Precision Gravitational Wave Spin Observables from EFT

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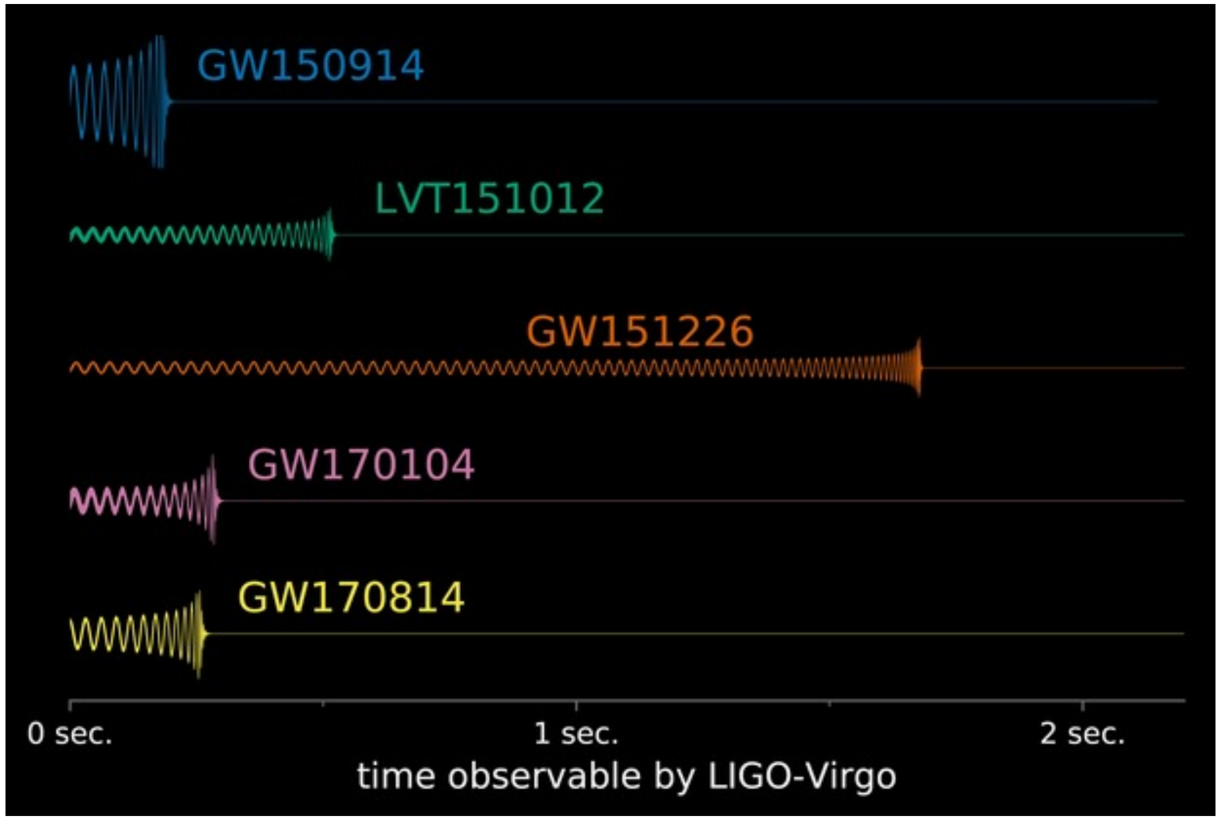
Outline of my talk:

- Motivation
- Overview of the EFT (Nonrelativistic GR) formalism
- Computing NLO Spin Effects
- Concluding remarks

Compact Object Binaries and LIGO/Virgo/LISA

- First GW detection from black hole merger by LIGO in 2015



LIGO/University of Oregon/Ben Farr

- Why?
  - Strong field G.R.
  - Structure of Neutron Stars
  - Multi-messenger physics
  - Populations in stellar graveyard (spins, masses, etc)
  - Others?

Big Picture: Producing precise templates for GW detectors

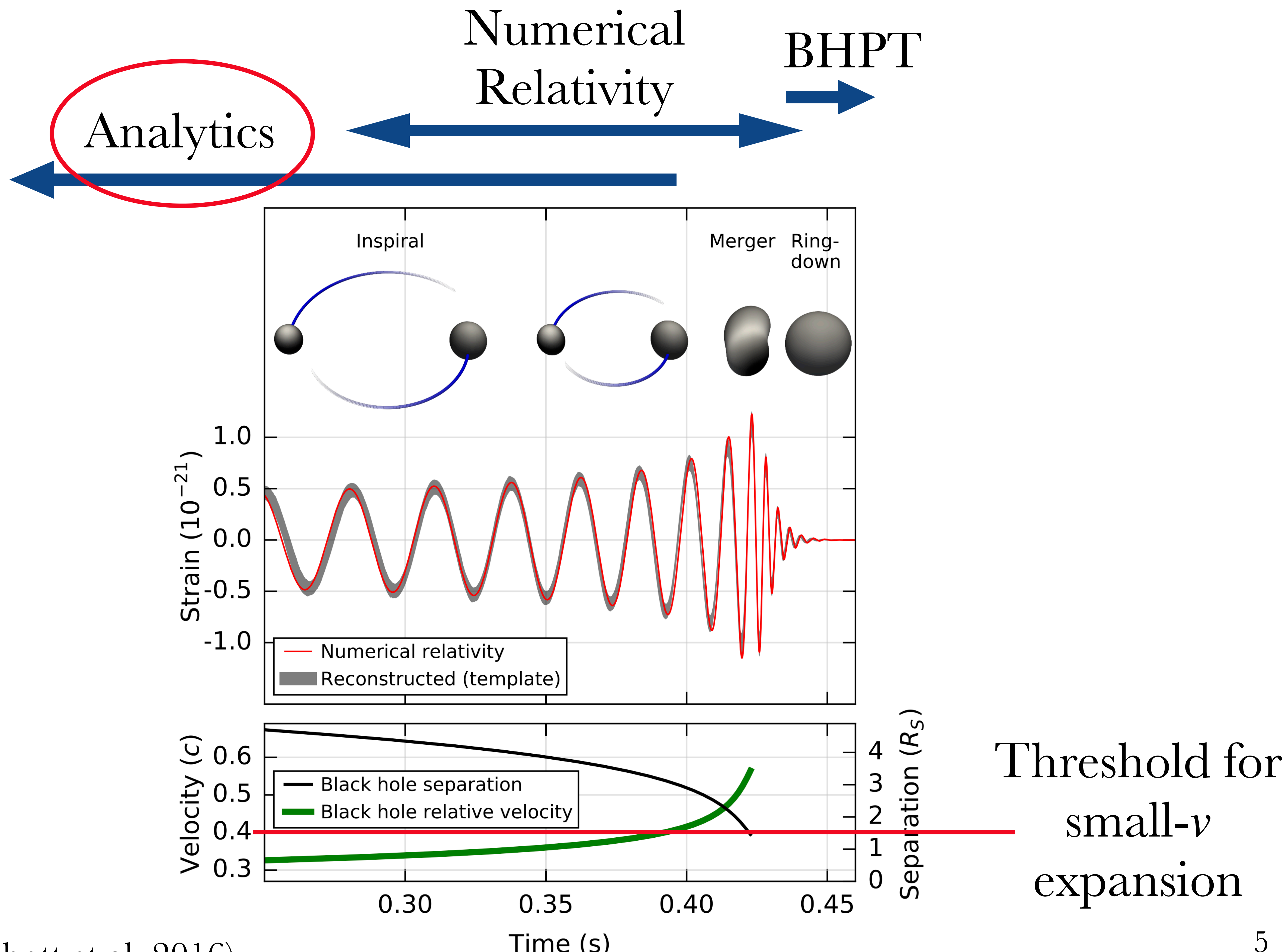
- Challenge: Solve Einstein's field equations

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G\, T_{\mu\nu}$$

Dynamical metric

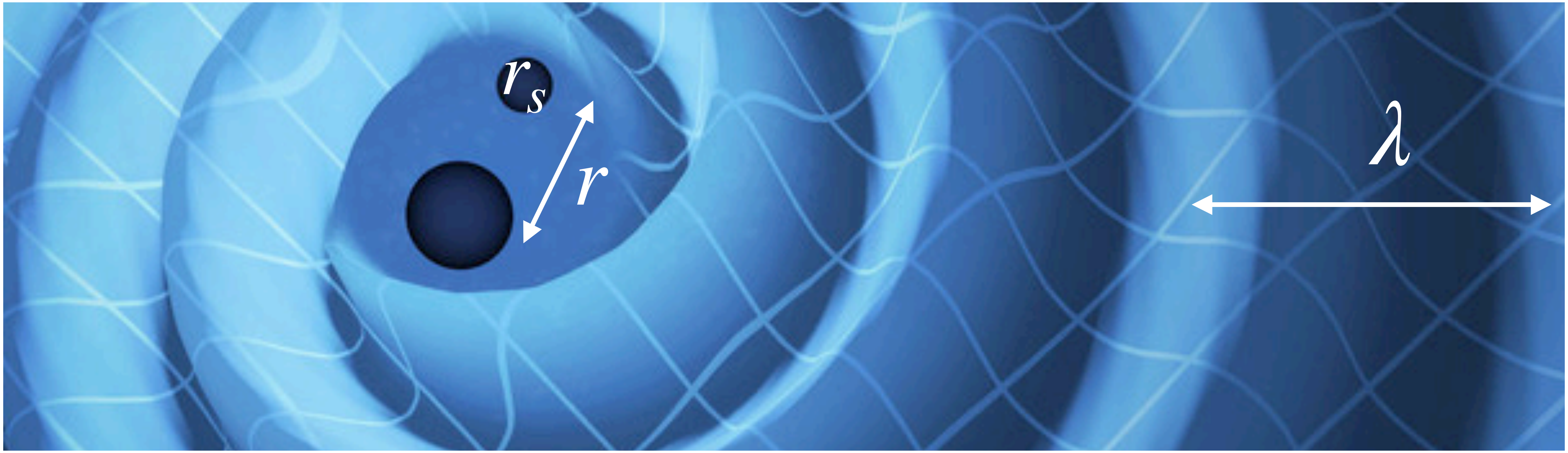
Realistic matter description

Solutions give waveforms to be used in templates and analysis



(B. P. Abbott et al. 2016)

EFT approach takes advantage of separation of scales



LIGO/T. Pyle

- Post-Newtonian Approximation: $\mathcal{O}(v^n) = \left(\frac{n}{2}\right)$ PN
- By Virial theorem: $\frac{G_N m}{r} = v^2 \quad \rightarrow \quad \frac{r_s}{r} \sim v^2, \; \frac{r}{\lambda} \sim v$

$r_s \equiv$ Size of Compact Object $\quad r \equiv$ Orbital Radius $\quad \lambda \equiv$ Radiation Wavelength

- Start with full GR coupled to point particles:

$$S = S_{\text{matter}}(\dot{x}_A, g, S) + S_{\text{Einstein–Hilbert}}(g) + S_{\text{gauge fix}}$$

- Expand metric:

$$g^{\mu\nu} = \eta^{\mu\nu} + \frac{h^{\mu\nu}}{M_{\text{Pl}}} = \eta^{\mu\nu} + \bar{h}^{\mu\nu} + H^{\mu\nu}$$

Radiation Modes: $\bar{h}^{\mu\nu}$

Potential Modes: $H^{\mu\nu}$

- Scaling: $\left(k^0, \boldsymbol{k}\right)_{\text{pot}} \sim (\, v/r \,, 1/r\,)$ and $\left(k^0, \boldsymbol{k}\right)_{\text{rad}} \sim (\, v/r \,, v/r\,)$

$$\exp\left[iS\left(x_A, \bar{h}\right)\right] = \int \mathscr{D}H_{\mu\nu} \exp\left[iS\left(x_A, \bar{h} + H\right)\right]$$

(Goldberger and Rothstein 2005)

- Action reproduces GR spin equations of motion:

$$\frac{Dp^\mu}{D\tau} = -\frac{1}{2}R^\mu_{\ \nu\alpha\beta}u^\nu S^{\alpha\beta}, \quad \frac{DS^{\mu\nu}}{D\tau} = p^\mu u^\nu - u^\mu p^\nu$$

- (Porto 2006) Use Routhian mechanics + tetrads: $\eta_{IJ} = e^\mu_I e^\nu_J g_{\mu\nu}$

$$\mathcal{R} = -p^\mu u_\mu - \frac{1}{2}\omega_\mu^{ab} S_{ab} u^\mu \qquad (\omega_\mu^{ab} \equiv e^b_\nu \nabla_\mu e^{a\nu})$$

- Compute equations of motion from:

$$\frac{\delta}{\delta x^\mu}\int \mathcal{R}\, d\sigma = 0, \qquad \frac{dS^{ab}}{d\sigma} = \left\{S^{ab}, \mathcal{R}\right\}$$

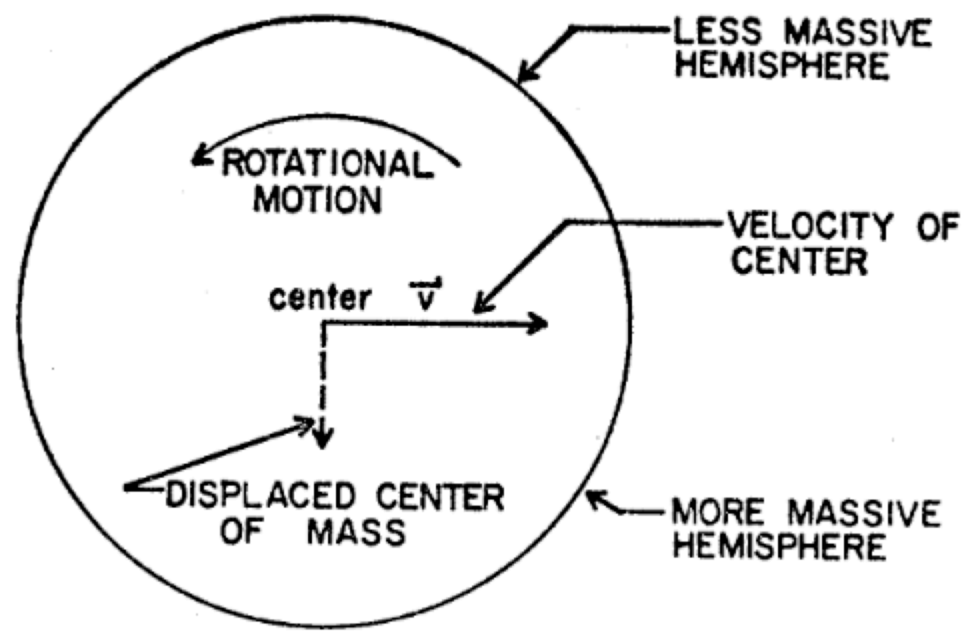
- Power counting for maximally rotating objects:

$$S \sim m v_{\rm rot} r_s < m r_s \sim L v$$

EFT spin defined in the locally-flat frame

Need additional constraint to go from tensor to vector, known as spin-supplementary condition:

- Covariant: $p_\mu S^{\mu\nu} = 0$
- Newton-Wigner: $S^{\mu 0} - S^{\mu j}\left(\frac{\tilde{p}^j}{\tilde{p}_0 + m}\right) = 0$



(Fleming 1964)

Appropriate spins in adiabatic approximation are *conserved norm spin vectors* (Will 2005) related to LFF vector by

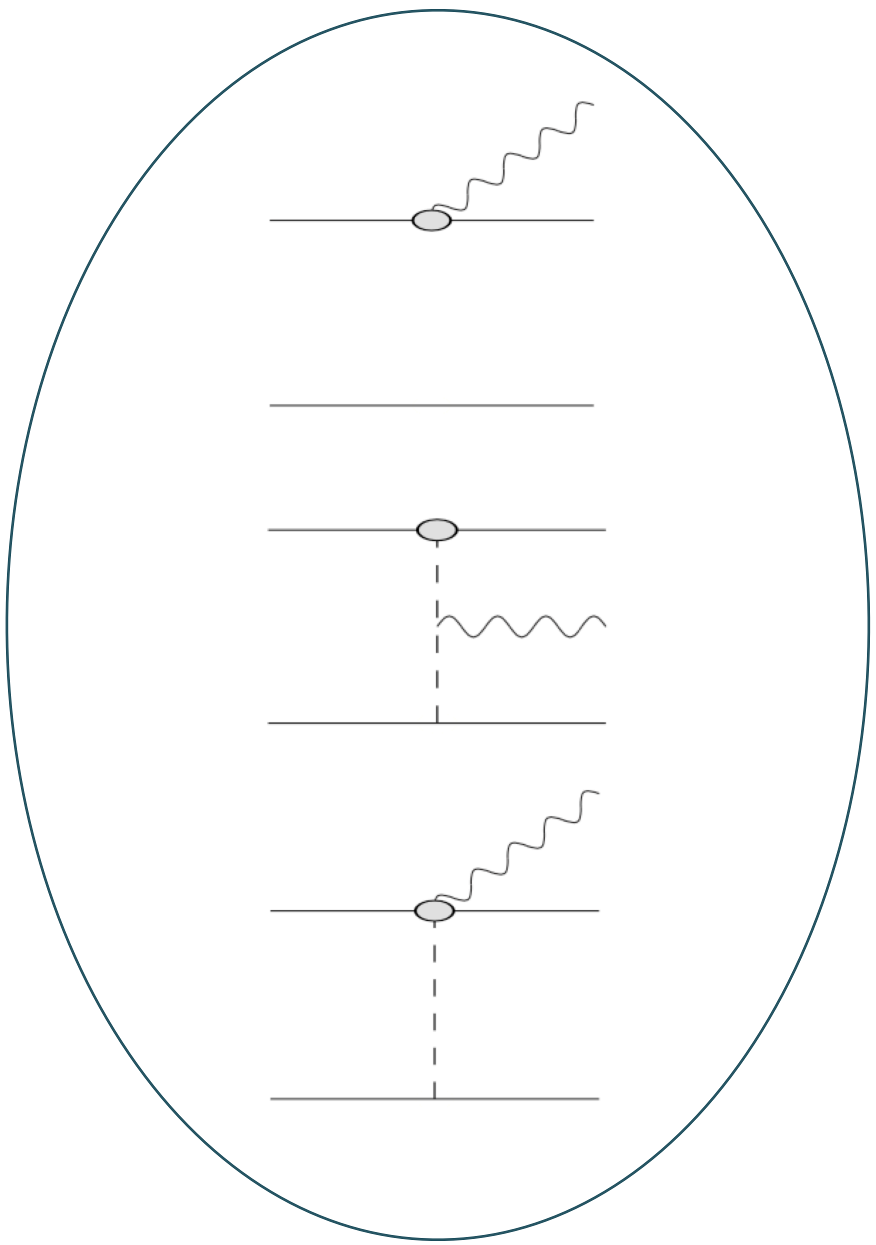
$$\mathbf{S}_A = \left(1 + \tfrac{1}{2}\mathbf{v}_A^2\right)\mathbf{S}_A^c - \tfrac{1}{2}\mathbf{v}_A\left(\mathbf{v}_A \cdot \mathbf{S}_A^c\right) + \cdots$$

Spin evolution takes precession form, i.e.,

$$\frac{d\mathbf{S}_A^c}{dt} = \mathbf{\Omega}_A \times \mathbf{S}_A^c$$

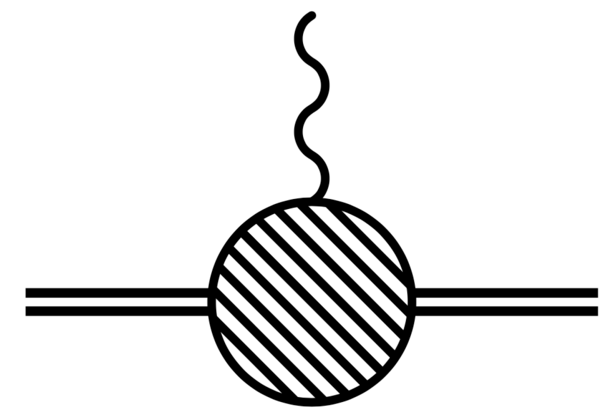
(Owen et al 1997, Faye et al 2006, Porto 2010)

Schematic of EFT



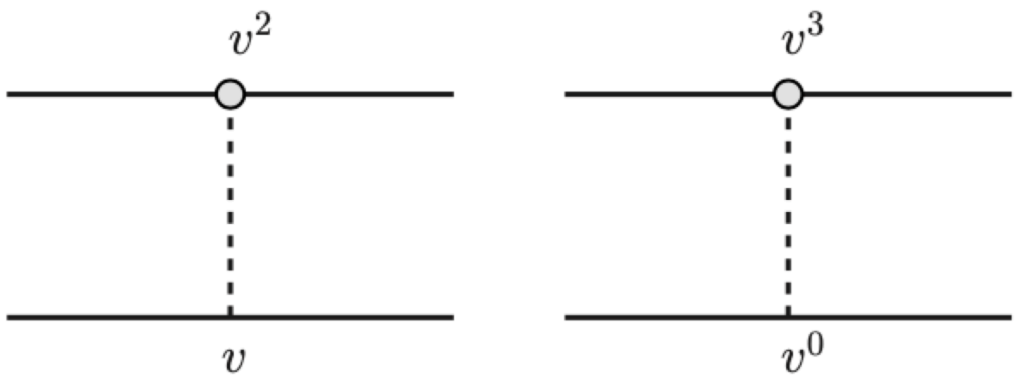
"Full" theory

Integrate out off-shell potential modes



$I_{ij}, J_{ij}, I_{ijk}, J_{ijk}, \ldots$

Effective theory



1.5PN potential from Lagrangian:

$$V^{\mathrm{SO}}_{1.5\mathrm{PN}} = \frac{G_N m_2}{r^3} x^j \left(S_1^{j0} + S_1^{jk} \left(v_1^k - 2 v_2^k \right) \right) + 1 \leftrightarrow 2$$

Euler-Lagrange

$$\mathbf{a}^{(\mathrm{cov})}_{\mathrm{SO}} = \frac{1}{r^3} \left\{ 6\hat{\mathbf{n}} \left[(\hat{\mathbf{n}} \times \mathbf{v}) \cdot \left(2\mathbf{S} + \frac{\delta m}{m} \boldsymbol{\Delta} \right) \right] - \left[\mathbf{v} \times \left(7\mathbf{S} + 3\frac{\delta m}{m} \boldsymbol{\Delta} \right) \right] + 3\dot{r} \left[\hat{\mathbf{n}} \times \left(3\mathbf{S} + \frac{\delta m}{m} \boldsymbol{\Delta} \right) \right] \right\}$$

In general: $\mathbf{a} = \mathbf{a}_{\mathrm{N}} + \mathbf{a}_{1\mathrm{PN}} + \mathbf{a}^{\mathrm{SO}}_{1.5\mathrm{PN}} + \mathbf{a}_{2\mathrm{PN}} + \mathbf{a}^{\mathrm{SS}}_{2\mathrm{PN}} + \mathbf{a}^{\mathrm{RR}}_{2.5\mathrm{PN}} + \cdots$

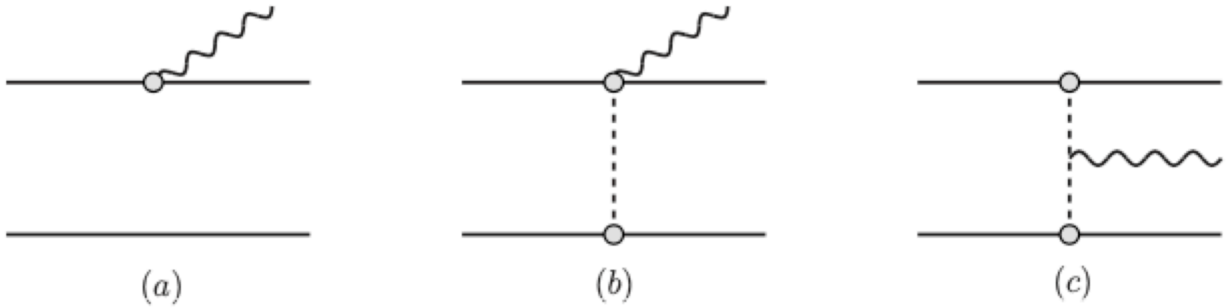
Effective action:

$$S_{\rm rad}\left[\bar{h}, x_a\right] = -\int dt\sqrt{\bar{g}_{00}}\left[M(t) + \sum_{l=2}^{\infty}\left(\frac{1}{l!}I^L\nabla_{L-2}E_{i_{l-1}i_l} - \frac{2l}{(2l+1)!}J^L\nabla_{L-2}B_{i_{l-1}i_l}\right)\right]$$

Coupling of one radiation mode and the stress-energy pseudotensor:

$$\Gamma[\bar{h}] = -\frac{1}{2m_{Pl}}\int d^4x T^{\mu\nu}(x)\bar{h}_{\mu\nu}$$

Compute single radiation mode diagrams to extract $T_{\mu\nu}$



Use general expressions for moments as functions of $T_{\mu\nu}$ (Ross 2013)

Next-to-leading order spin-orbit effects in the equations of motion, energy loss and phase evolution of binaries of compact bodies in the effective field theory approach

Brian Pardo[1] and Natália T. Maia[1] (arXiv:2009.05628)

Gravitational radiation from inspiralling compact objects: Spin-spin effects completed at the next-to-leading post-Newtonian order

Gihyuk Cho,[1] Brian Pardo,[2] and Rafael A. Porto[1] (arXiv:2103.14612)

Purpose: Complete NLO spin effects

- Equations of motion
- Adiabatic invariants
- Flux-Balance Laws
- Accumulated orbital phase and recoil velocity - circular orbits
- Compare with literature (Blanchet et al. 2006, Bohe et al. 2015, Racine et al. 2009, Faye et al. 2006, Blanchet et al. 2006, Bohe et al 2013)

- Potentials at NLO already computed by Porto and Rothstein
  - Use to compute accelerations and spin evolution
  - (ArXiv 1005.5730, 0804.0260, 0802.0720)

- Multipole moments at NLO already computed by Porto, Rothstein, and Ross
  - Use to compute flux-balance laws
  - (ArXiv 1007.1312)
  - We showed these are equivalent to literature with transformation to PN spins/conserved norm spins

Example: NLO Spin-Orbit Potential (Porto 2010):

$$V_{\rm SO}^{\rm NLO} = \frac{Gm_2}{r^3}\left[\left\{S_1^{i0}\left(2\mathbf{v}_2^2 - 2\mathbf{v}_1\cdot\mathbf{v}_2 - \frac{3}{2r^2}\left(\mathbf{v}_2\cdot\mathbf{r}\right)^2 - \frac{1}{2}\mathbf{a}_2\cdot\mathbf{r}\right) + \left(2\mathbf{v}_1\cdot\mathbf{v}_2 + \frac{3\left(\mathbf{v}_2\cdot\mathbf{r}\right)^2}{r^2} - 2\mathbf{v}_2^2 + \mathbf{a}_2\cdot\mathbf{r}\right)S_1^{ij}\mathbf{v}_2^j\right.\right.$$
$$-\left(\frac{3}{2r^2}\left(\mathbf{v}_2\cdot\mathbf{r}\right)^2 + \frac{1}{2}\mathbf{a}_2\cdot\mathbf{r}\right)S_1^{ij}\mathbf{v}_1^j + 2S_1^{ij}\mathbf{a}_2^j\mathbf{v}_2\cdot\mathbf{r} + r^2 S_1^{ij}\mathbf{a}_2^j\Big\}\mathbf{r}^i$$
$$+S_1^{i0}\left(\left(\mathbf{v}_1-\mathbf{v}_2\right)^i\mathbf{v}_2\cdot\mathbf{r} - \frac{3}{2}\mathbf{a}_2^i r^2\right) + S_1^{ij}\mathbf{v}_2^i\mathbf{v}_1^j\mathbf{v}_2\cdot\mathbf{r} - r^2 S_1^{ij}\mathbf{a}_2^j\mathbf{v}_2^i - \frac{1}{2}r^2 S_1^{ij}\mathbf{a}_2^j\mathbf{v}_1^i\Bigg]$$
$$+\frac{G^2 m_2}{r^4}\mathbf{r}^i\left[-\left(m_1+2m_2\right)S_1^{i0} + \left(m_1 - \frac{m_2}{2}\right)S_1^{ij}\mathbf{v}_1^j + \frac{5m_2}{2}S_1^{ij}\mathbf{v}_2^j\right] + 1 \leftrightarrow 2$$

Euler-Lagrange

$\mathbf{a}_{\rm reduced}$

CM corrections

SSC frame corrections

$$(\mathbf{a}^i)^{(2.5\rm PN)}_{\rm SO} = \frac{G}{m\nu r^4}\{\mathbf{n}^i\left[\mathbf{S}\cdot\mathbf{L}\left(-\frac{Gm}{r}(42+29\nu) + 3(-1+10\nu)\mathbf{v}^2 - 30\nu\dot r^2\right)\right.$$
$$\left.-\frac{\delta m}{m}\boldsymbol{\Sigma}\cdot\mathbf{L}\left(\frac{Gm}{r}\left(22+\frac{33}{2}\nu\right) + 3(1-5\nu)\mathbf{v}^2 + 15\nu\dot r^2\right)\right]$$
$$+3\dot r\mathbf{v}^i\left[3\mathbf{S}\cdot\mathbf{L}(-1+\nu) + \frac{\delta m}{m}\boldsymbol{\Sigma}\cdot\mathbf{L}(-1+2\nu)\right]$$
$$\left.-2\frac{Gm}{r}\mathbf{L}^i\left[\mathbf{S}\cdot\mathbf{n}(1+2\nu) + \frac{\delta m}{m}\boldsymbol{\Sigma}\cdot\mathbf{n}(1+\nu)\right]\right\}$$
$$+\frac{G}{r^3}\{(\mathbf{S}\times\mathbf{n})^i\dot r\left[\frac{Gm}{r}(26+25\nu) + \frac{3}{2}(1-15\nu)\mathbf{v}^2 + \frac{45}{2}\nu\dot r^2\right]$$
$$+\frac{\delta m}{m}(\boldsymbol{\Sigma}\times\mathbf{n})^i\dot r\left[\frac{Gm}{r}\left(10+\frac{27}{2}\nu\right) + \left(\frac{3}{2}-12\nu\right)\mathbf{v}^2 + 15\nu\dot r^2\right]$$
$$+(\mathbf{S}\times\mathbf{v})^i\left[-\frac{Gm}{r}(22+15\nu) + \frac{3}{2}(-1+11\nu)\mathbf{v}^2 - \frac{33}{2}\nu\dot r^2\right]$$
$$\left.-\frac{\delta m}{m}(\boldsymbol{\Sigma}\times\mathbf{v})^i\left[\frac{Gm}{r}\left(10+\frac{15}{2}\nu\right) + \left(\frac{3}{2}-8\nu\right)\mathbf{v}^2 + 9\nu\dot r^2\right]\right\}$$

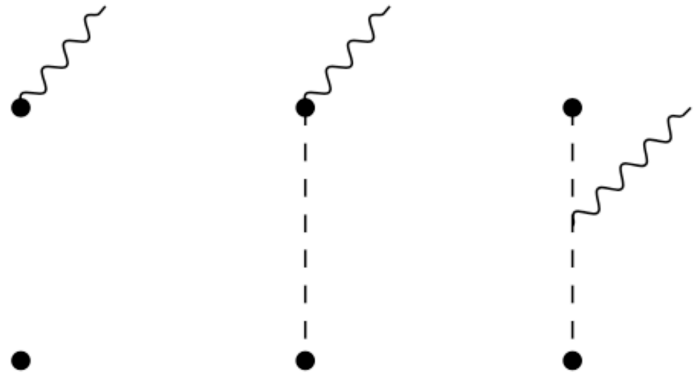
- Energy: $$E = \sum_{A=1}^{2} \sum_{n} \mathbf{p}_{\mathbf{x}_A^{(n)}} \cdot \mathbf{x}_A^{(n+1)} + V + E_{\text{reduced}}$$
$$\mathbf{p}_{\mathbf{x}_A^{(n)}} = - \sum_{A=1}^{2} \sum_{k=n+1} \left(-\frac{d}{dt} \right)^{k-n-1} \frac{\partial V}{\partial \mathbf{x}_A^{(k)}}$$

- Orbital angular momentum: $$\mathbf{L} = \epsilon^{ijk} \sum_{A=1}^{2} \sum_{n} \mathbf{x}_A^{j(n)} \mathbf{p}^k_{\mathbf{x}_A^{(n)}} + \mathbf{L}_{\text{reduced}}$$

Nontrivial check: $\frac{d}{dt}(\mathbf{L} + \mathbf{S}) = 0$

- Center of mass correction from 1-point function:

$$\mathbf{r}^i_{\text{cm}} = \frac{1}{m} \int d^3x \, \mathbf{x}^i \, T^{00}(\mathbf{x}, t) \qquad \sim$$



- Results with corresponding results in literature were in agreement!

- Energy Loss:

$$\frac{dE}{dt} = -\frac{G}{5}\left(I^{(3)}_{ij}I^{(3)}_{ij} + \frac{16}{9}J^{(3)}_{ij}J^{(3)}_{ij} + \frac{5}{189}I^{(4)}_{ijk}I^{(4)}_{ijk} + \frac{5}{84}J^{(4)}_{ijk}J^{(4)}_{ijk} + \cdots\right)$$

- Angular momentum loss:

$$\frac{d\boldsymbol{J}^i}{dt} = -G\epsilon^{iab}\left(\frac{2}{5}I^{(2)}_{aj}I^{(3)}_{bj} + \frac{32}{45}J^{(2)}_{aj}J^{(3)}_{bj} + \frac{1}{63}I^{(3)}_{ajk}I^{(4)}_{bjk} + \frac{1}{28}J^{(3)}_{ajk}J^{(4)}_{bjk} + \cdots\right)$$

- Momentum Loss:

$$\frac{d\boldsymbol{P}^i}{dt} = -G\left(\frac{2}{63}I^{(4)}_{ijk}I^{(3)}_{jk} + \frac{16}{45}\epsilon_{ijk}I^{(3)}_{jl}J^{(3)}_{kl} + \frac{1}{126}\epsilon_{ijk}I^{(4)}_{jlm}J^{(4)}_{klm} + \frac{4}{63}J^{(4)}_{ijk}J^{(3)}_{jk} + \cdots\right)$$

- Center of mass flux:

$$\frac{d\boldsymbol{G}^i}{dt} = \boldsymbol{P}^i - G\left(\frac{1}{21}\left(I^{(3)}_{ijk}I^{(3)}_{jk} - I^{(4)}_{ijk}I^{(2)}_{jk}\right) + \frac{2}{21}\left(J^{(3)}_{ijk}J^{(3)}_{jk} - J^{(4)}_{ijk}J^{(2)}_{jk}\right) + \cdots\right)$$

- Results consistent with literature for NLO spin-orbit and spin-spin!

NLO Spin Effects

- Reduction to circular, spin-(anti)aligned orbits:

$$\boldsymbol{S}_A \equiv \pm S_A \boldsymbol{\ell}, \qquad \dot{r} = 0, \qquad r\omega^2 = -\langle \mathbf{n} \cdot \mathbf{a} \rangle$$

$$\frac{\dot{\omega}}{\omega} = \frac{3}{2x} \left(\frac{dE[x]}{dx} \right)^{-1} \frac{dE[x]}{dt}, \quad \text{with } x \equiv (Gm\omega)^{2/3}$$

- Accumulated phase:

$$\phi = \int dt\, \omega = \int d\omega \, \frac{\omega}{\dot{\omega}}$$

$$\phi = \phi_0 - \frac{32}{\nu} \left\{ x^{-5/2} + x^{-3/2} \left[\frac{3715}{1008} + \frac{55}{12}\nu \right] + \frac{x^{-1}}{Gm^2} \left[\frac{125}{8} \frac{\delta m}{m} \Sigma_\ell^c + \frac{235}{6} S_\ell^c \right] \right.$$
$$\left. + x^{-1/2} \left[\frac{15293365}{1016064} + \frac{27145}{1008}\nu + \frac{3085}{144}\nu^2 \right] - \frac{\log x}{Gm^2} \left[\left(\frac{41745}{448} - \frac{15}{8}\nu \right) \frac{\delta m}{m} \Sigma_\ell^c + \left(\frac{554345}{2016} + \frac{55}{8}\nu \right) S_\ell^c \right] \right\}$$

- Recoil Velocity: $\mathbf{V}^i_{\text{kick}} = \frac{1}{m} \int_{-\infty}^{t} dt \left(\frac{d\mathbf{P}^i}{dt} \right)$

$$\left(V^i_{\text{kick}} \right)_{\text{SO}} = \frac{8\nu^2 x^{9/2}}{15Gm^2} \mathbf{n}^i \left\{ -2 \left(\Sigma_c \ell \right) + \frac{1}{21} x \left(-940\delta \left(S_c \ell \right) + 3(-67 + 412\nu)(\Sigma_c \ell) \right) \right\}$$

- Results consistent with literature up to NLO spin-orbit and spin-spin!

- Computed NLO spin effects, including phase and recoil velocity
- Computed NLO spin pieces needed for templates
- Completed pieces needed at NNLO
- Showed correspondence between literature results and EFT
  - Conservative sector
  - Radiative sector (new!)
- NNLO in the works!

Thank you!