Gravitational Synchrotron Radiation

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Introduction

- At energy scales much lower than the Planck scale, the Einstein field equation can be linearized. With the convention $G = c = \hbar = 1$, we write

$$\Box \psi_{\mu\nu} = 16\pi T_{\mu\nu} \quad ,$$

- where $\psi_{\mu\nu} = h_{\mu\nu} - \eta_{\mu\nu} h/2$ is the trace-reversed perturbation around the flat spacetime $\eta_{\mu\nu}$ with the curved metric

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \ \eta = \mathrm{diag}(1, -1, -1, -1),$$

  and $T_{\mu\nu}$ is the energy-momentum stress tensor.

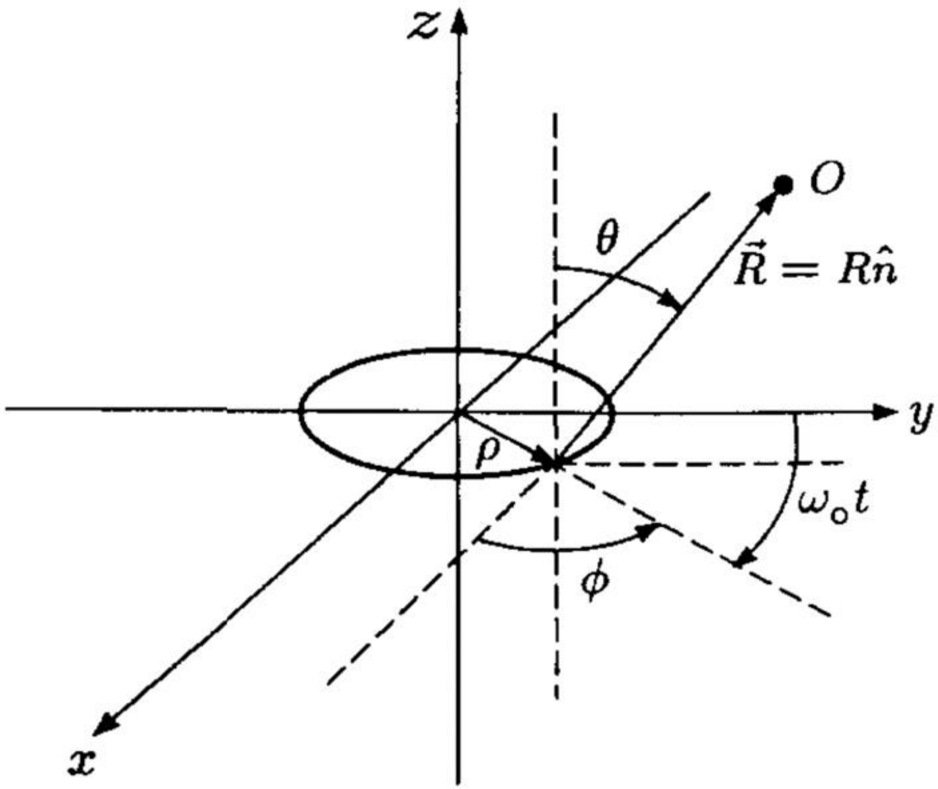
- Clearly this eq. provides solutions as propagating waves, i.e., the gravitational waves (GWs).

- We all know that the change of the quadrupole moment (in time) of a massive object can give rise to GWs.
- It was pointed out by Gertsenshtein (1962) that when an EM wave propagates through a static transverse background EM field, there is a nontrivial energy-momentum stress tensor, $T^{f}_{\mu\nu}$, which can resonantly excite GWs with the same frequency and propagates in the same direction.
- In the case where the dynamical EM wave is produced by a charged particle interacting with a background EM field, the stress tensor has contributions from both the particle and the radiation field:

$$T_{\mu\nu} = T^{p}_{\mu\nu} + T^{f}_{\mu\nu}.$$

Gravitational Synchrotron Radiation

- We now refer to the subset of GWs generated by. $T^{p}_{\mu\nu}$ the gravitational synchrotron radiation (GSR).



Coordinates involved in GSR

- As a general property of wave equations, we describe the solution in the radiation zone as

$$\psi_{\mu\nu}(\vec{R}) = -\frac{4}{R} e^{ikR} T_{\mu\nu}(\vec{k}) \quad ,$$

where $\vec{R} = R\vec{n}$ and R Is the distance to the observation point.

- The GW radiation power is

$$\begin{aligned} W_G(\omega) &= -\frac{R^2}{32\pi} \int d\Omega \left[\partial_0 \psi^{\mu\nu} \partial_i \psi_{\mu\nu} - \frac{1}{2} \partial_0 \psi^{\mu}_{\mu} \partial_i \psi^{\nu}_{\nu} \right] n^i \\ &= -\frac{\omega^2}{2\pi} \int d\Omega \, T^{\mu\nu}(\vec{k}) T_{\mu\nu}(-\vec{k}) \end{aligned} \quad ,$$

where n_i is the $i\text{th}$ component of $\vec{n}$.

- For GSR, it can be shown that the dominant contribution comes from the component doubly-transverse to the tangent of the circular orbit,

$$T^p_{\perp\perp}(k) = \frac{2\pi}{\omega_0}\gamma m e^{-i\nu(\pi/2-\phi)} \times \left\{ \sin^2\phi J_\nu(\xi) + 2i\sin\phi\cos\phi\left[-\frac{\nu}{\xi^2}J_\nu(\xi) + \frac{\nu}{\xi}J'_\nu(\xi)\right] - \cos 2\phi J''_\nu(\xi)\right\},$$

where ν is the harmonic number, $\xi = \nu\beta c\sin\theta, \omega_0 = c/\rho$ is the orbital frequency, and J_ν is the Bessel function.

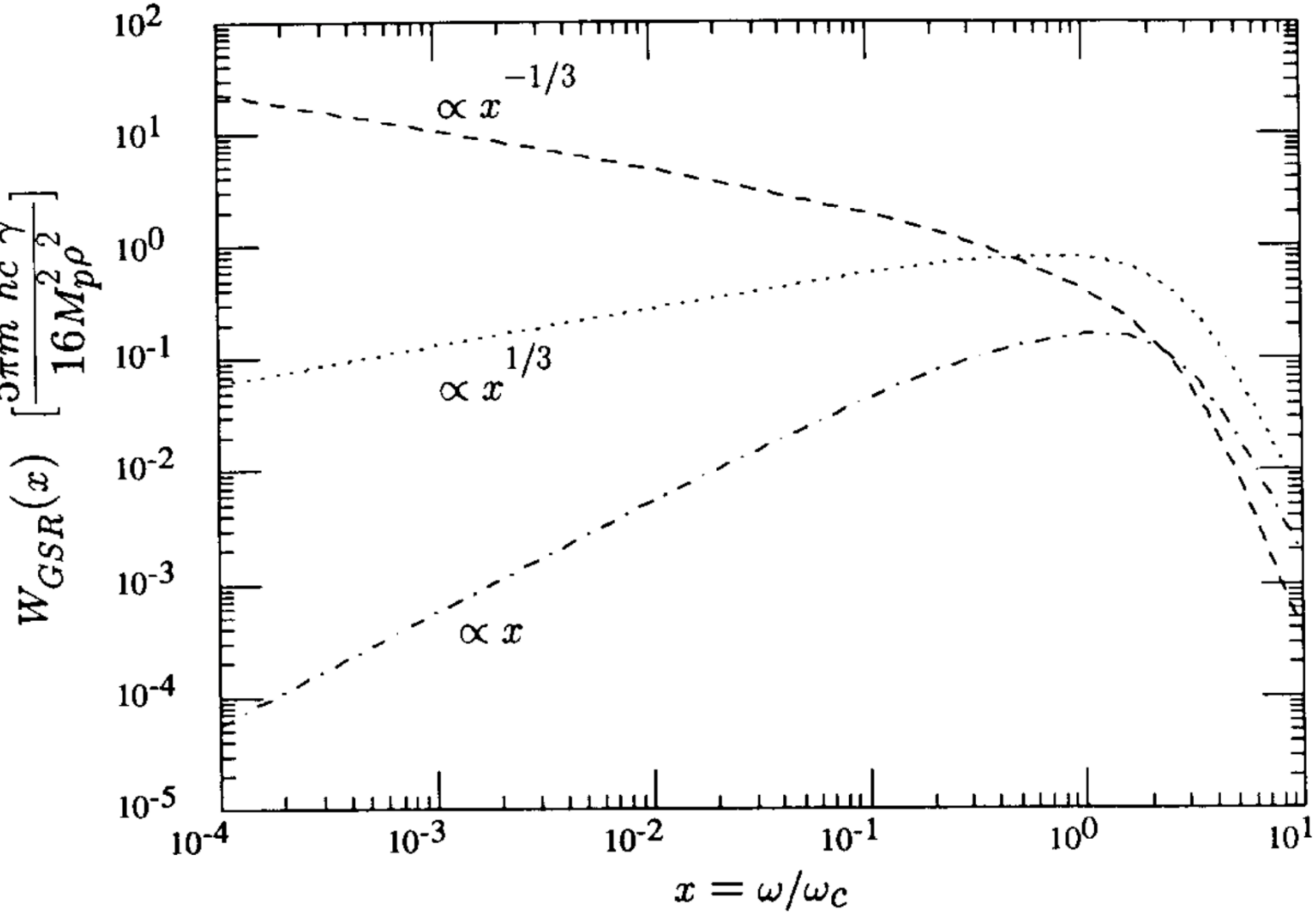
- Inserting $T^p_{\perp\perp}(k)$ into the previous equation, we obtain the GSR power spectrum

$$\frac{dW_{GSR}}{dx} = \frac{3\sqrt{\pi}}{32}\frac{Gm^2\gamma^4\omega_0^2}{c}\left[3x^{-1/3}\Phi(y) - 5x^{1/3}\Phi'(y) + 3x\Phi_2(y)\right] ,$$

where $x = \omega/\omega_c,\ y = x^{2/3},\ \omega_c = \gamma^3\omega_0$ is the critical frequency of the GSR, Φ is the Airy function, and

$$\Phi_2(y) = \frac{y^{1/2}}{2^{2/3}\pi^{1/2}} \int_{-\infty}^{\infty} dz \Phi^2(y(1+z^2)/2^{2/3}) \quad .$$

GSR Power Spectrum



Gravitational synchrotron radiation power spectrum

- At small x, the spectrum scales as $x^{-1/3}$, $x^{1/3}$, and x, respectively.

- Further integration over the power spectrum, we find the total power,

$$W_{GSR} = \frac{5\pi}{16}\frac{Gm^2c\gamma^4}{\rho^2} = \frac{5\pi}{16}\frac{m^2}{M_P^2}\frac{\hbar c^2\gamma^4}{\rho^2} \quad ,$$

where M_P is the Planck mass and ρ is the radius of the storage ring.

- Although the total power scales as γ^4, same as that of the conventional electromagnetic synchrotron radiation (EMSR), the GSR power is dominated by the fundamental frequency due to its scaling law $x^{-1/3}$. This is characteristically different from that of EMSR, where the dominant frequency is ω_c.

- Therefore, not only all N particle in a bunch in a storage ring radiate GSR coherently, all n_b bunches radiate coherently as long as they are NOT distributed symmetrically around the ring.
- The total GSR rate of 'graviton' emission is then

$$N_{GSR} \sim 5.6 n_b^2 N^2 \frac{m^2}{M_P^2} \frac{c\gamma^4}{\rho}.$$

Estimated GSR Graviton yields

Storage Rings	PEP-II	LEP-I	LEP-II	HERA	LHC
$\mathcal{E}$[GeV]	9	50	100	880	7000
$\gamma[10^3]$	18	100	200	7.5	0.88
$N[10^{10}]$	3.8	45	45	10	10
n_b	1700	4	4	210	2800
$l[cm]$	3.46	6.24	6.24	27.7	18.4
ρ[m]	500	4300	4300	1035	4300
Gravitational SR					
ω_0[kHz]	600	70	70	290	70
$N_{GSR}[10^{-7}\text{sec}^{-1}]$	1.3×10^3	38	150	6×10^6	1.8×10^{10}
Resonant Conversion					
$\omega_c[10^9\text{GHz}]$	3.5	70	560	0.12	4.8×10^{-5}
$N_{res}[10^{-7}\text{sec}^{-1}]$	0.1	0.1	0.3	10^3	2×10^5

Comments

- Note, however, this is the total yield around the ring. The collectable signals would be much reduced if the detector is localized in a specific direction.
- Furthermore, at such low frequencies (the fundamental frequency), the notion of gravitons as a discrete entity in the GW is questionable.
- We remind again that this is only a fraction of the total graviton yield from such an electromagnetic system where the EMSR can also convert to GSR through resonant conversion.

Resonant Conversion of EMSR to GSR

- For a radiation (dynamical) field F^b traversing a background (static) EM field F^0, the electromagnetic part of the stress energy-momentum tensor has the form

$$T^f \sim (F^b + F^0)(F^b + F^0).$$

- The square of the background field, $F^0 F^0$, bares no relation to the dynamics, and we will ignore it. The $F^b F^b$ term can be dropped since almost everywhere $F^b \ll F^0$ except at the source. But that has already been taken into account in the mass renormalization.
- So the contribution to GW is simply from $F^b F^0 + F^0 F^b$.

- It can be shown that, in general, such a resonant conversion gives

P Chen, Mod. Phys. Lett. 6, 1069 (1991).

$$W_G(\omega) = \frac{\pi}{4}\frac{1}{\alpha}\frac{m^2}{M_P^2}\left(\frac{L}{\lambda_c}\frac{B}{B_c}\right)^2\left[1 - \frac{\sin(\omega L)}{\omega L}\right]^2 W_{EM}(\omega) \quad ,$$

where λ_c is the Compton wavelength, and

$$B_c \equiv m^2c^3/e\hbar \sim 4.4\times10^{13}\,\text{Gauss}$$

is the Schwinger critical field.

- The square bracket represents the form factor from the Fourier spectrum of the background field, and is of the order unity for wavelengths $\lambda \lesssim 2L$, where the last zero at $\sin(2\pi L/\lambda) = \sin\pi$ occurs.
- As is well-known, the EMSR critical frequency, $\omega_c = \gamma^3\omega_0$. Thus it is a factor γ^3 higher than that of GSR. For LHC, $\sim\mu m \ll L \sim 14m$ (length of bending magnets). See Table 1.

GSR induced Spacetime Perturbation

- Recall that the spacetime metric perturbation is

$$\psi_{\mu\nu} = h_{\mu\nu} - \eta_{\mu\nu} h/2$$

and for GSR it has the form

$$\psi_{\mu\nu}(\vec{R}) = -\frac{4}{R} e^{ikR} T_{\mu\nu}(\vec{k}) \quad ,$$

with

$$T^{p}_{\perp\perp}(k) = \frac{2\pi}{\omega_0} \gamma m e^{-i\nu(\pi/2-\phi)} \times \left\{ \sin^2\phi J_\nu(\xi) + 2i \sin\phi \cos\phi \left[-\frac{\nu}{\xi^2} J_\nu(\xi) + \frac{\nu}{\xi} J'_\nu(\xi) \right] - \cos 2\phi J''_\nu(\xi) \right\},$$

- The spacetime metric perturbation has no dimension, whereas the stress energy tensor has the dimension L^{-4}. Transforming to the 4D Fourier space, $[T_{\mu\nu}(k)]$=L^0.

- Now the Einstein field equation has a Newton's constant on the LHS in front of the stress tensor, which, in our convention, $G = c = \hbar = 1$, has been omitted. Note that $[G] = L^2$. As a result, $[\psi_{\mu\nu}(\omega)]$=L^1.
- That is,

$$T^{p}_{\perp\perp}(k) \propto \frac{\gamma m}{\omega_0} = \frac{\gamma m c^2}{(h/2\pi)\omega_0} = \frac{\gamma m c^2}{hc/2\pi\rho}.$$

- We conclude that the spacetime perturbation in GSR is

$$h \sim |\psi_{\mu\nu}(x)| \sim \frac{\gamma}{\lambda_p} |\psi_{\mu\nu}(\omega)| \sim \gamma m_p \frac{G}{R} T^{p}_{\perp\perp}(k) \sim \frac{\gamma^2 m_p^2}{M_P^2} \frac{\rho}{R},$$

where λ_p is the Compton wavelength of the particle.
- For LHC and let $R \sim 10\ \rho$, we find $h \sim 10^{-31}$. Assuming $N \sim 10^{11}$ protons in a bunch and $n_b \sim 100$ bunches in a train, we find the total h to be

$$h_{GSR} \sim N n_b h \sim 10^{-18}.$$

Conclusion

- We have demonstrated that relativistic charged particles in storage rings can in principle emit gravitational synchrotron radiation (GSR).
- The radiation power of GSR is dominated by the 'fundamental frequency' of the ring, i.e., $\omega_0 = c/\rho$.
- All N particles in a bunch and all n_b bunches in a train radiate GSR coherently.
- Since the observation point can be located near the storage ring, we find that the spacetime metric perturbation can be $h_{GSR} \sim n_b N \times 10^{-31} \sim 10^{-18}$ for LHC. Would heavy ion mode be better off?
- Resonant conversion of EMSR to GSR in the case of LHC corresponds to bone fide 'gravitons' with wavelength about $\sim \mu m$, which, if detected, would be the first observation of gravitons.