SL(2,C)$$

$$p_\mu \qquad p_{\alpha\dot{\alpha}} = p_\mu \sigma^\mu_{\alpha\dot{\alpha}}$$

$$p^2 \longleftrightarrow \det p_{\alpha\dot{\alpha}}$$

$$p^2 = m^2 \longrightarrow p_{\alpha\dot{\alpha}} = \lambda^I_\alpha \tilde{\lambda}_{I\dot{\alpha}}$$

$$I = 1, 2$$

Massive Spinor-Helicity Variables

$$p_{\alpha\dot{\alpha}} = \lambda_\alpha^I \tilde{\lambda}_{I\dot{\alpha}} \equiv |\lambda^I\rangle[\tilde{\lambda}_I|$$

Little group transformation

$$\lambda_\alpha^I \to W_J^I \lambda_\alpha^J, \qquad \tilde{\lambda}_{I\dot{\alpha}} \to (W^{-1})_I^J \tilde{\lambda}_{J\dot{\alpha}}$$

Scattering amplitudes

$$\mathcal{M}(\lambda^{I_1} \cdots \lambda^{I_{2S}}) \to W_{J_1}^{I_1} \cdots W_{J_{2S}}^{I_{2S}} \mathcal{M}(\lambda^{J_1} \cdots \lambda^{J_{2S}})$$

N. Arkani-Hamed, T.C. Huang & Y.t. Huang '17

Unitarity and Locality

Unitarity

Consistent Factorization

$$\frac{\sum_h \mathcal{M}_3^h \mathcal{M}_3^{-h}}{p^2}$$

Locality

Simple poles

Higgs Mechanism

Higgs Mechanism

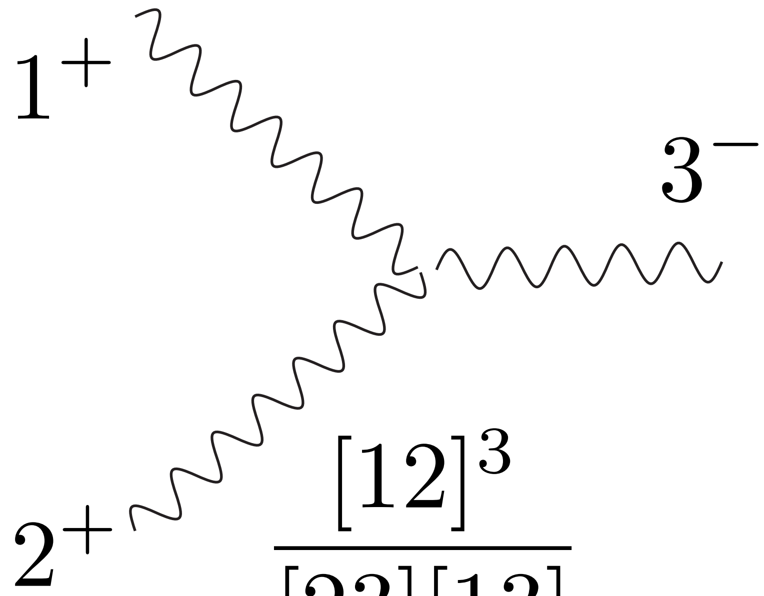
$$\partial_\mu A_\mu - \xi m_A \phi = 0$$

$$k_\mu \epsilon_\mu = 0, \quad \phi = 0$$

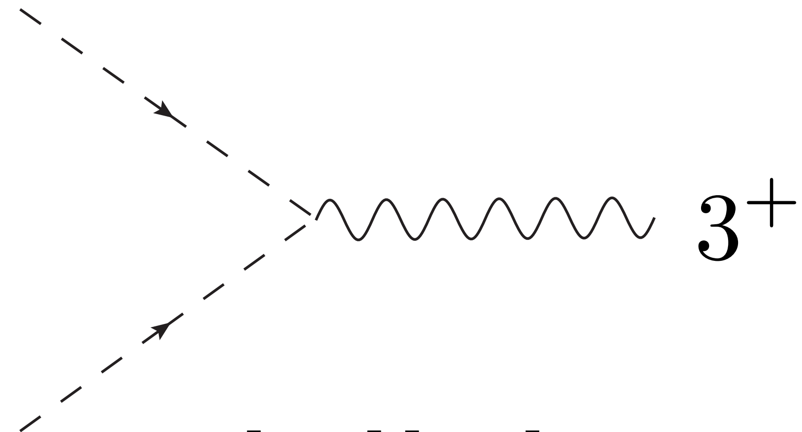
$$\frac{k_\mu}{m_A}\epsilon_\mu \sim \xi\phi$$

Sign of Goldstone Equivalence Theorem

Higgs Mechanism as IR Unification



$$\frac{[12]^3}{[23][13]}$$



$$\frac{[13][23]}{[12]}$$

$$\frac{[\mathbf{12}]\langle\mathbf{23}\rangle\langle\mathbf{13}\rangle}{m_1 m_2}$$

Non-local poles to mass singularity

N. Arkani-Hamed, T.C. Huang & Y.t. Huang '17

Tree-level Unitarity

$$\mathcal{M}_n(E_i, \theta_i)$$

Fixed angles

$$\mathcal{M}_n \lesssim E^{4-n}$$

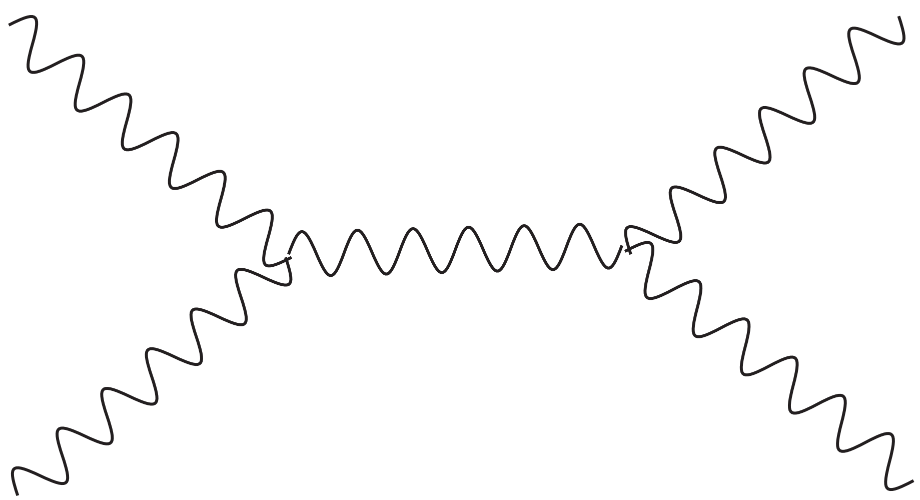
Up to logarithmic factor

J. Cornwall, D. Levin & G. Tiktopoulos '74

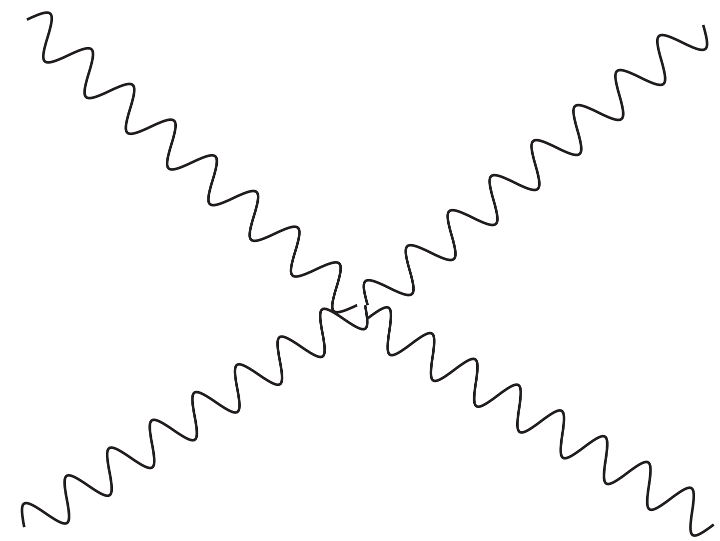
Gauge Boson Sector

$C_{abc}\partial_\mu A^a_\nu A^b_\mu A^c_\nu$

$D_{abcd}A^a_\mu A^b_\mu A^c_\nu A^d_\nu$



E^6 E^4 E^2



E^4 E^2

Gauge Boson Sector

$C_{abc}\partial_\mu A^a_\nu A^b_\mu A^c_\nu$ $D_{abcd}A^a_\mu A^b_\mu A^c_\nu A^d_\nu$

E^6 C_{abc} fully antisymmetric

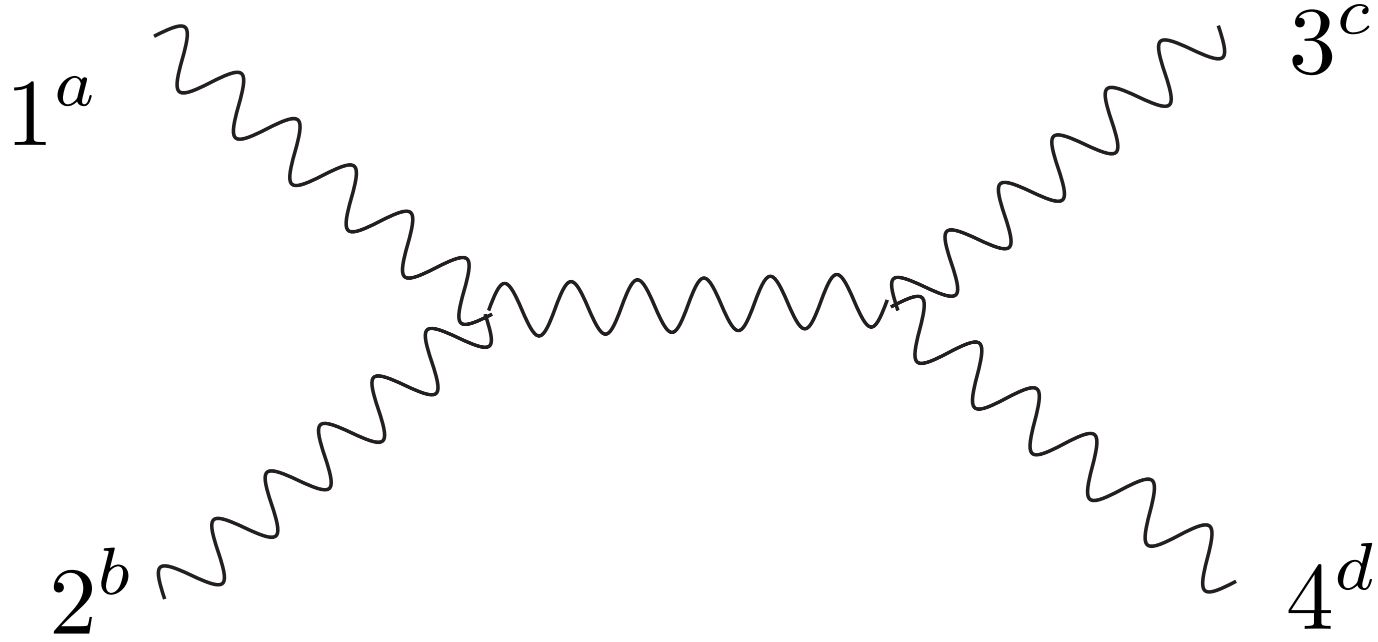
E^4 $D_{abcd} \propto C_{abf}C_{fcd}$

E^2 $C_{abf}C_{fcd} + C_{bcf}C_{fad} + C_{caf}C_{fbd} = 0$

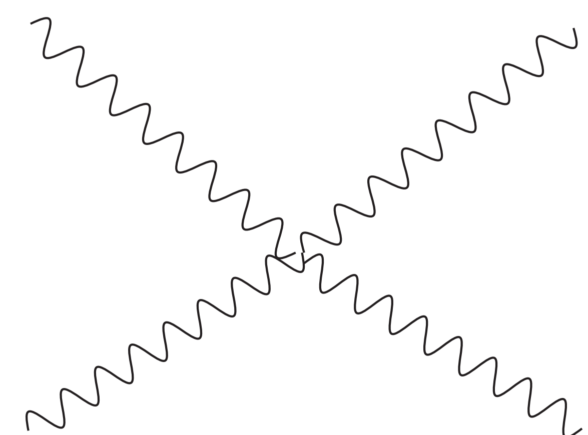
Gauge Boson Sector: On-shell

$$C_{abe}\frac{[\mathbf{12}]\langle\mathbf{2I}\rangle\langle\mathbf{1I}\rangle}{m_1 m_2}$$

$$C_{cde}\frac{\langle\mathbf{34}\rangle[\mathbf{3I}][\mathbf{4I}]}{m_3 m_4}$$



$$C_{abe}C_{cde}\frac{[\mathbf{12}]\langle\mathbf{34}\rangle\langle\mathbf{2}|p_1|\mathbf{3}]\langle\mathbf{1}|p_2|\mathbf{4}]}{(s-m_I^2)m_1 m_2 m_3 m_4}$$



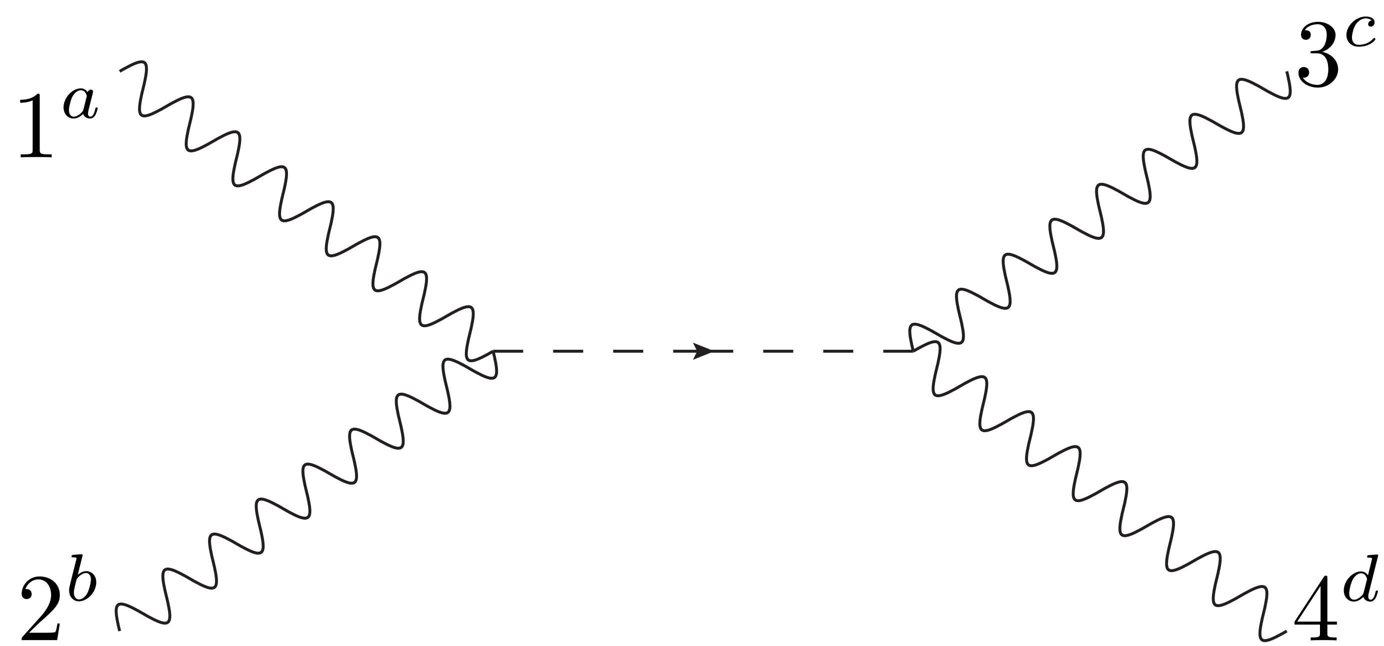
E^4

$$C_{abe}C_{cde}\frac{[\mathbf{12}]\langle\mathbf{12}\rangle\langle\mathbf{34}\rangle[\mathbf{34}]}{m_1 m_2 m_3 m_4}$$

DL, Z.Yin: work in progress

Gauge Boson Sector: On-shell

$$\frac{F^{ab\alpha}}{m_1 m_2}[\mathbf{12}]\langle\mathbf{12}\rangle$$



$$\frac{F^{cd\alpha}}{m_3 m_4}[\mathbf{34}]\langle\mathbf{34}\rangle$$

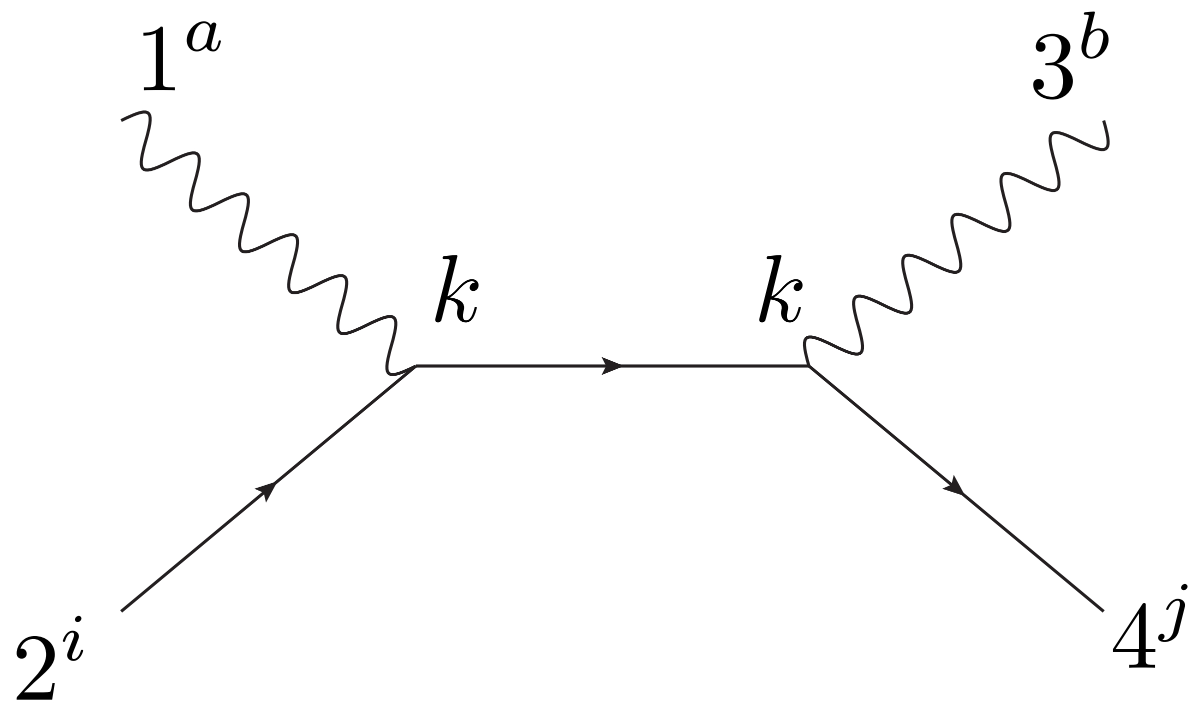
$$\frac{F^{ab\alpha}F^{cd\alpha}}{m_1 m_2 m_3 m_4}\frac{[\mathbf{12}]\langle\mathbf{12}\rangle[\mathbf{34}]\langle\mathbf{34}\rangle}{s - m_\alpha^2} \longrightarrow E^2$$

$F \propto m_V$ For $SU(2)\times U(1)$, see B. Bachu, A. Yelleshpur '19

Fermion Sector: s-channel

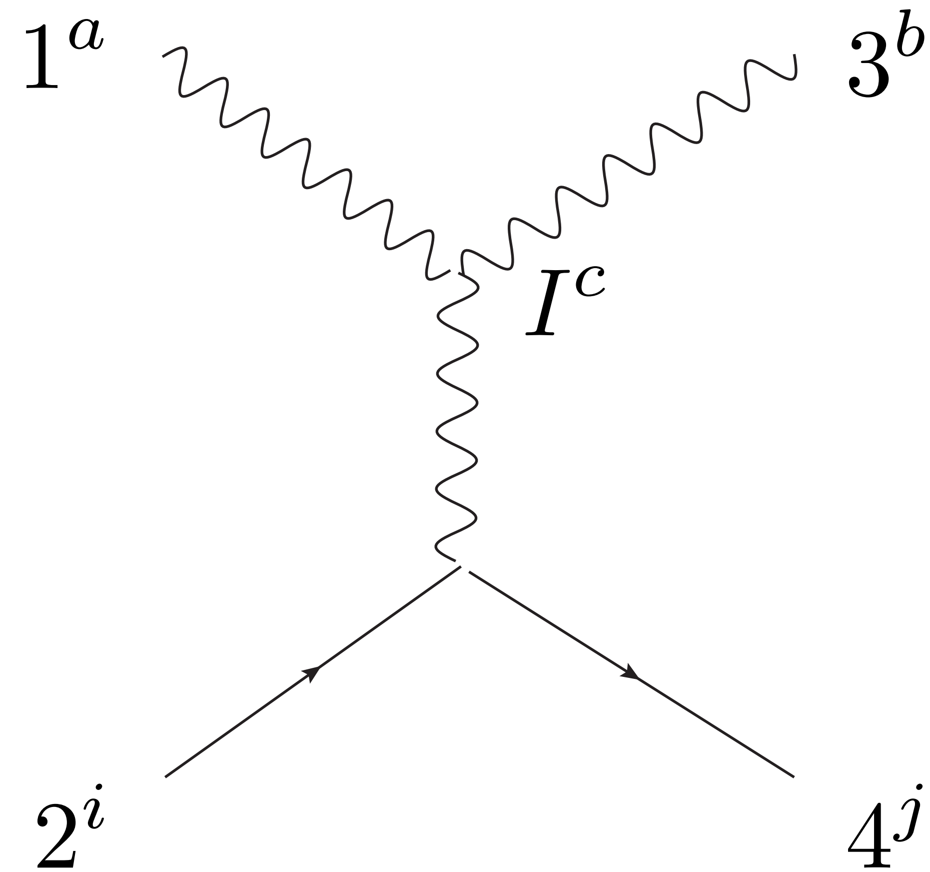
$R^{a}_{ki}\,\frac{[\mathbf{12}]\langle\mathbf{1I}\rangle}{m_1}$

$R^{b}_{jk}\,\frac{[\mathbf{3I}]\langle\mathbf{34}\rangle}{m_3}$



$$\frac{(R^b R^a)_{ji}}{m_1 m_3}\,\frac{[\mathbf{12}]\langle\mathbf{34}\rangle\langle\mathbf{1}|p_1+p_2|\mathbf{3}]}{s-m_k^2}$$

Fermion Sector: t-channel

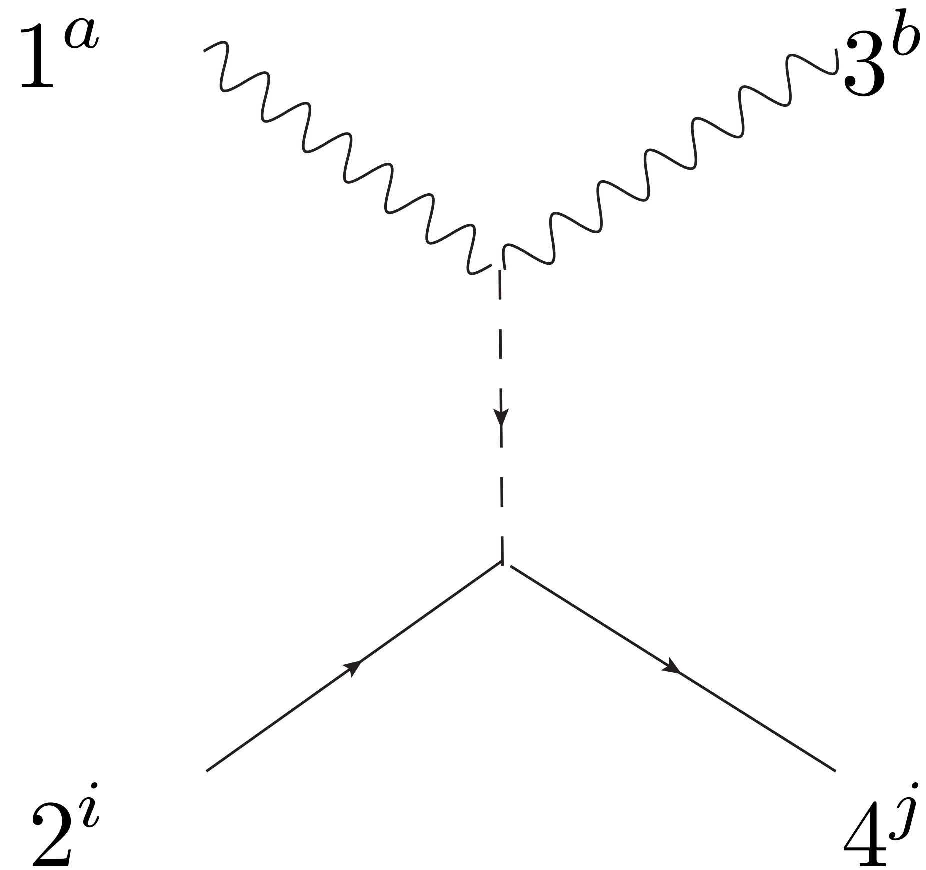


$$C_{abc}\frac{\langle\mathbf{1I}\rangle\langle\mathbf{3I}\rangle[\mathbf{13}]}{m_1 m_3}$$

$$R^c_{ji}\frac{[\mathbf{2I}]\langle\mathbf{4I}\rangle}{m_I}$$

$$\frac{C_{abc}R^c_{ji}}{m_1 m_3}\frac{\langle\mathbf{14}\rangle[\mathbf{13}]\langle\mathbf{3}|p_1+p_3|\mathbf{2}]}{t-m_I^2}$$

Fermion Sector: Higgs bosons



$$\frac{F^{ab\alpha}}{m_1 m_3}[\mathbf{13}]\langle\mathbf{13}\rangle$$

$$Y^{\alpha}_{ji}[\mathbf{24}] + Y^{\dagger\alpha}_{ji}\langle\mathbf{24}\rangle$$

$$\frac{F^{ab\alpha}Y^{\alpha}_{ji}}{m_1 m_3(t - m^2_{\alpha})}[\mathbf{13}]\langle\mathbf{13}\rangle[\mathbf{24}] \longrightarrow E$$

Fermion Sector

$$[L^a, L^b] = iC^{abc} L^c$$

$$E^2$$

$$[R^a, R^b] = iC^{abc} R^c$$

$$E$$

$$Y \propto \frac{m_\psi m_V}{g}$$

Lagrangian vs On-shell Amplitudes

✓ Principle of Stationary Action	✓ No Gauge Redundancy
✓ Wilsonian RGE	✓ Recursive Relations
✓ Locality Manifest	✓ Little Group Manifest

Conclusion

- ✓ A new understanding of Higgs Mechanism

- ✓ Open question: will new principle emerge from new understanding?