Revisiting Gravitational Wave Detection in an SRF Cavity

Sebastian A. R. Ellis
IPhT, CEA Saclay

Based on:
210x.xxxxx
A. Berlin, R. T. D'Agnolo, SARE

Roadmap

Historical context

Electromagnetism in General Relativity

Indirect signal — an SRF cavity as a resonant bar

Direct signal — cavity as a Gertsenshtein converter

Sources & experimental context

Noise in an SRF cavity

SRF figures of merit & goals

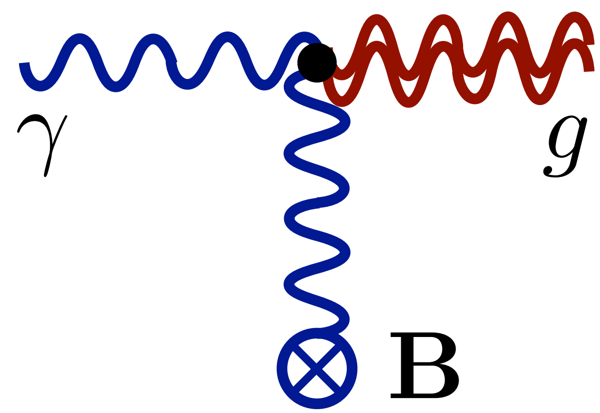
Potential sensitivity**

***ultra-preliminary*

GW interaction w/ EM

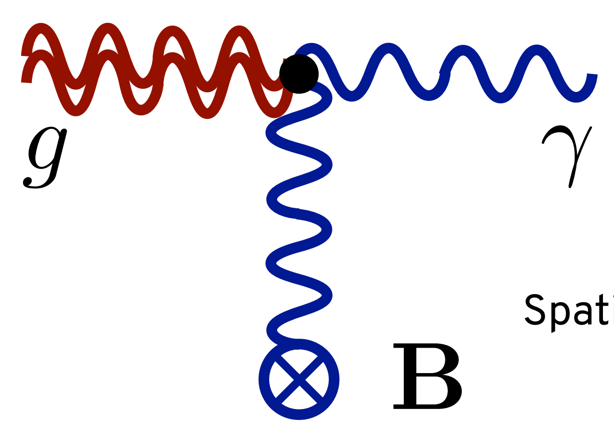
Gertsenshtein effect, 1962

Also Zeldovich 1973



$$\Box h_{\{+,\times\}} = 16\pi G_N B \partial_i A_{\{+,\times\}}$$

Naturally, inverse process also allowed



Spatial and temporal variations of graviton contribute

See e.g. Domcke & Garcia-Cely PRL126 (2021)

GW interaction w/ EM strategy: venerable history

Braginskii & Menskii, 1971

JETP LETTERS VOLUME 13, NUMBER 11 5 JUNE 1971

HIGH-FREQUENCY DETECTION OF GRAVITATIONAL WAVES

V. B. Braginskii and M. B. Menskii
Physics Department, Moscow State University
Submitted 18 March 1971
ZhETF Pis. Red. 13, No. 11, 585 - 587 (5 June 1971)

Pegoraro, Picasso & Radicati, 1978

J. Phys. A: Math. Gen., Vol. 11, No. 10, 1978. Printed in Great Britain

On the operation of a tunable electromagnetic detector for gravitational waves

F Pegoraro†, E Picasso‡ and L A Radicati‡§
†Scuola Normale Superiore, Pisa, Italy
‡CERN, Geneva, Switzerland

Received 6 December 1977, in final form 20 April 1978

Pegoraro, Radicati, Bernard & Picasso, 1978

Led to MAGO collaboration @ CERN early 2000's

ELECTROMAGNETIC DETECTOR FOR GRAVITATIONAL WAVES

F. PEGORARO, L.A. RADICATI
Scuola Normale Superiore, Pisa, Italy

and

Ph. BERNARD and E. PICASSO
CERN, Geneva, Switzerland

Received 29 June 1978

See also Caves 1979, Reece, Reiner & Melissinos 1982, 1984

GW interaction w/ EM

Electromagnetism in General Relativity $$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$F_{\mu\nu} = g_{\mu\alpha} g_{\nu\beta} F^{\alpha\beta}$$

Maxwell's equations:

$$\nabla_\mu F^{\mu\nu} = -J^\nu, \quad \nabla_{[\mu} F_{\nu\alpha]} = 0$$

GW interaction w/ EM

Electromagnetism in General Relativity

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$F_{\mu\nu} = g_{\mu\alpha} g_{\nu\beta} F^{\alpha\beta}$$

Maxwell's equations:

$$\nabla_\mu F^{\mu\nu} = -J^\nu, \quad \nabla_{[\mu} F_{\nu\alpha]} = 0$$

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \quad g^{\mu\nu} = \eta^{\mu\nu} - h^{\mu\nu}$$

$$\eta^{\mu\nu} = \eta_{\mu\nu} = (-, \vec{+})$$

$$\partial_\mu F^{\mu\nu} \simeq J^\nu \left(1 + \frac{h_\alpha{}^\alpha}{2}\right) - h^{\nu\alpha} J_\alpha + \frac{\partial_\mu (h_\alpha{}^\alpha F^{\mu\nu})}{2} + \partial_\mu \left(h^{\mu\alpha} F^\nu{}_\alpha + h^{\nu\alpha} F_\alpha{}^\mu\right)$$

GW interaction w/ EM

Electromagnetism in General Relativity

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$F_{\mu\nu} = g_{\mu\alpha} g_{\nu\beta} F^{\alpha\beta}$$

Maxwell's equations:

$$\nabla_\mu F^{\mu\nu} = -J^\nu, \quad \nabla_{[\mu} F_{\nu\alpha]} = 0$$

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \quad g^{\mu\nu} = \eta^{\mu\nu} - h^{\mu\nu}$$

$$\eta^{\mu\nu} = \eta_{\mu\nu} = (-, \vec{+})$$

$$\partial_\mu F^{\mu\nu} \simeq J^\nu \left(1 + \frac{h_\alpha{}^\alpha}{2}\right) - h^{\nu\alpha} J_\alpha + \frac{\partial_\mu (h_\alpha{}^\alpha F^{\mu\nu})}{2} + \partial_\mu \left(h^{\mu\alpha} F^\nu{}_\alpha + h^{\nu\alpha} F_\alpha{}^\mu\right)$$

Variations of metric (GWs) act as EM source terms

Framing the question

Proper detector frame

$$ds^2 \simeq -dt^2\left(1 + \overset{\textit{inert. acc.}}{2\mathbf{a}\cdot\mathbf{x}} + \overset{\textit{grav. redshift}}{(\mathbf{a}\cdot\mathbf{x})^2} - \overset{\textit{rotation t-dilate}}{(\boldsymbol{\Omega}\times\mathbf{x})^2} + R_{0i0j}x^i x^j\right)$$

$$+\, 2dt\,dx^i\left(\overset{\textit{Sagnac effect}}{\epsilon_{ijk}\Omega^j x^k} - \frac{2}{3}R_{0jik}x^j x^k\right) + dx^i dx^j\left(\delta_{ij} - \frac{1}{3}R_{ikjl}x^k x^l\right)$$

Framing the question

Proper detector frame

$$ds^2 \simeq -dt^2\left(1 + \overset{\textit{inert. acc.}}{2\mathbf{a}\cdot\mathbf{x}} + \overset{\textit{grav. redshift}}{(\mathbf{a}\cdot\mathbf{x})^2} - \overset{\textit{rotation t-dilate}}{(\mathbf{\Omega}\times\mathbf{x})^2} + R_{0i0j}x^i x^j\right)$$
$$+ 2dt\,dx^i\left(\overset{\textit{Sagnac effect}}{\epsilon_{ijk}\Omega^j x^k} - \frac{2}{3}R_{0jik}x^j x^k\right) + dx^i dx^j\left(\delta_{ij} - \frac{1}{3}R_{ikjl}x^k x^l\right)$$

Gravitational wave in TT gauge

$$\partial_\mu h^{\mu\nu} = 0, \quad h_\mu{}^\mu = 0, \quad h_{00} = h_{0i} = 0$$

Framing the question

Proper detector frame

$$ds^2 \simeq -dt^2\left(1 + \overset{\text{inert. acc.}}{2\mathbf{a}\cdot\mathbf{x}} + \overset{\text{grav. redshift}}{(\mathbf{a}\cdot\mathbf{x})^2} - \overset{\text{rotation t-dilate}}{(\boldsymbol{\Omega}\times\mathbf{x})^2} + R_{0i0j}x^ix^j\right)$$
$$+ 2dtdx^i\left(\overset{\text{Sagnac effect}}{\epsilon_{ijk}\Omega^jx^k} - \frac{2}{3}R_{0jik}x^jx^k\right) + dx^idx^j\left(\delta_{ij} - \frac{1}{3}R_{ikjl}x^kx^l\right)$$

Gravitational wave in TT gauge

$$\partial_\mu h^{\mu\nu} = 0, \quad h_\mu{}^\mu = 0, \quad h_{00} = h_{0i} = 0$$

Riemann takes simple form

$$\boxed{R_{0i0j} = -\frac{1}{2}\ddot{h}_{ij}^{\mathrm{TT}}}$$

Framing the question

Proper detector frame

$$ds^2 \simeq -dt^2\left(1 + \overset{\textit{inert. acc.}}{2\mathbf{a}\cdot\mathbf{x}} + \overset{\textit{grav. redshift}}{(\mathbf{a}\cdot\mathbf{x})^2} - \overset{\textit{rotation t-dilate}}{(\boldsymbol{\Omega}\times\mathbf{x})^2} + \boxed{R_{0i0j}x^i x^j}\right)$$
$$+ 2dt\,dx^i\left(\overset{\textit{Sagnac effect}}{\epsilon_{ijk}\Omega^j x^k} - \frac{2}{3}R_{0jik}x^j x^k\right) + dx^i dx^j\left(\delta_{ij} - \frac{1}{3}R_{ikjl}x^k x^l\right)$$

Gravitational wave in TT gauge

$$\partial_\mu h^{\mu\nu} = 0, \quad h_\mu{}^\mu = 0, \quad h_{00} = h_{0i} = 0$$

Riemann takes simple form

$$\boxed{R_{0i0j} = -\frac{1}{2}\ddot{h}_{ij}^{\mathrm{TT}}}$$

Framing the question

Proper detector frame

$$ds^2 \simeq -dt^2\left(1 - \frac{1}{2}\ddot{h}_{ij}^{\mathrm{TT}} x^i x^j\right) + dx^i dx^i$$

Framing the question

Proper detector frame

$$ds^2 \simeq -dt^2\left(1 - \frac{1}{2}\ddot{h}_{ij}^{\mathrm{TT}} x^i x^j\right) + dx^i dx^i$$

Maxwell's new and improved equations

$$\nabla \cdot \mathbf{E} = \rho(1 - h_{00}) + \nabla h_{00} \cdot \mathbf{E}$$

$$\nabla \times \mathbf{B} - \partial_t \mathbf{E} = \mathbf{J} + \partial_t (h_{00} \mathbf{E})$$

Framing the question

Proper detector frame

$$ds^2 \simeq -dt^2\left(1 - \frac{1}{2}\ddot{h}_{ij}^{\mathrm{TT}} x^i x^j\right) + dx^i dx^i$$

Maxwell's new and improved equations

$$\nabla \cdot \mathbf{E} = \rho(1 - h_{00}) + \nabla h_{00} \cdot \mathbf{E}$$

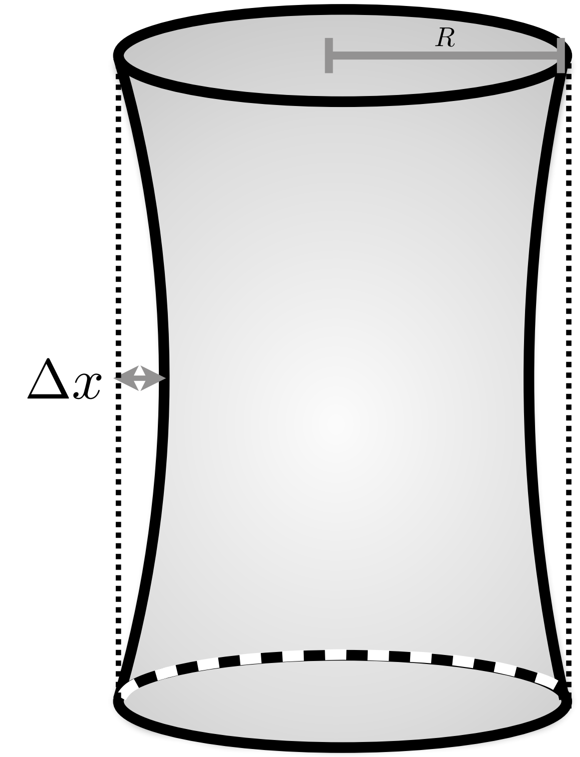
$$\nabla \times \mathbf{B} - \partial_t \mathbf{E} = \mathbf{J} + \partial_t(h_{00}\mathbf{E})$$

Generation of EM wave from GW and background field:

$$\Box \mathbf{E} = -\partial_t^2(h_{00}\mathbf{E}_0)$$

GW interaction w/ Cavity Walls

Indirect effect: GWs perturb cavity walls Δx

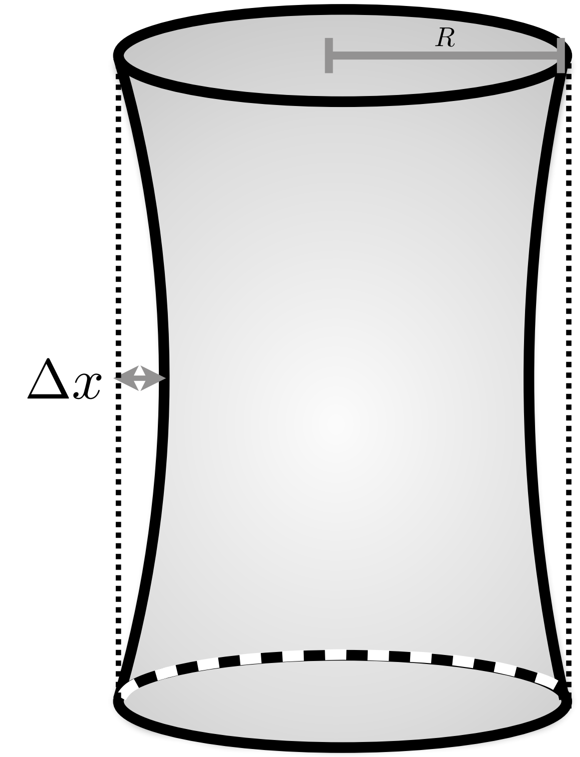


GW interaction w/ Cavity Walls

Indirect effect: GWs perturb cavity walls Δx

Cavity modes dependent on geometry

Small perturbations: $\omega_c \to \omega_c(1 + f(\Delta x))$

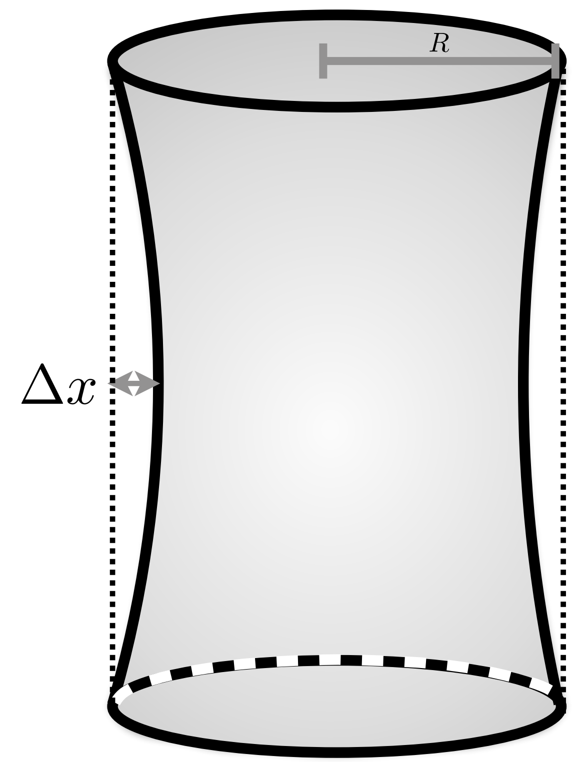


GW interaction w/ Cavity Walls

Indirect effect: GWs perturb cavity walls Δx

Cavity modes dependent on geometry

Small perturbations: $\omega_c \to \omega_c(1 + f(\Delta x))$



Proper detector frame, effect of GW is that of Newtonian force on a test mass:

$$F_i \simeq \frac{m}{2} \ddot{h}_{ij}^{\mathrm{TT}} x^j$$

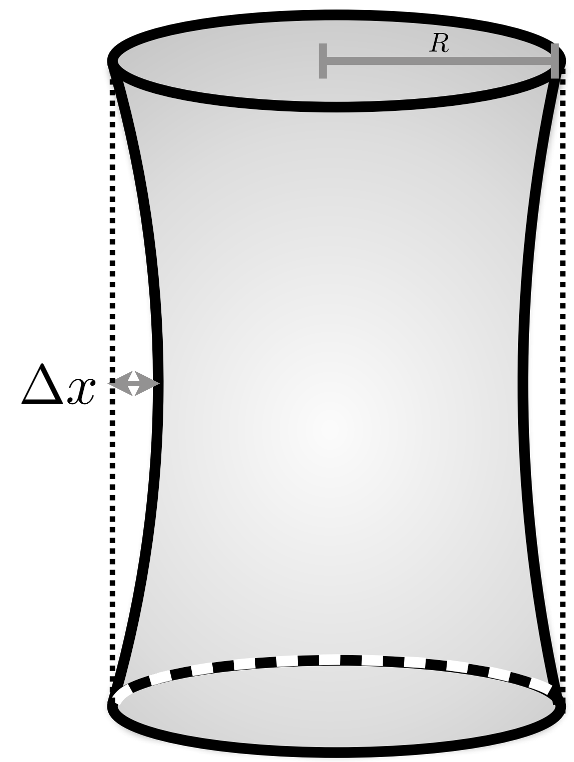
Passing gravitational wave will move walls, spreading power in frequency space

GW interaction w/ Cavity Walls

Indirect effect: GWs perturb cavity walls Δx

Cavity modes dependent on geometry

Small perturbations: $\omega_c \to \omega_c(1 + f(\Delta x))$



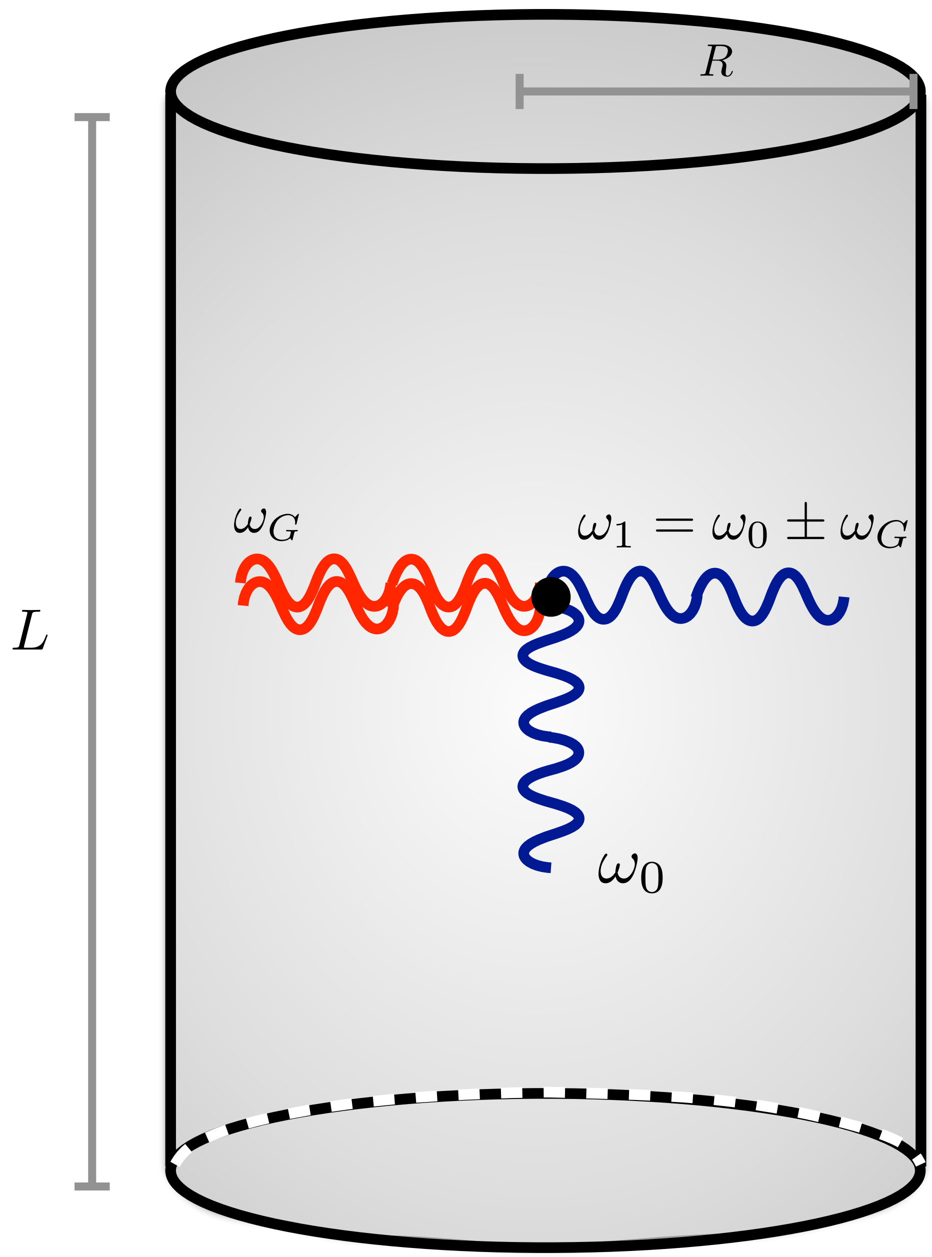
Proper detector frame, effect of GW is that of Newtonian force on a test mass:

$$F_i \simeq \frac{m}{2} \ddot{h}_{ij}^{\mathrm{TT}} x^j$$

Passing gravitational wave will move walls, spreading power in frequency space

Focus of MAGO collaboration @ CERN in early 2000s – *e.g. gr-qc/0502054*

Gravity Wave Resonant Frequency Conversion



Cylinder for illustrative purposes only!

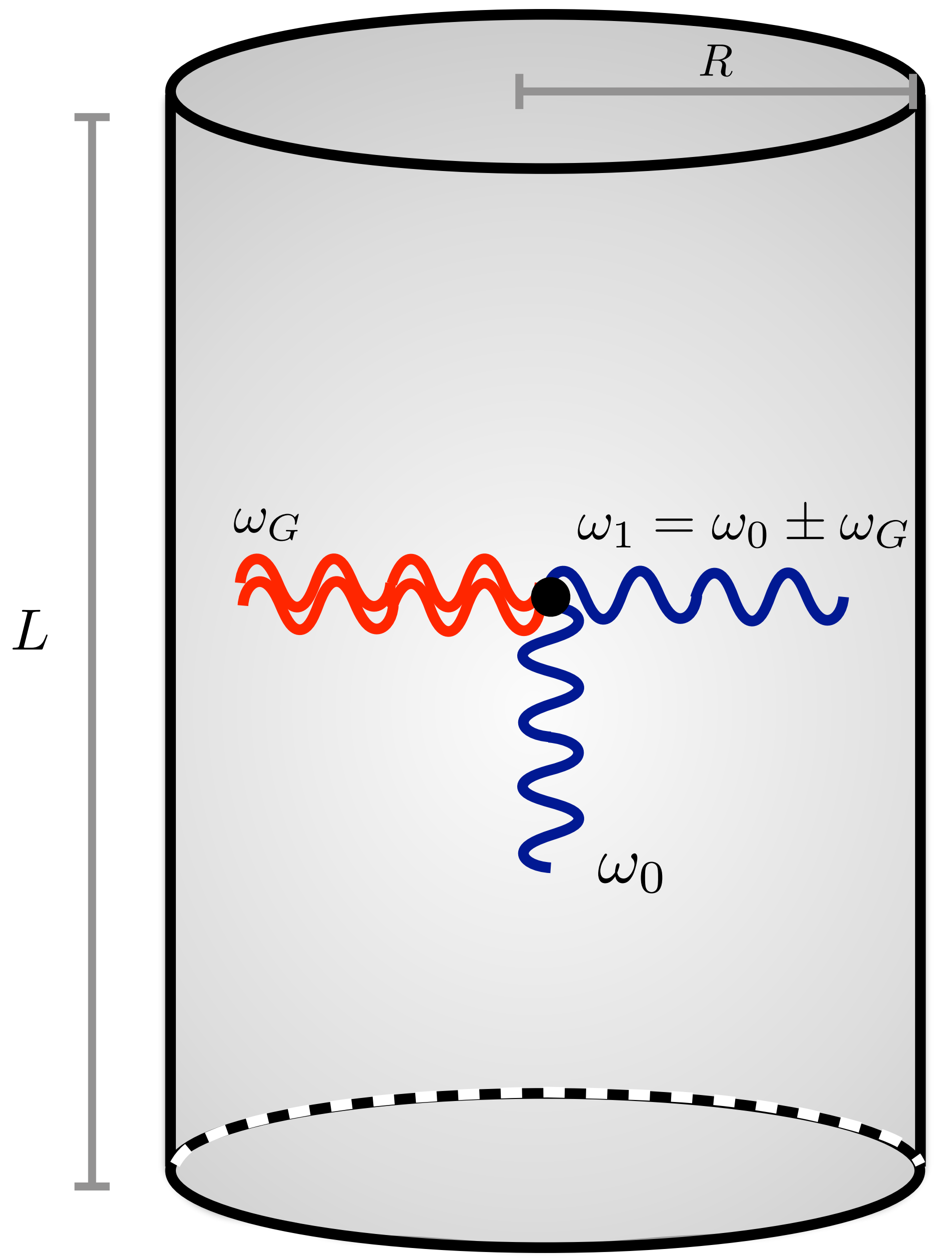
Superconducting RF Cavity

$$\omega_i \sim \mathrm{GHz}$$

$$Q_{\mathrm{int}} \sim 10^9 \div 10^{13}$$

Fields must have quadrupole moment

Gravity Wave Resonant Frequency Conversion



Cylinder for illustrative purposes only!

Superconducting RF Cavity

$$\omega_i \sim \mathrm{GHz}$$

$$Q_{\mathrm{int}} \sim 10^9 \div 10^{13}$$

Tunability:

$$\delta\omega \lesssim \ \mathrm{MHz} \quad \text{piezos}$$

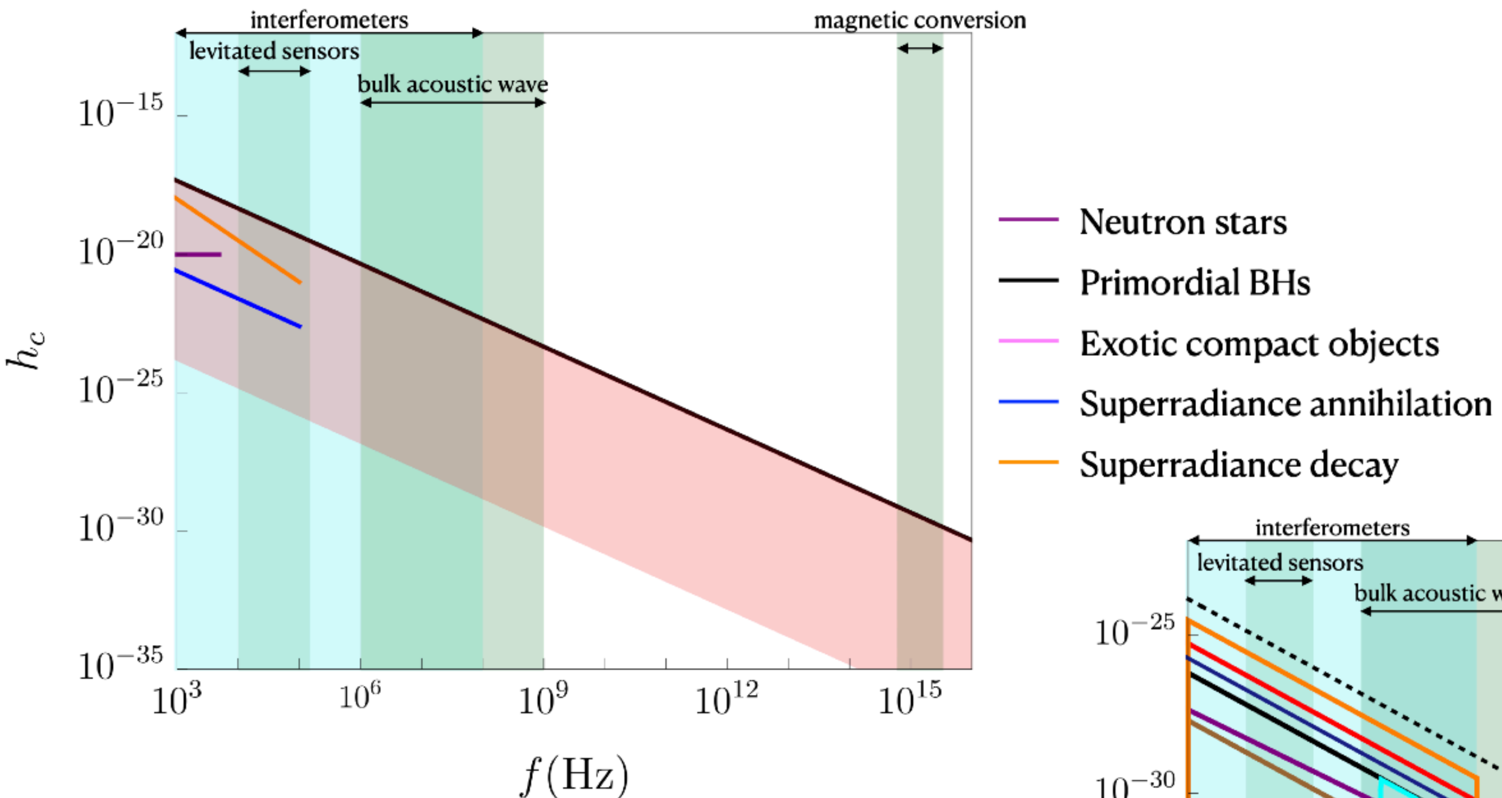
$$\delta\omega \gtrsim \ \mathrm{MHz} \quad \text{fins}$$

Fields must have quadrupole moment

Gravity Wave Resonant Frequency Conversion

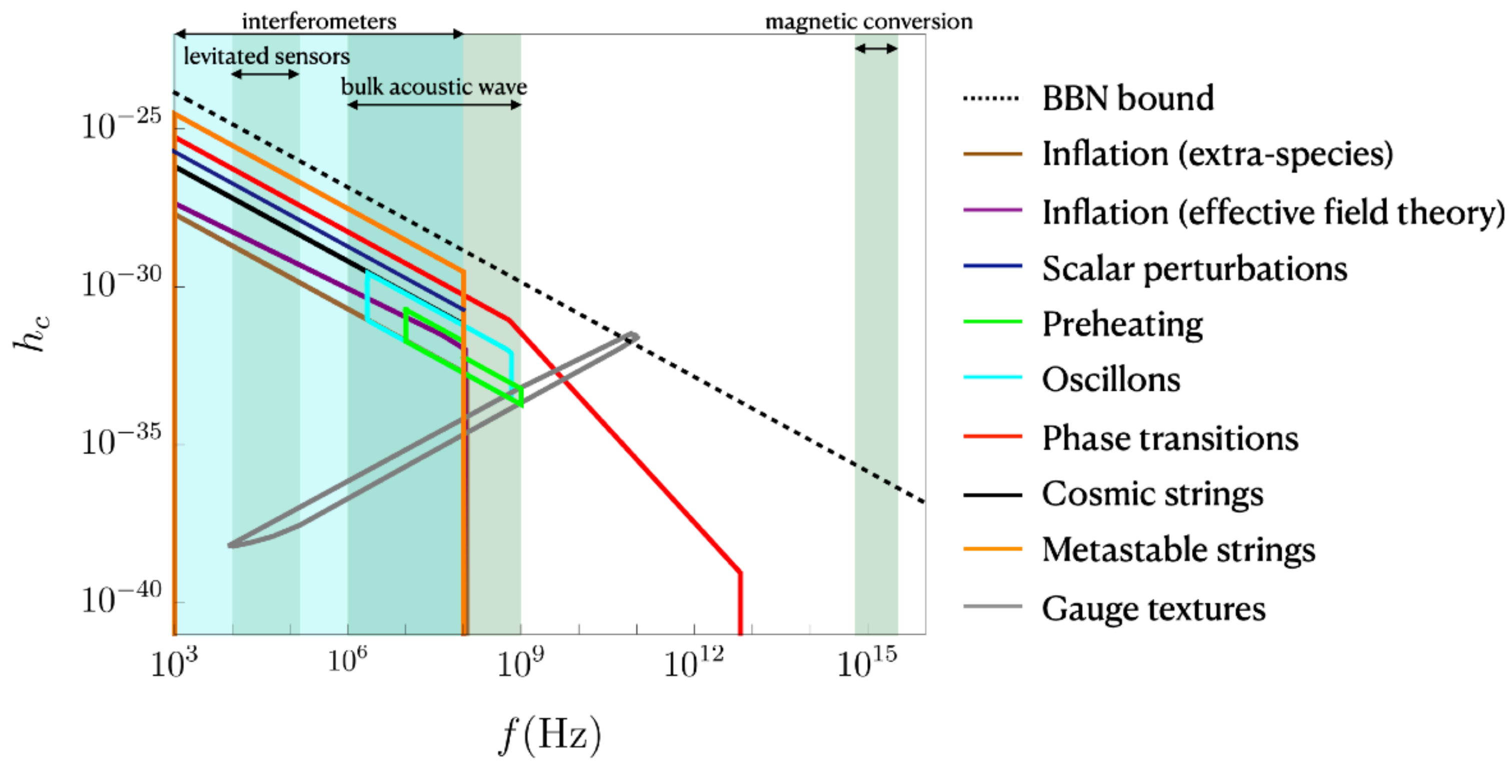
Superconducting RF Cavity $\omega_i \sim \mathrm{GHz}$

What about sources in MHz to GHz range?



Coherent

Stochastic



Aggarwal et al, 2011.12414

Experimental context

Technical concept	Frequency	Proposed sensitivity (dimensionless)	Proposed sensitivity $\sqrt{S_n(f)}$
Spherical resonant mass, Sec. 4.1.3 [282]			
Mini-GRAIL (built) [289]	2942.9 Hz	10^{-20} $2.3 \cdot 10^{-23}\,(*)$	$5 \cdot 10^{-20}\,\mathrm{Hz}^{-\frac{1}{2}}$ $10^{-22}\,\mathrm{Hz}^{-\frac{1}{2}}\,(*)$
Schenberg antenna (built) [286]	3.2 kHz	$2.6 \cdot 10^{-20}$ $2.4 \cdot 10^{-23}\,(*)$	$1.1 \cdot 10^{-19}\,\mathrm{Hz}^{-\frac{1}{2}}$ $10^{-22}\,\mathrm{Hz}^{-\frac{1}{2}}\,(*)$
Laser interferometers			
NEMO (devised), Sec. 4.1.1 [25, 272]	$[1-2.5]$ kHz	$9.4 \cdot 10^{-26}$	$10^{-24}\,\mathrm{Hz}^{-\frac{1}{2}}$
Akutsu's proposal (built), Sec. 4.1.2 [277, 328]	100 MHz	$7 \cdot 10^{-14}$ $2 \cdot 10^{-19}\,(*)$	$10^{-16}\,\mathrm{Hz}^{-\frac{1}{2}}$ $10^{-20}\,\mathrm{Hz}^{-\frac{1}{2}}\,(*)$
Holometer (built), Sec. 4.1.2 [279]	$[1-13]$ MHz	$8 \cdot 10^{-22}$	$10^{-21}\,\mathrm{Hz}^{-\frac{1}{2}}$
Optically levitated sensors, Sec. 4.2.1 [59]			
1-meter prototype (under construction)	$(10-100)$ kHz	$2.4 \cdot 10^{-20} - 4.2 \cdot 10^{-22}$	$(10^{-19} - 10^{-21})\,\mathrm{Hz}^{-\frac{1}{2}}$
100-meter instrument (devised)	$(10-100)$ kHz	$2.4 \cdot 10^{-22} - 4.2 \cdot 10^{-24}$	$(10^{-21} - 10^{-23})\,\mathrm{Hz}^{-\frac{1}{2}}$

Aggarwal et al, 2011.12414

Experimental context

Resonant polarization rotation, Sec. 4.2.4 [307]			
Cruise's detector (devised) [308]	$(0.1 - 10^5)$ GHz	$h \simeq 10^{-17}$	×
Cruise & Ingley's detector (prototype) [309, 310]	100 MHz	$8.9 \cdot 10^{-14}$	$10^{-14}\,\mathrm{Hz}^{-\frac{1}{2}}$
Enhanced magnetic conversion (theory), Sec. 4.2.5 [311]	5 GHz	$h \simeq 10^{-30} - 10^{-26}$	×
Bulk acoustic wave resonators (built), Sec. 4.2.6 [316, 317]	(MHz – GHz)	$4.2 \cdot 10^{-21} - 2.4 \cdot 10^{-20}$	$10^{-22}\,\mathrm{Hz}^{-\frac{1}{2}}$
Superconducting rings, (theory), Sec. 4.2.7 [318]	10 GHz	$h_{0,n,\mathrm{mono}} \simeq 10^{-31}$	×
Microwave cavities, Sec. 4.2.8			
Caves' detector (devised) [320]	500 Hz	$h \simeq 2 \cdot 10^{-21}$	×
Reece's 1st detector (built) [321]	1 MHz	$h \simeq 4 \cdot 10^{-17}$	×
Reece's 2nd detector (built) [322]	10 GHz	$h \simeq 6 \cdot 10^{-14}$	×
Pegoraro's detector (devised) [323]	$(1 - 10)$ GHz	$h \simeq 10^{-25}$	×
Graviton-magnon resonance (theory), Sec. 4.2.9 [324]	$(8 - 14)$ GHz	$9.1 \cdot 10^{-17} - 1.1 \cdot 10^{-15}$	$(10^{-22} - 10^{-20})\,\mathrm{Hz}^{-\frac{1}{2}}$

Table 1: Summary of existing and proposed detectors with their respective sensitivities. See Sec. 4.3 for details.

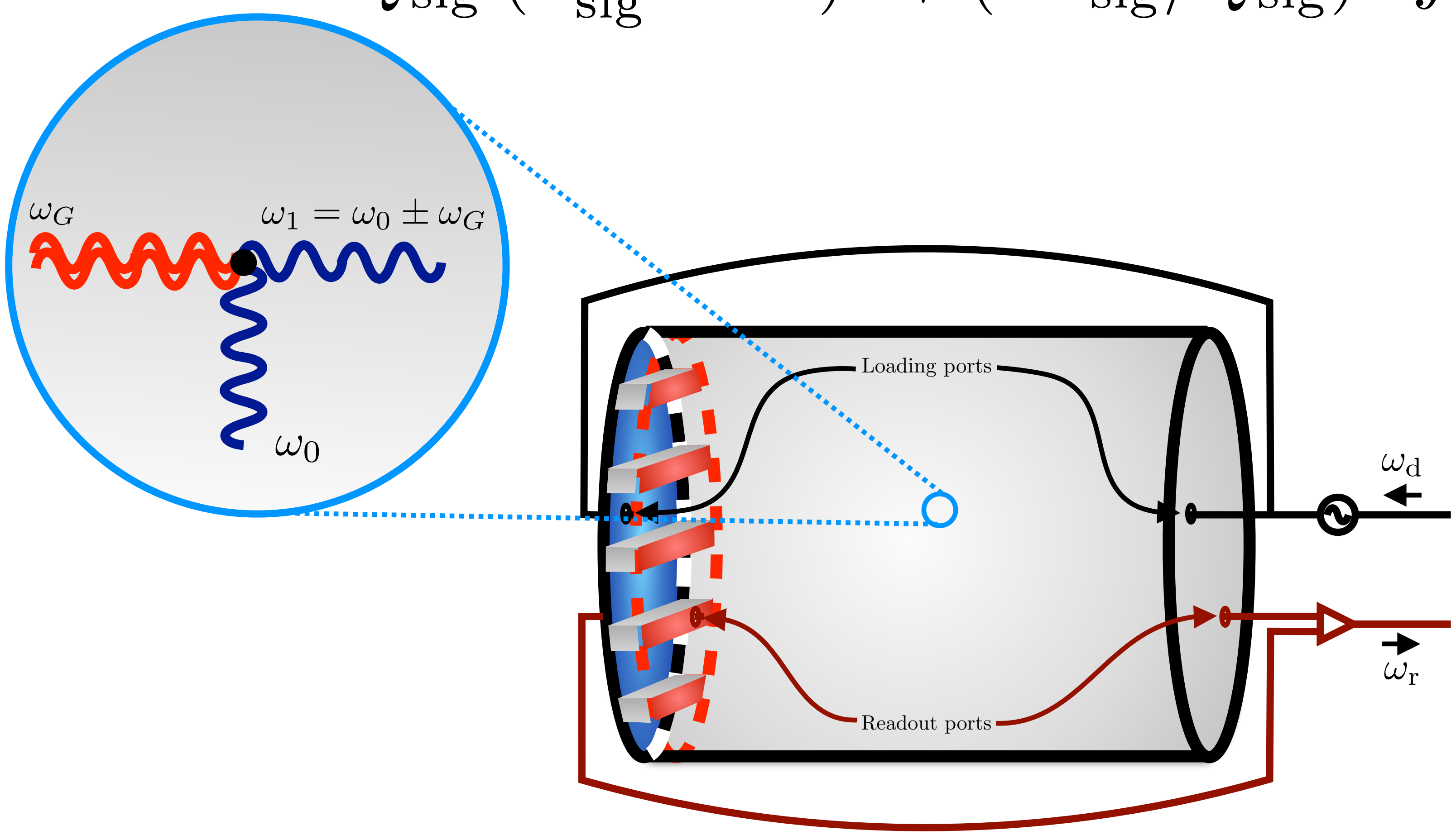
Aggarwal et al, 2011.12414

Gravitational wave signal

Power Spectral Density:

$$S_{\mathrm{sig}}(\omega) = \frac{\omega_{\mathrm{sig}}}{Q_{\mathrm{sig}}} \frac{\omega^4(\eta E_0 h_0)^2 V}{(\omega_{\mathrm{sig}}^2 - \omega^2)^2 + (\omega\omega_{\mathrm{sig}}/Q_{\mathrm{sig}})^2} \int \frac{d\omega'}{2\pi} S_H(\omega - \omega') S_{e_0}(\omega')$$

Convolution of GW PSD and loaded mode PSD

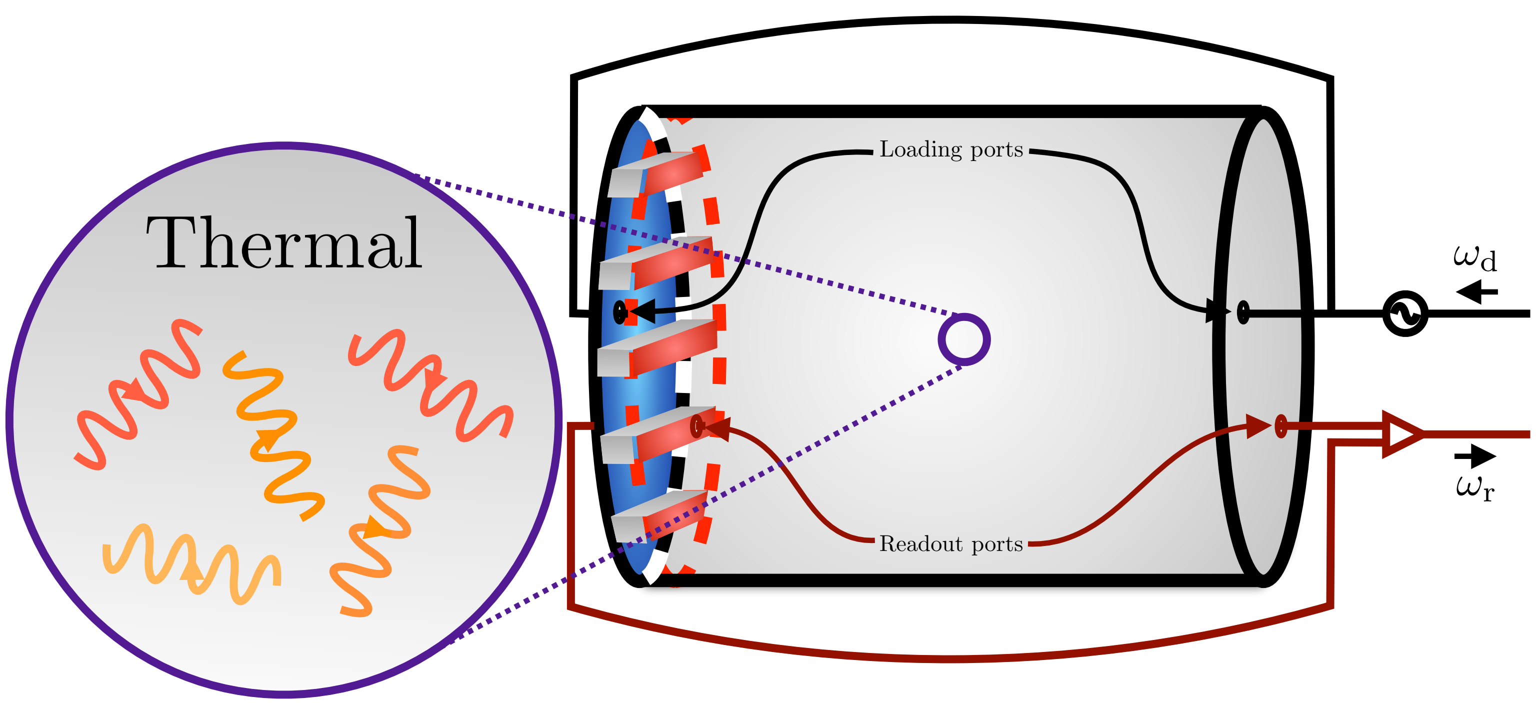


Signal PSD will depend on spectrum of GW source e.g. monochromatic, flat, neither…

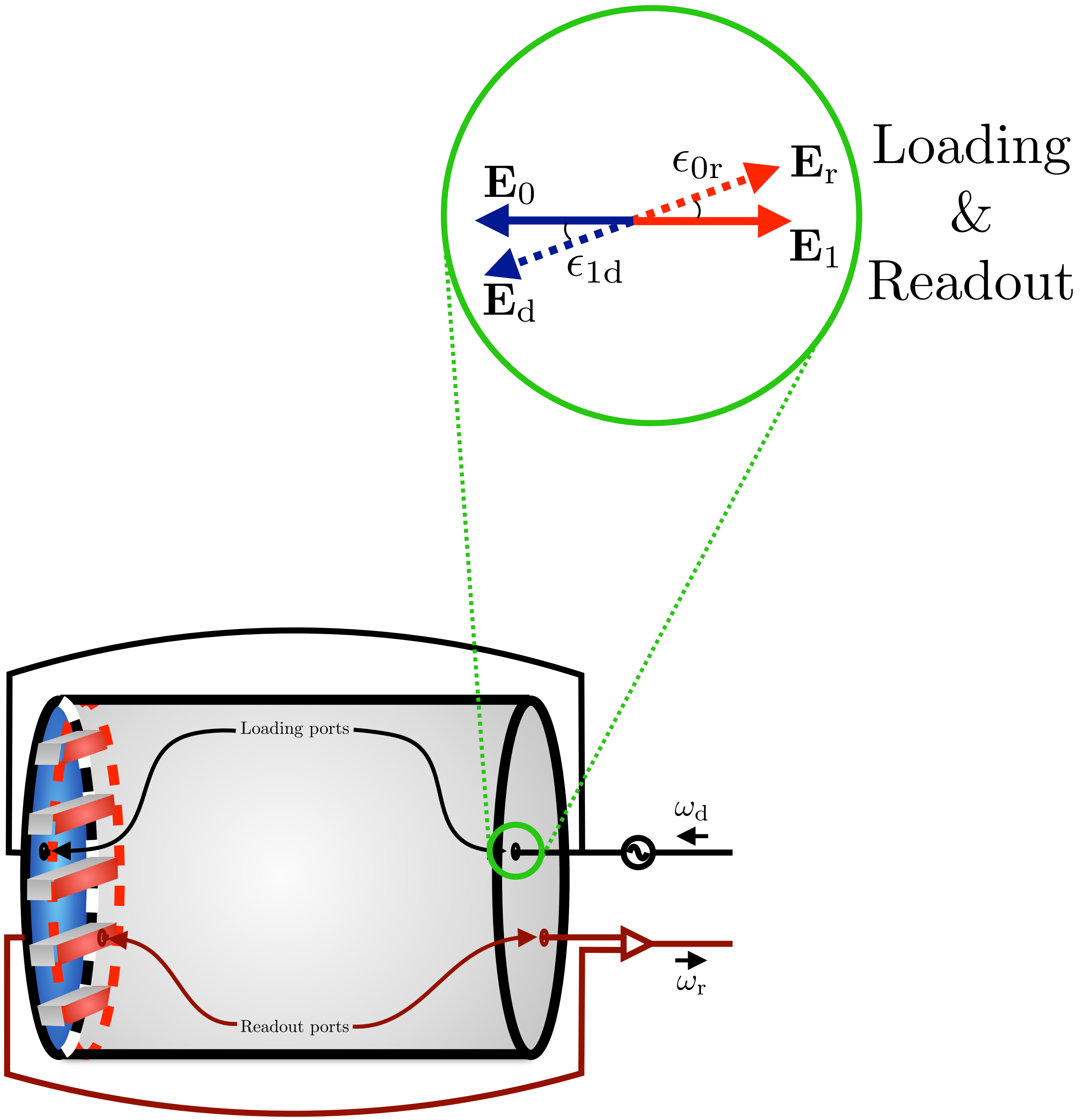
Standard Noise Sources: Thermal Noise

Power Spectral Density:

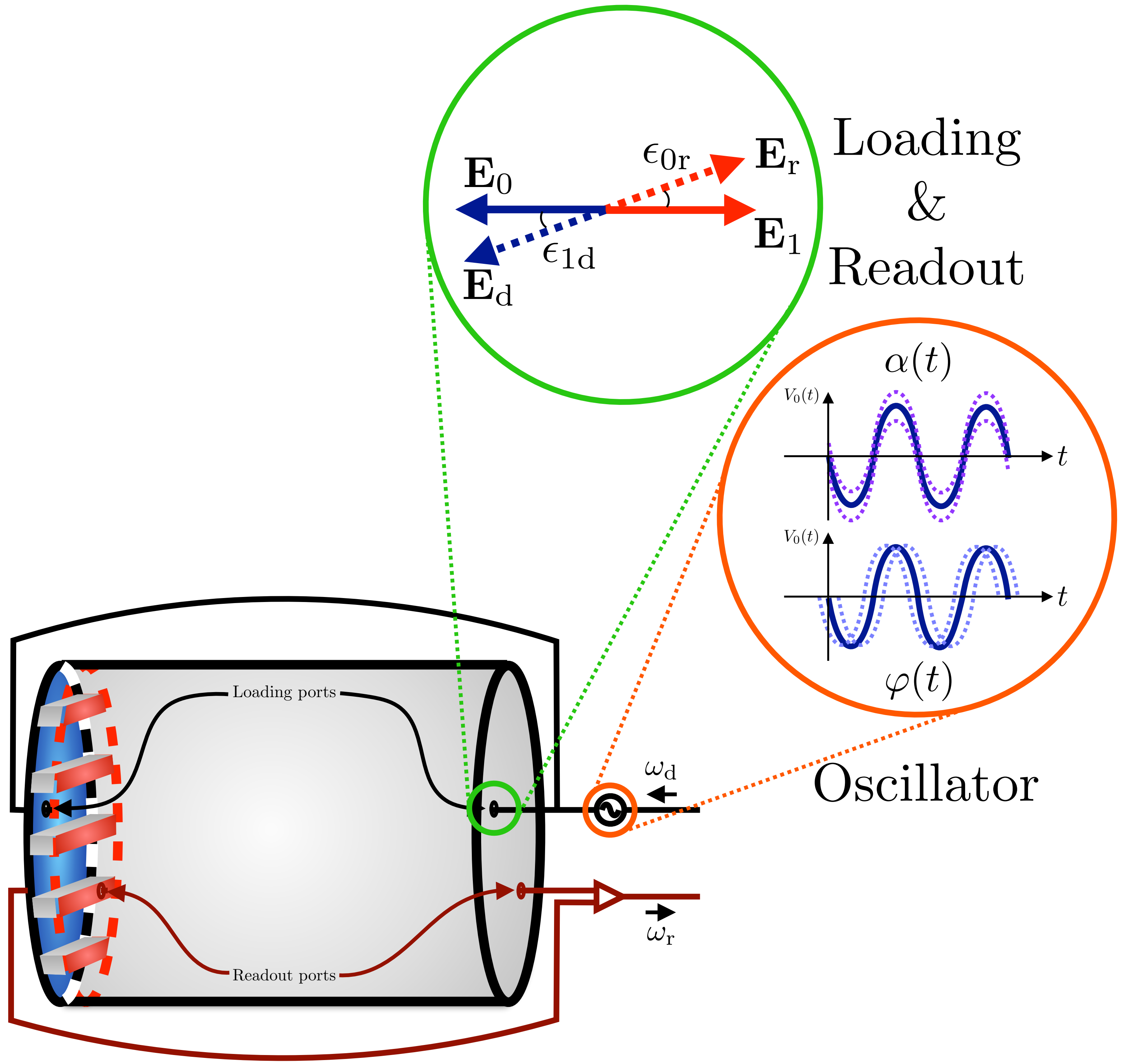
$$S_{\rm th}(\omega) = \frac{Q_1}{Q_{\rm int}} \frac{4\pi T \, (\omega\,\omega_1/Q_1)^2}{(\omega^2 - \omega_1^2)^2 + (\omega\,\omega_1/Q_1)^2}$$



Non-standard Noise Sources: Phase Noise



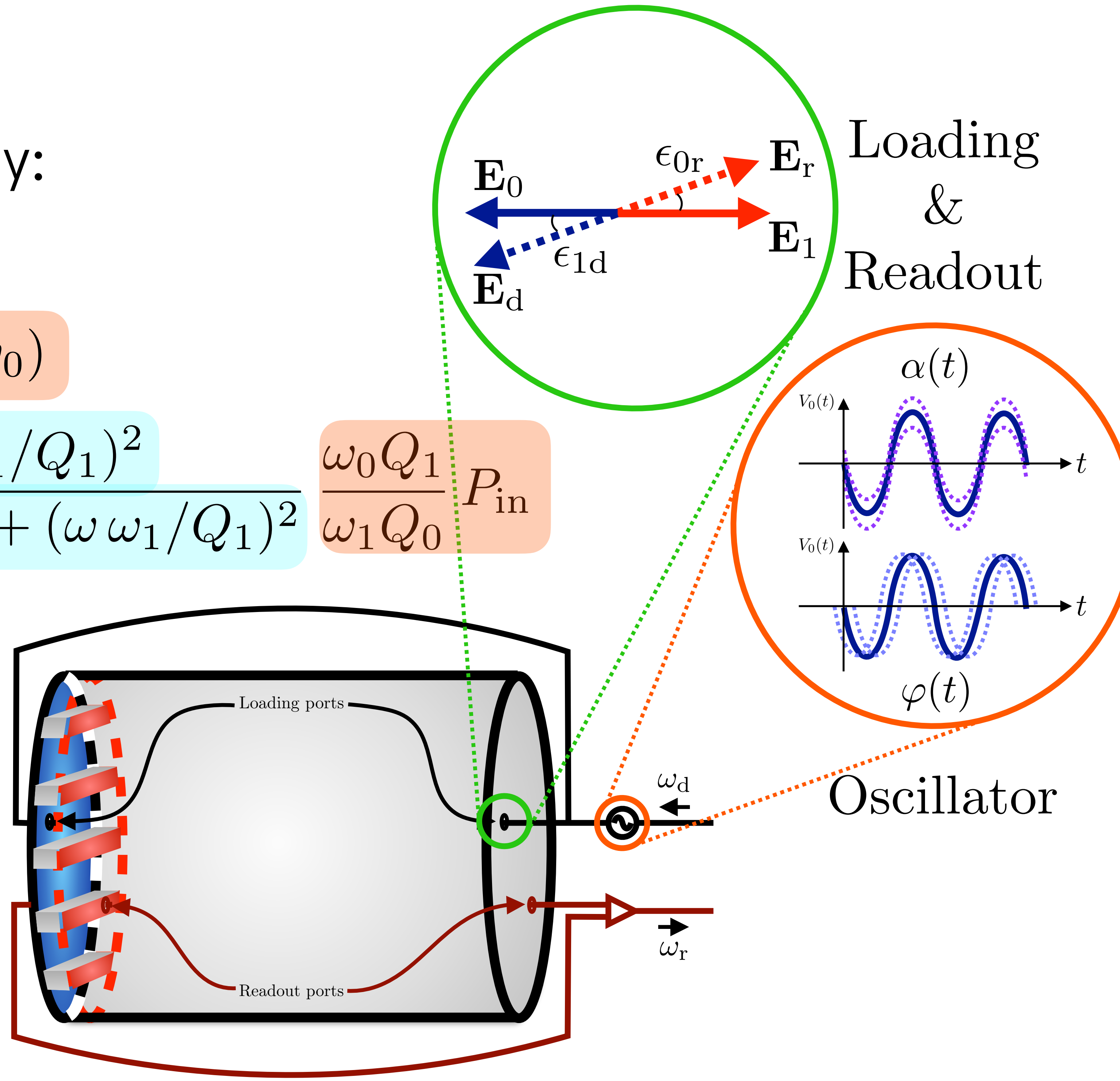
Non-standard Noise Sources: Phase Noise



Non-standard Noise Sources: Phase Noise

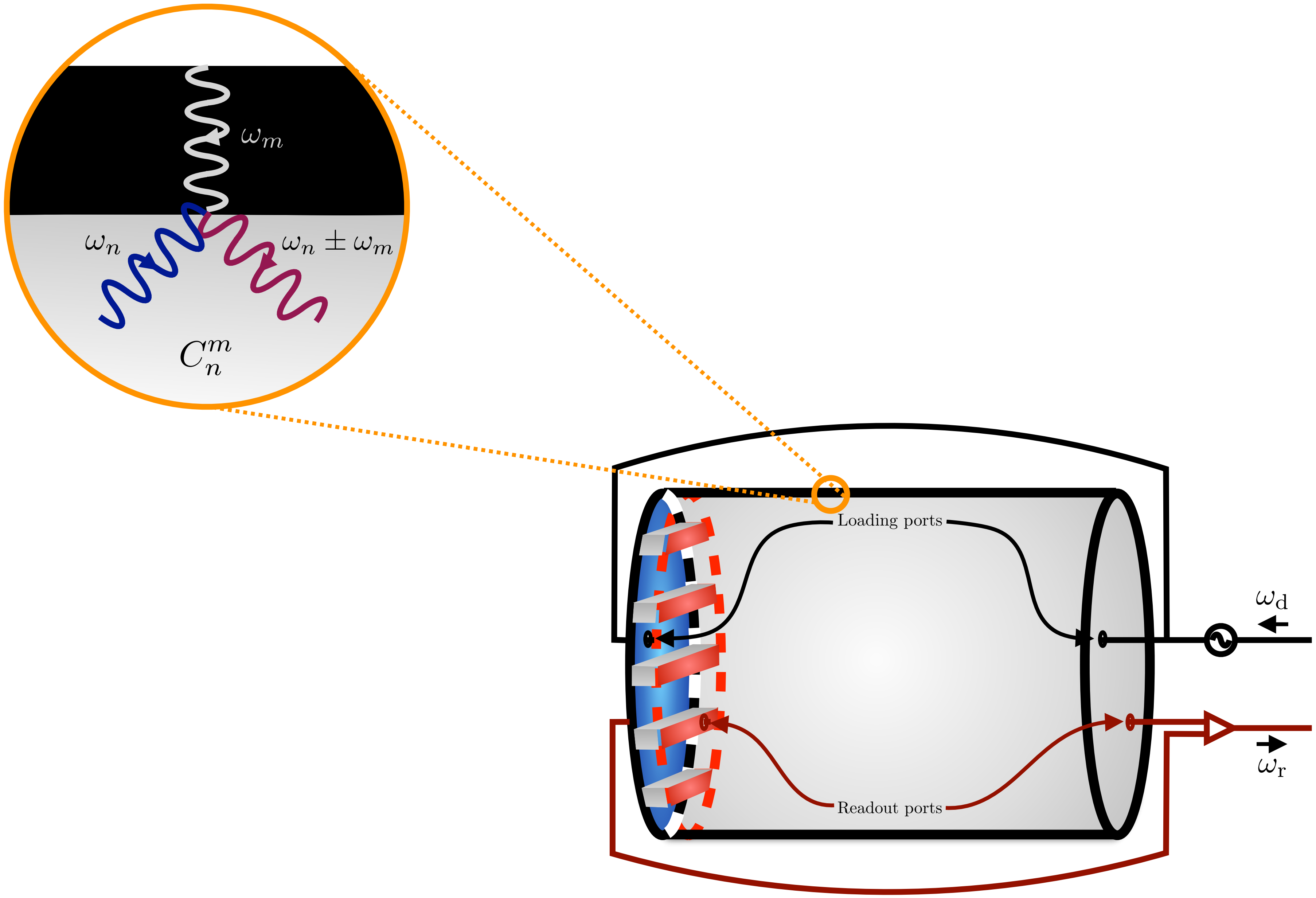
Power Spectral Density:

$$S_{\text{phase}}(\omega) \simeq \frac{1}{2}\,\epsilon_{1\text{d}}^2\, S_\varphi(\omega-\omega_0) \times \frac{(\omega\,\omega_1/Q_1)^2}{(\omega^2-\omega_1^2)^2+(\omega\,\omega_1/Q_1)^2}\,\frac{\omega_0 Q_1}{\omega_1 Q_0} P_{\text{in}}$$



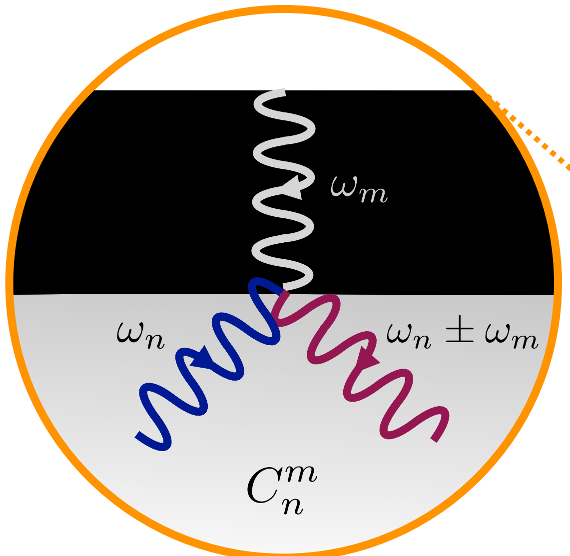
Non-standard Noise Sources: Wall Vibrations

Vibrations



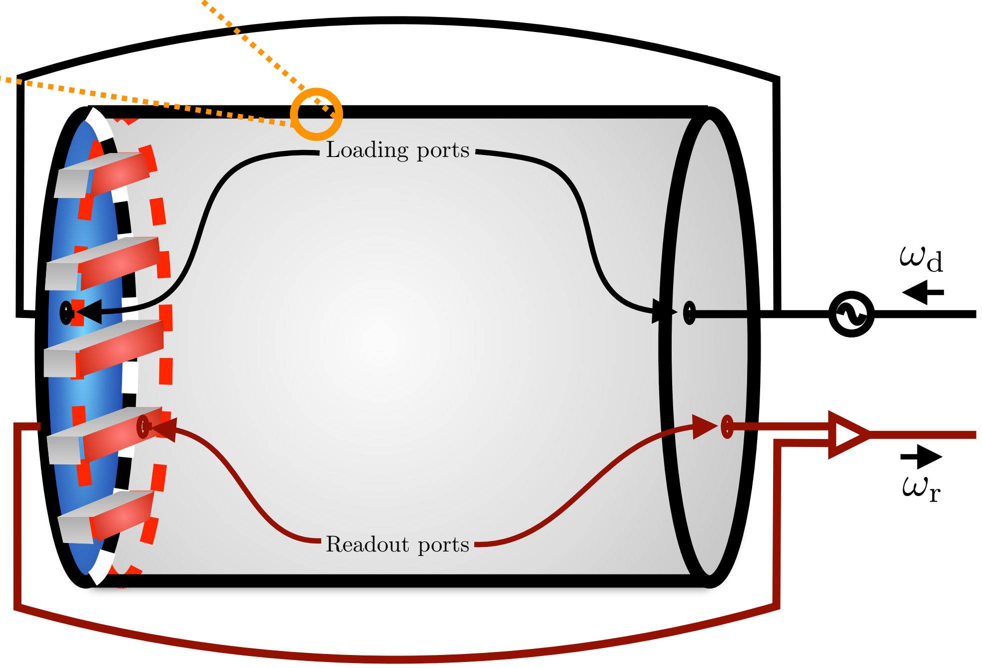
Non-standard Noise Sources: Wall Vibrations

Vibrations



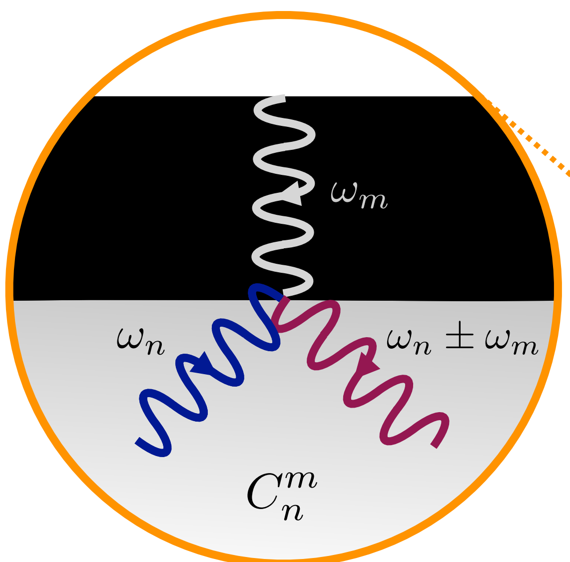
Power Spectral Density:

$$S_{\rm mech}(\omega) \simeq \frac{\epsilon_{1d}^2}{4} \frac{\omega_0}{Q_0} P_{\rm in} \times \sum_{n=0,1} \frac{\left(S_{q_m}(\omega-\omega_0)/V^{2/3}\right)(\omega_n/Q_n)\,\omega_n^4\,\omega^2}{\left[(\omega^2-\omega_n^2)^2+(\omega\,\omega_n/Q_n)^2\right]\left[(\omega_0^2-\omega_n^2)^2+(\omega_0\,\omega_n/Q_n)^2\right]}$$



Non-standard Noise Sources: Wall Vibrations

Vibrations



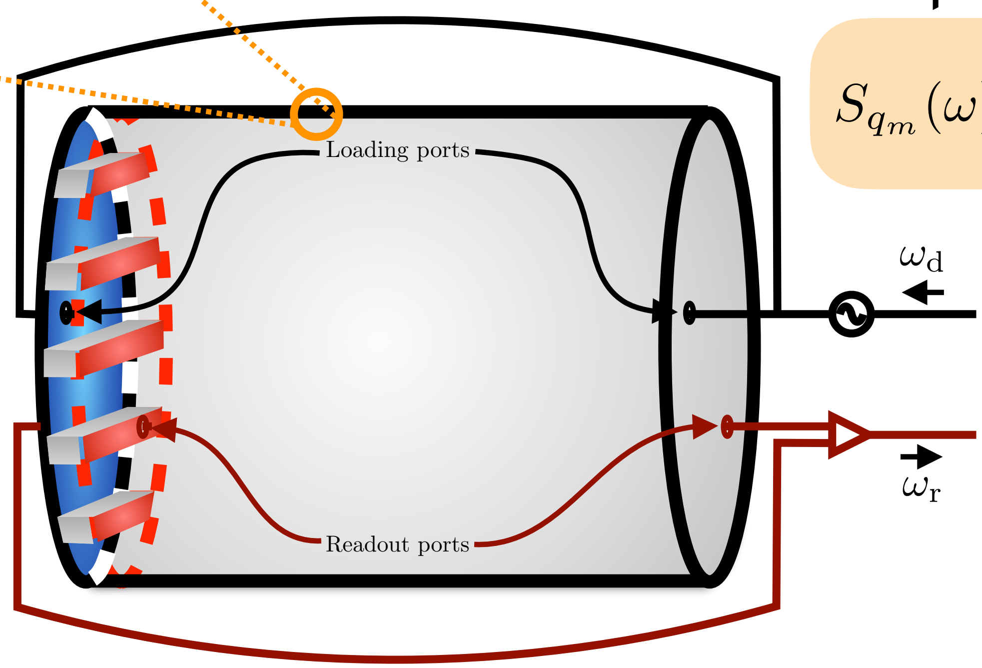
Power Spectral Density:

$$S_{\rm mech}(\omega) \simeq \frac{\epsilon_{1d}^2}{4} \frac{\omega_0}{Q_0} P_{\rm in} \times$$

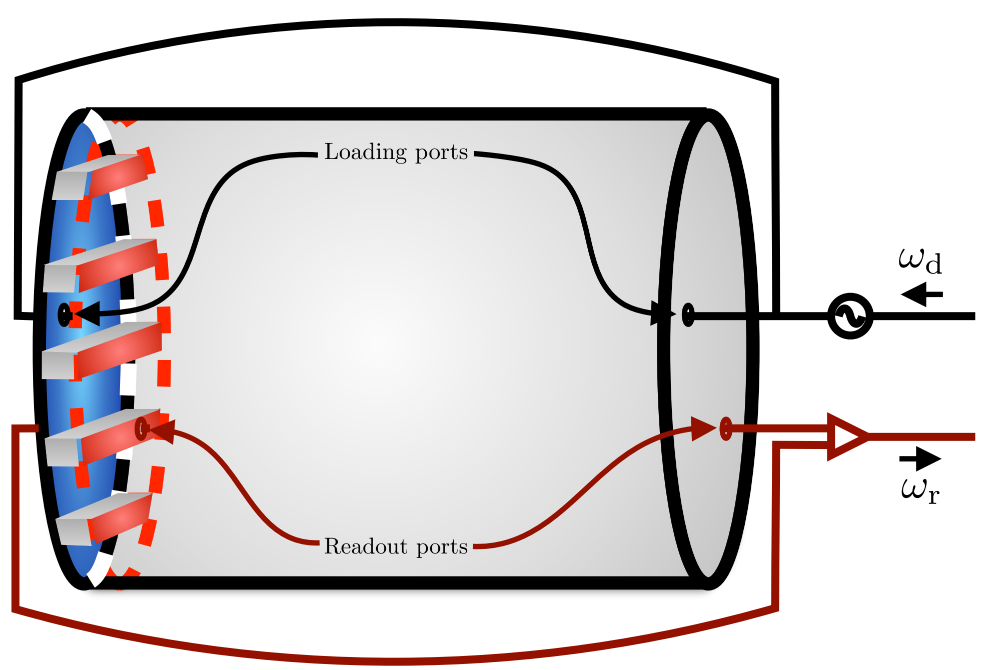
$$\sum_{n=0,1} \frac{\left(S_{q_m}(\omega-\omega_0)/V^{2/3}\right)(\omega_n/Q_n)\,\omega_n^4\,\omega^2}{\left[(\omega^2-\omega_n^2)^2+(\omega\,\omega_n/Q_n)^2\right]\left[(\omega_0^2-\omega_n^2)^2+(\omega_0\,\omega_n/Q_n)^2\right]}$$

Displacement PSD:

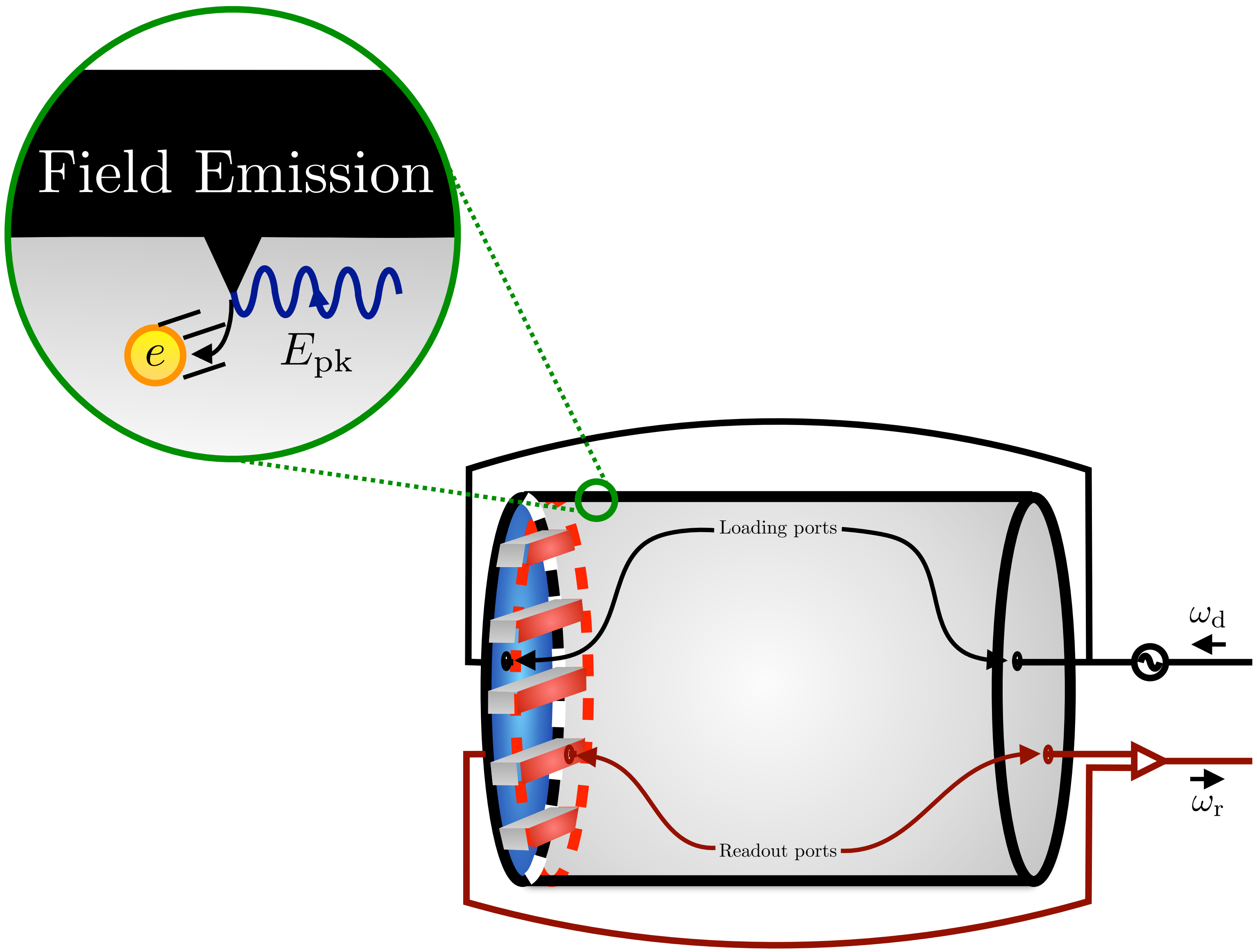
$$S_{q_m}(\omega) \simeq \frac{1}{M^2} \frac{S_{f_m}(\omega)}{(\omega^2-\omega_m^2)^2+(\omega_m\omega/Q_m)^2}$$



Non-standard Noise Sources: Field Emission



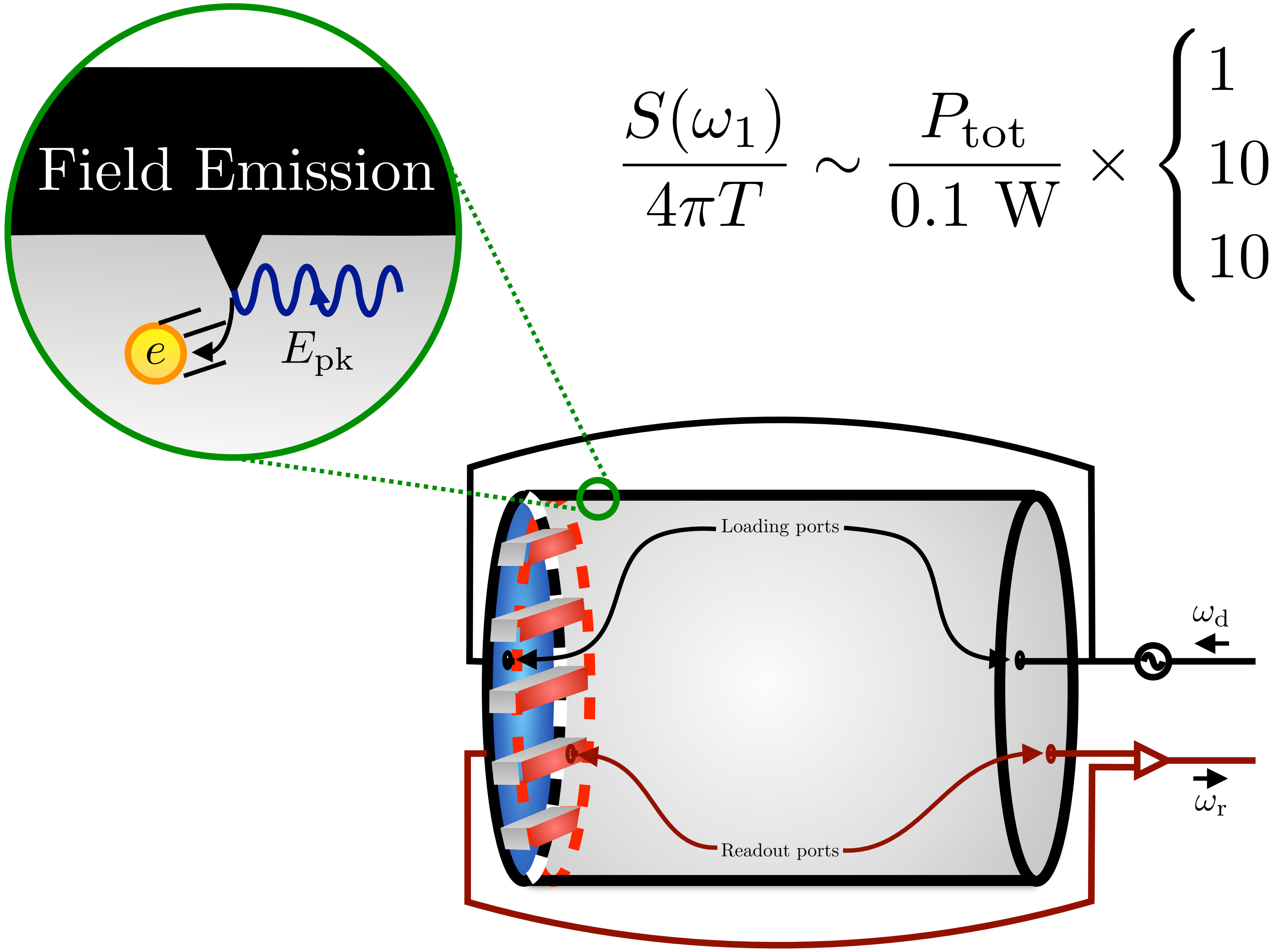
Non-standard Noise Sources: Field Emission



Non-standard Noise Sources: Field Emission

Power Spectral Density:

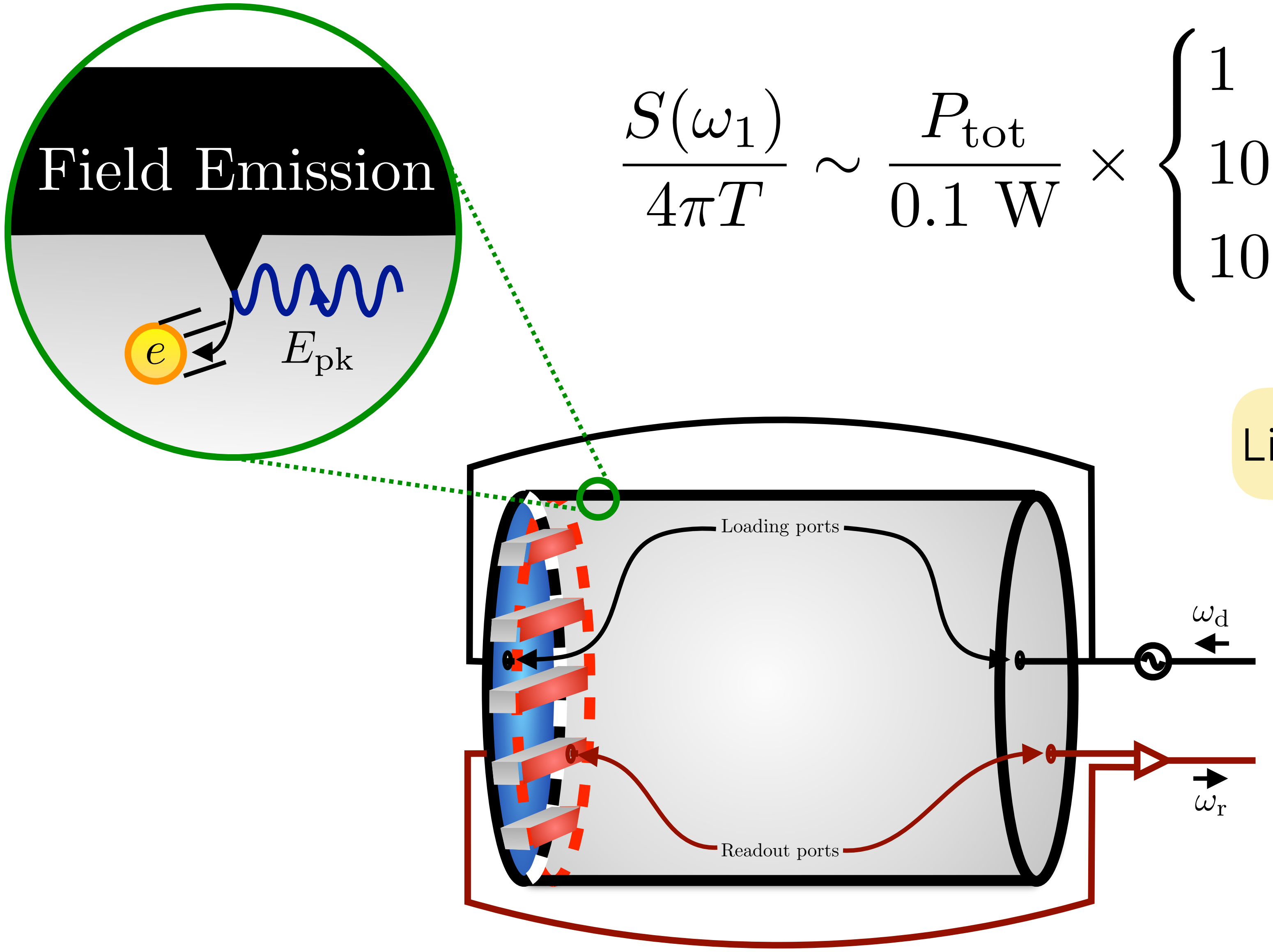
$$\frac{S(\omega_1)}{4\pi T} \sim \frac{P_{\rm tot}}{0.1~\rm W} \times \begin{cases} 1 & \text{synchrotron} \\ 10^{-6} & \text{transition} \\ 10^{-5} & \text{Bremsstrahlung}\,, \end{cases}$$



Non-standard Noise Sources: Field Emission

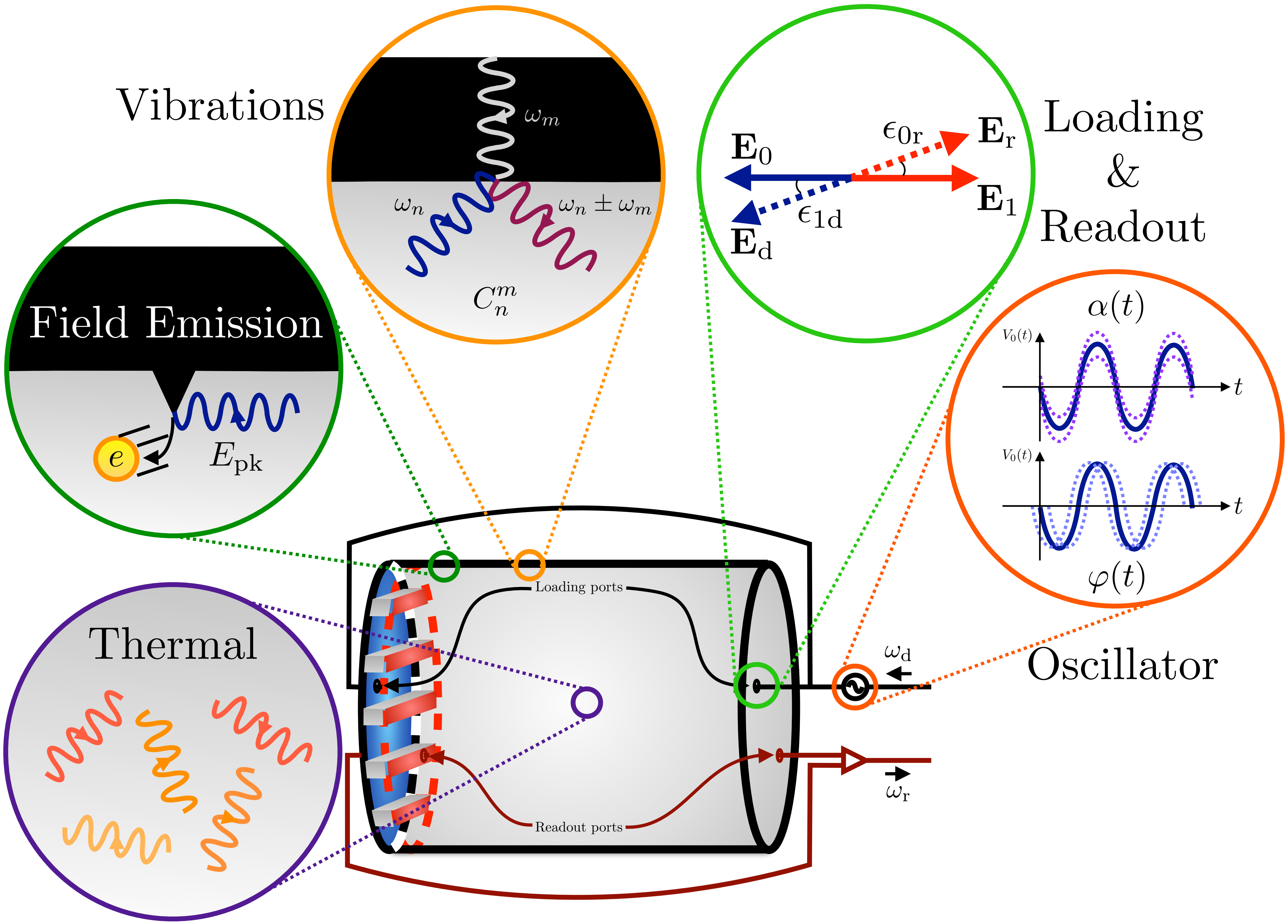
Power Spectral Density:

$$\frac{S(\omega_1)}{4\pi T} \sim \frac{P_{\rm tot}}{0.1\ {\rm W}} \times \begin{cases} 1 & \text{synchrotron} \\ 10^{-6} & \text{transition} \\ 10^{-5} & \text{Bremsstrahlung}\,, \end{cases}$$

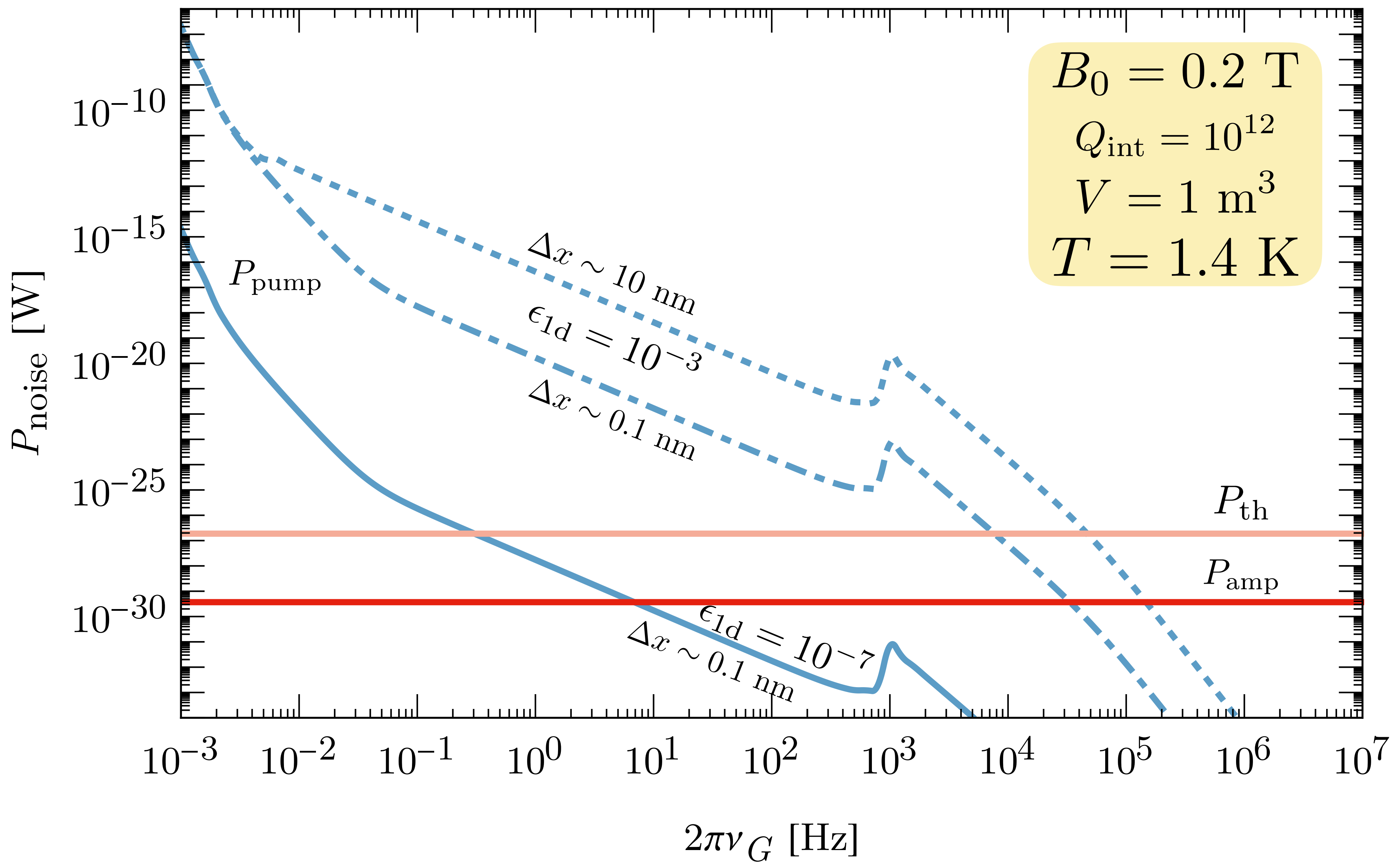


Limits max B-field ~ 0.2T

All Noise Sources



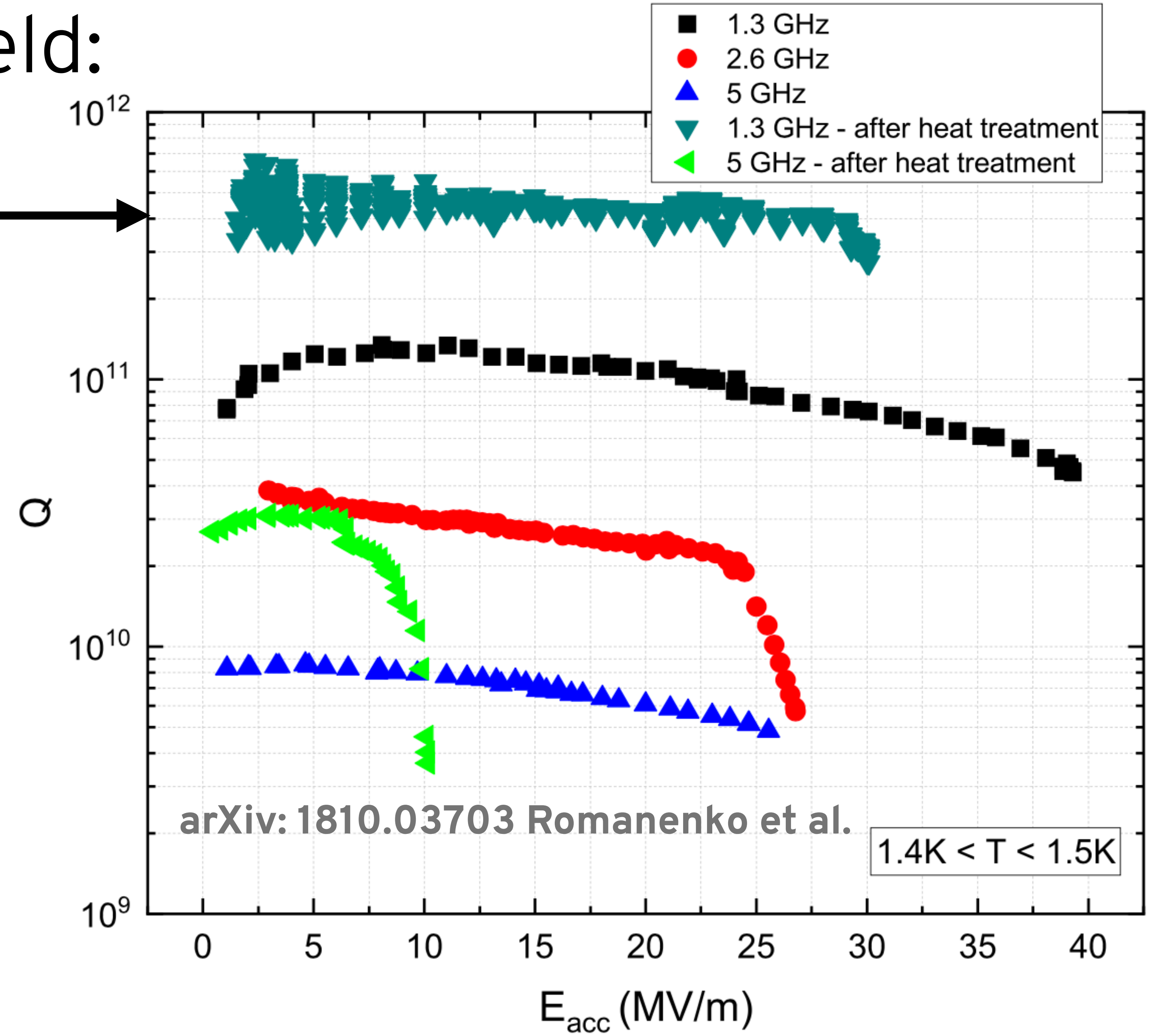
All Noise Sources



Experimental precedent

Q-factor & B-field:

Q~4x10[11] @ B~0.1T



Experimental precedent

Mode rejection:

Ɛ = 10^{-7} achieved

gr-qc/0502054 Ballantini et al.

physics/0004031 Bernard, Gemme, Parodi, Picasso

Low-frequency seismic noise:

$$\Delta\omega/\omega \sim \delta \sim 10^{-10}$$

DarkSRF (2020)

Experimental precedent

Mode rejection:

$\epsilon = 10^{-7}$ achieved

gr-qc/0502054 Ballantini et al.

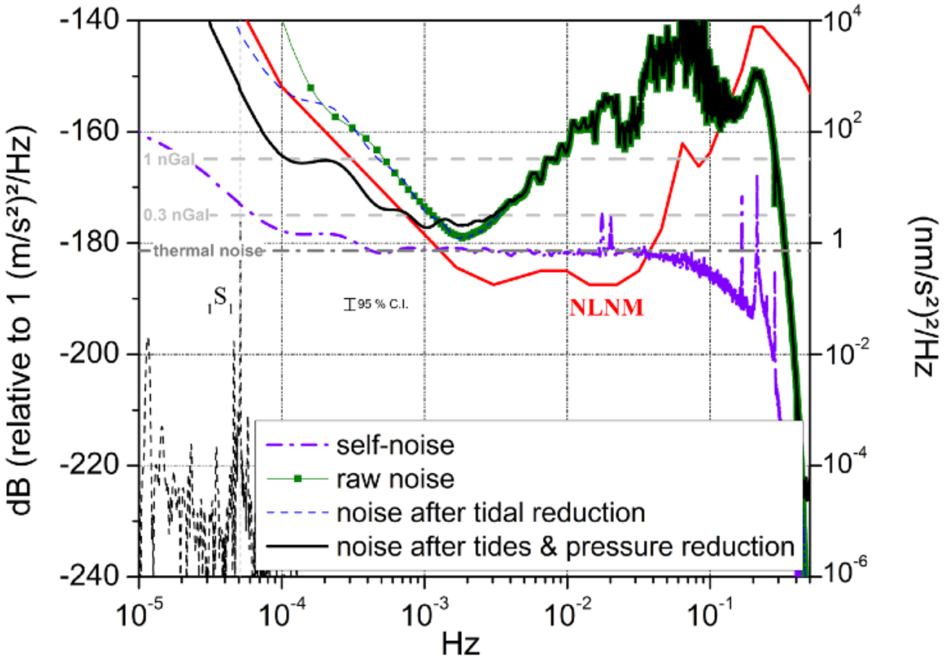
physics/0004031 Bernard, Gemme, Parodi, Picasso

Low-frequency seismic noise:

$$\Delta\omega/\omega \sim \delta \sim 10^{-10}$$

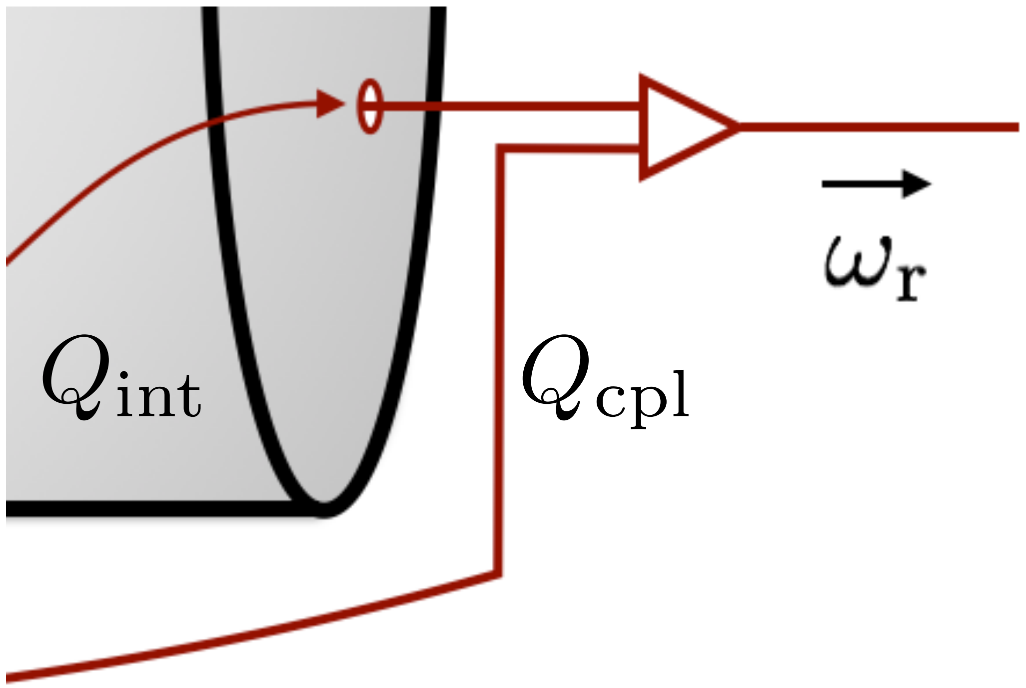
DarkSRF (2020)

Scientific Reports 8, 15324 (2018) Rosat & Hinderer



Signal to Noise: *readout & overcoupling*

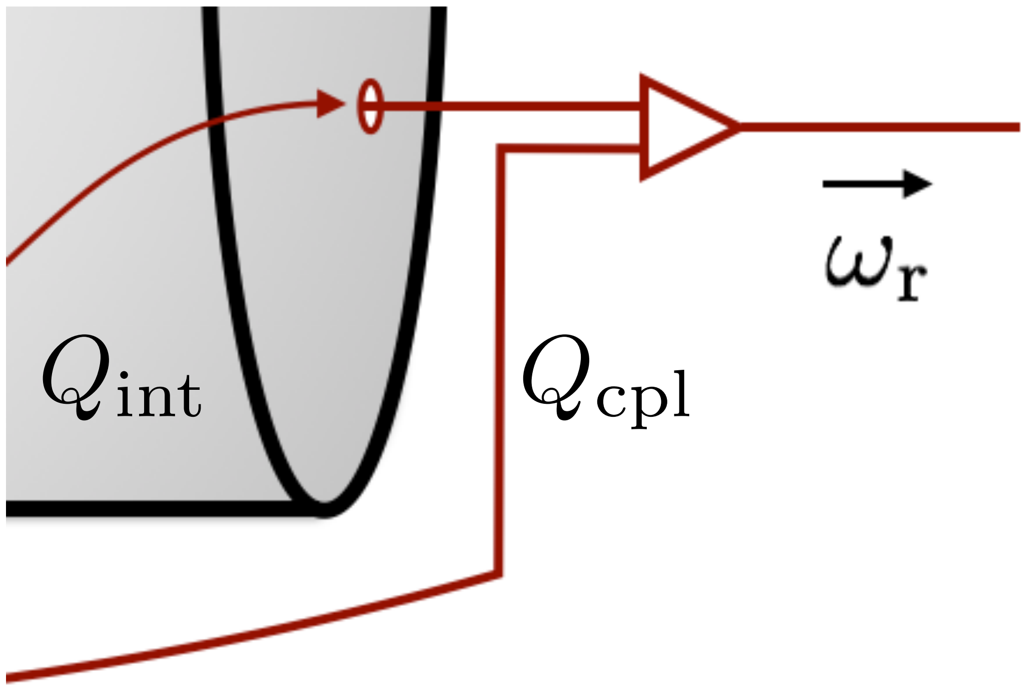
Readout:



$$Q_1 = \frac{Q_{\rm int} Q_{\rm cpl}}{Q_{\rm int} + Q_{\rm cpl}}$$

Signal to Noise: *readout & overcoupling*

Readout:



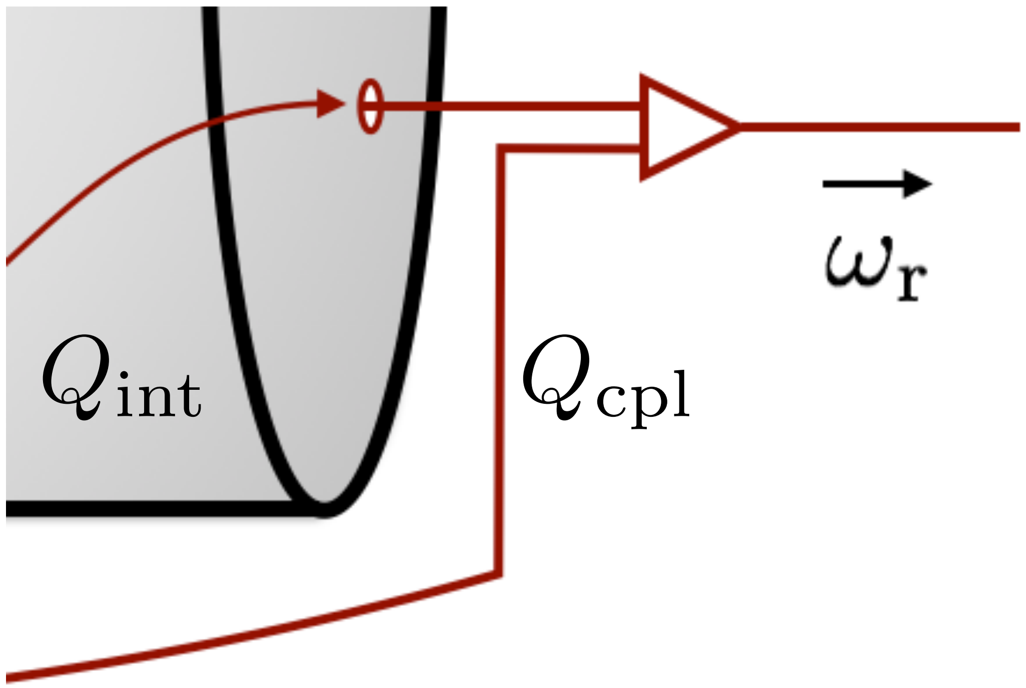
$$Q_1 = \frac{Q_{\rm int} Q_{\rm cpl}}{Q_{\rm int} + Q_{\rm cpl}}$$

Signal:

$$S_{\rm sig}(\omega) \to \frac{Q_1}{Q_{\rm cpl}} S_{\rm sig}(\omega)$$

Signal to Noise: *readout & overcoupling*

Readout:



$$Q_1 = \frac{Q_{\rm int} Q_{\rm cpl}}{Q_{\rm int} + Q_{\rm cpl}}$$

Signal:

$$S_{\rm sig}(\omega) \to \frac{Q_1}{Q_{\rm cpl}} S_{\rm sig}(\omega)$$

Noise:

$$S_{\rm noise}(\omega) = S_{\rm ql}(\omega) + \frac{Q_1}{Q_{\rm cpl}} \left(S_{\rm th}(\omega) + S_{\rm phase}(\omega) + S^{(1)}_{\rm mech}(\omega) \right) + \frac{Q_0}{Q_{\rm cpl}} S^{(0)}_{\rm mech}(\omega)$$

Signal to Noise

Roughly:

$$(\mathrm{SNR})^2 \simeq t_{\rm int} \int_0^\infty d\omega \left(\frac{S_{\rm sig}(\omega)}{S_{\rm noise}(\omega)} \right)^2$$

Monochromatic:

$$S_{\rm sig}^{\rm MC}(\omega) = \frac{\omega_{\rm sig}}{Q_{\rm sig}} \frac{\omega^4 (\eta E_0 h_0)^2 V}{(\omega_{\rm sig}^2 - \omega^2)^2 + (\omega\omega_{\rm sig}/Q_{\rm sig})^2} S_{e_0}(\omega - \omega_G)$$

$$h_0 \sim \omega_G^2 V^{2/3} h$$

Flat:

$$S_{\rm sig}^{\rm Flat}(\omega) = \frac{\omega_{\rm sig}}{Q_{\rm sig}} \frac{\omega^4 (\eta E_0)^2 V}{(\omega_{\rm sig}^2 - \omega^2)^2 + (\omega\omega_{\rm sig}/Q_{\rm sig})^2} \frac{3H_0^2}{8} \left(\frac{\Omega_{\rm GW}(\omega - \omega_0)}{(\omega - \omega_0)^3} + \frac{\Omega_{\rm GW}(\omega + \omega_0)}{(\omega + \omega_0)^3} \right)$$

$$\Omega_{\rm GW} \sim \frac{1}{3H_0^2} \omega^2 h_{\rm sto}^2$$

Design params:

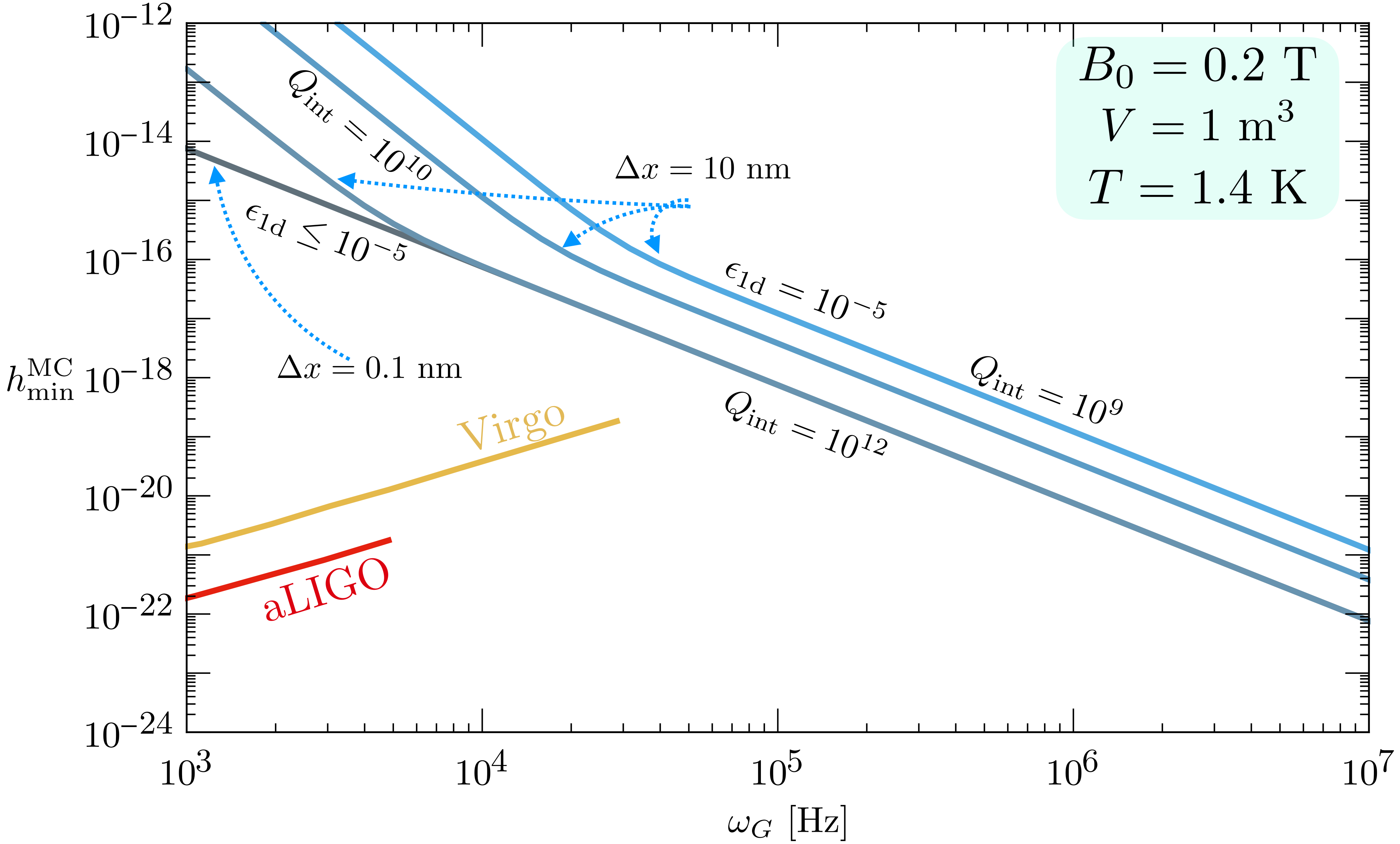
$$S_{\rm noise}(\omega) \sim S_{\rm th}(\omega) = \frac{Q_1}{Q_{\rm int}} \frac{4\pi T \, (\omega\, \omega_1/Q_1)^2}{(\omega^2 - \omega_1^2)^2 + (\omega\, \omega_1/Q_1)^2}$$

$$h_{\rm min}^{\rm MC} \sim \frac{1}{\omega_G^2} \left(\frac{T}{\omega_1 Q_1} \right)^{1/2} \left(\frac{\delta\omega}{t_{\rm int}} \right)^{1/4} \frac{1}{E_0 V^{7/6}} \sim 10^{-22} \left(\frac{10^7 \ {\rm Hz}}{\omega_G} \right)^2$$

Reach – Monochromatic source**

***ultra-preliminary*

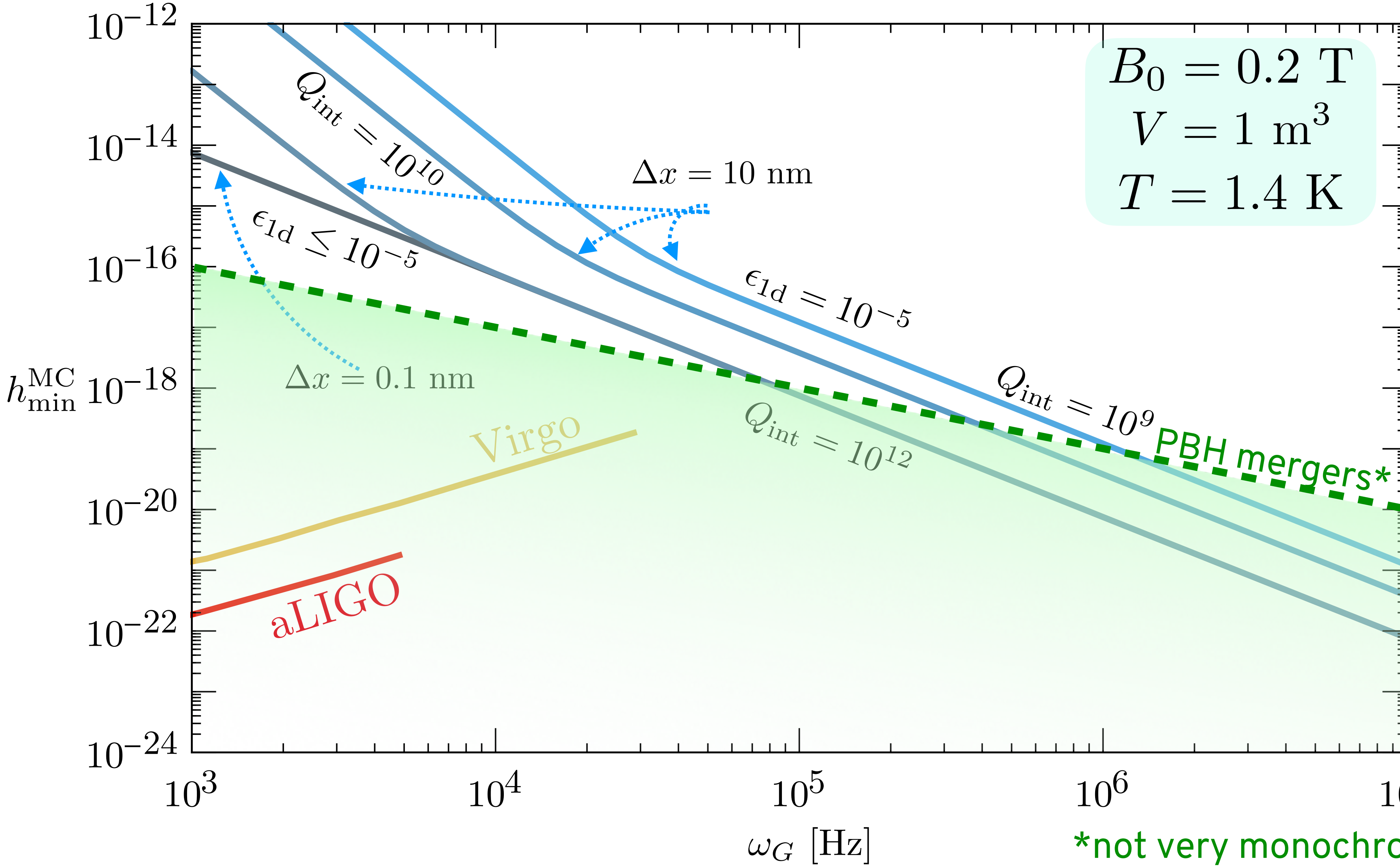
$$h_{\min}^{\rm MC} \sim \frac{1}{\omega_G^2}\left(\frac{T}{\omega_1 Q_1}\right)^{1/2}\left(\frac{\delta\omega}{t_{\rm int}}\right)^{1/4}\frac{1}{E_0 V^{7/6}} \sim 10^{-22}\left(\frac{10^7\ {\rm Hz}}{\omega_G}\right)^2$$



Reach — Monochromatic source**

***ultra-preliminary*

$$h_{\rm min}^{\rm MC} \sim \frac{1}{\omega_G^2}\left(\frac{T}{\omega_1 Q_1}\right)^{1/2}\left(\frac{\delta\omega}{t_{\rm int}}\right)^{1/4}\frac{1}{E_0 V^{7/6}} \sim 10^{-22}\left(\frac{10^7\ {\rm Hz}}{\omega_G}\right)^2$$

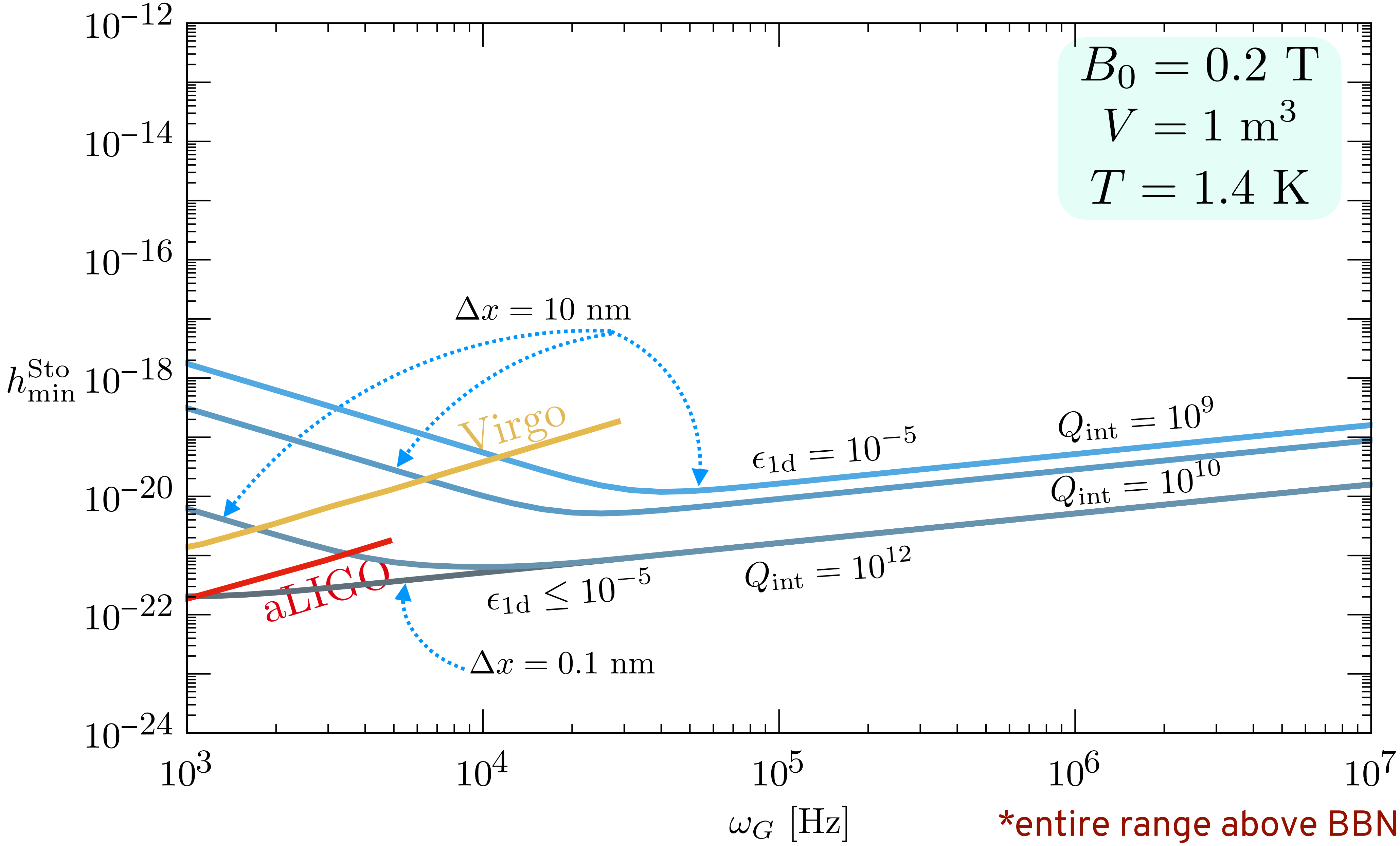


*not very monochromatic

Reach — Stochastic source**

***ultra-preliminary*

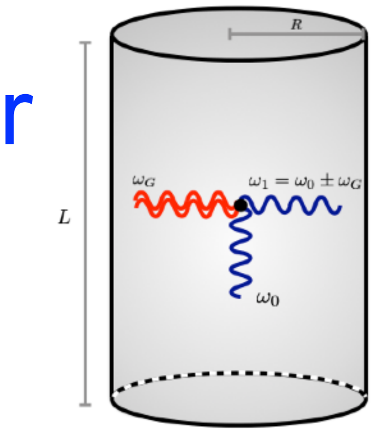
$$h_{\rm min}^{\rm Sto} \sim \left(\frac{T\omega_G}{\omega_1 Q_1 V}\right)^{1/2}\left(\frac{1}{t_{\rm int}\delta\omega}\right)^{1/4}\frac{1}{E_0} \sim 10^{-20}\left(\frac{\omega_G}{10^7\ \rm Hz}\right)^{1/2}$$



*entire range above BBN bound

Conclusion

Direct signal — cavity as a Gertsenshtein converter

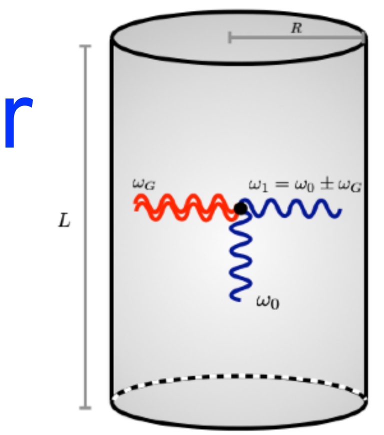


Cylinder for illustrative purposes only!

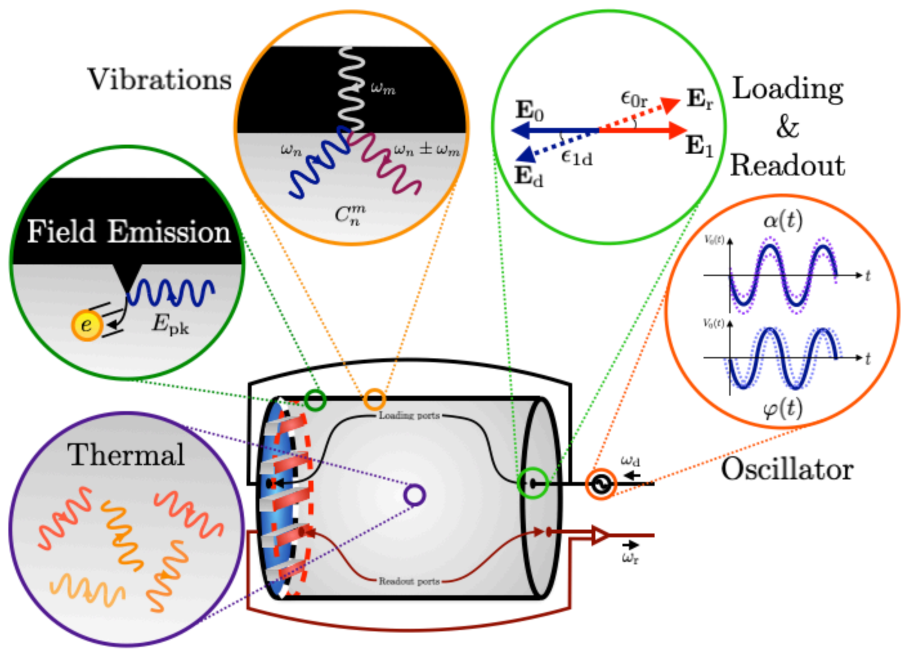
Bonus! Technology useful for axion DM direct detection

Conclusion

Direct signal — cavity as a Gertsenshtein converter



Cylinder for illustrative purposes only!



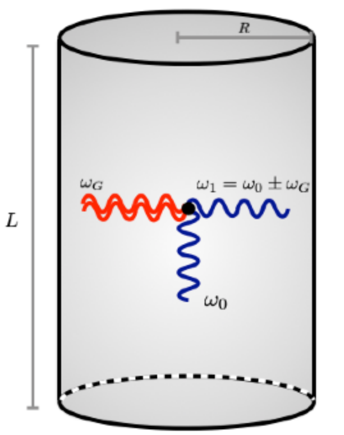
Noise in SRF Cavity requires precise Cavity control:

- **Careful loading**
- **vibration control**
- **mode isolation**

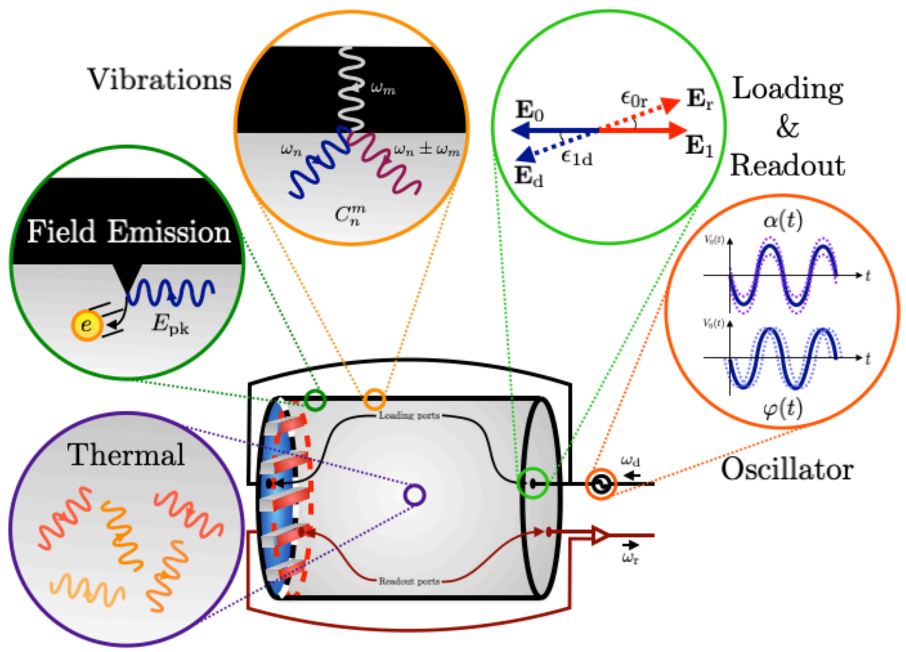
Bonus! Technology useful for axion DM direct detection

Conclusion

Direct signal — cavity as a Gertsenshtein converter



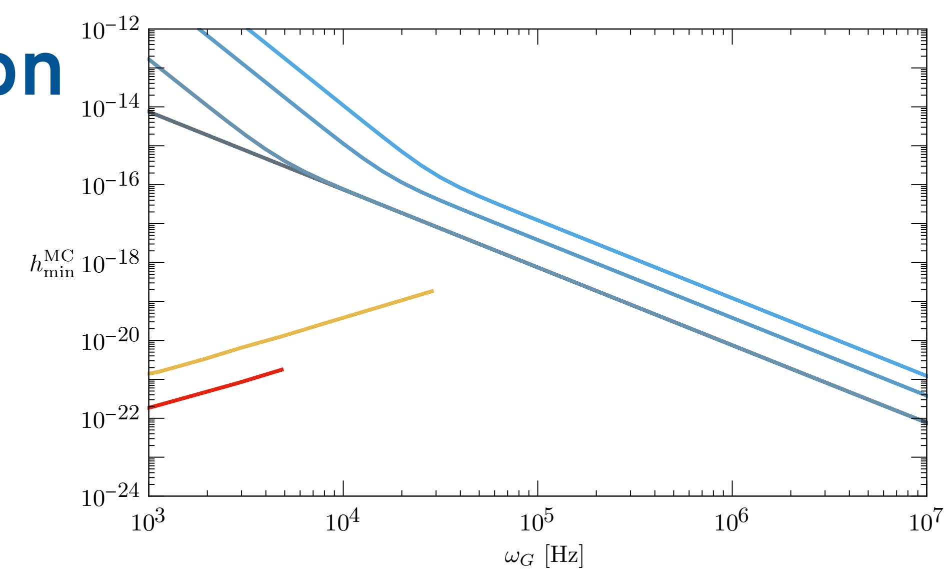
Cylinder for illustrative purposes only!



Noise in SRF Cavity requires precise Cavity control:

- **Careful loading**
- **vibration control**
- **mode isolation**

Strain sensitivity up to $h \sim 10^{-22}$–10^{-20}



Bonus! Technology useful for axion DM direct detection

Backup

Statistical treatment

Both signal and noise exponentially distributed:

$$L[\tilde{d}] = \prod_i \frac{e^{-|\tilde{d}_i|^2/(S_s(\omega_i)+S_n(\omega_i))}}{\pi(S_s(\omega_i)+S_n(\omega_i))}$$

Test statistic:

$$q(g_{a\gamma\gamma}) = -2\,\log\left(\frac{L(g_{a\gamma\gamma},\hat{\hat{\theta}}_s,\hat{\hat{\theta}}_n)}{L(\hat{g}_{a\gamma\gamma},\hat{\theta}_s,\hat{\theta}_n)}\right)\Theta(g^2_{a\gamma\gamma}-\hat{g}^2_{a\gamma\gamma})$$

For $t_{\rm int} \gg \tau_a$ Wilks' theorem implies

$$q(g_{a\gamma\gamma}) \simeq \sum_i \left(\frac{g^2_{a\gamma\gamma}\,\lambda_{s,i}(\hat{\hat{\theta}}_s)}{\lambda_{n,i}(\hat{\hat{\theta}}_n)}\right)^2 \simeq \frac{t_{\rm int}}{2\pi}\int_0^\infty d\omega\,\left(\frac{S_s(\omega)}{S_n(\omega)}\right)^2 \qquad \mathrm{SNR}(t_{\rm int}\gg\tau_a) \gtrsim \begin{cases} 1.3 & 90\%\ \mathrm{C.L.} \\ 1.6 & 95\%\ \mathrm{C.L.}\ , \end{cases}$$

For $t_{\rm int} \ll \tau_a$ GW signal in single DFT bin

$$q(g^2_{a\gamma\gamma},S) = 2\times\begin{cases} 0 & g^2_{a\gamma\gamma}\lambda_s+\lambda_n<S \\ \frac{S}{g^2_{a\gamma\gamma}\lambda_s+\lambda_n}-1+\log\frac{g^2_{a\gamma\gamma}\lambda_s+\lambda_n}{S} & \lambda_n\le S\le g^2_{a\gamma\gamma}\lambda_s+\lambda_n \\ \frac{S}{g^2_{a\gamma\gamma}\lambda_s+\lambda_n}-\frac{S}{\lambda_n}+\log\frac{g^2_{a\gamma\gamma}\lambda_s+\lambda_n}{\lambda_n} & S<\lambda_n \end{cases} \qquad \mathrm{SNR}(t_{\rm int}\ll\tau_a) \gtrsim \begin{cases} 5.6 & 90\%\ \mathrm{C.L.} \\ 12.5 & 95\%\ \mathrm{C.L.} \end{cases}$$