Scattering amplitudes of massive spin-2 Kaluza-Klein particles in Stabilized Extra Dimensions

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Summary

How does bad high energy growth get tamed in Extra Dimensional Models ?

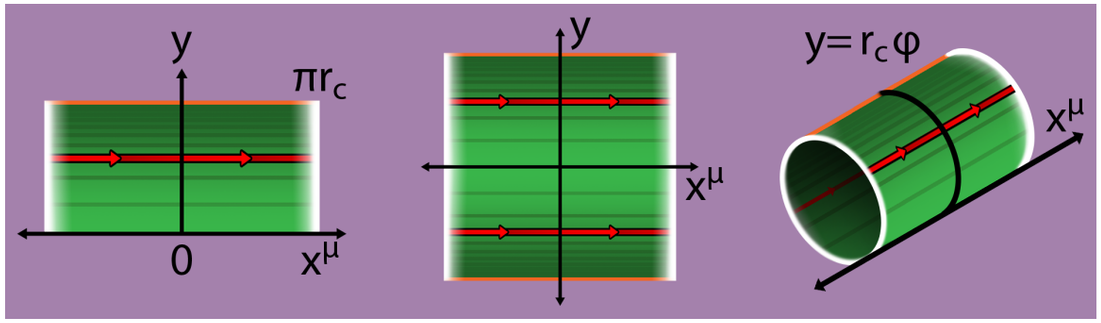
MASSIVE SPIN-2 PARTICLES AND ENERGY GROWTHS $\rightarrow \frac{s^5}{m^8 M_{pl}^2}$

MATRIX ELEMENTS AND CANCELLATIONS IN TOROIDAL AND AdS COMPACTIFICATIONS $\rightarrow \boxed{s/M_{Pl}^2} \rightarrow$ Boundary value problem, Sum rules

STABILIZING THE EXTRA DIMENSION VIA GOLDBERGER-WISE MECHANISM $\rightarrow \boxed{\frac{s^2 m_r^2}{\Lambda_\pi^2 m_n^4}} \rightarrow$ Naive re-introduction of bad high energy behavior

MATRIX ELEMENTS AND CANCELLATIONS IN STABILIZED MODELS $\rightarrow$ Sum rules for the Stabilized model

Stabilization of the extra dimension



Choose even solutions on $[-\pi r_c, \pi r_c]$ $\Longrightarrow$ massless particles

- In the original RS formulation, the vev of the radion field is associated with the length r_c of the extra-dimension - However, there is no dynamics (no potential) to generate this.

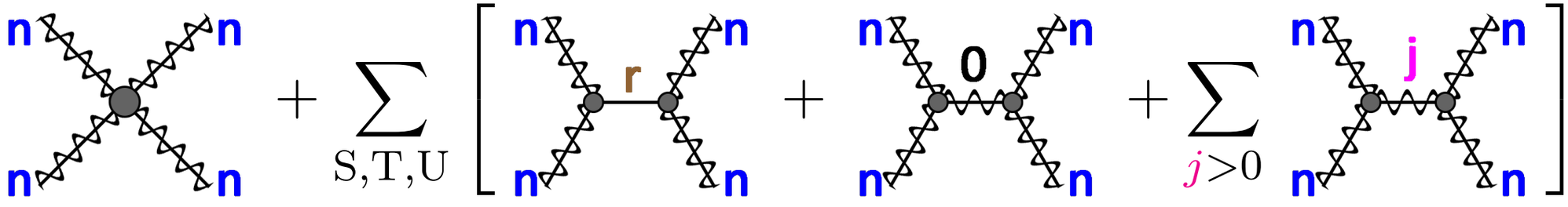
- The radion is massless and leads to a Brans-Dicke (Scalar-Tensor) theory of gravity and contradicts experiments when there is matter on the TeV Brane.

- The Goldberger -Wise mechanism stabilizes the extra-dimension and makes the radion massive.

Goldberger, Wise 2000
De-Wolfe, Freedman, Karch, Gubser 2001
Peloso, Koffman 2001
Tanaka, Garriga 2001
Smolyakov, Volobuev 2002

How scattering amplitudes behave for a massive radion

Contributions to the matrix Element

$\mathcal{M} =$ 

Add a naive mass term for the radion $\longrightarrow$ **massive radion propagator**

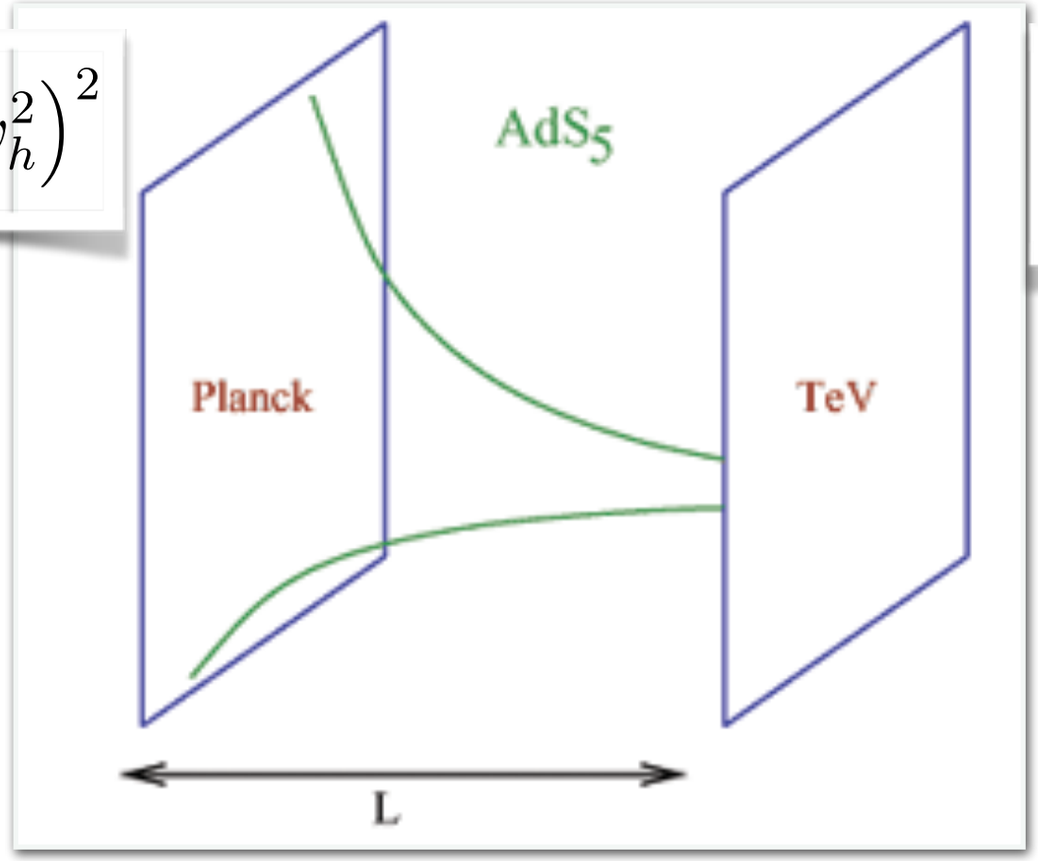
$$\boxed{\mathcal{M} \propto \frac{s^2 m_r^2}{\Lambda_\pi^2 m_n^4}} \longrightarrow$$ **Re-introduction of a low energy cut-off**

Conjecture : Contribution must cancel from from the dynamics of the larger problem so that the validity of the EFT must be $\sim \Lambda_\pi$

Any mechanism of mass generation cannot simply provide a radion mass without adding additional dynamics. What can this additional dynamics be? - Goldberger-Wise mechanism.

Goldberger Wise Mechanism in a nutshell

$$\sqrt{-g_h}\lambda_h\left(\Phi^2 - v_h^2\right)^2$$



$$\sqrt{-g_v}\lambda_v\left(\Phi^2 - v_v^2\right)^2$$

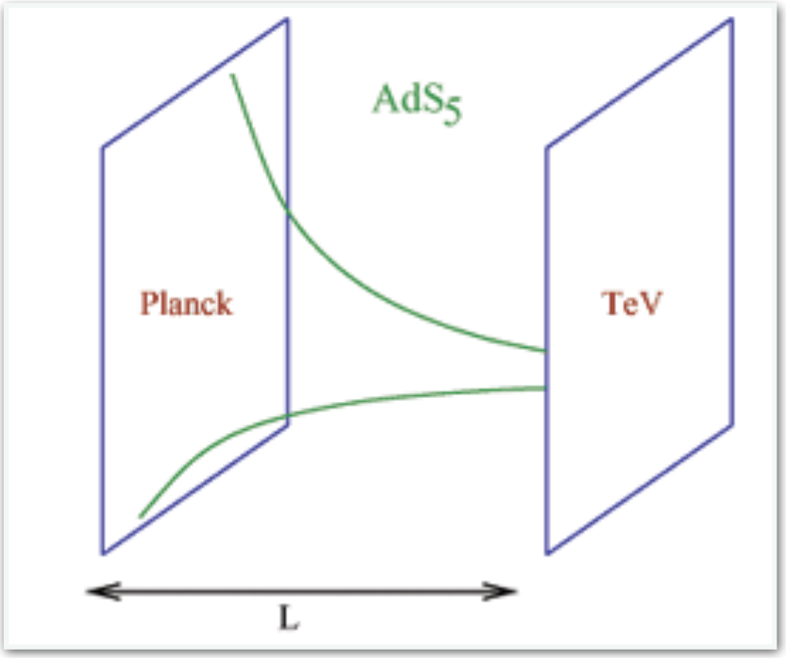
$$S_b = \frac{1}{2}\int d^4x \int_{-\pi}^{\pi} d\phi\sqrt{G}\left(G^{AB}\partial_A\Phi\partial_B\Phi - m^2\Phi^2\right)$$

- **The vevs at the two branes are different.**
- **The vev $\phi_0(y)$ interpolates the two different vevs at the the two branes and must be a non-trivial function of the 5D coordinate.**
- **After mixing and applying the gauge fixing constraint, we are left with a potential $V(\langle\hat{r}\rangle)$, which, after minimization leads to a non-zero vev for the radion field.**

$$V_\Phi(r_c) \xrightarrow{\textbf{Minimize}} kr_c = \left(\frac{4}{\pi}\right)\frac{k^2}{m^2}\ln\left[\frac{v_h}{v_v}\right]$$

Stabilization from Dynamics

Goldberger-Wise Mechanism in a nutshell



$$\mathcal{L}_{5D} \equiv \mathcal{L}_{EH} + \mathcal{L}_{\Phi\Phi} + \mathcal{L}_{pot} \ ,$$

$$\mathcal{L}_{EH} \equiv -\frac{2}{\kappa^2}\sqrt{G}\,R$$

$$\mathcal{L}_{\Phi\Phi} \equiv \sqrt{G}\left[\frac{1}{2}\,\tilde{G}^{MN}\,(\partial_M\hat{\Phi})\,(\partial_N\hat{\Phi})\right] \ .$$

$$\mathcal{L}_{pot} \equiv -\frac{4}{\kappa^2}\left[\sqrt{G}\,V[\hat{\Phi}] + \sqrt{\overline{G}}\,V_1[\hat{\Phi}]\delta(\varphi) + \sqrt{\overline{G}}\,V_2[\hat{\Phi}]\delta(\varphi-\pi)\right] \ .$$

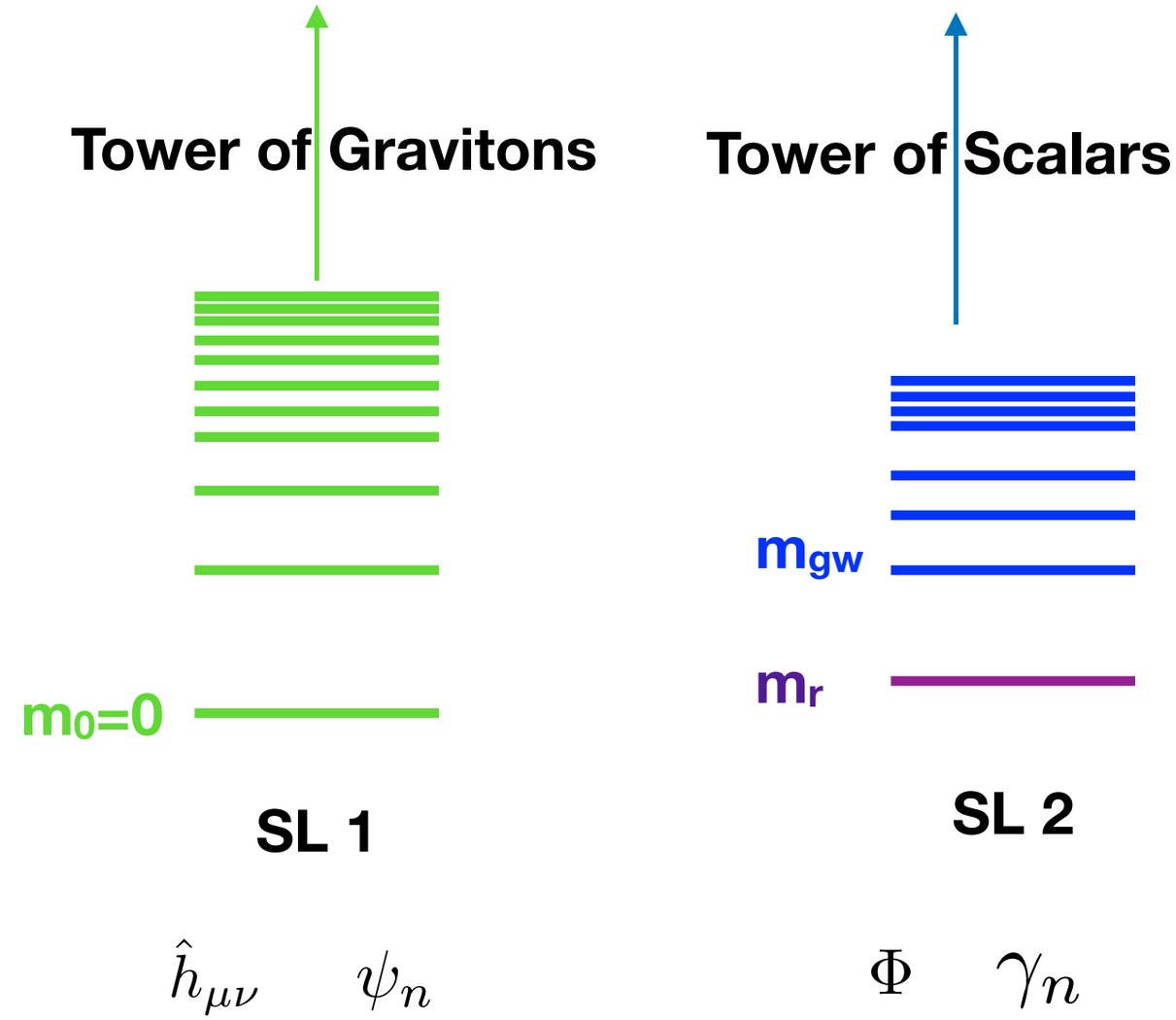
$$\hat{\Phi}(x,y) \equiv \frac{1}{\kappa}\hat{\phi}(x,y) \equiv \frac{1}{\kappa}\left[\phi_0(y) + \hat{f}(x,y)\right] \ , \quad g_{\mu\nu} \to \eta_{\mu\nu} + \kappa h_{\mu\nu}$$

- $\phi_0(y)$ **is the non trivial, y dependent vev of the scalar field. Without this there is no mixing between the modulus and the bulk scalar.**

$$\mathcal{L}_{\Phi\Phi} \supset \frac{\sqrt{G}}{2}\tilde{G}^{55}(\partial_y\hat{\Phi})(\partial_y\hat{\Phi}) \quad \Rightarrow \quad \hat{r}(\partial_y\phi_0)(\partial_y\hat{f}) \ .$$

-
- **Residual gauge invariance of the action:** $\hat{r} \to \hat{r} - 2\partial_y\xi_y \ , \quad \hat{f} \to \hat{f} - \partial_y\phi_0\xi_y$
- **Fix this gauge with the choice:** $(\partial_\varphi\phi_0)\,\hat{f}(x,y) \equiv \sqrt{6}\,e^{2A}\left[\kappa\,(\partial_\varphi\hat{r})\right]$
- **Radion and GW-scalars described by a single 5D field** $\Longrightarrow$ **One tower of 4D scalars**

4D Field content of the Stabilized model



- **Two Sturm-Liouville problems**
- ***Radion*** **is identified as the lightest mode of the tower of scalars**
- **We refer to the heavier modes as *"GW scalars"***
- **Radion contribution to the amplitude must cancel with a combination GW scalars and gravitons**

Dynamics of the Stabilized model

- **Step 1 : Obtain the vacuum solutions for the potential V**
- **Step 2 : Perturb around the vacuum solution**
- **Step 3 : Expand the Lagrangian, obtain Canonical quadratic and interaction terms**
- **Step 4 : Obtain the relevant SL problem and couplings (overlap integrals)**

$$\frac{1}{\kappa^2}\int d^5x\sqrt{|G|}R$$

Yields a massless graviton + a spin-2 tower + *a radion*

$$\frac{1}{\kappa^2}\int d^5x\sqrt{|G|}\left(G^{MN}(\partial_M\Phi)(\partial_N\Phi)+V(\Phi)\right)$$

Yields a tower of scalars (GW scalars) that mix with the radion

$$\sum_i\int d^5x\ \sqrt{|G|}\delta(y-y_i)\lambda_i(\Phi)+\text{Boundary}\quad\text{Terms}$$

Generates a non-trivial VEV : Mixes radion and scalar

Constraint and gauge condition : $(\partial_\varphi\phi_0)\,\hat{f}(x,y)\equiv\sqrt{6}\,e^{2A}\left[\kappa\,(\partial_\varphi\hat{r})\right]$

Sturm Liouville Problem of the two towers

Graviton Sector

Scalar Sector

$$\hat{h}_{\mu\nu}(x, y = r_c\varphi) = \frac{1}{\sqrt{\pi r_c}} \sum_{n=0}^{+\infty} \hat{h}^{(n)}_{\mu\nu}(x)\, \psi_n(\varphi)$$

Mode Decomposition

$$\hat{r}(x, y = r_c\varphi) = \frac{1}{\sqrt{\pi r_c}} \sum_{i=0}^{+\infty} \hat{r}^{(i)}(x)\, \gamma_i(\varphi)$$

$$\partial_\varphi \left[e^{-4A} (\partial_\varphi \psi_n) \right] = -\mu_n^2 e^{-2A} \psi_n$$

SL equation

$$\partial_\varphi \left[\frac{e^{2A}}{(\partial_\varphi \phi_0)^2} (\partial_\varphi \gamma_n) \right] - \frac{e^{2A}}{6} \gamma_n = -\mu_{(n)}^2 \frac{e^{4A}}{(\partial_\varphi \phi_0)^2} \gamma_n$$

$$(\partial_\varphi \psi_n) = 0 \text{ at } \varphi \in \{0, \pi\}$$

Boundary Conditions

$$\left\{ \left[2\frac{d^2 V_1}{d\hat{\phi}^2} - \frac{(\partial_\varphi^2 \hat{\phi})}{(\partial_\varphi \hat{\phi})} \right] (\partial_\varphi \gamma_n) + \mu_{(n)}^2 e^{2A} \gamma_n \right\} \Big|_{\hat{\phi}=\phi_0,\, \varphi=0^+} = 0 \,,$$

$$\left\{ \left[2\frac{d^2 V_2}{d\hat{\phi}^2} + \frac{(\partial_\varphi^2 \hat{\phi})}{(\partial_\varphi \hat{\phi})} \right] (\partial_\varphi \gamma_n) - \mu_{(n)}^2 e^{2A} \gamma_n \right\} \Big|_{\hat{\phi}=\phi_0,\, \varphi=\pi^-} = 0 \,.$$

$$\int_0^\pi d\theta\; e^{-2A(\theta)} \psi_n(\theta) \psi_m(\theta) = \pi \delta_{n,m}$$

Normalization

$$\frac{6}{\pi} \int_{-\pi}^{\pi} d\varphi \left[\frac{e^{2A}}{(\partial_\varphi \phi_0)^2} (\partial_\varphi \gamma_m)(\partial_\varphi \gamma_n) + \frac{e^{2A}}{6} \gamma_m \gamma_n \right] = \delta_{mn}$$

Simplification in the "Stiff Wall" limit

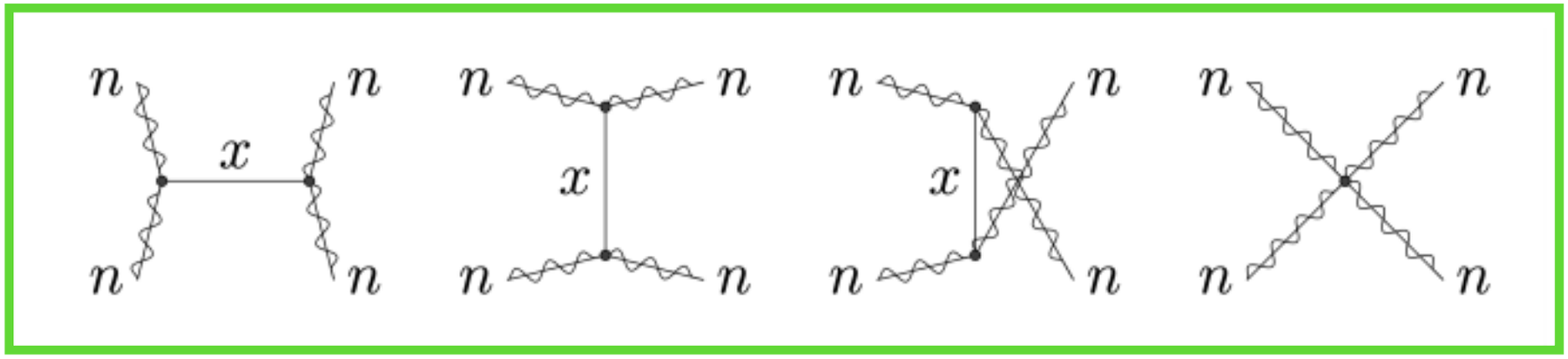
$$\left\{ \left[2\frac{d^2 V_1}{d\hat{\phi}^2} - \frac{(\partial_\varphi^2 \hat{\phi})}{(\partial_\varphi \hat{\phi})} \right] (\partial_\varphi \gamma_n) + \mu_{(n)}^2 e^{2A} \gamma_n \right\} \Big|_{\hat{\phi}=\phi_0,\, \varphi=0^+} = 0 \,,$$

$$\left\{ \left[2\frac{d^2 V_2}{d\hat{\phi}^2} + \frac{(\partial_\varphi^2 \hat{\phi})}{(\partial_\varphi \hat{\phi})} \right] (\partial_\varphi \gamma_n) - \mu_{(n)}^2 e^{2A} \gamma_n \right\} \Big|_{\hat{\phi}=\phi_0,\, \varphi=\pi^-} = 0 \,.$$

$$\frac{d^2 V_1}{d\hat{\phi}^2}\Big|_{\varphi=0^+} \quad \text{and} \quad \frac{d^2 V_2}{d\hat{\phi}^2}\Big|_{\varphi=\pi^-} \to +\infty \,. \implies \textbf{Neumann b.c.}$$

Boos et al 2002,2005
Peloso, Koffman, Frolov 2001
Tanaka, Garriga, Montes 2003
Chivukula, Foren, Mohan, Sengupta, Simmons 2021

Interactions and sum rules of the stabilized model



x **includes :**
1.Radion
2.graviton
3.massive spin2
4.GW- scalars

$$\overline{\mathcal{L}}_{B:hhr} = \sqrt{\frac{3}{8}}\left[\hat{r}\left(\llbracket \hat{h}'\hat{h}' \rrbracket - \llbracket \hat{h}' \rrbracket^2\right)\right]$$

The Lorentz structure of the r hh vertex remains unchanged

O(s^5) & O(s^4) Sum Rules are the same

$$a_{nnnn} = \sum_j a_{nnj}^2 \qquad \sum_j m_j^2 a_{nnj}^2 = \frac{4}{3} m_n^2 a_{nnnn}$$

- **Cubic and quartic interactions of gravitons follow the same pattern as the unstabilized case — and are true for arbitrary warping $A(y)$.**
- **Difference : All radion-graviton interactions replaced by radion + scalar tower - these are only relevant at O(s^3) and lower.**
- **We are able to prove the O(s^5) & O(s^4) and a linear combination of O(s^3) & O(s^2) sum rules analytically and for arbitrary warping $A(y)$ that are consistent with Einsteins equations.**

O(s³)& O(s²) Sum Rules

$$c_{nnnn} \equiv \frac{1}{\pi}\int d\varphi\ \varepsilon^{-6}\,(\partial_\varphi\psi_n)^4$$

$$a_{n'n'(i)} \equiv \frac{1}{\pi}\int_{-\pi}^{+\pi} d\varphi\ \varepsilon^{-2}\,(\partial_\varphi\psi_n)\,(\partial_\varphi\psi_n)\,\gamma_i$$

Stabilized

Sum over GW scalars + radion

$$\sum_j \mu_j^6\, a_{jnn}^2 = \frac{1}{3}\left[-27\sum_i \mu_{(i)}^2\left(e^{-kr_c\pi}\, a_{n'n'(i)}\right)^2 + 60\,\mu_n^2\, c_{nnnn} + 4\,\mu_n^6\, a_{nnnn}\right]$$

$$\sum_i \left[e^{-kr_c\pi}\, a_{n'n'(i)}\right]^2 = \frac{1}{27}\left[15\, c_{nnnn} + 3\,\mu_n^4\, a_{nn0}^2 + \mu_n^4\, a_{nnnn}\right]$$

Compare to unstabilized

$$\sum_j \mu_j^6\, a_{jnn}^2 = \frac{1}{3}\left[60\,\mu_n^2\, c_{nnnn} + 4\,\mu_n^6\, a_{nnnn}\right]$$

$$b_{n'n'r}^2 = \left[e^{-kr_c\pi}\, a_{n'n'(0)}\right]^2 = \frac{1}{27}\left[15\, c_{nnnn} + 3\,\mu_n^4\, a_{nn0}^2 + \mu_n^4\, a_{nnnn}\right]$$

Verification of sum rules for an explicit (DFGK) model

- **A class of stabilization models inspired by Superpotentials**
- **Exactly Solvable, with a back reaction that can be treated in perturbation theory**

$$\mathcal{L}_{\rm pot} \equiv -\frac{4}{\kappa^2}\left[\sqrt{G}\,V[\hat{\Phi}] + \sqrt{G}\,V_1[\hat{\Phi}]\delta(\varphi) + \sqrt{G}\,V_2[\hat{\Phi}]\delta(\varphi-\pi)\right] . \qquad V r_c^2 = \frac{1}{8}\left(\frac{dW}{d\hat{\phi}}\right)^2 - \frac{W^2}{24}$$

$$V_1 r_c = +\frac{W}{2} + \beta_1^2\left[\hat{\phi}(\varphi) - \phi_1\right]^2 \qquad V_2 r_c = -\frac{W}{2} + \beta_2^2\left[\hat{\phi}(\varphi) - \phi_2\right]^2 .$$

$$W[\hat{\phi}(\varphi)] = 12\tilde{k}r_c + \frac{\sqrt{6}}{\phi_1}\left[\phi_1^2 - \hat{\phi}(\varphi)^2\right]\epsilon$$

$$\phi_0(\varphi) = \phi_1 \exp\left(-\frac{2\sqrt{6}}{\phi_1}\epsilon|\varphi|\right) ,$$

VEV of the bulk scalar

- **ϵ is a perturbative parameter related to the bulk mass of the scalar field.**
- **Extra dimension is stabilized at $\mathcal{O}(\epsilon^2)$ or greater**
- **$\partial_y \phi_0(y)$ must be a non-constant function of the 5D coordinate since it interpolates the vevs at the two branes. Simplest possible function: ϵ^2**
- **Unstabilized limit : $\epsilon \to 0$**

Flat-Stablized model : Warp factor generated from the bulk potential

$$A(\varphi) = \tilde{k}r_c|\varphi| + \frac{\phi_1^2}{48}\left[\exp\left(-\frac{4\sqrt{6}}{\phi_1}\epsilon|\varphi|\right) - 1\right] + \frac{\phi_1}{2\sqrt{6}}\epsilon|\varphi| ,$$
$$= \tilde{k}r_c|\varphi| + \epsilon^2|\varphi|^2 + \mathcal{O}(\epsilon^3) .$$

$$\xrightarrow{\tilde{k} \to 0} \quad A(\varphi) = \epsilon^2|\varphi|^2 + \mathcal{O}(\epsilon^3)$$

Wave-Function and spectrum for the flat-stabilized model

- Solve for wave-function and masses to leading order in ϵ^2
- We are able to derive closed form analytic expressions
- We are able to show explicit verification of the sum rules analytically by plugging in these expressions into the sum rules.
- Graviton scattering amplitudes for the flat stabilized model grow at O(s).
- Also, this works as a good cross-check of our calculations.

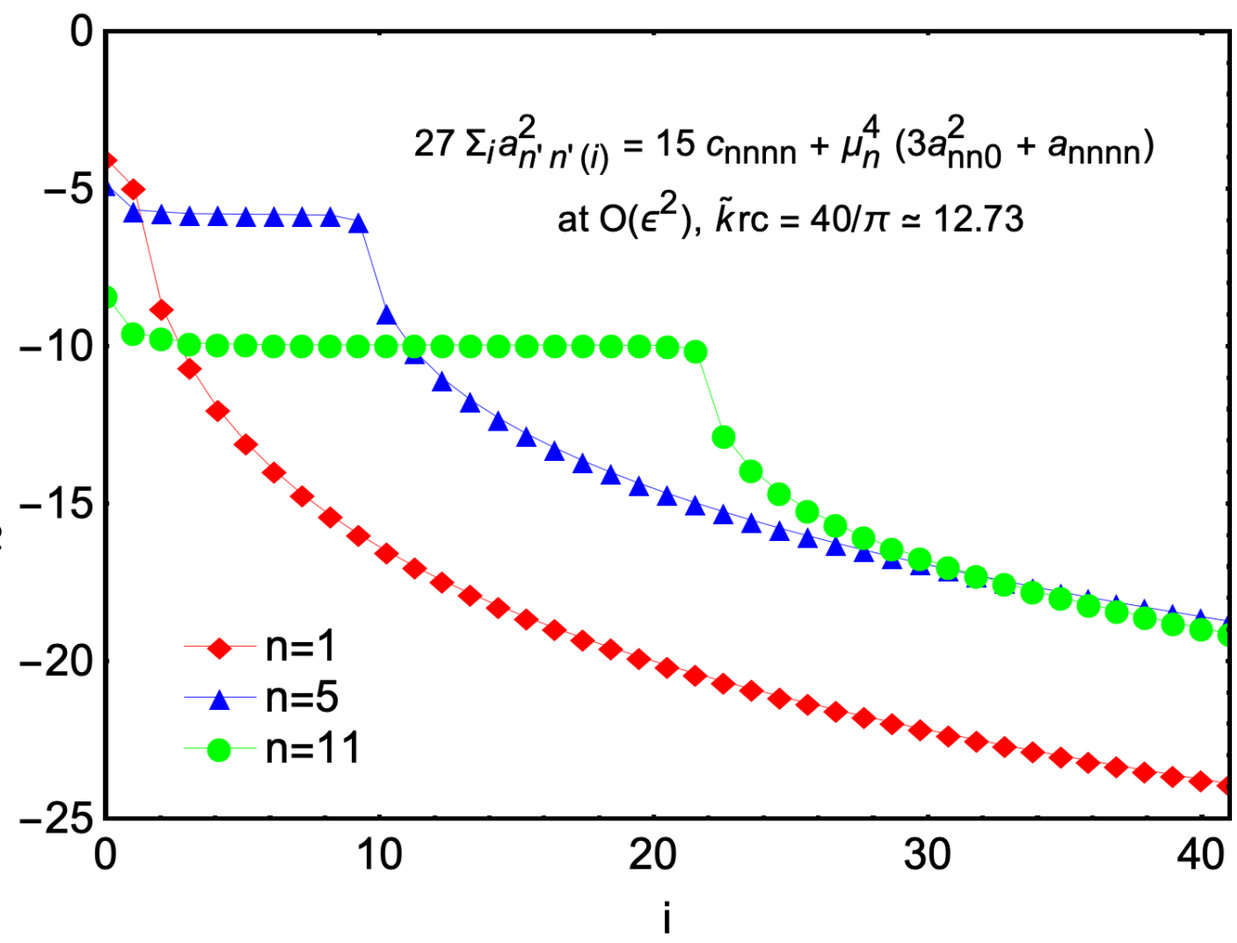
		$\mathcal{O}(1)$	$\mathcal{O}(\epsilon)$	$\mathcal{O}(\epsilon^2)$
graviton wfxn	ψ_0	$\frac{1}{\sqrt{2}}$	-	$\frac{\pi^2}{3\sqrt{2}}$
graviton mass^2	μ_0^2	-	-	-
spin-2 KK wfxn	ψ_n	$-c_{n\varphi}$	-	$\frac{1}{6n^2}\left[9+n^2(\pi^2-9\varphi^2)\right]c_{n\varphi}$ $+\frac{\varphi}{3n}\left[9+n^2(\varphi^2-\pi^2)\right]s_{n\varphi}$
spin-2 KK mass^2	μ_n^2	n^2	-	$3-\frac{2n^2\pi^2}{3}$
radion wfxn	γ_0	$\frac{1}{\sqrt{2}}$	-	$-\frac{\pi^2}{3\sqrt{2}}$
radion mass^2	$\mu_{(0)}^2$	-	-	4
GW scalar wfxn	γ_n	-	$-\frac{2}{n}c_{n\varphi}$	$\frac{4\sqrt{6}}{n^2\phi_1}\left[(n\varphi)c_{n\varphi}-s_{n\varphi}\right]$
GW scalar mass^2	$\mu_{(n)}^2$	n^2	-	$1-\frac{2n^2\pi^2}{3}+\frac{24}{\phi_1^2}$

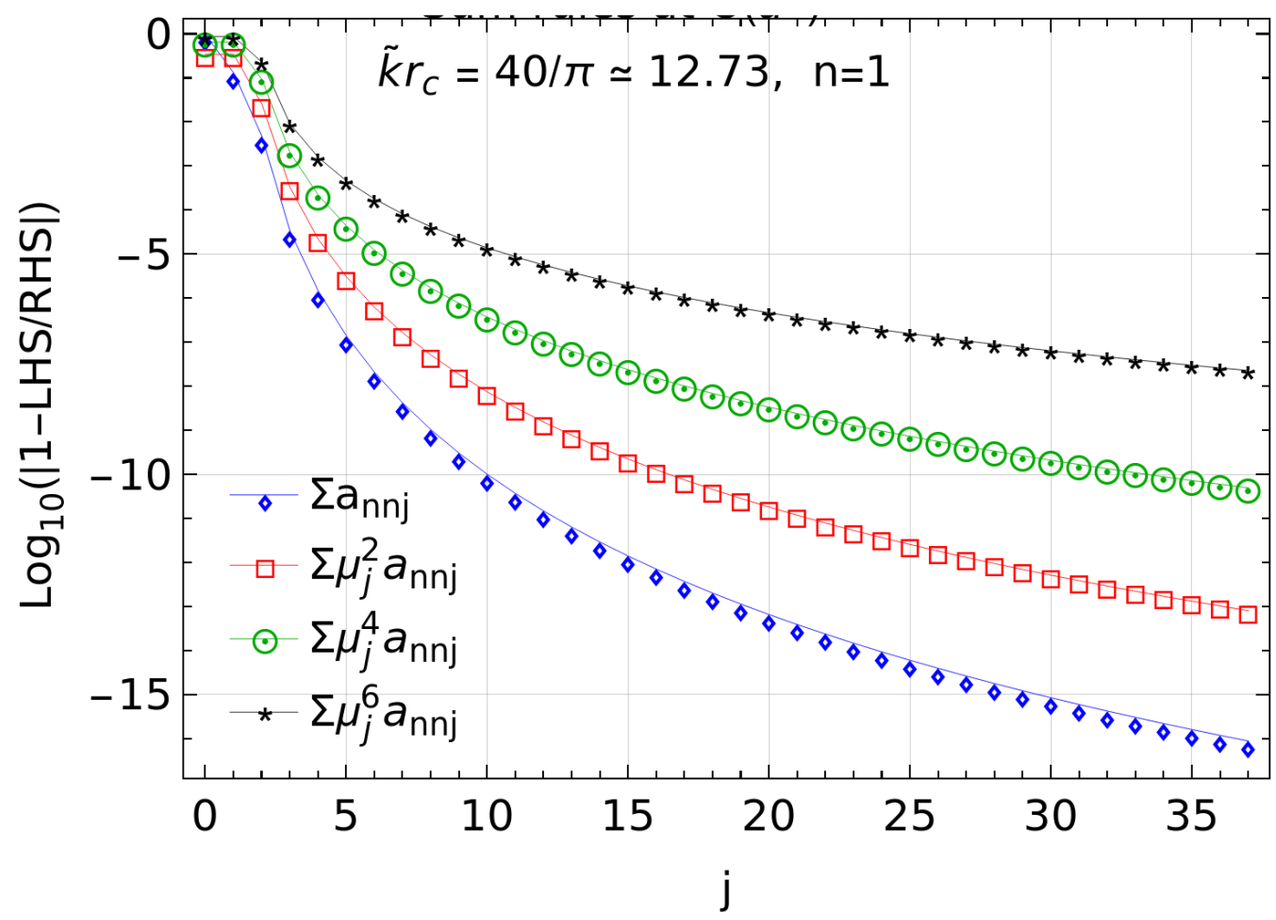
Chivukula, Foren, Mohan, Sengupta, Simmons 2021

Checking sum rules for the full DFGK model for a warped Geometry

$$A(\varphi) = \tilde{k}r_c|\varphi| + \frac{\phi_1^2}{48}\left[\exp\left(-\frac{4\sqrt{6}}{\phi_1}\epsilon|\varphi|\right) - 1\right] + \frac{\phi_1}{2\sqrt{6}}\epsilon|\varphi| \ ,$$
$$= \tilde{k}r_c|\varphi| + \epsilon^2|\varphi|^2 + \mathcal{O}(\epsilon^3) \ .$$

- ***Analyze the sum-rules in perturbation theory to*** $\mathcal{O}(\epsilon^2)$
- ***Calculate perturbed graviton sector and scalar sector wave-function and masses***
- ***We have closed form expressions - but these are too ungainly to be shown here.***





Chivukula, Foren, Mohan, Sengupta, Simmons: In preparation

Summary -again

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STABILIZING THE EXTRA DIMENSION VIA GOLDBERGER-WISE MECHANISM $\rightarrow \boxed{\frac{s^2 m_r^2}{\Lambda_\pi^2 m_n^4}} \rightarrow$ Naive re-introduction of bad high energy behavior

MATRIX ELEMENTS AND CANCELLATIONS IN STABILIZED MODELS $\rightarrow$ ✔ Two towers and sum rules for the Stabilized model

Summary

- **Determined the Lagrangian of the stabilized model up to quartic order - this is non-trivial**
- **First calculation to involve full dynamics of a stabilized model.**

- **Cancellations due to different diagrams reduce $O(s^5)$ growth to $O(s)$.**

- **Compactified theories of extra dimensions -> No low energy cut-off**

- **Validity of 4D EFT - does not depend on modulus stabilization**
  - **5D Planck Scale for flat stabilized models**
  - Λ_π **for warped RS model**

- **Unstabilized model: Cancellations between single tower of states and couplings.**
- **Stabilized model: Cancellations between two towers.**

- **Uncovered sum rules that ensure this cancellation. This also provides a cross-check for our wavefunctions, couplings and masses- important for implementing in phenomenological applications.**