Radiative M1 Decays of Heavy Flavor Baryons in Effective Mass Scheme

Avijit Hazra* in collaboration with Dr. Rohit Dhir
*avijithr@srmist.edu.in
rohitdhv@srmist.edu.in
## Outline

1. Introduction
2. Theoretical framework
3. Numerical Results
4. Summary and Conclusion
5. Acknowledgment
6. Bibliography
Introduction

Despite the fact that the Standard Model is a well established framework to study interactions of fundamental particles, significant discrepancies can be seen between theoretical predictions and experimental results. However, within the past few decades, several theoretical approaches have been put forth to diminish the inconsistency between theory and experiments.

In the present work, we have calculated magnetic and transition moments, and M1 decay widths of ground-state singly, doubly, and triply heavy baryons. Also, we have given the estimates for the magnetic moments in an improved manner by determining hyperfine (one gluon exchange) interaction terms for $s^-$, $c^-$, and $b^-$ flavors from precise experimental values of baryon masses within the same flavor sector.

Following the EMS [1, 2, 3], we have calculated the constituent and effective masses of quarks inside a baryon for both $J^P = \frac{1}{2}^+$ and $J^P = \frac{3}{2}^+$ baryons.
In the EMS, the baryon mass can be written as the sum of the constituent quark masses and the spin-dependent hyperfine interaction among them [1, 2, 3],

\[ M_B = \sum_i m_i^\xi = \sum_i m_i + \sum_{i<j} b_{ij} s_i \cdot s_j, \]  

(1)

where, \( m_i^\xi \) represents the effective mass of the quark inside the baryon; \( s_i \) and \( s_j \) denote the spin operators of the \( i^{th} \) and \( j^{th} \) quark, respectively.

The \( b_{ij} \) for baryons \( B(qqq) \), is given by

\[ b_{ij} = \frac{16\pi\alpha_s}{9m_im_j} \langle \psi_0 | \delta^3(\vec{r}) | \psi_0 \rangle, \]  

(2)

where, \( \psi_0 \) is the baryon wave function at the origin.
For $(112)$-type $J^P = \frac{1}{2}^+$ baryons, we can write

\[ m_1^\xi = m_2^\xi = m + \alpha b_{12} + \beta b_{13}, \]

\[ m_3^\xi = m_3 + 2\beta b_{13}, \tag{3} \]

where, $m_1 = m_2 = m$ and $b_{13} = b_{23}$. The $\alpha$ and $\beta$ parameters are to be determined as follows:

\[ M_{B_1^+} = 2m + m_3 + \frac{b_{12}}{4} - b_{13}, \tag{4} \]

for

\[ s_1.s_2 = \frac{1}{4}, s_1.s_3 = s_2.s_3 = -\frac{1}{2}, \tag{5} \]

thus giving

\[ \alpha = \frac{1}{8} \text{ and } \beta = -\frac{1}{4}. \tag{6} \]
Theoretical frame work (continues)

Therefore, equation (1) may be generalized for $J^P = \frac{1}{2}^+$ baryons as

$$M_{\frac{1}{2}^+} = m_1 + m_2 + m_3 + \frac{b_{12}}{4} - \frac{b_{23}}{2} - \frac{b_{13}}{2}.$$  \hspace{1cm} (7)

By using equation (3), we will obtain more general expressions for effective masses of the quarks inside the baryon, as follows:

1. For (112)-type $J^P = \frac{1}{2}^+$ baryons with quarks 1 and 2 being identical,

$$m_{\xi}^1 = m_{\xi}^2 = m + \frac{b_{12}}{8} - \frac{b_{13}}{4},$$  \hspace{1cm} (8)

and

$$m_{\xi}^3 = m_3 - \frac{b_{13}}{2} \text{ for } 1 = 2 \neq 3.$$  \hspace{1cm} (9)
Theoretical frame work (continues)

2. The baryonic states with three different quarks flavor \((123)\) can have both anti-symmetric \(\Lambda_{123}\)-type and symmetric \(\Sigma_{123}\)-type flavor configuration under the exchange of quarks 1 and 2. (a) For \((123)\) \(\Lambda\)-type, \(J^P = \frac{1}{2}^+\) baryons,

\[
m_1^\xi = m_1 - \frac{3b_{12}}{8},
\]

\[
m_2^\xi = m_2 - \frac{3b_{21}}{8},
\]

and

\[
m_3^\xi = m_3 \text{ for } 1 \neq 2 \neq 3.
\]

(b) For \((123)\) \(\Sigma\)-type, \(J^P = \frac{1}{2}^+\) baryons,

\[
m_1^\xi = m_1 + \frac{b_{12}}{8} - \frac{b_{13}}{4},
\]

\[
m_2^\xi = m_2 + \frac{b_{12}}{8} - \frac{b_{23}}{4},
\]

and

\[
m_3^\xi = m_3 - \frac{b_{23}}{4} - \frac{b_{13}}{4} \text{ for } 1 \neq 2 \neq 3.
\]
Following the similar procedure described for $J^P = \frac{1}{2}^+$ baryons, the generalized mass formula for different flavor configuration of $J^P = \frac{3}{2}^+$ baryons is given by

$$M_{\mathcal{B}_{\frac{3}{2}^+}} = m_1 + m_2 + m_3 + \frac{b_{12}}{4} + \frac{b_{23}}{4} + \frac{b_{13}}{4},$$

for $\alpha = \beta = \frac{1}{8}$.

(14)

Throughout the above discussions 1, 2, 3 represents $u, d, s, c,$ and $b$ quarks.

1. For (112)-type $J^P = \frac{3}{2}^+$ baryons,

$$m_1^\xi = m_2^\xi = m + \frac{b_{12}}{8} + \frac{b_{13}}{8},$$

(15)

and

$$m_3^\xi = m_3 + \frac{b_{13}}{4} \text{ for } 1 = 2 \neq 3.$$
Theoretical frame work (continues)

2. For \( J^P = \frac{3}{2}^+ \) baryons,

\[
m_1^\xi = m_1 + \frac{b_{12}}{8} + \frac{b_{13}}{8},
\]
\[
m_2^\xi = m_2 + \frac{b_{23}}{8} + \frac{b_{12}}{8},
\]

and
\[
m_3^\xi = m_3 + \frac{b_{13}}{8} + \frac{b_{23}}{8}.
\]

(17)

3. For \( J^P = \frac{3}{2}^+ \) baryons,

\[
m_1^\xi = m_2^\xi = m_3^\xi = m + \frac{b_{12}}{4},
\]

and
\[
b_{12} = b_{23} = b_{13}.
\]

(20)
The values of constituent quark masses and hyperfine interaction terms $b_{ij}$ are obtained from the experimentally observed baryon masses [4]. In order to obtain the effective quark masses, especially in the charm and bottom sector, we have calculated the interaction contribution of single gluon exchange term from the corresponding flavor sector. We proceed to obtain,

$$m_u = m_d = 362 \text{ MeV}, \quad m_s = 538 \text{ MeV},$$

$$b_{uu} = b_{ud} = b_{dd} = 195 \text{ MeV}. \quad (21)$$

from $N, \Delta$, and $\Lambda$ up to strange sector.

In the charm sector, from $\Omega_c$ and $\Xi_c^{(\ast)}$, we obtain:

$$m_c = 1646 \text{ MeV}, \quad b_{us} = b_{ds} = 153 \text{ MeV},$$

$$b_{ss} = 81 \text{ MeV}, \quad (22) \Sigma_c^{(\ast)} \text{ will gives}$$

$$b_{uc} = b_{dc} = 43 \text{ MeV}, \quad (23)$$

and $\Omega_c^{(\ast)}$, and $\Xi_{cc}^{++}$ yields

$$b_{sc} = 47 \text{ MeV}, \quad (24)$$
and

\[ b_{cc} = 40 \text{ MeV}. \tag{25} \]

In the bottom sector, \( \Lambda_b, \Sigma_b^{(*)}, \) and \( \Xi_b^{(*)} \) leads to

\[ m_b = 5043 \text{ MeV}, \quad b_{ub} = b_{db} = 13 \text{ MeV}, \quad b_{sb} = 17 \text{ MeV}. \tag{26} \]

For the first time we have estimated \( b_{cb} \) and \( b_{bb} \) from the hyperfine interaction terms \( b_{sc} \) and \( b_{sb} \) that are obtained from the experimentally known masses. We use the symmetry relations [1, 2, 3] to get,

\[ b_{cb} = \left( \frac{m_s}{m_b} \right) b_{sc} = 5.0 \text{ MeV}, \quad \text{and} \quad b_{cb} = \left( \frac{m_s}{m_c} \right) b_{sb} = 5.4 \text{ MeV}, \tag{27} \]

since both the values are roughly same, we use

\[ b_{cb} \approx 5 \text{ MeV}. \tag{28} \]

Furthermore, we get

\[ b_{bb} = \left( \frac{m_s m_c}{m_b m_b} \right) b_{sc} = 1.6 \text{ MeV}, \quad \text{and} \quad b_{bb} = \left( \frac{m_s m_b}{m_b m_b} \right) b_{sb} = 1.8 \text{ MeV}. \tag{29} \]

which is approximated to

\[ b_{bb} \approx 2 \text{ MeV}. \tag{30} \]
Theoretical framework (continues)

- Effective quark masses for $J^P = \frac{1}{2}^+$ baryons
  - For singly heavy baryons,

\[
\begin{align*}
  m_{\Lambda}^{u} &= m_{\Lambda}^{d} = 289 \text{ MeV}, m_{c}^{u} = 1646 \text{ MeV}; \\
  m_{\Lambda}^{c} &= m_{\Sigma}^{u} = m_{\Sigma}^{d} = 376 \text{ MeV}, m_{c}^{c} = 1625 \text{ MeV}; \\
  m_{\Xi}^{u} &= m_{\Xi}^{d} = 370 \text{ MeV}, m_{s}^{c} = 545 \text{ MeV}, m_{c}^{c} = 1624 \text{ MeV}; \\
  m_{\Xi}^{c} &= m_{\Xi}^{d} = 305 \text{ MeV}, m_{s}^{c} = 481 \text{ MeV}, m_{c}^{c} = 1646 \text{ MeV}; \\
  m_{\Omega}^{u} &= m_{\Omega}^{d} = 289 \text{ MeV}, m_{c}^{u} = m_{\Xi}^{d} = 5043 \text{ MeV}; \\
  m_{\Omega}^{c} &= 536 \text{ MeV}, m_{c}^{c} = 1622 \text{ MeV}; \\
  m_{\Xi}^{b} &= m_{\Xi}^{d} = 305 \text{ MeV}, m_{s}^{b} = 481 \text{ MeV}, m_{b}^{b} = 5043 \text{ MeV}; \\
  m_{\Xi}^{b} &= m_{\Xi}^{d} = 544 \text{ MeV}, m_{c}^{u} = m_{\Xi}^{d} = 5035 \text{ MeV}; \\
  m_{\Xi}^{b} &= m_{\Xi}^{d} = 383 \text{ MeV}, m_{s}^{b} = 5036 \text{ MeV}; \\
  m_{u}^{u} &= m_{u}^{u} = 378 \text{ MeV}, m_{s}^{b} = 553 \text{ MeV}, m_{b}^{b} = 5036 \text{ MeV}.
\end{align*}
\]
Effective quark masses for $J^P = \frac{3}{2}^+$ baryons

For singly heavy baryons,

$m_{u^c} = m_{d^c} = 392 \text{ MeV}, m_{c^c} = 1657 \text{ MeV};$

$m_{u^b} = m_{d^b} = 386 \text{ MeV}, m_{s^c} = 563 \text{ MeV}, m_{c^c} = 1657 \text{ MeV};$

$m_{s^c} = 554 \text{ MeV}, m_{c^c} = 1658 \text{ MeV};$

$m_{u^b} = m_{b^b} = 388 \text{ MeV}, m_{c^b} = 5046 \text{ MeV};$

$m_{u^b} = m_{d^b} = 383 \text{ MeV}, m_{s^b} = 559 \text{ MeV}, m_{c^b} = 5047 \text{ MeV};$

$m_{s^b} = 550 \text{ MeV}, m_{b^b} = 5047 \text{ MeV}.$
Magnetic Moments of Heavy Flavor Baryons

- Magnetic Moments of \((J^P = \frac{1}{2}^+)\) and \((J^P = \frac{3}{2}^+)\) Baryons

\[
\mu(B) = \frac{1}{3}(4\mu_1 - \mu_2),
\]
\[
\mu(B) = \mu_3,
\]
\[
\mu(B') = \frac{1}{3}(2\mu_1 + 2\mu_2 - \mu_3),
\]
\[
\mu(B^*) = \mu_1 + \mu_2 + \mu_3.
\]

(31)

where, \(\mu_i = \frac{e_i}{2m_i}\) denote the effective magnetic moments of first, second and third quarks, respectively. We adopt the convention that \([q_1 q_2]\) denotes anti-symmetric (\(S = 0\)) and \(\{q_1 q_2\}\) denote symmetric (\(S = 1\)) combinations of quark flavor indices (with respect to the interchange of \(q_1\) and \(q_2\)):

\[
|B\rangle = \left| [q_1 q_2]^{S=0} q_3, J = \frac{1}{2} \right>
\]
\[
|B'\rangle = \left| \{q_1 q_2\}^{S=1} q_3, J = \frac{1}{2} \right>
\]
\[
|B^*\rangle = \left| \{q_1 q_2\}^{S=1} q_3, J = \frac{3}{2} \right>
\]

(32)
Transition Moments Relations

\[ \mu_{1/2} \rightarrow 1/2^+ = \sqrt{\frac{1}{3}} \left[ \mu^\xi(2) - \mu^\xi(1) \right], \]

\[ \mu_{3/2} \rightarrow 1/2^+ = \sqrt{\frac{2}{3}} \left[ \mu^\xi(1) - \mu^\xi(2) \right], \]

\[ \mu_{3/2} \rightarrow 1/2' = \sqrt{\frac{2}{3}} \left[ \mu^\xi(1) + \mu^\xi(2) - 2\mu^\xi(3) \right]. \]

To evaluate \( \mu_{1/2} \rightarrow 1/2^+ \) and \( \mu_{3/2} \rightarrow 1/2^+ \) transition moments, we take the geometric mean of effective quark masses of the constituent quarks of initial and final state baryons.

\[ m_i^\xi (B_J^* \rightarrow B_J) = \sqrt{m_i^\xi (B_J^* ) m_i^\xi (B_J) } = \sqrt{m_i^\xi (B_J^* ) m_i^\xi (B_J) }, \]

where, symbols have their usual meaning.
Using the Eqs. (8) - (13) and (15) - (20), we calculate the transition masses of baryons as follows:

\[ m_{\Lambda}^{c} = m_{c}^{d} = 336 \text{ MeV}, \]
\[ m_{\Xi}^{c} = m_{d}^{c} = 343 \text{ MeV}, \]
\[ m_{\Sigma}^{c} = m_{d}^{c} = 384 \text{ MeV}, \]
\[ m_{\Xi'}^{c} = m_{d}^{c} = 378 \text{ MeV}, \]
\[ m_{s}^{\Omega} = 545 \text{ MeV}, \]
\[ m_{c}^{\Omega} = 1640 \text{ MeV}; \]
We will continue our presentation with the analysis for M1 partial widths of the ground state heavy baryons. We ignore the transition of type E2 which is expected to be much smaller in magnitude [5, 6] when compared to M1. The radiative decay widths of the decay type \( \mathcal{B}_J^\prime(\ast) \rightarrow \mathcal{B}_J\gamma \) (Ref. [7, 8]) is given by

\[
\Gamma(\mathcal{B}_J^\prime(\ast) \rightarrow \mathcal{B}_J\gamma) = \frac{\alpha \omega^3}{m_p^2} \frac{2}{(2J + 1)} |\mu(\mathcal{B}_J^\prime(\ast) \rightarrow \mathcal{B}_J)|^2,
\]

(35)

where,

\[
\omega = \frac{M^2_{\mathcal{B}_J^\prime(\ast)} - M^2_{\mathcal{B}}}{2M_{\mathcal{B}_J^\prime(\ast)}},
\]

(36)

is the photon momentum in the center-of-mass system of the initial baryon states. Here, \( \mu(\mathcal{B}_J^\prime(\ast) \rightarrow \mathcal{B}_J) \) is the transition magnetic moment (in \( \mu_N \)), \( J \) is the spin quantum number for parent state, and \( M_{\mathcal{B}_J^\prime(\ast)} \) and \( M_{\mathcal{B}} \) are the masses of initial and final baryon state, respectively.
Theoretical frame work

Numerical Results

Table: I. Magnetic moments (in nuclear magneton, $\mu_N$) of $J^P = 1/2^+$ charm baryons.

<table>
<thead>
<tr>
<th>Baryons</th>
<th>EMS</th>
<th>[8]</th>
<th>[10]</th>
<th>[14]</th>
<th>[12]</th>
<th>[15]</th>
<th>[16]</th>
<th>[17]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Lambda_c^+$</td>
<td>0.380</td>
<td>0.335</td>
<td>-</td>
<td>-</td>
<td>0.421</td>
<td>-0.382</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$\Sigma_c^{++}$</td>
<td>2.091</td>
<td>2.280</td>
<td>2.15 ± 0.1</td>
<td>2.027</td>
<td>1.831</td>
<td>1.604</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$\Sigma_c^+$</td>
<td>0.427</td>
<td>0.487</td>
<td>0.46 ± 0.03</td>
<td>-</td>
<td>0.380</td>
<td>0.100</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$\Sigma_c^0$</td>
<td>-1.238</td>
<td>-1.310</td>
<td>-1.24 ± 0.05</td>
<td>-1.117</td>
<td>-1.091</td>
<td>-1.403</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$\Xi_c^+$</td>
<td>0.380</td>
<td>0.142</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.233</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$\Xi_c^0$</td>
<td>0.380</td>
<td>0.346</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.193</td>
<td>-</td>
</tr>
<tr>
<td>$\Xi_c'{}^+$</td>
<td>0.615</td>
<td>0.825</td>
<td>0.60 ± 0.02</td>
<td>-</td>
<td>0.523</td>
<td>0.559</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$\Xi_c'{}^0$</td>
<td>-1.074</td>
<td>-1.130</td>
<td>-1.05 ± 0.04</td>
<td>-1.012</td>
<td>-1.077</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$\Omega_c^0$</td>
<td>-0.906</td>
<td>-0.950</td>
<td>-0.85 ± 0.05</td>
<td>-0.639</td>
<td>-1.179</td>
<td>-0.748</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$\Xi_{cc}^{++}$</td>
<td>-0.104</td>
<td>-0.110</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-0.23 ± 0.05</td>
</tr>
<tr>
<td>$\Xi_{cc}^+$</td>
<td>0.815</td>
<td>0.719</td>
<td>0.425</td>
<td>0.392</td>
<td>0.43 ± 0.09</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Omega_{cc}^+$</td>
<td>0.711</td>
<td>0.645</td>
<td>0.413</td>
<td>0.397</td>
<td>0.39 ± 0.09</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Avijit Hazra
SRM Institute of Science and Technology, Department of Physics and Nanotechnology, Kattankulathur, Tamil Nadu - 603203, India
Radiative M1 Decays of Heavy Flavor Baryons in Effective Mass Scheme
### Numerical Results

Table: II. Magnetic moments (in $\mu_N$) of $J^P = 3/2^+$ charm baryons.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Sigma^{*++}_c$</td>
<td>3.570</td>
<td>3.980</td>
<td>2.410</td>
<td>3.22 ± 0.15</td>
<td>4.81 ± 1.22</td>
<td>3.232</td>
<td>-</td>
</tr>
<tr>
<td>$\Sigma^{*+}_c$</td>
<td>1.176</td>
<td>1.250</td>
<td>0.670</td>
<td>0.68 ± 0.04</td>
<td>2.00 ± 0.46</td>
<td>1.136</td>
<td>-</td>
</tr>
<tr>
<td>$\Sigma^{*0}_c$</td>
<td>-1.219</td>
<td>-1.490</td>
<td>-1.070</td>
<td>-1.86 ± 0.07</td>
<td>-0.81 ± 0.20</td>
<td>-1.044</td>
<td>-</td>
</tr>
<tr>
<td>$\Xi^{*+}_c$</td>
<td>1.440</td>
<td>1.470</td>
<td>0.810</td>
<td>0.90 ± 0.04</td>
<td>1.68 ± 0.42</td>
<td>1.333</td>
<td>-</td>
</tr>
<tr>
<td>$\Xi^{*0}_c$</td>
<td>-0.987</td>
<td>-1.200</td>
<td>-0.900</td>
<td>-1.57 ± 0.06</td>
<td>-0.68 ± 0.18</td>
<td>-0.837</td>
<td>-</td>
</tr>
<tr>
<td>$\Omega^{*0}_c$</td>
<td>-0.752</td>
<td>-0.936</td>
<td>-0.700</td>
<td>-1.28 ± 0.08</td>
<td>-0.62 ± 0.18</td>
<td>-1.129</td>
<td>-</td>
</tr>
<tr>
<td>$\Xi^{*++}_{cc}$</td>
<td>2.433</td>
<td>2.350</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>2.940</td>
</tr>
<tr>
<td>$\Xi^{*+}_{cc}$</td>
<td>-0.084</td>
<td>-0.178</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-0.670</td>
</tr>
<tr>
<td>$\Omega^{*+}_{cc}$</td>
<td>0.186</td>
<td>0.048</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-0.520</td>
</tr>
<tr>
<td>$\Omega^{*++}_{ccc}$</td>
<td>1.133</td>
<td>0.989</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>
**Numerical Results**

**Table:** III. Magnetic $\mu_{3/2}^{+} \rightarrow 1/2^{+}$ transition moments (in $\mu_N$) of charm baryons.

<table>
<thead>
<tr>
<th>Transition</th>
<th>EMS</th>
<th>[18]</th>
<th>[8]</th>
<th>[19]</th>
<th>[6]</th>
<th>[20]*</th>
<th>[12]</th>
<th>[23]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Sigma^*_c \rightarrow \Lambda^+_c$</td>
<td>2.277</td>
<td>2.400</td>
<td>2.070</td>
<td>2.000</td>
<td>$-2.18 \pm 0.08$</td>
<td>1.48 $\pm 0.55$</td>
<td>1.758</td>
<td>-</td>
</tr>
<tr>
<td>$\Sigma^*<em>{c} \rightarrow \Sigma^{++}</em>{c}$</td>
<td>1.177</td>
<td>$-1.370$</td>
<td>1.340</td>
<td>1.070</td>
<td>1.52 $\pm 0.07$</td>
<td>1.06 $\pm 0.38$</td>
<td>0.988</td>
<td>-</td>
</tr>
<tr>
<td>$\Sigma^*<em>{c} \rightarrow \Sigma^{+}</em>{c}$</td>
<td>0.025</td>
<td>$-0.003$</td>
<td>0.102</td>
<td>0.190</td>
<td>0.33 $\pm 0.02$</td>
<td>0.45 $\pm 0.11$</td>
<td>0.009</td>
<td>-</td>
</tr>
<tr>
<td>$\Sigma^0_{c} \rightarrow \Sigma^{0}_{c}$</td>
<td>$-1.128$</td>
<td>1.480</td>
<td>$-1.140$</td>
<td>$-0.690$</td>
<td>$-0.87 \pm 0.03$</td>
<td>0.19 $\pm 0.08$</td>
<td>1.013</td>
<td>-</td>
</tr>
<tr>
<td>$\Xi^*<em>c \rightarrow \Xi^{+}</em>{c}$</td>
<td>1.979</td>
<td>2.080</td>
<td>1.860</td>
<td>1.050</td>
<td>1.69 $\pm 0.08$</td>
<td>1.47 $\pm 0.66$</td>
<td>0.985</td>
<td>-</td>
</tr>
<tr>
<td>$\Xi^0_{c} \rightarrow \Xi^{0}_{c}$</td>
<td>$-0.253$</td>
<td>$-0.500$</td>
<td>$-0.249$</td>
<td>$-0.310$</td>
<td>$-0.29 \pm 0.04$</td>
<td>0.16 $\pm 0.075$</td>
<td>0.253</td>
<td>-</td>
</tr>
<tr>
<td>$\Omega^0_{c} \rightarrow \Omega^{0}_{c}$</td>
<td>0.154</td>
<td>$-0.230$</td>
<td>0.066</td>
<td>0.230</td>
<td>0.43 $\pm 0.02$</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$\Omega^0_{c} \rightarrow \Omega^{0}_{c}$</td>
<td>$-1.015$</td>
<td>1.240</td>
<td>$-0.994$</td>
<td>$-0.590$</td>
<td>$-0.74 \pm 0.03$</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$\Xi^{<em>++}_{cc} \rightarrow \Xi^{</em>++}_{cc}$</td>
<td>$-0.900$</td>
<td>0.960</td>
<td>$-0.892$</td>
<td>$-0.490$</td>
<td>$-0.60 \pm 0.04$</td>
<td>-</td>
<td>0.872</td>
<td>-</td>
</tr>
<tr>
<td>$\Xi^{<em>++}_{cc} \rightarrow \Xi^{</em>++}_{cc}$</td>
<td>$-1.298$</td>
<td>1.330</td>
<td>$-1.210$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>$-2.350$</td>
</tr>
<tr>
<td>$\Xi^{<em>++}_{cc} \rightarrow \Xi^{</em>++}_{cc}$</td>
<td>$1.185$</td>
<td>$-1.410$</td>
<td>1.070</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>1.550</td>
</tr>
<tr>
<td>$\Omega^{<em>++}_{cc} \rightarrow \Omega^{</em>++}_{cc}$</td>
<td>0.912</td>
<td>$-0.890$</td>
<td>0.869</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>1.540</td>
</tr>
</tbody>
</table>

*T. M. Aliev, K. Azizi and A. Ozpineci have given their results in natural magneton ($e\hbar/2cM_B$), however, to convert to nuclear magneton we multiply the entire magnetic moments with $2m_N/(M_{B_{3/2}}^2 + M_{B_{1/2}}^2)$. 

---

Avijit Hazra  
SRM Institute of Science and Technology, Department of Physics and Nanotechnology, Kattankulathur, Tamil Nadu - 603203, India  
Radiative M1 Decays of Heavy Flavor Baryons in Effective Mass Scheme
### Numerical Results

**Table:** IV. Photon momenta, $\omega$ of charm and bottom baryons.

<table>
<thead>
<tr>
<th>Transition</th>
<th>$\omega$ (in MeV)</th>
<th>[8]</th>
<th>Transition</th>
<th>$\omega$ (in MeV)</th>
<th>[8]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Sigma^+_c \rightarrow \Lambda^+_c$</td>
<td>161</td>
<td></td>
<td>$\Sigma^0_b \rightarrow \Lambda^0_b$</td>
<td>193</td>
<td>190</td>
</tr>
<tr>
<td>$\Xi^0_c \rightarrow \Xi^0_c$</td>
<td>106</td>
<td>105</td>
<td>$\Xi^0'_b \rightarrow \Xi^0_b$</td>
<td>141</td>
<td>135</td>
</tr>
<tr>
<td>$\Xi^+_c \rightarrow \Xi^+_c$</td>
<td>108</td>
<td>106</td>
<td>$\Xi^-_b \rightarrow \Xi^-_b$</td>
<td>136</td>
<td>138</td>
</tr>
<tr>
<td>$\Sigma^{*+}_c \rightarrow \Lambda^{+}_c$</td>
<td>220</td>
<td>220</td>
<td>$\Sigma^{*0}_b \rightarrow \Lambda^{0}_b$</td>
<td>209</td>
<td>210</td>
</tr>
<tr>
<td>$\Sigma^{*0}_c \rightarrow \Sigma^{0}_c$</td>
<td>64</td>
<td>64</td>
<td>$\Sigma^{*0}_b \rightarrow \Sigma^{0}_b$</td>
<td>17</td>
<td>20</td>
</tr>
<tr>
<td>$\Sigma^{*+}_c \rightarrow \Sigma^{+}_c$</td>
<td>64</td>
<td>64</td>
<td>$\Sigma^{<em>-}_b \rightarrow \Sigma^{</em>-}_b$</td>
<td>19</td>
<td>20</td>
</tr>
<tr>
<td>$\Sigma^{*++}_c \rightarrow \Sigma^{++}_c$</td>
<td>64</td>
<td>64</td>
<td>$\Sigma^{*+}_b \rightarrow \Sigma^{+}_b$</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>$\Xi^{*0}_c \rightarrow \Xi^{0}_c$</td>
<td>170</td>
<td>169</td>
<td>$\Xi^{*0}_b \rightarrow \Xi^{0}_b$</td>
<td>158</td>
<td>155</td>
</tr>
<tr>
<td>$\Xi^{*+}_c \rightarrow \Xi^{+}_c$</td>
<td>172</td>
<td>172</td>
<td>$\Xi^{<em>-}_b \rightarrow \Xi^{</em>-}_b$</td>
<td>156</td>
<td>158</td>
</tr>
<tr>
<td>$\Xi^{*0}_c \rightarrow \Xi^{*0}_c$</td>
<td>66</td>
<td>67</td>
<td>$\Xi^{*0}_b \rightarrow \Xi^{*0}_b$</td>
<td>17</td>
<td>20</td>
</tr>
<tr>
<td>$\Xi^{<em>+}_c \rightarrow \Xi^{</em>+}_c$</td>
<td>66</td>
<td>69</td>
<td>$\Xi^{<em>-}_b \rightarrow \Xi^{</em>-}_b$</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>$\Omega^{*0}_c \rightarrow \Omega^{0}_c$</td>
<td>70</td>
<td>70</td>
<td>$\Omega^{<em>-}_b \rightarrow \Omega^{</em>-}_b$</td>
<td>39</td>
<td>20</td>
</tr>
</tbody>
</table>
### Numerical Results

**Table**: V. The radiative decay widths (in KeV) of charm baryons.

<table>
<thead>
<tr>
<th>Transition</th>
<th>EMS</th>
<th>[24]</th>
<th>[8]</th>
<th>[19]</th>
<th>[6]†</th>
<th>[20, 21, 22]</th>
<th>[25]</th>
<th>[26]</th>
<th>[12]</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Sigma^+_c \rightarrow \Lambda^+_c \gamma )</td>
<td>93.22</td>
<td>46.10</td>
<td>74.10</td>
<td>65.60</td>
<td>81.05</td>
<td>50.00</td>
<td>80.60</td>
<td>97.98</td>
<td>66.66</td>
</tr>
<tr>
<td>( \Xi^0_c \rightarrow \Xi^0_c \gamma )</td>
<td>0.338</td>
<td>0.002</td>
<td>0.185</td>
<td>0.460</td>
<td>0.432</td>
<td>0.270</td>
<td>0000</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>( \Xi^+_c \rightarrow \Xi^+_c \gamma )</td>
<td>21.36</td>
<td>10.20</td>
<td>18.60</td>
<td>5.430</td>
<td>14.78</td>
<td>8.500</td>
<td>42.30</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>( \Sigma^{*+}_c \rightarrow \Lambda^+_c \gamma )</td>
<td>230.3</td>
<td>126.0</td>
<td>190.0</td>
<td>161.6</td>
<td>211.1</td>
<td>130.0</td>
<td>373.0</td>
<td>244.4</td>
<td>135.3</td>
</tr>
<tr>
<td>( \Sigma^{<em>++}_c \rightarrow \Sigma^{</em>+}_c \gamma )</td>
<td>1.480</td>
<td>0.826</td>
<td>1.960</td>
<td>1.200</td>
<td>2.468</td>
<td>2.650</td>
<td>3.940</td>
<td>1.980</td>
<td>2.060</td>
</tr>
<tr>
<td>( \Sigma^{*0}_c \rightarrow \Sigma^0_c \gamma )</td>
<td>1.376</td>
<td>1.080</td>
<td>1.410</td>
<td>0.490</td>
<td>0.818</td>
<td>0.080</td>
<td>3.430</td>
<td>1.440</td>
<td>2.162</td>
</tr>
<tr>
<td>( \Sigma^{*+}_c \rightarrow \Sigma^+_c \gamma )</td>
<td>0.001</td>
<td>0.004</td>
<td>0.011</td>
<td>0.040</td>
<td>0.174</td>
<td>0.400</td>
<td>0.040</td>
<td>0.011</td>
<td>4 × 10^{-5}</td>
</tr>
<tr>
<td>( \Xi^{*0}_c \rightarrow \Xi^0_c \gamma )</td>
<td>1.299</td>
<td>0.908</td>
<td>0.745</td>
<td>1.840</td>
<td>1.707</td>
<td>0.660</td>
<td>0000</td>
<td>1.150</td>
<td>0.811</td>
</tr>
<tr>
<td>( \Xi^{*+}_c \rightarrow \Xi^+_c \gamma )</td>
<td>82.18</td>
<td>44.30</td>
<td>81.60</td>
<td>21.60</td>
<td>59.93</td>
<td>52.00</td>
<td>139.0</td>
<td>99.94</td>
<td>15.69</td>
</tr>
<tr>
<td>( \Xi^{*0}_c \rightarrow \Xi^0_c \gamma )</td>
<td>1.247</td>
<td>1.030</td>
<td>1.330</td>
<td>0.420</td>
<td>0.663</td>
<td>2.142</td>
<td>3.030</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>( \Xi^{*+}_c \rightarrow \Xi^+_c \gamma )</td>
<td>0.028</td>
<td>0.011</td>
<td>0.063</td>
<td>0.070</td>
<td>0.218</td>
<td>0.274</td>
<td>0.004</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>( \Omega^{*0}_c \rightarrow \Omega^0_c \gamma )</td>
<td>1.143</td>
<td>1.070</td>
<td>1.130</td>
<td>0.320</td>
<td>0.508</td>
<td>0.932</td>
<td>0.890</td>
<td>0.820</td>
<td>0.464</td>
</tr>
<tr>
<td>( \Xi^{*++}_c \rightarrow \Xi^{++}_c \gamma )</td>
<td>2.427</td>
<td>1.430</td>
<td>2.790</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>( \Xi^{*+}_c \rightarrow \Xi^+_c \gamma )</td>
<td>2.026</td>
<td>2.080</td>
<td>2.170</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>( \Omega^{*+}_c \rightarrow \Omega^+ \gamma )</td>
<td>1.978</td>
<td>0.949</td>
<td>1.600</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

†The values given in the column are calculated from the transition magnetic moments given by G. S. Yang and H. C. Kim in their results.

Avijit Hazra  
SRM Institute of Science and Technology, Department of Physics and Nanotechnology, Kattankulathur, Tamil Nadu - 603203, India

Radiative M1 Decays of Heavy Flavor Baryons in Effective Mass Scheme
Summary and Conclusion

In the present work, we have primarily focused on the prediction of magnetic properties of heavy flavor baryons in the framework of EMS. The EMS takes into account the modification based on hyperfine interaction between constituent quarks via one gluon exchange inside the baryon. Another unique feature of EMS is that it is parameter independent and, in addition, we have also incorporated symmetry breaking (through masses), which is a desirable feature for consistent predictions of baryon properties. We have calculated the magnetic and transition moments involving low-lying heavy baryons containing up to three heavy quarks, and consequently, have predicted M1 radiative decay widths for $\frac{1}{2}' \rightarrow \frac{1}{2}$ and $\frac{3}{2} \rightarrow \frac{1}{2}$ baryon states. Also, we have compared our results with existing predictions from other theoretical models. In order to make robust predictions, we have utilized precisely measured experimental values of baryon masses, and have used LQCD estimates in the case of unobserved baryons.

Following the current approach, we have accomplished two improvements. Firstly, symmetry breaking (through masses and interaction terms) is partially incorporated, and secondly, a more reliable calculation of effective masses has been achieved. In addition, we have tried to limit the uncertainties in photon momenta by mostly relying on experimental information. In the light of preceding arguments, it is therefore expected that our results would provide reasonably accurate predictions of magnetic (transition) moments and M1 radiative decay widths.
Bibliography


P. A. Zyla et al. [Particle Data Group], PTEP 2020, no.8, 083C01 (2020) doi:10.1093/ptep/ptaa104


